AE-16

The Dancoff correction in various geometries

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Summary:

The mutual shadowing of fuel lumps in a moderator is studied assuming that the source density of resonance neutrons is constant in the moderator, that one collision with a moderator atom removes a resonance neutron from the resonance interval and that the fuel is black to resonance neutrons.

Formulae for the shadowing factor are given for square or circular tubes, parallel plates of finite or infinite width and parallel circular cylinders, and some numerical results are presented.

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Introduction.

The resonance escape probability in a reactor lattice is generally calculated by means of an effective resonance integral, which is composed of a volume and a surface term. In close-packed lattices, i. e. when the distance between neighbouring lumps is not sufficiently large compared to the mean free path in the moderator, the surface effect is reduced because of shadowing by adjacent lumps.

The calculation of the shadowing in different geometries can be quite laborious. Dancoff and Ginsburg[1] have given a few curves for the case of parallel circular rods. However, the values of the parameters are such as to restrict the applicability to close packed light water lattices. Since we are interested in other types of lattices, especially rod cluster elements in heavy water reactors, we have extended the calculations for parallel circular cylinders. We have also treated other geometries, namely tubes and parallel plates.

We consider only the geometrical aspects, and it is outside the scope of this paper to discuss such points as for example the validity of the physical model used, or what formula to use for the resonance integral and what value to use for the mean free path in different moderators. Neither do we consider the effect of the lumps being partially transparent to resonance neutrons.

1. Assumptions and formulation of the problem.

We consider an infinite moderator in which fuel lumps are inserted. The following assumptions are made:

a) The fuel is black for neutrons of a particular resonaance energy or small energy interval.

b) The source density of resonance neutrons is constant in the moderator.

c) A single collision with a moderator atom removes a neutron from the resonance interval, so that the neutron escapes absorption in the particular resonance.

For points on the surface of a lump, from which it is possible to see other lumps, part of the sources are shadowed. The problem is to calculate for any surface element of one lump the reduction of the inward neutron current due to the presence of a second lump, and then to calculate the average reduction for the total surface of the first lump.

2. Reduction of the problem to two dimensions.

Since only general cylindrical systems will be considered here, the axial coordinate can be eliminated from the beginning. This was also done by Dancoff and Ginsburg. We give the calculation here since we derive a general formula, which we need for all cylindrical geometries.

Consider a surface element dS of the first lump and a cylindrical volume source of source density q and small cross section dA (fig. 1). The current of neutrons through dS coming directly from a source element of height dz can be written

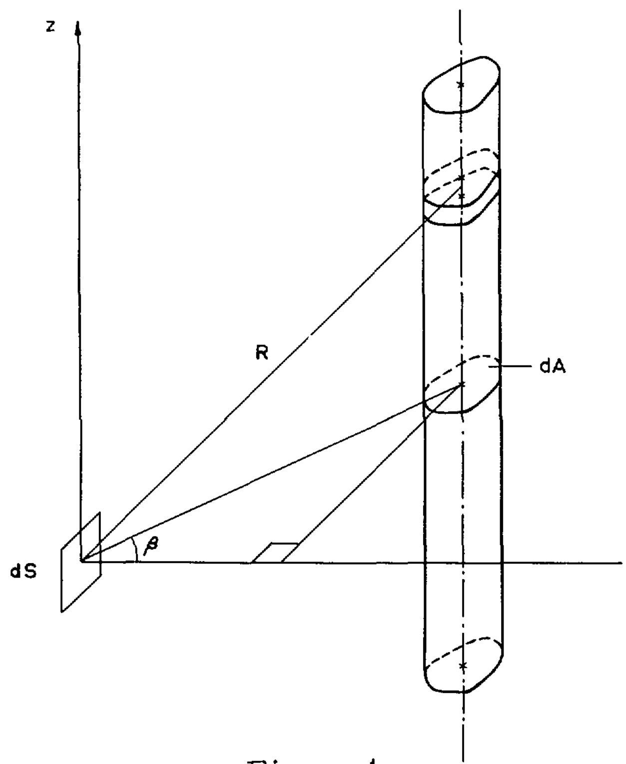

Figure 1

$$dS \;\; dj = q \, dA \, dz \, dS \, \frac{r \cos \beta}{R} \, \frac{e^{-\Sigma R}}{4 \pi R^2} \qquad (1)$$

where j = current per unit area
$\Sigma = \frac{1}{\lambda}$ = the moderator macroscopic cross section and the geometrical lengths and the angle β are explained by fig. 1.

Equation (1) is divided by dS and integrated over z

$$j = \frac{q\, dA\, r \cos\beta}{4\pi} \int_{-\infty}^{+\infty} \frac{e^{-\Sigma R}}{R^3}\, dz \tag{2}$$

With the substitution R = r cosh u and z = r sinh u we get

$$j = \frac{q}{2\pi} \frac{Ki_2(\Sigma r)}{r} \cos\beta\, dA \tag{3}$$

The functions $Ki_n(x)$ are generally called Bickley functions[2].
They are defined as

$$Ki_n(x) = \int_0^{\infty} \frac{e^{-x \cosh u}}{\cosh^n u}\, du \tag{4}$$

and some properties are

$$\frac{d}{dx} Ki_n(x) = - Ki_{n-1}(x)$$

$$\int_x^{\infty} Ki_n(x)\, dx = Ki_{n+1}(x) \tag{5}$$

$$Ki_o(x) = K_o(x)$$

where $K_o(x)$ is the modified Bessel function.

By integrating equation (3) over some area in the plane perpendicular to the z-axis, we obtain the current from the corresponding cylindrical volume source, J

$$J = \frac{q}{2\pi} \int_A \frac{Ki_2(\Sigma r)}{r} \cos\beta\, dA \tag{6}$$

In the special case of no shadowing the surface element is exposed to a half-space and the integration is easily carried out. The current per unit area in this case is called J_o. With $dA = r\,dr\,d\beta$

$$J_o = \frac{q}{2\pi} \int_0^{\infty} Ki_2(\Sigma r)\,dr \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\beta\,d\beta = \frac{q}{4\Sigma} \qquad (7)$$

Here we have used $Ki_3(0) = \frac{\pi}{4}$ and $Ki_3(\infty) = 0$.

In the case of shadowing we calculate the shadowed part of the total current. This current is called J_s, and is obtained by evaluating the integral in eq. (6) over the hatched area of fig. 2.

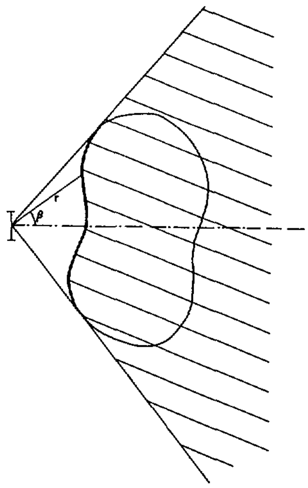

Figure 2

Putting as before $dA = r\,dr\,d\beta$ and integrating over r from r to infinity we get

$$J_s = \frac{q}{2\pi} \int \cos\beta\,d\beta \int_r^{\infty} Ki_2(\Sigma r)\,dr = \frac{q}{2\pi\Sigma} \int Ki_3(\Sigma r) \cos\beta\,d\beta \qquad (8)$$

In the resulting integral over β, r is a function of β, and the integration shall be performed over the angles under which the second lump can be seen from the surface element of the first one.

The relative reduction in current through the surface element is called C, in accordance with the notation of Dancoff and Ginsburg.

$$C = \frac{J_s}{J_o} = \frac{2}{\pi} \int Ki_3 (\Sigma r) \cos \beta \; d\beta \tag{9}$$

C refers to a certain point on the surface of the first lump. It has to be averaged over the total surface of the lump except for cases of high symmetry where all points on the surface are equivalent.

C can be considered as a reduction in effective surface, and a shadowed surface should be multiplied by 1 - C before being used in any formula for the calculation of the effective resonance integral.

Now we proceed to calculate C for certain geometries. Although some of these calculations are well-known, we include them for completeness. We start in most cases from equation (9). But to make the equations shorter from now on we use the mean free path in the moderator as unit length, and therefore the basic equation is

$$C = \frac{2}{\pi} \int Ki_3(r) \cos \beta \; d\beta \tag{10}$$

3. Calculations for different geometries.

3.1. Holes inside the fuel.

For moderator regions inside the fuel the effectiveness of the surface can be related to the escape probability defined by Case, de Hoffman and Plazcek[3]. The hole is assumed to be convex and filled with moderator. Let the cross section of the cylindrical hole be A and the surface per unit length be S. The escape probability is called P_o as in Case et al. Then the average current into the fuel per unit area is $\frac{qA}{S} P_o$. This shall be divided by the current through an unshadowed surface, which is q/4 from equation (7) with $\Sigma = 1$. Thus

$$1 - C = \frac{4A}{S} P_o \tag{11}$$

3.1.1. Circular hole.

With the radius of the hole equal to a, $A = \pi a^2$ and $S = 2\pi a$ so that

$$1 - C = 2a\, P_o (a) \qquad (12)$$

The function $P_c (a) = 1 - P_o (a)$ has been tabulated by Case et al.

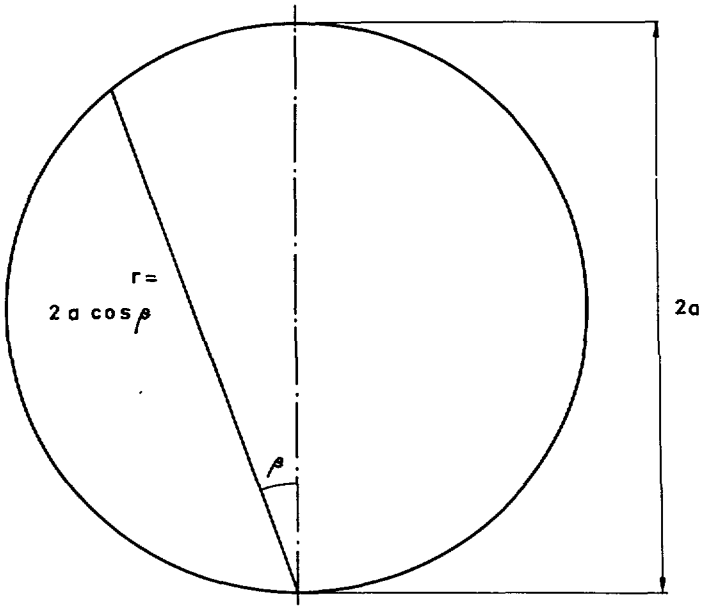

Figure 3

Using equation (10) we get for this case (see fig. 3)

$$C = \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} Ki_3(2a \cos\beta)\cos\beta \; d\beta \qquad (13)$$

This equation can easily be integrated to give the same expression as is obtained from the function P_o given by Case et al. (see appendix 1).

3.1.2. Square hole.

For a square hole with side d the following expression is obtained after some algebra

$$C = \frac{4}{\pi d} \left[\frac{1}{3} + \frac{1}{2} Ki_4 (d\sqrt{2}) - Ki_4 (d) + \frac{d}{\sqrt{2}} Ki_3 (d\sqrt{2}) + d\, F (d, d\sqrt{2}) \right] \qquad (14)$$

where the function

$$F (x, y) = \int_x^y Ki_2 (u) \frac{\sqrt{u^2 - x^2}}{u} \, du \qquad (15)$$

has to be evaluated numerically (see table 1).

3.2. Shadowing lumps.

Here we treat two types of geometries, parallel plates of infinite or finite width and parallel circular cylinders.

3.2.1. Parallel infinite plates.

Equation (10) gives immediately, if the distance between the plates is d

$$C = 2 \frac{2}{\pi} \int_0^{\frac{\pi}{2}} Ki_3 \left(\frac{d}{\cos\beta}\right) \cos\beta \; d\beta \tag{16}$$

Integrating by parts and substituting $\frac{d}{\cos\beta} = u$ we obtain

$$C = \frac{4}{\pi} \int_d^{\infty} Ki_2(u) \frac{\sqrt{u^2 - d^2}}{u} du = \frac{4}{\pi} F(d, \infty) \tag{17}$$

It is well known that in this case

$$C = 2 E_3(d) \tag{18}$$

where

$$E_n(x) = \int_1^{\infty} \frac{e^{-xt}}{t^n} dt \tag{19}$$

Comparing equations (17) and (18) we find

$$\int_d^{\infty} Ki_2(u) \frac{\sqrt{u^2 - d^2}}{u} du = \frac{\pi}{2} E_3(d) \tag{20}$$

This equation is shown in a more direct way in appendix 2.

3. 2. 2. Parallel plates of finite width.

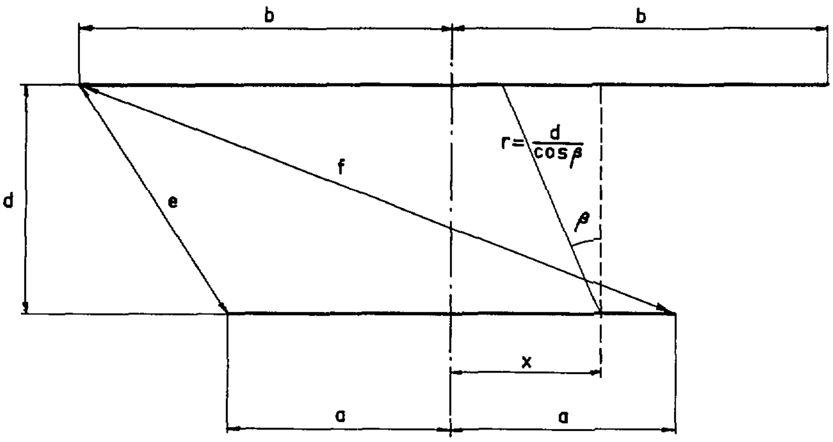

Figure 4

The case when the centers of the two plates are situated on the same perpendicular is considered (see fig. 4). Using equation (10) C is obtained as the double integral

$$C = 2 \frac{2}{\pi} \frac{1}{2a} \int_{-a}^{+a} dx \int_{0}^{\mathrm{arctg}\frac{b-x}{d}} Ki_3\left(\frac{d}{\cos\beta}\right) \cos\beta \, d\beta \qquad (21)$$

After some intermediate steps which are omitted here the result can be expressed in the following form

$$C = \frac{2}{\pi a} \Big\{ (b+a)\, F(d,f) - |b-a|\, F(d,e) + f\, Ki_3(f) - e\, Ki_3(e) +$$

$$+ d^2 \Big[K_o(e) - K_o(f) + Ki_2(f) - Ki_2(e) \Big] \Big\} \qquad (22)$$

where 2a is the width of the shadowed plate and 2b the width of the shadowing plate, e is the "short diagonal" and f is the "long diagonal". F(x,y) is defined by equation (15).

3. 2. 3. Parallel cylinders of equal radius. Finite mean free path in the moderator.

This is the case treated by Dancoff and Ginsburg[1]. However, by using another expression for C, we have obtained a simpler integral, which we think is better suited for numerical computation.

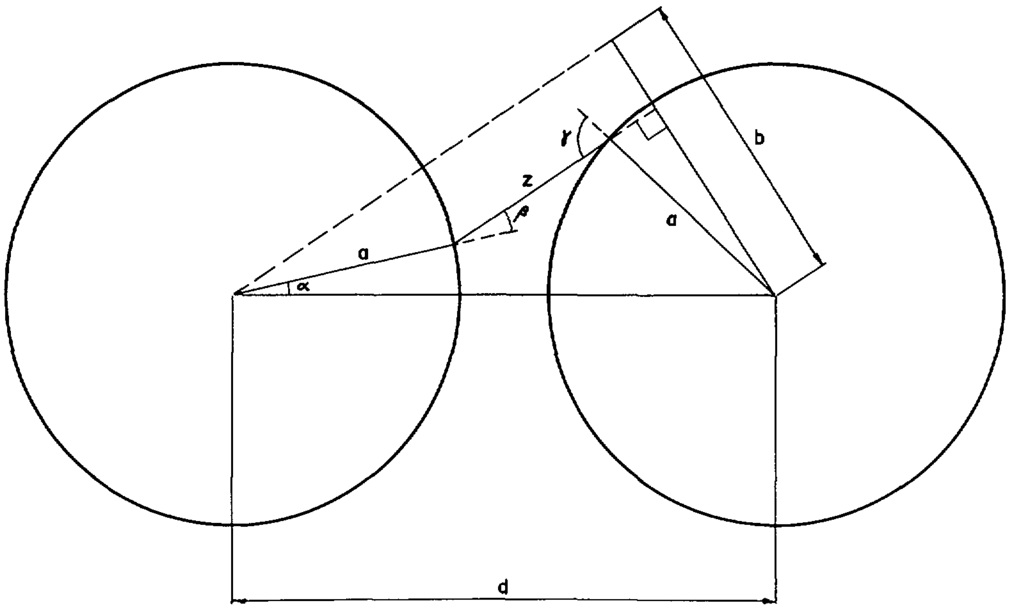

Figure 5

Consider two parallel cylinders of radii a and with a distance d between the centers. Using equation (10) we get with the angles given in figure 5

$$C = \frac{1}{2\pi a} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} a \, d\alpha \; \frac{2}{\pi} \int Ki_3(z) \cos\beta \, d\beta \tag{23}$$

It can easily be seen in the figure that the distance z between the points on the cylinder surfaces is given by

$$z = d \cos(\alpha+\beta) - a \cos\beta - a \cos\gamma \tag{24}$$

The limits for the integration over β depends upon α. It is, however, more convenient to use a double integral with constant limits for both variables. This can be achieved by using the angles β and γ as variables of integration, for in that case the limits for γ are $-\frac{\pi}{2}$ and $+\frac{\pi}{2}$ for any β, and vice versa.

A relation between the three angles can be obtained by considering the length of the perpendicular b (see fig. 5)

$$b = d \sin(\alpha+\beta) = a \sin\beta + a \sin\gamma \tag{25}$$

Keeping β constant we get

$$d \cos(\alpha+\beta)\, d\alpha = a \cos\gamma\, d\gamma \tag{26}$$

$$d\alpha = \frac{a \cos\gamma\, d\gamma}{\sqrt{d^2 - a^2 (\sin\beta + \sin\gamma)^2}} \tag{27}$$

Thus the following equation is obtained for C

$$C = \frac{1}{\pi^2} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\beta \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\gamma \frac{Ki_3(z) \cos\beta \cos\gamma}{\sqrt{\left(\frac{d}{a}\right)^2 - (\sin\beta + \sin\gamma)^2}} \tag{28}$$

With

$$\left.\begin{aligned} u &= \sin\beta \\ v &= \sin\gamma \end{aligned}\right\} \tag{29}$$

another form is

$$C = \frac{1}{\pi^2} \int_{-1}^{+1} du \int_{-1}^{+1} dv \frac{Ki_3(z)}{\sqrt{\left(\frac{d}{a}\right)^2 - (u+v)^2}} \tag{30}$$

where

$$z = a\left[\sqrt{\left(\frac{d}{a}\right)^2 - (u+v)^2} - \sqrt{1 - u^2} - \sqrt{1 - v^2}\right] \qquad (31)$$

When these formulae are applied to clusters of rods, the C for a particular rod is the sum of the C:s originating from all other rods, if the other rods do not shield each other. Otherwise for some rods the domain of integration has to be reduced due to the fact that some directions of incoming current are already shadowed by more closely situated rods. As an example consider the situation in figure 6. We want

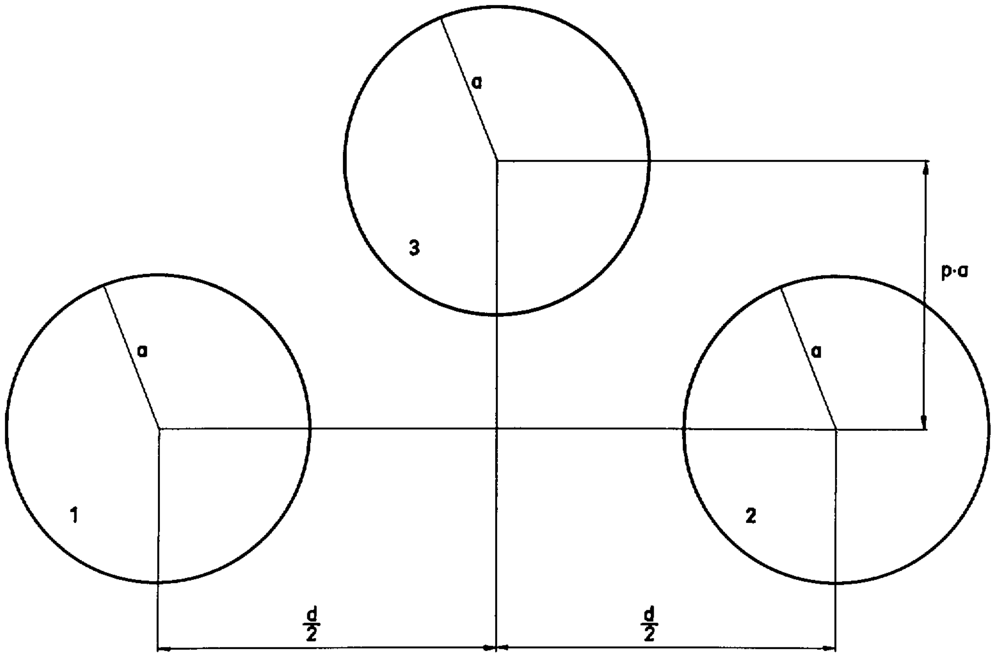

Figure 6

to calculate C_{2-1}, i. e. the shadowing of cylinder 1, exerted by 2, in the presence of 3. The area to be excluded is situated around the point u = -1, v = 1, and the boundary of this area is given by

$$\left(\frac{a}{d}\right)^2 (u + v)^2 + \frac{1}{p^2}\left(1 - \frac{u-v}{2}\right)^2 = 1 \qquad (32)$$

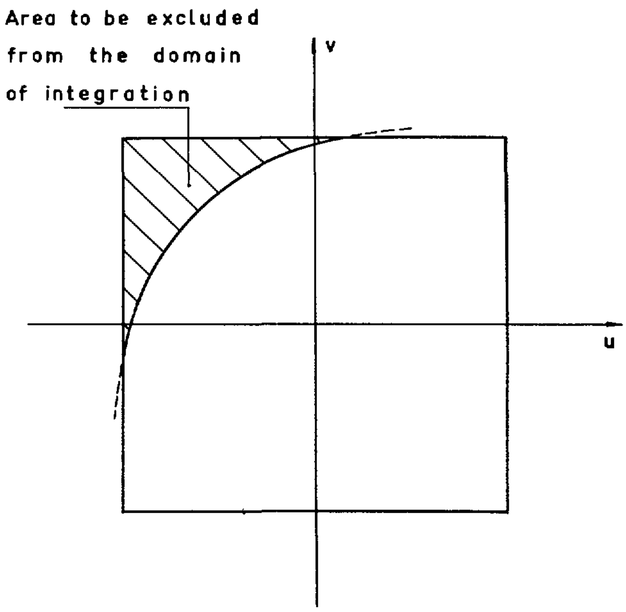

Figure 7

3.2.4. Parallel cylinders of equal radius. Infinite mean free path in the moderator.

We shall also consider the case of parallel cylinders when the mean free path in the moderator is infinite. Then all lengths are of course zero when measured with the mean free path as a unit. However, the ratio of lengths is not changed. Equation (30) becomes, since $Ki_3(0) = \frac{\pi}{4}$

$$C = \frac{1}{4\pi} \int_{-1}^{+1} du \int_{-1}^{+1} \frac{dv}{\sqrt{\left(\frac{d}{a}\right)^2 - (u+v)^2}} \qquad (33)$$

This equation can easily be integrated to

$$C = \frac{1}{\pi} \left[\arcsin \frac{2a}{d} - \tan \left(\frac{1}{2} \arcsin \frac{2a}{d} \right) \right] \qquad (34)$$

Thie[4] has also treated this case. He considers a ring of touching cylinders equidistant from a central cylinder. The total shadowing of the central cylinder is 1, and therefore the shadowing by each cylinder in the ring should be $C = \frac{1}{\pi} \arcsin \frac{a}{d}$. This expression is not quite correct, since no account is taken of the fact that the cylinders in the ring partly shield each other.

In the case of infinite mean free path it is also possible to perform the integration when there are other cylinders between the two cylinders considered. The integrations are elementary but tedious and the results will not be reproduced here. In the special case of a cluster of cylinders arranged in such a way that it is impossible to draw a line between the cylinders without penetrating or touching them, it is found that the total effective surface is equal to the so-called rubber band surface. It should perhaps be pointed out that for finite mean free path in the moderator the effective surface is of course larger than the rubber band surface.

4. Numerical results.

The function F(x, y) defined in equation (15) was computed numerically. The results are given in table 1.

The shadowing C for a circular hole, a square hole and two infinite parallel plates is given in fig. 8 as a function of d/λ where λ is the mean free path in the moderator. d is in each case a characteristic length, namely the diameter of the circle, the side of the square and the distance between the plates. C is obtained from formulae (12), (14) and (18) respectively. To compute these values we have used tables of P_c for a cylinder ($P_o = 1 - P_c$) and tables of E_3 given by Case et al.[3]. The Ki-functions were obtained from the Kj-functions tabulated by Muller[5].

C for two parallel cylinders was calculated from equation (30) on the electronic digital computer BESK. The tables are too extensive to be reproduced here, but the results are given in fig. 9.

5. Acknowledgements.

We very much appreciate the cooperation of A. Rentze, who carried out most of the numerical work for table 1 and fig. 8, and of the mathematical group which performed the machine computations. K. Loimaranta gave valuable advice and L. Persson programmed and carried out the machine calculations.

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Appendix 1

The integral in equation (13) is to be solved. Since the integral is an even function of β we can write

$$C = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} Ki_3 (2a \cos \beta) \cos \beta \; d\beta \qquad \text{(A. 1. 1)}$$

Integration by parts gives

$$C = \frac{4}{\pi} \left\{ \Big[Ki_3 (2a \cos\beta) \sin \beta \Big]_0^{\frac{\pi}{2}} - 2a \int_0^{\frac{\pi}{2}} Ki_2 (2a \cos\beta) \sin^2\beta \; d\beta \right\} \qquad \text{(A. 1. 2)}$$

Now we use the recurrence formula for the Bickley functions[2], and also use $Ki_3(0) = \frac{\pi}{4}$ to get

$$1 - C = \frac{8a}{\pi} \int_0^{\frac{\pi}{2}} \Big[K_1(2a \cos\beta) - Ki_1(2a \cos\beta) \Big] 2a \cos\beta \; \sin^2\beta \; d\beta \qquad \text{(A. 1. 3)}$$

We integrate again by parts, in the first term of the integrand the factor $K_1(2a \cos\beta)$ $2a \sin\beta$ and in the second term the factor $\cos\beta \; \sin^2\beta$. We also use $Ki_1(0) = \frac{\pi}{2}$ and $\sin^4\beta = \frac{3}{8} - \frac{\cos 2\beta}{2} + \frac{\cos 4\beta}{8}$ to get

$$1 - C = \frac{8a}{\pi} \left[\int_0^{\frac{\pi}{2}} K_0(2a \cos\beta) \left(- \cos 2\beta + \frac{a^2}{2} - \frac{2a^2}{3} \cos 2\beta + \frac{a^2}{6} \cos 4\beta \right) d\beta - \frac{\pi a}{3} \right] \qquad \text{(A. 1. 4)}$$

A special case of a formula given by Watson[6] p. 441 is

$$\int_0^{\frac{\pi}{2}} K_0(2z \cos \theta) \cos 2m\theta \; d\theta = (-1)^m \frac{\pi}{2} I_m(z) K_m(z) \qquad \text{(A. 1. 5)}$$

Thus the integration can be carried out

$$1 - C = \frac{8a}{\pi}\left[\frac{\pi}{2} I_1K_1 + \frac{\pi a^2}{4} I_0K_0 + \frac{\pi a^2}{3} I_1K_1 + \frac{\pi a^2}{12} I_2K_2 - \frac{\pi a}{3}\right] \quad (A.1.6)$$

Here the argument of all the modified Bessel functions is a. After making use of the recurrence formulae for the modified functions and rearranging the following expression is obtained

$$C = 1 - \frac{4a^2}{3}\left\{2\left[a\left(I_0K_0 + I_1K_1\right) - 1\right] + I_0K_1 - I_1K_0 + \frac{I_1K_1}{a}\right\} \quad (A.1.7)$$

The quantity $\frac{1 - C}{2a}$ is, as it should, identical with the expression for $P_0(a)$ given by Case et al.[3] p. 33, except for some slight misprints in their formula.

Appendix 2

The following equality is to be proved

$$\int_z^\infty Ki_2(u) \frac{\sqrt{u^2 - z^2}}{u} = \frac{\pi}{2} E_3(z) \tag{A. 2. 1}$$

We start with the more general form

$$I_{n,m}(z) = \int_z^\infty Ki_n(u) \frac{(u^2 - z^2)^m}{u^{n-1}} \, du$$

The definition of $Ki_n(u)$ is introduced

$$I_{n,m}(z) = \int_z^\infty du \int_0^\infty dt \, \frac{e^{-u \cosh t}}{\cosh^n t} \, \frac{(u^2 - z^2)^m}{u^{n-1}} \tag{A. 2. 2}$$

With the new variables

$$\begin{cases} v = \dfrac{u}{z} \cosh t \\ s = \dfrac{u}{z \cosh t} \end{cases} \tag{A. 2. 3}$$

$$dv\, ds = \begin{vmatrix} \dfrac{u \sinh t}{z} & \dfrac{\cosh t}{z} \\ -\dfrac{u \sinh t}{z \cosh^2 t} & \dfrac{1}{z \cosh t} \end{vmatrix} du\, dt = \frac{2}{z} \sqrt{s(v - s)} \, du\, dt \tag{A. 2. 4}$$

we get

$$I_{n,m}(z) = \frac{z^{2m-n+2}}{2} \int_1^\infty e^{-zv} \, v^{-(n-\frac{1}{2})} \, dv \int_{\frac{1}{v}}^{v} \frac{(vs - 1)^m}{\sqrt{v-s}} \, ds \tag{A. 2. 5}$$

A further substitution is used for s

$$r = \frac{vs - 1}{v^2 - 1} \qquad s = \frac{r(v^2 - 1) + 1}{v} \qquad ds = \frac{v^2 - 1}{v}\, dr \tag{A. 2. 6}$$

Now the inner integral becomes

$$\int_{\frac{1}{v}}^{v} \frac{(vs - 1)^m}{v - s}\, ds = (v^2 - 1)^{m+\frac{1}{2}}\, v^{-\frac{1}{2}} \int_0^1 r^m (1 - r)^{-\frac{1}{2}}\, dr =$$

$$= (v^2 - 1)^{m+\frac{1}{2}}\, v^{-\frac{1}{2}}\, B(m+1, \tfrac{1}{2}) \tag{A. 2. 7}$$

where B is the beta-function.

In the special case n=2, $m=\frac{1}{2}$ the original integral is, since $B(\frac{3}{2}, \frac{1}{2}) = \frac{\pi}{2}$

$$I_{2, \frac{1}{2}}(z) = \frac{\pi z}{4} \int_1^{\infty} \frac{e^{-zv}}{v^2} (v^2 - 1)\, dv = \frac{\pi}{2}\, E_3(z) \tag{A. 2. 8}$$

Table 1

The function

$$F(x, y) = \int_x^y Ki_2(u) \sqrt{1 - \left(\frac{x}{u}\right)^2}\, du$$

y \ x	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
0	0										
0,1	0,0929	0									
0,2	0,1733	0,0546	0								
0,3	0,2435	0,1186	0,0385	0							
0,4	0,3050	0,1775	0,0885	0,0290	0						
0,5	0,3590	0,2301	0,1368	0,0690	0,0227	0					
0,6	0,4066	0,2769	0,1811	0,1087	0,0551	0,0183	0				
0,7	0,4486	0,3183	0,2210	0,1459	0,0881	0,0448	0,0149	0			
0,8	0,4857	0,3551	0,2567	0,1799	0,1194	0,0724	0,0369	0,0123	0		
0,9	0,5185	0,3877	0,2886	0,2105	0,1483	0,0989	0,0601	0,0307	0,0103	0	
1,0	0,5476	0,4166	0,3170	0,2381	0,1747	0,1236	0,0826	0,0503	0,0258	0,0086	0
1,1	0,5733	0,4422	0,3424	0,2628	0,1985	0,1462	0,1037	0,0695	0,0424	0,0218	0,0073
1,2	0,5962	0,4651	0,3649	0,2849	0,2200	0,1668	0,1233	0,0877	0,0589	0,0360	0,0185
1,3	0,6166	0,4853	0,3850	0,3047	0,2393	0,1855	0,1411	0,1045	0,0745	0,0501	0,0306
1,4	0,6347	0,5034	0,4029	0,3223	0,2565	0,2023	0,1573	0,1200	0,0890	0,0635	0,0428
1,5	0,6508	0,5194	0,4188	0,3381	0,2720	0,2174	0,1719	0,1340	0,1024	0,0761	0,0544
1,6	0,6651	0,5337	0,4330	0,3521	0,2858	0,2309	0,1852	0,1468	0,1147	0,0878	0,0653
1,7	0,6779	0,5465	0,4457	0,3647	0,2982	0,2431	0,1970	0,1584	0,1259	0,0985	0,0755
1,8	0,6892	0,5578	0,4570	0,3759	0,3093	0,2540	0,2077	0,1688	0,1360	0,1082	0,0848
1,9	0,6994	0,5680	0,4671	0,3859	0,3192	0,2638	0,2173	0,1782	0,1451	0,1171	0,0934
2,0	0,7084	0,5770	0,4761	0,3948	0,3281	0,2725	0,2259	0,1866	0,1534	0,1251	0,1011
2,5	0,7411	0,6096	0,5086	0,4272	0,3602	0,3043	0,2574	0,2176	0,1838	0,1550	0,1303
3,0	0,7598	0,6283	0,5272	0,4457	0,3786	0,3227	0,2756	0,2357	0,2017	0,1726	0,1476
3,5	0,7705	0,6390	0,5379	0,4564	0,3893	0,3333	0,2861	0,2461	0,2121	0,1829	0,1578
4,0	0,7767	0,6452	0,5441	0,4626	0,3955	0,3394	0,2923	0,2522	0,2181	0,1889	0,1638
4,5	0,7803	0,6488	0,5477	0,4662	0,3991	0,3430	0,2958	0,2558	0,2217	0,1924	0,1673
5,0	0,7824	0,6509	0,5498	0,4683	0,4012	0,3451	0,2979	0,2579	0,2237	0,1945	0,1694
∞	0,7854	0,6539	0,5528	0,4713	0,4041	0,3481	0,3009	0,2608	0,2267	0,1975	0,1723

The shadowing in the case of a circular hole (1), a square hole (2) and two infinite parallel plates (3)

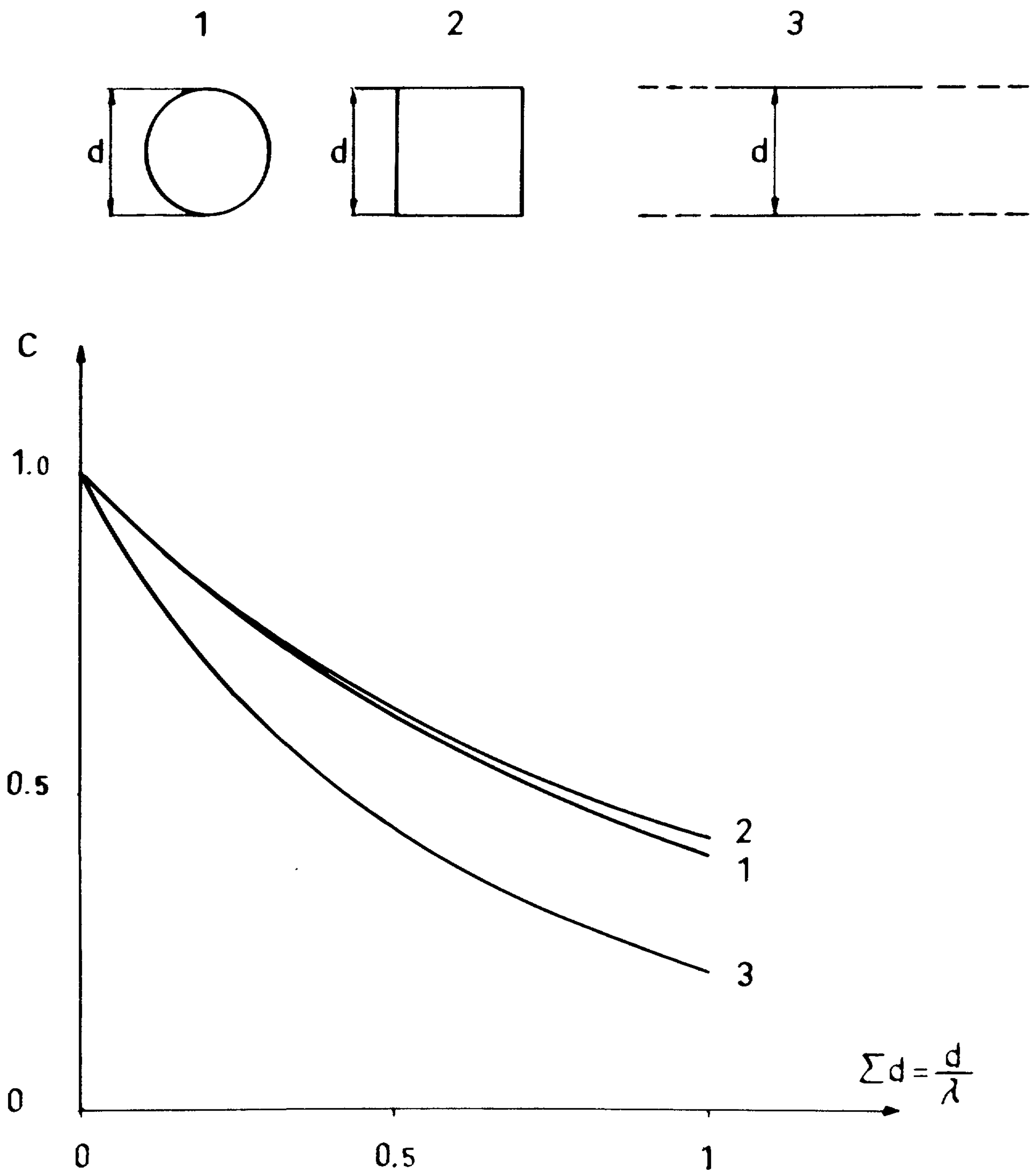

Figure 8

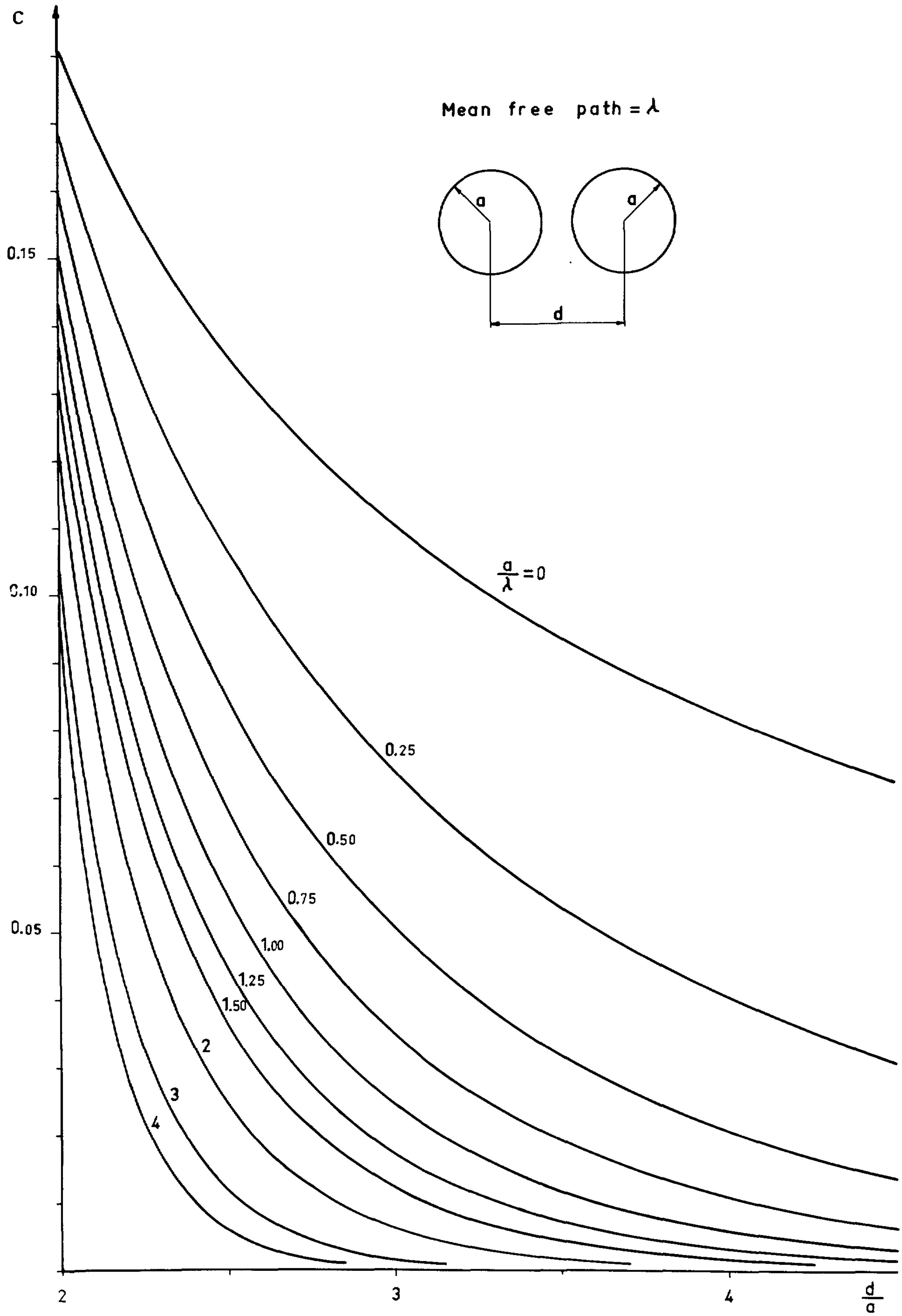
C
Mean free path = λ
a
a
d
a/λ = 0
0.25
0.50
0.75
1.00
1.25
1.50
2
3
4
0.15
0.10
0.05
2
3
4
d/a

Figure 9

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