AE-21

The fast fission effect in a cylindrical fuel element

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AKTIEBOLAGET ATOMENERGI

STOCKHOLM · SWEDEN · 1959

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Summary:

A new formula for the fast fission factor is derived, which takes proper account to fast capture. The fission neutron spectrum is divided into two groups with constant fission cross section in one group and zero fission cross section in the other. The average total, elastic, inelastic and capture cross sections in the two groups are calculated. Different assumptions regarding anisotropic and inelastic scattering are investigated. The effects of backscattering from the moderator and fast fission in neighbouring fuel elements are pointed out. Formulas for the fast fission ratio and for the fast conversion ratio are derived. The calculated fast fission ratios are compared with ecperimental values. Curves are given for the fast fission factor in uranium metal and uranium oxide.

Completion of manuscript November 1956
Printed June 1959

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The fast fission effect in a cylindrical fuel element

1. Introduction.

The commonly used formula for calculation of the fast fission factor ε is

$$\varepsilon - 1 = \frac{\left[(\nu - 1) - \frac{\sigma_c}{\sigma_f}\right] \frac{\sigma_f}{\sigma_t} P}{1 - \frac{\nu \sigma_f + \sigma_e}{\sigma_t} P'} \tag{1}$$

See, for example, Reactor Handbook (1) Ch. 1. 5, p 479.

In the derivation of this equation one examines neutrons of different generations and calculates the total number of neutrons which are either slowed down below the threshold energy for fission in U^{238} or which leave the fuel. In this way ε is obtained as an infinite series. Every term is a function of the cross-sections and it contains also, as a factor, the collision probability for the neutron generation in question. For all the neutron generations other than the first the collision probability is put equal to the collision probability at constant source density, P', thus making it possible to sum the series. P is the first generation collision probability, which depends on the distribution of thermal flux.

The cross-sections in equation (1) shall be the mean values for fission neutrons. The Reactor Handbook quotes values for natural uranium which give agreement with experimental results. Since these values were released, further material has been published and with curves from BNL-325 [2] it is possible to obtain almost the same values if we form the mean values according to

$$\bar{\sigma} = \frac{\int_{E_0}^{\infty} N(E)\,\sigma(E)\,dE}{\int_{0}^{\infty} N(E)\,dE} \tag{2}$$

in which N(E) indicates the spectral distribution of the fission neutrons and E_0 is about 1 MeV. In this way, however, the mean cross-sections become smaller than in reality. In equation (1) for example we use $\sigma_t = 4.3$ b, whereas σ_t (E) for uranium is greater than 6.8 b right up to 7 MeV according to BNL-325. If we examine the consequences of this on the simplifying assumption that $\sigma_t(e)$ is constant, we find that equation (2) assumes that the collision probability satisfies

$$k \cdot P(x) = P(kx)$$

in which k is that fraction of fission neutrons which are above threshold energy for fast fission and x is the product of the total macroscopic cross-section and a characteristic length of the uranium lump, for example the radius of a cylinder. This relation is approximately true for small x but evidently wrong for large x.

These complications do not matter with uranium metal, where the cross-sections are adapted to give the right result, but with a practical fuel element, since we must use a homogenising process, it is doubtful which kind of mean cross-section we should use for the other materials contained. We have therefore put forward a theoretically better justified formula for ε in which all cross-sections are taken from BNL-325. This formula gives the same results as equation (1) for uranium cylinders and does not cause difficulties in principle with homogenised fuel elements.

2. Definition of the fast fission factor ε.

Glasstone & Edlund [3] have two definitions for ε, the first on p. 83:

"the ratio of the total number of fast neutrons produced by fissions due to neutrons of all energies to the number resulting from thermal-neutron fissions".

If we used this definition strictly we should take into consideration neither capture above the threshold energy nor the fact that a neutron is lost with every fast fission.

The numerator in equation (1) should thus become

$$\nu \frac{\sigma_f}{\sigma_t} P$$

The other definition is on p. 276:

"the number of neutrons slowing down past the fission threshold of uranium^{238} per primary fission neutron, i. e. per neutron produced by thermal fission".

This definition is the basis of equation (1).

One may ask why capture shall be included in ε only down to the threshold energy for fast fission, but not below. We have instead chosen to include capture down to a lower boundary energy, in such a manner that practically the complete fission spectrum lies above the boundary level. We have set this boundary level at 0.1 MeV. This value is open to discussion, but it should not be particularly critical.

We thus define ε as the number of neutrons which are either slowed down below 0.1 MeV in the fuel or leave the fuel, per primary neutron from thermal fission.

3. Two-group calculation of ε.

In the first case we examine now uranium metal, and when explicit forms for the collision probability are needed we assume that the fuel has the shape of circular cylinders.

The fission cross-section of U^{238} increases rapidly mainly between 1 and 2 MeV [2] . The measured values go up to 3 MeV. With the guidance of curves concerning fission cross-sections for Np^{237}, U^{236}, U^{234} and Pa^{231}, together with consideration of the form of the fission spectrum it has been thought legitimate to put σ_f constant above 2 MeV and equal to 0,545 b. Below 1 MeV we put $\sigma_f = 0$.

It is natural to divide the fission neutrons into two groups 1 and 2 so that $\sigma_{1f} = 0$ in group 1 and $\sigma_{2f} = 0.545$ in group 2. The borderline between group 1 and group 2 shall be decided so that

$$\sigma_{2f} \int_{E_2}^{\infty} N(E)\, dE = \int_{0}^{\infty} N(E)\, \sigma_f(E)\, dE \tag{3}$$

4. Fission spectrum.

It is assumed that the energy distribution is the same for fast and thermal fission.

For the fission neutrons´ energy distribution the following formula is used, which is given by Leachman [4] .

$$N(E) = c \cdot \sqrt{E} \exp\left(-\frac{E}{1.29}\right) \tag{4}$$

with E in MeV. The distribution function is normalised with

$$c = \frac{2}{(1.29)^{3/2}\sqrt{\pi}} = \frac{1}{1.299}$$

The fraction of neutrons between E_1 and E_2 ($> E_1$) is

$$\int_{E_1}^{E_2} N(E)\, dE = \frac{2}{\sqrt{\pi}} \left\{ \sqrt{\frac{E_1}{1.29}}\, e^{-\frac{E_1}{1.29}} - \sqrt{\frac{E_2}{1.29}}\, e^{-\frac{E_2}{1.29}} \right\} + \phi\left(\sqrt{\frac{E_2}{1.29}}\right) - \phi\left(\sqrt{\frac{E_1}{1.29}}\right) \tag{5}$$

where ϕ (x) is the error integral according to Jahnke and Emde [5] p. 24:

$$\phi(x) = E_2(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \tag{6}$$

With the help of equation (5) the number of fission neutrons in suitably chosen intervals has been calculated (table 1), and the corresponding histogram is given in figure 1.

With this distribution and with $\sigma_f(E)$ taken from BNL-325, E_2 is obtained according to equation (3).

$$E_2 = 1.49 \text{ MeV}$$

5. Group constants.

Neutrons with energy below $E_1 = 0.1$ MeV belong to group 0, with energies between $E_1 = 0.1$ MeV and $E_2 = 1.49$ MeV to group 1 and with energies above $E_2 = 1.49$ MeV to group 2.

The following mean cross-sections are needed:

σ_{1c} and σ_{2c} = capture cross-sections for groups 1 and 2 respectively

σ_{11} and σ_{22} = cross-sections which state the probability that a neutron in group 1 or group 2 respectively is still in the same group after a collision.

$\sigma_{10}, \sigma_{21}, \sigma_{20}$ = cross-sections which state the probability that a colliding neutron is transferred from group 1 to group 0, from group 2 to group 1 and from group 2 to group 0 respectively.

σ_{2f} = fission cross-section for group 2.

$\sigma_1 = \sigma_{1c} + \sigma_{11} + \sigma_{10}$ = total cross-section for group 1.

$\sigma_2 = \sigma_{2c} + \sigma_{22} + \sigma_{21} + \sigma_{20} + \sigma_{2f}$ = total cross-section for group 2.

Further needed are

Σ_1 and Σ_2 = the total macroscopic cross-sections.

ν_0, ν_1 and ν_2 = number of fission neutrons in groups 0, 1 and 2 resp.

$\nu = \nu_0 + \nu_1 + \nu_2$ = number of fission neutrons per fast fission.

We have calculated mean cross-sections for the materials commonly involved in a fuel element (table 2). Since the data used are not very complete, the table must be considered to have a provisional character, and the contents at several points are quite approximate.

ν for fast fission in U^{238} with fission neutrons has been calculated with ν (E) according to Leachman ([4] , fig. 13). Approximately we have

$$\nu(E) = 2.40 + 0.119\,E \qquad (E \text{ in MeV}) \tag{7}$$

The mean value for fissions caused by fission neutrons becomes

$$\nu = \frac{\int_{1.49}^{\infty} \nu(E)\, N(E)\, dE}{\int_{1.49}^{\infty} N(E)\, dE} = 2.76 \tag{8}$$

According to table 1 it is found that

$$\nu_0 = 0.043 \qquad \nu_1 = 1.307 \qquad \nu_2 = 1.410$$

6. Assumptions.

1. The form of the fission spectrum is assumed to be the same for thermal and fast fission. On the other hand the ν -values need not be equal since only ν for U^{238} is included in ε.

2. It is assumed that the energy distribution alters but little with collisions, so that the same mean cross-section can be used for all neutrons, independent of whether they originate direct from fission or whether they have collided.

3. The source density is assumed to be constant for all neutron generations other than the first, which originates direct from thermal fission, and all sources are taken to have isotropic velocity distribution.

7. Formula for ε.

The number of collisions per unit time in group 1 and group 2 are called N_1 and N_2 respectively, and the number of neutrons per second emitted from thermal fission is taken to be 1. ε is the number of neutrons per second which either leave the fuel element or are slowed down to below 0.1 MeV. This number can be expressed with the aid of N_1 and N_2:

$$\varepsilon = \frac{\nu_0}{\nu} + \frac{\nu_1}{\nu}(1-P_1) + \frac{\nu_2}{\nu}(1-P_2) + \frac{\sigma_{10}}{\sigma_1} N_1 + \frac{\sigma_{20} + \nu_0 \sigma_f}{\sigma_2} N_2$$

$$+ \left(N_1 - P_1 \frac{\nu_1}{\nu}\right) \frac{1-P'_1}{P'_1} + \left(N_2 - P_2 \frac{\nu_2}{\nu}\right) \frac{1-P'_2}{P'_2} \tag{9}$$

Further we have the continuity equations for both groups:

$$\left.\begin{aligned} N_1 &= \frac{\nu_1}{\nu} P_1 + N_1 \frac{\sigma_{11}}{\sigma_1} P'_1 + N_2 \frac{\sigma_{21} + \nu_1 \sigma_f}{\sigma_2} P'_1 \\ N_2 &= \frac{\nu_2}{\nu} P_2 + N_2 \frac{\sigma_{22} + \nu_2 \sigma_f}{\sigma_2} P'_2 \end{aligned}\right\} \tag{10}$$

In equation (9) and (10) N_1 and N_2 can be eliminated and we obtain

$$\varepsilon - 1 = -\frac{\nu_1}{\nu} \frac{P_1}{1-\frac{\sigma_{11}}{\sigma_1} P'_1} \frac{\sigma_{1c}}{\sigma_1} + \frac{\nu_2}{\nu} \frac{P_2}{1-\frac{\sigma_{22}+\nu_2\sigma_f}{\sigma_2} P'_2} \frac{(\nu-1)\sigma_f-\sigma_{2c}}{\sigma_2} - \qquad (11)$$

$$-\frac{\nu_2}{\nu} \frac{P_2}{1-\frac{\sigma_{22}+\nu_2\sigma_f}{\sigma_2} P'_2} \frac{\sigma_{21}+\nu_1\sigma_f}{\sigma_2} \frac{P'_1}{1-\frac{\sigma_{11}}{\sigma_1} P'_1} \frac{\sigma_{1c}}{\sigma_1}$$

Cross-sections are to be taken from table 2 and ν -values from equation (8).

8. Collision probabilities.

As already said we set the collision probabilities equal to P,′ for the second and following neutron generations, P′ being the collision probability for constant source strength. This approximation gives a quite small error. For the first generation the source strength is proportional to the thermal flux, and consequently largest at the fuel surface. For the following generations the source strength approaches an asymptotic distribution (apart from a constant factor) in which the source strength is smallest at the surface. The asymptotic distribution is not much different from constant source strength, and for the generations immediately after the first the difference is still smaller. One has in many cases for the second generation even very nearly constant source density.

For a cylinder, P′ is a function of the product of the total macroscopic cross-section Σ_1 and Σ_2 respectively and the radius a. This function has been tabulated by Case, de Haffmann and Placzek, whose table is reproduced in AEF 69 ([6] , table 1). There P′ is called P_c and Σ_a is denoted by $\frac{a}{l}$.

For the first generation we also often set P = P.′ We can, however, easily correct for the varying source density. It is sufficient to approximate the source density, which is proportional to the thermal flux, by a function of the type $\alpha + \beta r^2$. Then we obtain

$$P = P' - (F-1)(P' - P_r) \qquad (12)$$

in which P_r is the collision probability for a cylinder where the source density is proportional to r^2 (see AEFI-41 [7]), and F is the ratio between the thermal surface-flux and the thermal mean flux in the cylinder. $P' - P_r$ is represented as a function of $\Sigma \cdot a$ in diagram 2.

By varying $\frac{\beta}{\alpha}$ from 0 to ∞ F is made to vary from 1 to 2. If F is greater than 2 it is not sufficient to use a second degree function for the source density; further terms of higher degree must be used. However, in practice F is seldom greater than 1. 5, so that the treatment given here is sufficient. Moreover, one can frequently put $P = P'$

9. Coupling effect.

The collision probability is increased somewhat for the reason that a neutron can pass through the moderator and collide again with the original fuel element or another. We shall only consider here the case of a neutron going from one fuel element to another without colliding in the moderator. The emitting fuel element is approximated by a line source with isotropic angular distribution. Other fuel elements are assumed to have small cross sectional area (A per fuel element) in comparison with the moderator. The distance between the emitting fuel element and the nth of the others is called d_n. The total macroscopic cross-sections in the fuel element and in the moderator are called Σ_u and Σ_m respectively. Then the probability that a neutron, emitted from a fuel element, will collide with another fuel element without colliding in the moderator can be written

$$P_k = \sum_{n=1}^{N} \frac{A_n \Sigma_u}{2\pi d_n} Ki_1 (\Sigma_m d_n) \qquad (13)$$

The function $Ki_1(x)$ has been defined by Bickley and Nayler [8] .

$$Ki_1(x) = \int_0^{\infty} \frac{e^{-x \cosh u}}{\cosh u} du \qquad (14)$$

A table of $Ki_1(x)$ can be found in [9] .

10. The fast fission ratio and the fast conversion ratio.

The fast fission ratio R is defined as the ratio between the number of fissions caused by fast neutrons and the number of fissions caused by thermal neutrons, or the ratio between the number of fissions in U^{238} and the number of fissions in U^{235}.

The fast conversion ratio (FCR) is defined as the number of Pu-nuclei formed through capture of fast neutrons per U^{235} nucleus consumed.

The number of secondary neutrons per thermal fission and per fast fission are called ν_t and ν_f respectively (earlier merely ν) and the ratio of capture cross-section to fission cross-section for thermal neutrons is called α_t.

$$R = \frac{N_2 \cdot \frac{\sigma_f}{\sigma_2}}{\frac{1}{\nu_t}} = \nu_t \cdot \frac{\nu_2}{\nu_f} \frac{\sigma_f}{\sigma_2} \frac{P_2}{1 - \frac{\sigma_{22} + \nu_2 \sigma_f}{\sigma_2} P_2'} \tag{15}$$

Also

$$FCR = \frac{N_1 \frac{\sigma_{1c}}{\sigma_1} + N_2 \frac{\sigma_{2c}}{\sigma_2}}{\frac{1}{\nu_t}(1+\alpha_t)} = \frac{R(\nu_f - 1) - \nu_t(\varepsilon - 1)}{1 + \alpha_t} \tag{16}$$

11. Discussion.

We have given a method for calculation of ε which is undeniably more complicated than the usual. The advantage is that we have only used measured cross-sections according to BNL-325, so that the method can be used without difficulties in principle on homogenised fuel elements. We have calculated values of ε for cylinders of uranium metal and uranium oxide (figures 3 and 4 respectively). For uranium metal we obtain the same results as Guggenheim and Pryce [10]. This should not be taken as proof that the method is correct, however.

The uncertain points with calculation of ε according to this method are the following:

a) The effect of back-scattering from the moderator.

b) Choice of the lower boundary energy for group 1.

c) The value of ν_f, the number of secondary neutrons in fast fission.

d) Energy loss and cross-section in inelastic scattering by heavy nuclei.

e) The effect of anisotropic scattering at high energies.

a) Back-scattering and b) Lower boundary energy.

We have chosen to include absorption in U^{238} down to 0.1 MeV in ε. The question is whether this absorption should be included in ε or whether it is included in the resonance escape factor p. In p appears V_0/V_1 which together with a cross-section factor can be taken to represent the ratio between probabilities for collision in the fuel and collision in the moderator. Before a neutron leaves the fuel it has greater probability of colliding in the fuel, and we can therefore imagine the absorption split up in such a manner that, as long as the neutron has not left the fuel, the absorption is included in ε, but as soon as it has collided in the moderator the absorption is included in p. This way of reasoning justifies also the inclusion in the coupling effect of only the case where the neutron goes from one fuel element to another without colliding in the moderator.

The situation is different if a neutron which has collided in the moderator passes into a fuel element with sufficient energy to cause fast fission. This effect is not further considered here but may be taken up later.

c) ν_f.

In general on uses a value for ν_f of about 2.5, whereas we use 2.76. The variation with energy, which is given by Leachman [4], ought to be reasonably correct. If $\nu(E)$ varies linearly with energy we find that $\bar{\nu} = \nu(\bar{E})$. The value 2.5 is obtained by taking the mean value over the whole of the fission spectrum with $\bar{E} \sim 1.9$. It is more correct to take the mean value of ν for neutrons above threshold energy for fission with $\bar{E} \sim 3$ MeV. We obtain thus a significantly higher ν.

The value of ν_f is obviously important for ε. For very small radii

$$\varepsilon - 1 = \frac{4}{3} a \left[(\nu_f - 1) \bar{\Sigma}_f - \bar{\Sigma}_c \right] \quad (\Sigma a << 1) \tag{17}$$

in which the mean values are taken over the whole spectrum, and also for greater radii $\varepsilon - 1$ is approximately proportional to $\nu_f - 1$.

d) Heavy nuclei like uranium have a fairly large inelastic scattering cross-section, σ_{inel}. This cross-section gives rise to an uncertainty in ε partly because the cross-section is not particularly well known, and partly because the energy spectrum of the scattered neutrons is not known with great accuracy. In the calculation of table 2 we have assumed for uranium that a neutron in an inelastic collision loses almost all its energy, so that $\sigma_{1\ inel}$ is included in σ_{10} and $\sigma_{2\ inel}$ is included in σ_{20}. We have, however, in comparison with experiment, also used other hypotheses, namely that the spectrum of the inelastically scattered neutrons has the form

A) $c \cdot dE$

B) $c \cdot E \cdot dE$

In the various cases σ_{inel} thus divides in different ways, $\sigma_{1\ inel}$ between σ_{11} and σ_{10}, and $\sigma_{2\ inel}$ between σ_{22}, σ_{21} and σ_{20}.

e) In elastic scattering at high energies against heavy nuclei there is a strong anisotropy, so that forward scattering is predominant. This means that, especially in group 2, we should use a smaller elastic cross-section. We have examined the effect of this in comparison with experiment, by examining whether the result is changed if we firstly put $\sigma_{2\ el} = 0$, corresponding to scattering without any change of direction, and for the second part multiply $\sigma_{1\ el}$ by $(1 - \overline{\cos\theta})$, in which $\overline{\cos\theta}$ is calculated for 1 MeV with an angular distribution according to BNL-325.

12. Comparison with experiments.

In those cases where measured values for ε have been given, one has measured the fast fission ratio R and then calculated ε. The results thus obtained depend on which values one has taken for ν_t, ν_f and certain cross-sections. We have therefore compared our calculated values for R with experimental values of R for uranium cylinders. In this comparison we have used various hypotheses for the energy spectrum of the inelastically scattered neutrons and also examined the effect of forward scattering according to the points d) and e) above respectively. We have calculated R for altogether seven different combinations of cross-sections and with $\nu_t = 2.47$ [11]. In cases 1 to 5 the inelastic cross-sections have been divided in different ways, and the elastically scattered neutrons are assumed to have isotropic angular distribution. In cases 6 and 7 the elastic cross-sections have been treated as discribed under e), and two hypotheses for the inelastic cross-sections are used. This should be immediately clear from the following scheme.

Case	$\sigma_{1\ el}$	$\sigma_{2\ el}$	$\sigma_{1\ inel}$	$\sigma_{2\ inel}$
1	isotropic	isotropic	$= \sigma_{10}$	$= \sigma_{20}$
2	"	"	$= \sigma_{10}$	$= \sigma_{21}$
3	"	"	according to B	according to B
4	"	"	according to B	according to A
5	"	"	$= \sigma_{11}$	$= \sigma_{21}$
6	$\overline{\cos\theta} = 1$	$\overline{\cos\theta} = 0.37$	according to B	according to B
7	$\overline{\cos\theta} = 1$	$\overline{\cos\theta} = 0.37$	according to B	according to A

Corresponding sets of cross-sections are found in table 3.

In figure 5 curves of R as a function of the cylinder radius have been drawn for the 7 different cases. Available experimental results have also been plotted [12], [13], [14]. The spread in these values is so great that one cannot find any hypotheses more probable than the others. No reason is found therefore as yet to abondon the values in table 2. Better measurements of R with rather large radii are much to be desired, so that one can distinguish between the hypotheses.

Besides R, $\varepsilon - 1$ and FCR have also been calculated for some different rod radii with the different hypotheses, table 4, 5 and 6. $\varepsilon - 1$ has also been plotted in figure 6.

13. References.

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IC/AJ
29. 5. 1959

Table 1

Fission spectrum

Energy, MeV	Fraction of neutrons in the range	Fraction of neutrons in the group
0		
	0.0155	Group 0 0.0155
0.1		
	0.0263	
0.2		
	0.0664	
0.4		
	0.0736	
0.6		
	0.0748	
0.8		Group 1 0.4736
	0.0725	
1.0		
	0.0692	
1.2		
	0.0640	
1.4		
	0.0268	
1.49		
	0.0320	
1.6		
	0.0539	
1.8		
	0.0486	
2.0		
	0.1012	
2.5		
	0.0760	
3.0		Group 2 0.5109
	0.0561	
3.5		
	0.0408	
4.0		
	0.0296	
4.5		
	0.0212	
5.0		
	0.0515	
∞		

The distribution function used is that of Leachman (4) :

$$N(E) = \frac{1}{1.299} \sqrt{E} \exp\left(-\frac{E}{1.29}\right) \qquad (E \text{ in MeV})$$

Table 2. Calculated mean cross-sections

All values in barns

	Group 1				Group 2					
	σ_{11}	σ_{10}	σ_{1c}	σ_1	σ_{22}	σ_{21}	σ_{20}	σ_{2c}	σ_{2f}	σ_2
D	2.67	0.30	0	2.97	0.93	1.34	0	0	0	2.27
C	2.96	0.04	0	3.00	1.30	0.23	0	0	0	1.53
O	4.16	0.02	0	4.18	1.43	0.21	0	0	0	1.64
Al	3.74	0.03	0.01	3.78	2.12	0.62	0	0.01	0	2.75
Zr	7.69	0.01	0	7.70	4.08	0.44	0	0	0	4.52
U_{nat}	6.58	1.30	0.13	8.01	4.83	0	1.997	0.038	0.545	7.41
D_2O	9.50	0.62	0	10.12	3.29	2.89	0	0	0	6.18
UO_2	14.90	1.34	0.13	16.37	7.69	0.42	1.997	0.038	0.545	10.69

Table 3. Cross-section sets with various hypotheses for σ_{el} and σ_{inel}

Case	σ_{11}	σ_{10}	σ_{1c}	σ_1	σ_{22}	σ_{21}	σ_{20}	σ_{2c}	σ_{2f}	σ_2
1	6.58	1.30	0.13	8.01	4.830	0	1.997	0.038	0.545	7.410
2	6.58	1.30	0.13	8.01	4.830	1.997	0	0.038	0.545	7.410
3	7.80	0.08	0.13	8.01	6.076	0.747	0.004	0.038	0.545	7.410
4	7.80	0.08	0.13	8.01	5.661	1.088	0.078	0.038	0.545	7.410
5	7.88	0	0.13	8.01	4.830	1.997	0	0.038	0.545	7.410
6	5.43	0.08	0.13	5.64	1.246	0.747	0.004	0.038	0.545	2.580
7	5.43	0.08	0.13	5.64	0.831	1.088	0.078	0.038	0.545	2.580

Table 4. R for various rod radii and hypotheses

Radius	Hypothesis						
a cm	1	2	3	4	5	6	7
0.5	0.0200	0.0200	0.0207	0.0205	0.0200	0.0208	0.0206
1.0	0.0384	0.0384	0.0412	0.0402	0.0383	0.0406	0.0397
1.5	0.0554	0.0554	0.0617	0.0594	0.0554	0.0599	0.0578
2.0	0.0710	0.0710	0.0819	0.0779	0.0710	0.0785	0.0749
2.5						0.0960	0.0906
3.0	0.0977	0.0977	0.1200	0.1117	0.0977		

Table 5. $\varepsilon - 1$ for various rod radii and hypotheses

Radius	Hypothesis						
a cm	1	2	3	4	5	6	7
0.5	0.0117	0.0116	0.0121	0.0119	0.0115	0.0123	0.0121
1.0	0.0225	0.0221	0.0241	0.0233	0.0218	0.0237	0.0230
1.5	0.0325	0.0315	0.0358	0.0340	0.0308	0.0345	0.0330
2.0	0.0414	0.0399	0.0470	0.0438	0.0384	0.0448	0.0420
2.5						0.0541	0.0501
3.0	0.0579	0.0548	0.0678	0.0609	0.0501		

Table 6. FCR for various rod radii and hypotheses

Radius a cm	Hypothesis 1	2	3	4	5	6	7
0.5	0.0053	0.0056	0.0055	0.0057	0.0057	0.0052	0.0054
1.0	0.0101	0.0110	0.0110	0.0111	0.0115	0.0109	0.0111
1.5	0.0145	0.0166	0.0171	0.0174	0.0181	0.0171	0.0171
2.0	0.0192	0.0223	0.0237	0.0244	0.0254	0.0232	0.0237
2.5						0.0298	0.0302
3.0	0.0244	0.0309	0.0369	0.0390	0.0407		

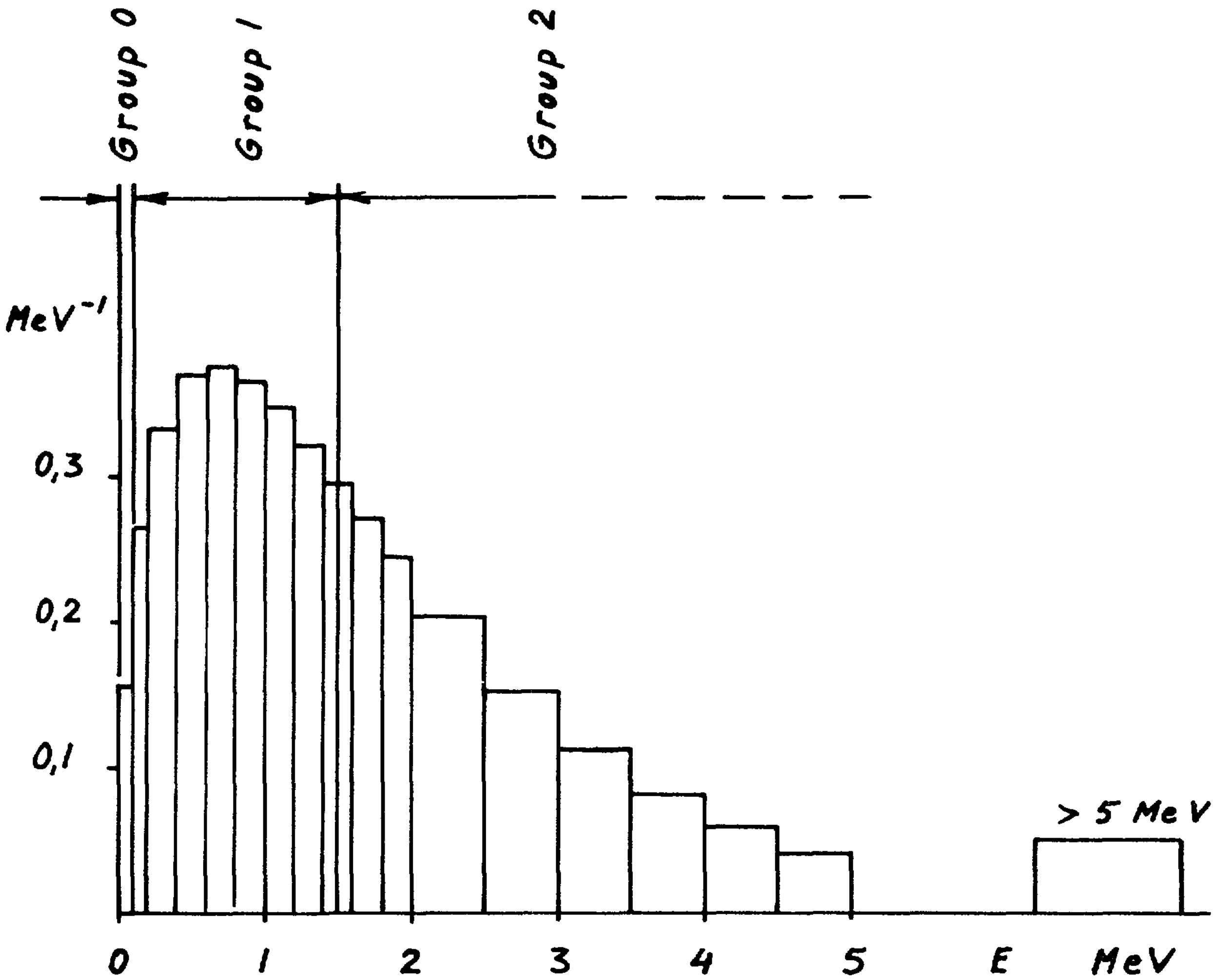

Fig 1. Normalised histogram of energy distribution of fission neutrons. Distribution function acc. to Leachman [4].

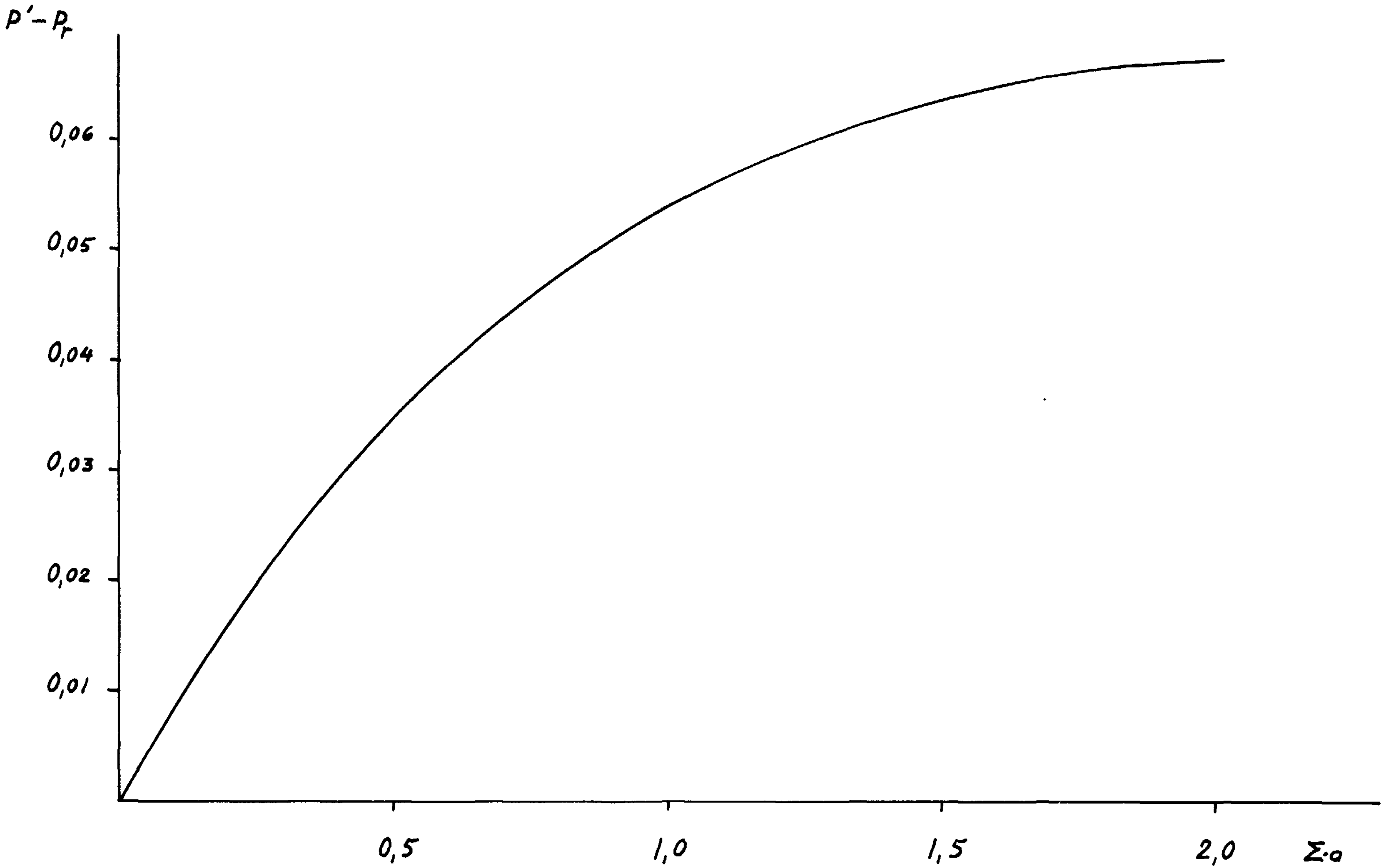

Fig 2. $P' - P_r$ as a function of $\Sigma \cdot a$.

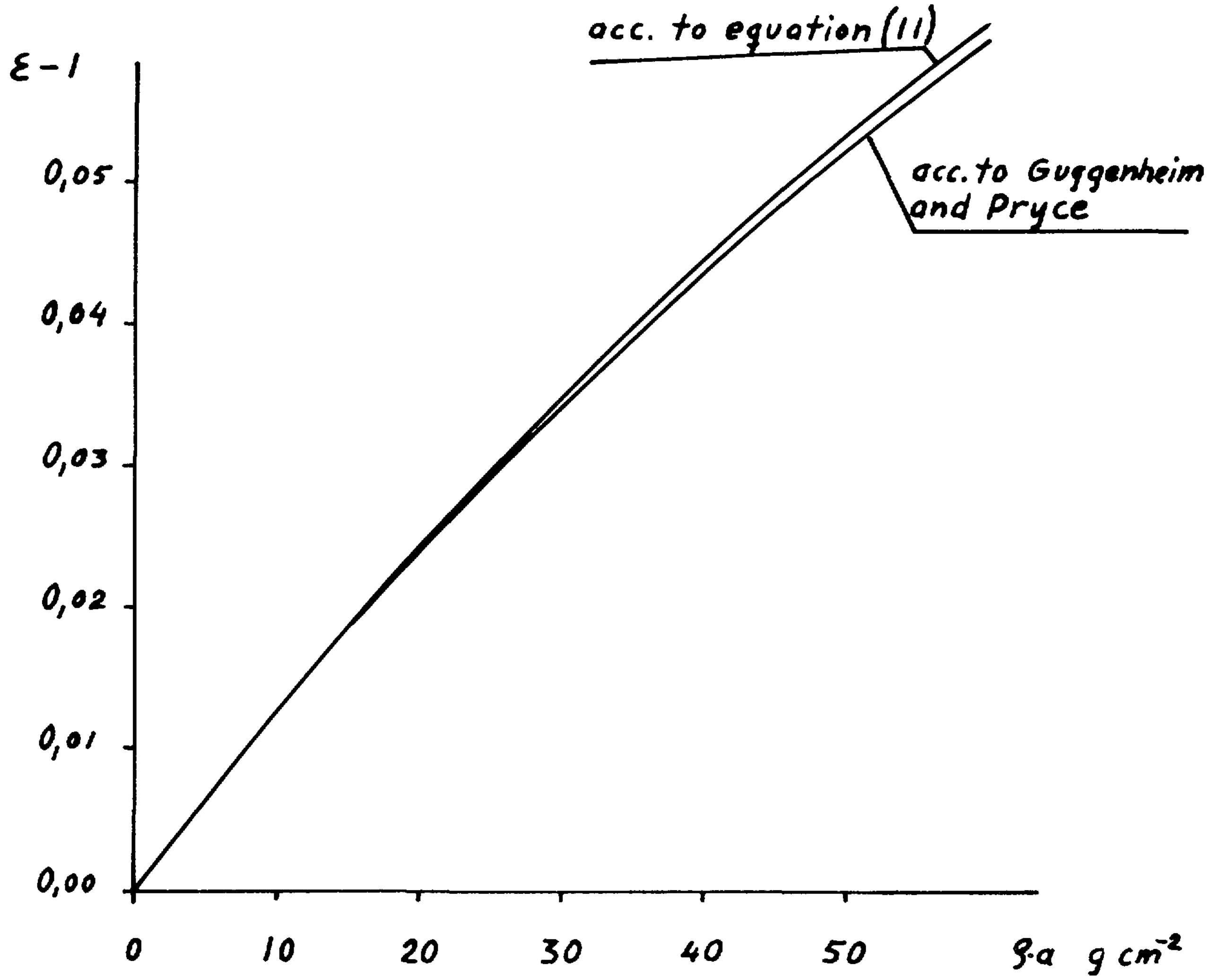

Fig. 3. $\varepsilon - 1$ for cylinders of uranium metal as a function of density times radius.

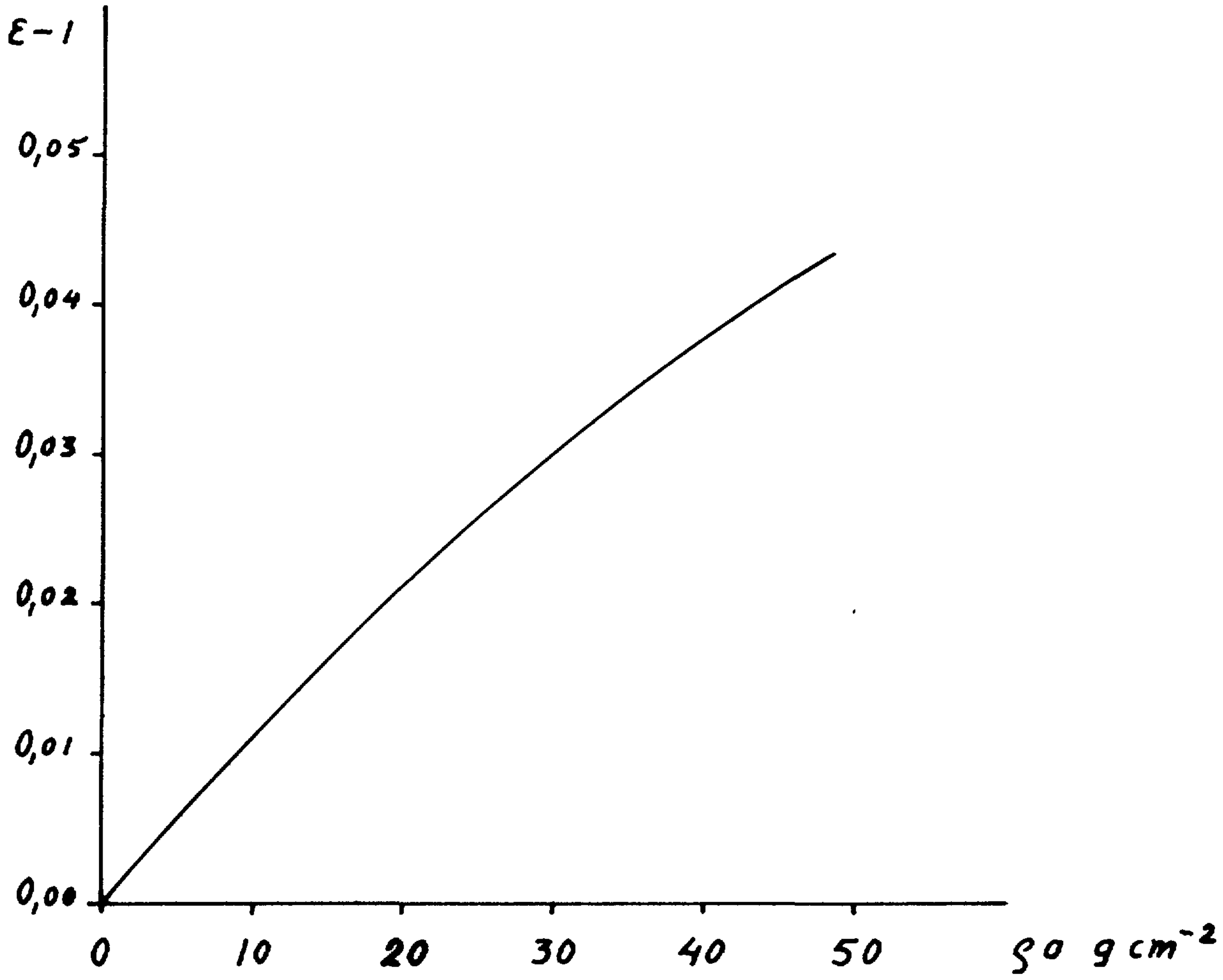

Fig. 4. ε−1 for cylinders of uranium oxide as a function of density times radius.

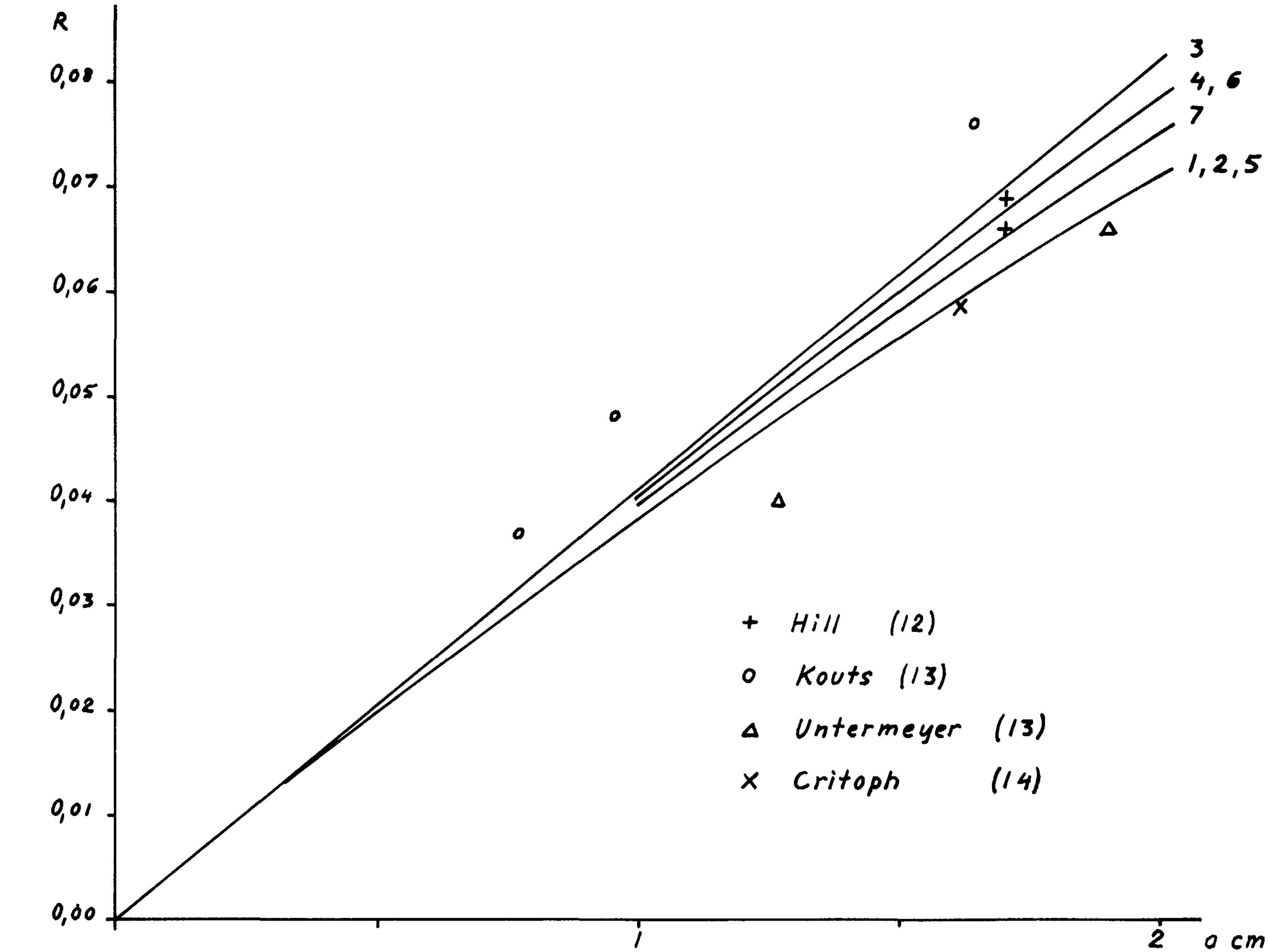

Fig. 5. R for uranium cylinders as a function of radius with various hypotheses for σ_{el} and σ_{inel}.

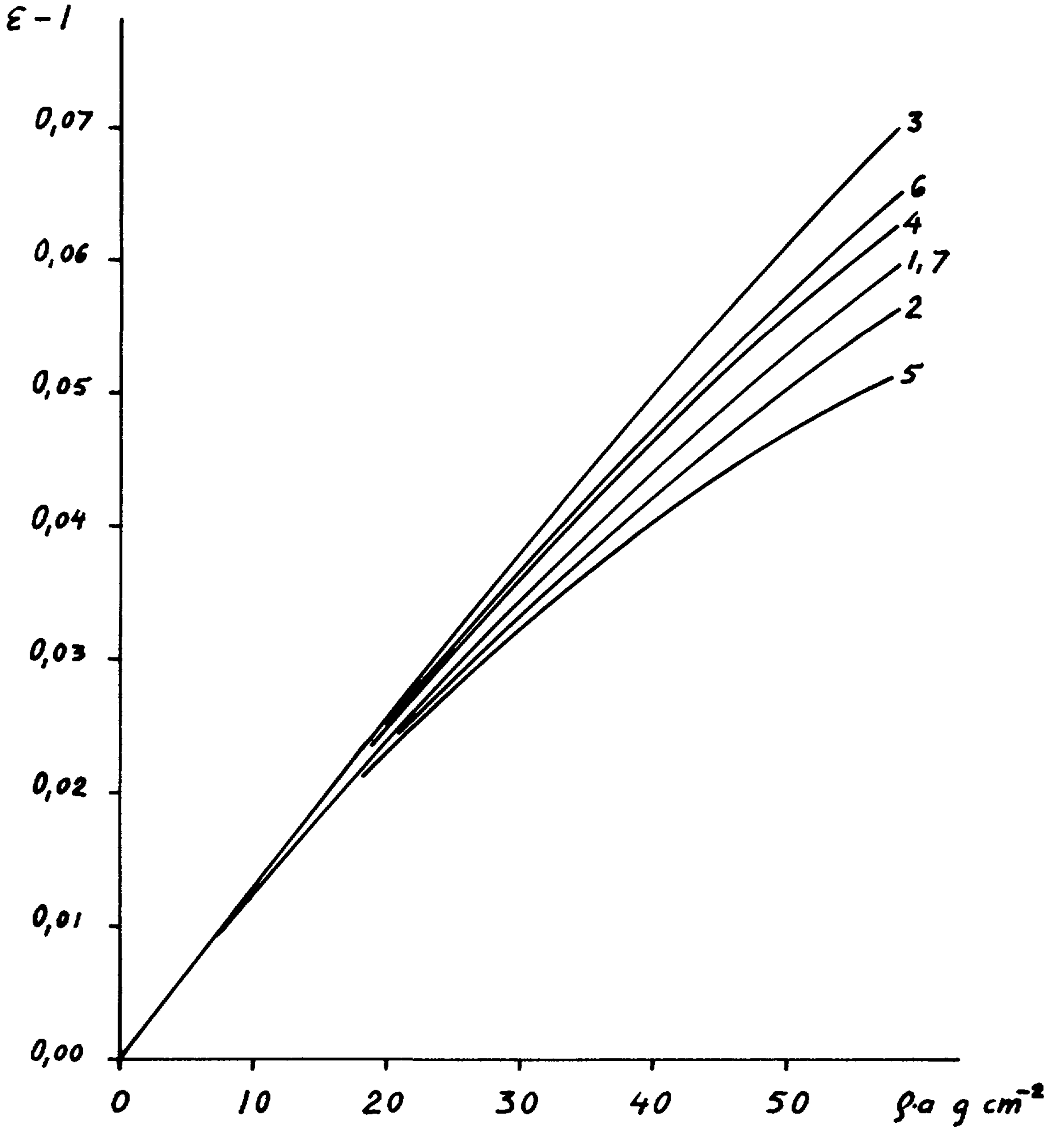

Fig. 6. $\varepsilon - 1$ for uranium cylinders as a function of radius with various hypotheses for σ_{el} and σ_{inel}.

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