AE-281
UDC 621.039.51.12
539.125.523.348

AE-281

Collision Probabilities for Finite Cylinders and Cuboids

I. Carlvik

AKTIEBOLAGET ATOMENERGI

STOCKHOLM, SWEDEN 1967

COLLISION PROBABILITIES FOR FINITE CYLINDERS AND CUBOIDS

by

I. Carlvik

SUMMARY

Analytical formulae have been derived for the collision probabilities of homogeneous finite cylinders and cuboids. The formula for the finite cylinder contains double integrals, and the formula for the cuboid only single integrals. Collision probabilities have been calculated by means of the formulae and compared with values obtained by other authors. It was found that the calculations using the analytical formulae are much quicker and give higher accuracy than Monte Carlo calculations.

Printed and distributed in May 1967

LIST OF CONTENTS

1. INTRODUCTION

The collision probability of a homogeneous body is the probability that particles from an isotropic constant source inside the body collide in the body in their first flight. Collision probabilities have been used to a great extent for the calculation of the fast effect and of the resonance absorption in nuclear reactors, but they are also important in other applications, for example neutron flux measurement by means of activation of small samples, calculation of the reactivity effect of small samples in a fast reactor, and the attenuation of gamma rays in a gamma source.

Analytical expressions have been given by Case et al. (1) for infinite slabs, infinite cylinders, spheres, hemispheres, oblate spheroids and oblate hemispheroids. In practice a sample is often given the form of a cuboid or a finite cylinder. These shapes are more complicated and collision probabilities for them cannot be expressed in simple functions only.

A general approach is to use the Monte Carlo method, and two such codes have been written. The CEP code by Foell, Berner and Tong (2) calculates the collision probability of a finite cylinder or annulus, which may be empty or not, and the code by Gubbins (3), which is also called CEP, calculates the collision probability of a finite cylinder or a cuboid.

McLeod (4) has derived an analytical formula for the collision probability of a finite cylinder. The formula contains only double integrals. All three papers contain numerical results.

We have derived a formula for the collision probability of a cuboid. For this geometry it is possible to express the collision probability in single integrals only. We were not aware of the work of McLeod, so we also derived an expression for the collision probability of a finite cylinder.

McLeod's approach and ours are so different that we hope that our derivation may still be of interest. McLeod utilizes formulae for chord distributions, whereas we use a more elementary method and calculate the currents out of the cylinder for a constant source distribution. Our final expression looks simpler than McLeod's, but this may be due to the fact that his formula is written to allow a general absorption law. McLeod's expression covers explicitly only cases with height to diameter larger than one, but the generalization should be simple. It can no doubt be proved that the two formulas are equivalent.

In the next two paragraphs the formulae are derived. The last paragraph shows sample calculations and comparisons with results of the other authors.

A typical computing time for a collision probability of a cylinder with an accuracy of 10^{-4} is something like one second on IBM 7044. The computing time for a cuboid is shorter. This is very fast when compared to Monte Carlo calculations. On the other hand a Monte Carlo method is more general and can easily be extended to still more complicated geometries or to inclusion of non-flat source distributions.

2. COLLISION PROBABILITY FOR A FINITE CYLINDER

Consider a cylinder of radius a (diameter d = 2a) and height h, measured in mean free paths. Let the collision probability be P_c, and set

$$P_c = 1 - P_e - P_m \tag{1}$$

where P_e is the probability of escape through the two end surfaces and P_m is the probability of escape through the curved surface.

P_e is calculated first. With notations from figure 1

$$P_e = \frac{2}{\pi a^2 h} \int_0^a \rho d\rho \int_0^{2\pi} d\varphi \int_0^h d\zeta \int_0^a r dr \int_0^{2\pi} d\psi \frac{e^{-R}}{4\pi R^2} \frac{\zeta}{R} \tag{2}$$

$\frac{\zeta}{R}$ is the cosine of the angle between the inward normal at the escape point and the neutron path.

Integration over one angle can be performed directly. With $\varphi - \psi = \alpha$

$$P_e = \frac{1}{\pi a^2 h} \int_0^a \rho d\rho \int_0^{2\pi} d\alpha \int_0^a r dr \int_0^h \zeta d\zeta \frac{e^{-R}}{R^3} \tag{3}$$

where

$$R^2 = t^2 + \zeta^2 ; \quad t^2 = r^2 + \rho^2 - 2r\rho \cos\alpha \tag{4}$$

Integrate over R instead of ζ ; $\zeta d\zeta = RdR$

$$\int_0^h \zeta d\zeta \frac{e^{-R}}{R^3} = \int_t^{\sqrt{t^2+h^2}} dR \frac{e^{-R}}{R^2} = \frac{E_2(t)}{t} - \frac{E_2(\sqrt{t^2+h^2})}{\sqrt{t^2+h^2}} \tag{5}$$

where $E_2(z)$ is an exponential integral

$$E_n(z) = \int_1^\infty \frac{e^{-zu}}{u^n} du \tag{6}$$

Set

$$P_e = \frac{1}{\pi a^2 h} \left[I(0) - I(h) \right] \tag{7}$$

$$I(z) = \int_0^a \rho d\rho \int_0^a r dr \int_0^{2\pi} d\alpha \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} \tag{8}$$

The last two integrals over r and α represent an integration over the surface of a circle. Integrate instead over t and β of figure 2.

$$\int_0^a \rho d\rho \int_0^a r dr \int_0^{2\pi} d\alpha \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} = \int_0^a \rho d\rho \int_0^{a+\rho} t dt \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} \int_{-\beta_1}^{+\beta_1} d\beta \tag{9}$$

For $0 \leq t \leq a - \rho \qquad \beta_1 = \pi$

$a - \rho \leq t \leq a + \rho \qquad \beta_1 = \arccos\left[\frac{\rho^2 + t^2 - a^2}{2\rho t}\right]$ (10)

$$I(z) = \int_0^a \rho d\rho \left\{ 2\pi \int_0^{a-\rho} t dt \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} + \int_{a-\rho}^{a+\rho} t dt \frac{E_2(\sqrt{t^2+z^2})}{\sqrt{t^2+z^2}} \cdot 2\arccos\left[\frac{\rho^2+t^2-a^2}{2\rho t}\right]\right\} \quad (11)$$

After some algebra one obtains

$$I(z) = \pi a^2 E_3(z) - 2\int_0^a \rho d\rho \int_{a-\rho}^{a+\rho} \frac{dt}{t} E_3(\sqrt{t^2+z^2}) \frac{t^2+a^2-\rho^2}{\sqrt{4\rho^2 t^2 - (\rho^2+t^2-a^2)^2}} \quad (12)$$

Exchange the order of integration. According to figure 3 and observing that the integrand is a function of $s = \rho^2$, one obtains

$$\int_0^a 2\rho d\rho \int_{a-\rho}^{a+\rho} dt\, f(t, \rho^2) = \int_0^a dt \int_{a-t}^a 2\rho d\rho\, f + \int_a^d dt \int_{t-a}^a 2\rho d\rho\, f =$$
$$= \int_0^a dt \int_{(a-t)^2}^{a^2} ds\, f + \int_a^d dt \int_{(a-t)^2}^{a^2} ds\, f = \int_0^d dt \int_{(a-t)^2}^{a^2} ds\, f \quad (13)$$

$$f = \frac{1}{t} E_3(\sqrt{t^2+z^2}) \frac{t^2+a^2-s}{\sqrt{4t^2 s - (s+t^2-a^2)^2}} \quad (14)$$

Set

$$\left.\begin{aligned} &t^2 + a^2 - s = 2\,atu \quad , \quad ds = -2\,at\,du \\ &f\,ds = -\frac{1}{t} E_3(\sqrt{t^2+z^2}) \frac{2\,at\,u\,du}{\sqrt{1-u^2}} \end{aligned}\right\} \quad (15)$$

$$I(z) = \pi a^2 E_3(z) - \int_0^d dt\, E_3(\sqrt{t^2 + z^2}) \int_{\frac{t}{d}}^{1} \frac{2\, au\, du}{\sqrt{1 - u^2}} \tag{16}$$

$$I(z) = \pi a^2 E_3(z) - \int_0^d dt\, E_3(\sqrt{t^2 + z^2})\, \sqrt{d^2 - t^2} \tag{17}$$

$$P_e = \frac{1}{h}\left[E_3(0) - E_3(h)\right] - \frac{1}{\pi a^2 h} \int_0^d dt \left[E_3(t) - E_3(\sqrt{t^2 + h^2})\right] \sqrt{d^2 - t^2} \tag{18}$$

For P_m, the probability of leakage through the cylindrical surface, one obtains with notations from figure 4

$$P_m = \frac{1}{\pi a^2 h} \int_0^h 2\pi a\, dz \int_0^a \rho d\rho \int_0^{2\pi} d\varphi \int_0^h d\zeta \frac{e^{-R}}{4\pi R^2} \cos \alpha' \tag{19}$$

Change the integration over ρ and φ to t and α as indicated in figure 5:

$$\left.\begin{aligned} \rho d\rho\, d\varphi &= t\, dt\, d\alpha \\ R^2 &= t^2 + (z - \zeta)^2 \\ \cos \alpha' &= \frac{t \cos \alpha}{R} \end{aligned}\right\} \tag{20}$$

$$P_m = \frac{1}{2\pi a h} \int_0^h dz \int_0^h d\zeta \int_0^d t dt \cdot 2 \int_0^{\arccos \frac{t}{d}} d\alpha\, \frac{e^{-R}}{R^2}\, \frac{t \cos \alpha}{R} =$$

$$= \frac{1}{\pi a h} \int_0^h dz \int_0^h d\zeta \int_0^d t^2 dt\, \frac{e^{-R}}{R^3} \sqrt{1 - \frac{t^2}{d^2}} \tag{21}$$

Set $u = z - \zeta$ (22)

The integrand depends on z and ζ only as a function of u^2. It is easily found that

$$\int_0^h dz \int_0^h d\zeta\, f(u^2) = 2\int_0^h (h-u)\, f(u^2)\, du \tag{23}$$

$$\left.\begin{aligned} P_m &= \frac{2}{\pi a h}\int_0^h (h-u)\, du \int_0^d t^2 dt\, \frac{e^{-R}}{R^3}\sqrt{1-\frac{t^2}{d^2}} \\ R^2 &= t^2 + u^2 \end{aligned}\right\} \tag{24}$$

The complete formula is then

$$p_c^{cylinder} = 1 - \frac{1}{h}\left[E_3(0) - E_3(h)\right] + \frac{4}{\pi d^2 h}\int_0^d dt\left[E_3(t) - E_3(\sqrt{t^2-h^2})\right]\sqrt{d^2-t^2} - \frac{4}{\pi d^2 h}\int_0^h (h-u)\, du \int_0^d t^2 dt\, \frac{e^{-\sqrt{u^2+t^2}}}{(t^2+u^2)^{3/2}}\sqrt{d^2-t^2} \tag{25}$$

It is possible to elaborate this expression further, although we do not believe that it gives any improvement. However, it will be pointed out how the last term can be expressed formally in single integrals.

Consider the integral

$$I_1 = I_2 - I_3$$

$$I_2 = h\int_0^h du \int_0^d t^2 dt\, \frac{e^{-R}}{R^3}\sqrt{d^2-t^2} \tag{26}$$

$$I_3 = \int_0^h u\, du \int_0^d t^2 dt\, \frac{e^{-R}}{R^3}\sqrt{d^2-t^2}$$

One integration can be performed directly in I_3

$$I_3 = \int_0^d t^2 \sqrt{d^2 - t^2} \int_0^h \frac{e^{-R}}{R^3} u \, du \tag{27}$$

$$R^2 = u^2 + t^2 \quad ; \quad u \, du = R \, dR \tag{28}$$

$$I_3 = \int_0^d t^2 \sqrt{d^2 - t^2} \left[\frac{E_2(t)}{t} - \frac{E_2(\sqrt{h^2 + t^2})}{\sqrt{h^2 + t^2}} \right] \tag{29}$$

In I_2 set

$$u = R \sin \varphi \, , \quad t = R \cos \varphi \, , \quad du \, dt = R \, dR \, d\varphi \tag{30}$$

The integration is split up in integrations over three areas; I, II, and III, according to figure 6:

$$I_2 = I_I + I_{II} - I_{III} \tag{31}$$

$$I_I = \int_0^d d R \, e^{-R} \int_0^{\frac{\pi}{2}} \cos^2 \varphi \sqrt{d^2 - R^2 \cos^2 \varphi} \, d\varphi \tag{32}$$

$$I_{II} = \int_d^{\sqrt{d^2 + h^2}} d R \, e^{-R} \int_{\arccos \frac{d}{R}}^{\frac{\pi}{2}} \cos^2 \varphi \sqrt{d^2 - R^2 \cos^2 \varphi} \, d\varphi \tag{33}$$

$$I_{III} = \int_h^{\sqrt{d^2 + h^2}} d R \, e^{-R} \int_{\arcsin \frac{h}{R}}^{\frac{\pi}{2}} \cos^2 \varphi \sqrt{d^2 - R^2 \cos^2 \varphi} \, d\varphi \tag{34}$$

The integrals over φ are elliptic. In the case of I_I and I_{II} they can be expressed in terms of complete elliptic integrals, but the elliptic integral of I_{III} is incomplete.

A computer routine COLCYL was written based on equation (25). In order to temove the singularity at u = t = 0, the last integral was written

$$\int_0^h (h-u)\,du \int_0^d t^2 dt\, \frac{e^{-R}}{R^3} \sqrt{d^2 - t^2} =$$

$$= \int_0^h (h-u)\,du \int_0^d \frac{t^2 dt}{R^3} \left[e^{-R} \sqrt{d^2 - t^2} - d \right] +$$

$$+ \frac{h}{2} \left[\left(\frac{d}{h}\right)^2 + \frac{d}{h} \sqrt{1 + \left(\frac{d}{h}\right)^2} + \ln \left(\frac{d}{h} + \sqrt{1 + \left(\frac{d}{h}\right)^2}\,\right) \right] \tag{35}$$

Further, in the numerical integration $\sqrt{d-t}$ was used as a variable to remove the infinite derivative of the integrand at t = d.

3. COLLISION PROBABILITY FOR A CUBOID

As in the case of the cylinder, the mean free path is used as the unit of length. Let the cuboid have sides a, b, and c (figure 7), and assume an isotropic, constant source density inside the cuboid. If P_{xy} denotes the escape probability through a surface with sides x and y, the collision probability P_c is

$$P_c = 1 - 2P_{ab} - 2P_{bc} - 2P_{ca} \tag{36}$$

With notations from figure 1, one gets for P_{ab}, that is the probability for escape through the bottom surface,

$$P_{ab} = \frac{1}{abc} \int_0^a d\xi \int_0^b d\eta \int_0^c d\zeta \int_0^a \int_0^b \frac{dx\,dy}{4\pi R^2} \frac{\zeta}{R} e^{-R} \tag{37}$$

Integrate first over ζ

$$\left.\begin{aligned} R^2 &= (x - \xi)^2 + (y - \eta)^2 + \zeta^2 \\ R\,dR &= \zeta d\zeta \end{aligned}\right\} \tag{38}$$

$$\int_0^c d\zeta \,\frac{\zeta}{R^3}\, e^{-R} = \int_{R_1}^{R_2} \frac{e^{-R}\,dR}{R^2} = \frac{E_2(R_1)}{R_1} - \frac{E_2(R_2)}{R_2} \tag{39}$$

where

$$\begin{aligned} R_1 &= \sqrt{(x - \xi)^2 + (y - \eta)^2} \\ R_2 &= \sqrt{(x - \xi)^2 + (y - \eta)^2 + c^2} \end{aligned} \tag{40}$$

As in paragraph 2

$$E_n(z) = \int_1^\infty \frac{e^{-zu}}{u^n}\, du \tag{41}$$

Define

$$I_{ab}(z) = \int_0^a dx \int_0^b dy \int_0^a d\xi \int^b d\eta \,\frac{E_2\left(\sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}\right)}{\sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2}} \tag{42}$$

Then

$$P_{ab} = \frac{1}{4\pi\, abc}\left[I_{ab}(0) - I_{ab}(c)\right] \tag{43}$$

Consider the integration over y and η and set

$$y - \eta = u \quad , \qquad dy = du \tag{44}$$

The integrand is a function of u^2 only. One obtains (compare equation 23)

$$\int_0^b dy \int_0^b d\eta\, f(u^2) = 2 \int_0^b (b - u)\; f(u^2)\, du \tag{45}$$

The integrations over x and ξ can be handled in the same way and the result is

$$I_{ab}(z) = 4 \int_0^a (a - v)\, dv \int_0^b (b - u)\, du \;\frac{E_2(\sqrt{u^2+v^2+z^2})}{\sqrt{u^2 + v^2 + z^2}} \tag{46}$$

Now $(a - v)(b - u) = ab - au - bv + uv$ and the corresponding four terms in $I_{ab}(z)$ are

$$I_1 = 4\,ab \int_0^a dv \int_0^b du \;\frac{E_2(\sqrt{u^2+v^2+z^2})}{\sqrt{u^2+v^2+z^2}}$$

$$I_2 = 4\,a \int_0^a dv \left[E_3(\sqrt{v^2+z^2}) - E_3(\sqrt{v^2+b^2+z^2})\right]$$

$$I_3 = 4\,b \int_0^b du \left[E_3(\sqrt{u^2+z^2}) - E_3(\sqrt{u^2+a^2+z^2})\right] \tag{47}$$

$$I_4 = 4 \cdot \frac{1}{2} \left\{\int_0^a v\, dv \left[E_3(\sqrt{v^2+z^2}) - E_3(\sqrt{v^2+b^2+z^2})\right] + \right.$$

$$\left. + \int_0^b u\, du \left[E_3(\sqrt{u^2+z^2}) - E_3(\sqrt{u^2+a^2+z^2})\right]\right\}$$

In I_1 set

$$u = r \sin\phi\;, \quad v = r\cos\phi\;, \quad du\,dv = r\,dr\,d\phi \tag{48}$$

According to figure 8 and with

$$g(r) = \frac{E_2(\sqrt{r^2+z^2})}{\sqrt{r^2+z^2}}$$

one gets

$$\int_0^a dv \int_0^b du\, g(r) = \int_0^{\sqrt{a^2+b^2}} r\,dr\,g(r) \int_0^{\frac{\pi}{2}} d\phi - \int_a^{\sqrt{a^2+b^2}} r\,dr\,g(r) \int_0^{\arccos\frac{a}{r}} d\phi - \int_b^{\sqrt{a^2+b^2}} r\,dr\,g(r) \int_{\arcsin\frac{b}{r}}^{\frac{\pi}{2}} d\phi \qquad (49)$$

and after some algebra

$$I_1 = 4\,ab\left\{\frac{\pi}{2}\,E_3(z) - a\int_a^{\sqrt{a^2+b^2}} E_3(\sqrt{r^2+z^2})\frac{dr}{r\sqrt{r^2-a^2}} - b\int_b^{\sqrt{a^2+b^2}} E_3(\sqrt{r^2+z^2})\frac{dr}{r\sqrt{r^2-b^2}}\right\} \qquad (50)$$

Set in the first integral

$$r^2 - a^2 = u^2 \quad ; \quad \frac{dr}{r\sqrt{r^2-a^2}} = \frac{du}{a^2+u^2} \qquad (51)$$

and analogously in the second integral

$$I_1 = 4\,ab\left\{\frac{\pi}{2}\,E_3(z) - a\int_0^b E_3(\sqrt{a^2+u^2+z^2})\frac{du}{a^2+u^2} - b\int_0^a E_3(\sqrt{v^2+b^2+z^2})\frac{dv}{b^2+v^2}\right\} \qquad (52)$$

Combining all terms one gets

$$I_{ab}(z) = 2\pi\,ab\,E_3(z) - 2\int_0^a \left[(2a-v)\,E_3(\sqrt{v^2+z^2}) + \frac{v^2-2av+b^2}{b^2+v^2}\,v\,E_3(\sqrt{v^2+b^2+z^2})\right]dv$$

$$- 2\int_0^b \left[(2b-u)\,E_3(\sqrt{u^2+z^2}) + \frac{u^2-2bu+a^2}{a^2+u^2}\,u\,E_3(\sqrt{a^2+u^2+z^2})\right]du \qquad (53)$$

and the collision probability P is expressed in single integrals as

$$P_c^{cuboid} = 1 - \frac{1}{2\pi abc}\left[I_{ab}(0) - I_{ab}(c) + I_{bc}(0) - I_{bc}(a) + I_{ca}(0) - I_{ca}(b)\right] \qquad (54)$$

It is in fact possible to do some more analytical integrations in equation 53 before treating it numerically. However, the gain in computing time, if any, would be rather small, and it is also possible that some accuracy would be lost. In particular, it is preferable to work with exponential integrals of one single order as in equation 53.

4. NUMERICAL CALCULATIONS

Two simple FORTRAN routines were written for the calculation of collision probabilities, COLCYL for the cylinder and COLCUB for the cuboid. Gaussian quadrature was used for the integrations. COLCYL is based on equation 25 with the remarks following the equation, and COLCUB is based on equation 54. The number of points in the integrations can be varied, and the calculations were performed with increasing number of points in order to study the convergence.

The approximations for the exponential integrals were taken from Handbook of Mathematical Functions (5), formulae 5.1.53 and 5.1.56.

In tables 1 to 4 we compare collision probabilities calculated by means of COLCUB and COLCYL with results given by the other authors for selected values of the parameters.

Table 1 gives values for cubes. They are compared with Monte Carlo results by Gubbins (3). In table 2 values for cylinders with height to diameter equal to one are compared with the corresponding Monte Carlo values by Gubbins, and in table 3 values for cylinders with height to diameter equal to two are compared with Monte Carlo values of Foell et al. (2).

The agreement with Monte Carlo values is as good as can be expected. Foell et al. do not quote explicitly the accuracy of their results. The numbers include five decimal places, but the number of significant decimals is probably three to four.

Table 4 finally shows the comparison with McLeod's results for cylinders. The agreement is good except for the very largest cylinders. We have used Gaussian 16-, 20-, and 24-point formulae over both integration ranges 0 to Σd and 0 to Σh, d being the diameter and h the height of the cylinder. One can see from the convergence in the tables that the 24-point formula does not give more than three correct decimal digits in cases when both the diameter and the height are more than 20 mean free paths. If COLCYL - or COLCUB - were to be used for such large bodies, it would be advisable to divide the integration ranges in subregions, so that more points could be used.

On the other hand, for cylinders of diameter less than five mean free paths, all results agree within 0.00002 except in one case where the difference is 0.00006.

5. CONCLUSIONS

Collision probabilities for cuboids and finite cylinders have been calculated by means of the FORTRAN codes COLCUB and COLCYL, which are based on analytical expressions. An absolute accuracy of about 5×10^{-5} can be obtained in one or two seconds on IBM 7044, if the linear dimensions of the bodies are not too large measured in mean free paths. The corresponding computing time with a Monte Carlo code would be at least several minutes, so one should resort to Monte Carlo methods only for more complicated geometries.

6. REFERENCES

1. CASE K M, DE HOFFMANN F, and PLACZEK G,
Introduction to the theory of neutron diffusion. Vol. I.
Los Alamos Scientific Library, Los Alamos, 1953.

2. FOELL W K, BERNER C W, and TONG S,
CEP. A Monte Carlo program for the calculation of first-flight escape probabilities for finite cylinders. 1965.
(IDO-17075.)

3. GUBBINS M E,
A Monte Carlo computer programme CEP for calculating collision probabilities in cuboids and finite cylinders. 1964.
(AEEW-M 442.)

4. McLEOD R I,
On the capture probability of radiation in a finite rod. 1963.
(TRG Report 573(D).)

5. Handbook of mathematical functions with formulas, graphs and mathematical tables.
Ed. by M. Abramowitz and I.A. Stegun.
Wash. 1964. National Bureau of Standards.
Appl. Mathem. Ser. 55.

TABLE 1

Collision probabilities for cubes
Comparison with Monte Carlo results of Gubbins

Σa	COLCUB			CEP Gubbins
	8-point formula	12	16	
0.1	0.04337	0.04337	0.04337	0.0434
0.4	0.15770	0.15770	0.15770	0.157
1.1	0.35426	0.35427	0.35427	0.356
3.0	0.62585	0.62585	0.62585	0.626
8.0	0.83145	0.83147	0.83147	0.833
14.0	0.89915	0.89918	0.89918	0.8993
20.0	0.92809	0.92812	0.92812	0.9280

TABLE 2

Collision probabilities for cylinders
with height equal to diameter.

Comparison with Monte Carlo results of Gubbins

$\Sigma d = \Sigma h$	COLCYL				CEP (Gubbins)
	8-point formula	16	24	P_c	Standard deviation
0.1	0.04069	0.04066	0.04065	0.0406	0.00084
0.4	0.14901	0.14891	0.14890	0.149	0.0030
1.1	0.33948	0.33922	0.33920	0.340	0.0061
3.0	0.61338	0.61265	0.61260	0.612	0.0086
8.0	0.82927	0.82719	0.82707	0.826	0.0070
17.0	0.92024	0.91517	0.91491	0.915	0.0057

TABLE 3

Collision probabilities for cylinders with height/diameter equal to 2.

Comparison with Monte Carlo results of Foell et al.

Σd	Σh	COLCYL			CEP (Foell, Berner, Tong)
		8-point formula	16	24	
0.8	1.6	0.30366	0.30369	0.30369	0.30646
2.0	4.0	0.54499	0.54503	0.54503	0.54682
4.0	8.0	0.72522	0.72524	0.72524	0.72649
6.0	12.0	0.80684	0.80682	0.80682	0.80691
8.0	16.0	0.85191	0.85179	0.85179	0.85220
12.0	24.0	0.89964	0.89920	0.89920	0.89932

TABLE 4

Collision probabilities for cylinders.
Comparison with results of McLeod.

Σd	Σh	COLCYL			McLeod
		16-point formula	20	24	
20.0	20.0	0.927596	0.927369	0.927287	0.92656
"	60.0	0.941347	0.941975	0.942219	0.94187
"	200.0	0.945675	0.947362	0.947696	0.94723
5.0	5.0	0.738986	0.738932	0.738912	0.73889
"	15.0	0.783825	0.783994	0.784058	0.78412
"	50.0	0.799964	0.799989	0.799986	0.79997
1.0	1.0	0.316346	0.316335	0.316331	0.31633
"	3.0	0.372257	0.372292	0.372304	0.37232
"	10.0	0.394533	0.394512	0.394505	0.39450
0.2	0.2	0.078916	0.078914	0.078913	0.078912
"	0.6	0.099236	0.099243	0.099245	0.099249
"	2.0	0.109865	0.109851	0.109847	0.10985
0.05	0.05	0.020638	0.020637	0.020637	0.020637
"	0.15	0.026408	0.026409	0.026410	0.026411
"	0.50	0.029801	0.029788	0.029785	0.029784
0.01	0.01	0.004178	0.004178	0.004178	0.0041781
"	0.03	0.005373	0.005373	0.005373	0.0053740
"	0.10	0.006115	0.006103	0.006100	0.0060939
0.001	0.001	$0.0^{3}420$	$0.0^{3}420$	$0.0^{3}420$	$0.0^{3}41896$
"	0.003	$0.0^{3}540$	$0.0^{3}540$	$0.0^{3}540$	$0.0^{3}53950$
"	0.10	$0.0^{3}629$	$0.0^{3}617$	$0.0^{3}614$	$0.0^{3}61322$

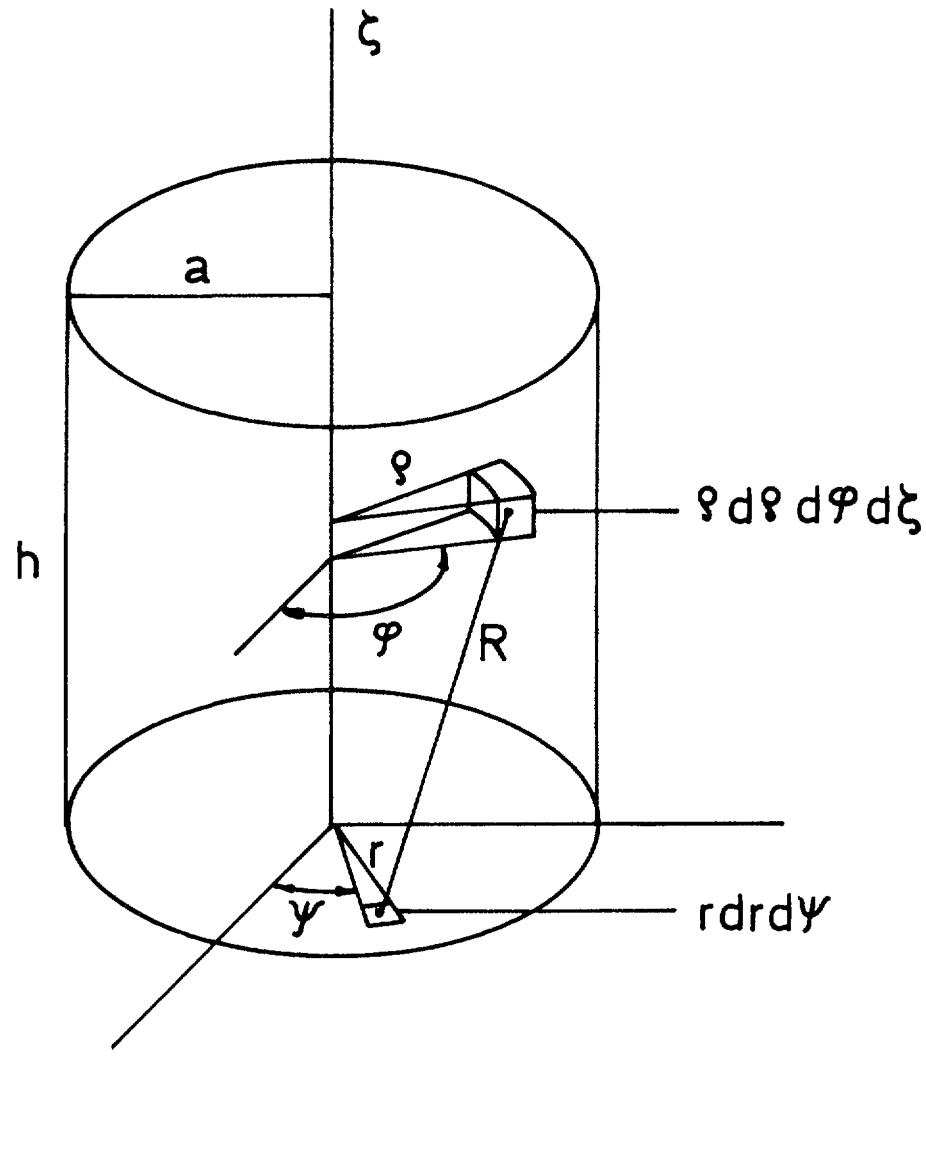

Figure 1

Notations for the calculation of escape through the bottom surface of a cylinder

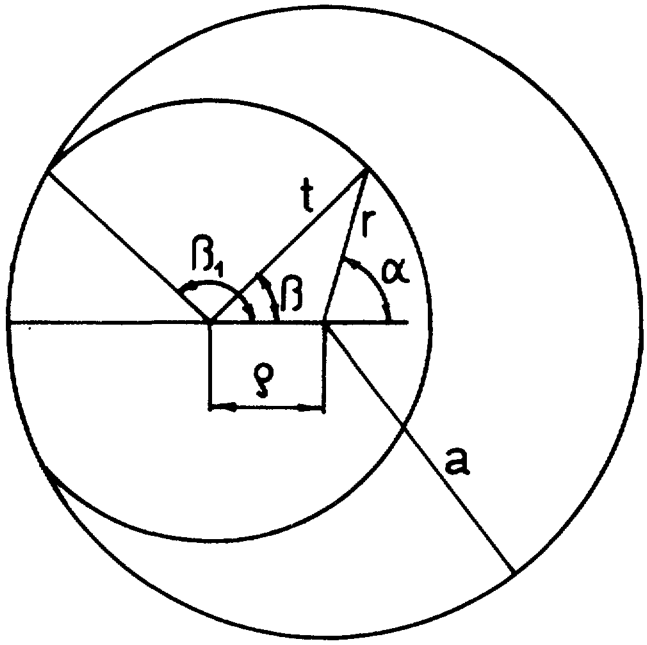

Figure 2

Change of variables (equation 9)

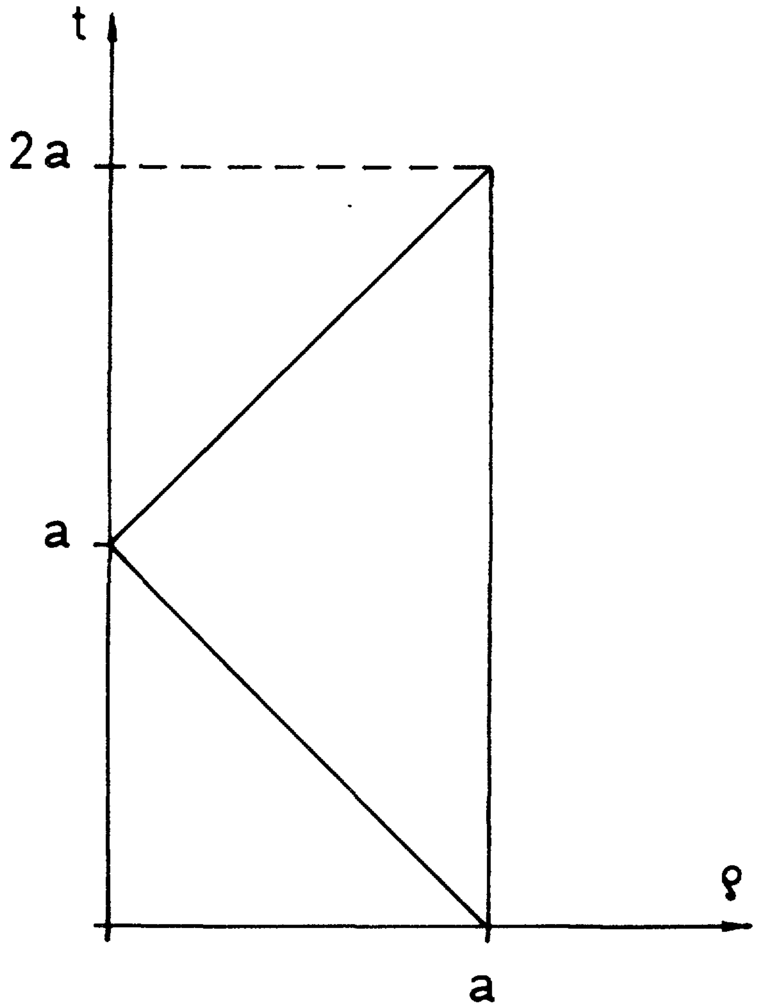

Figure 3

Change of variables (equation 13)

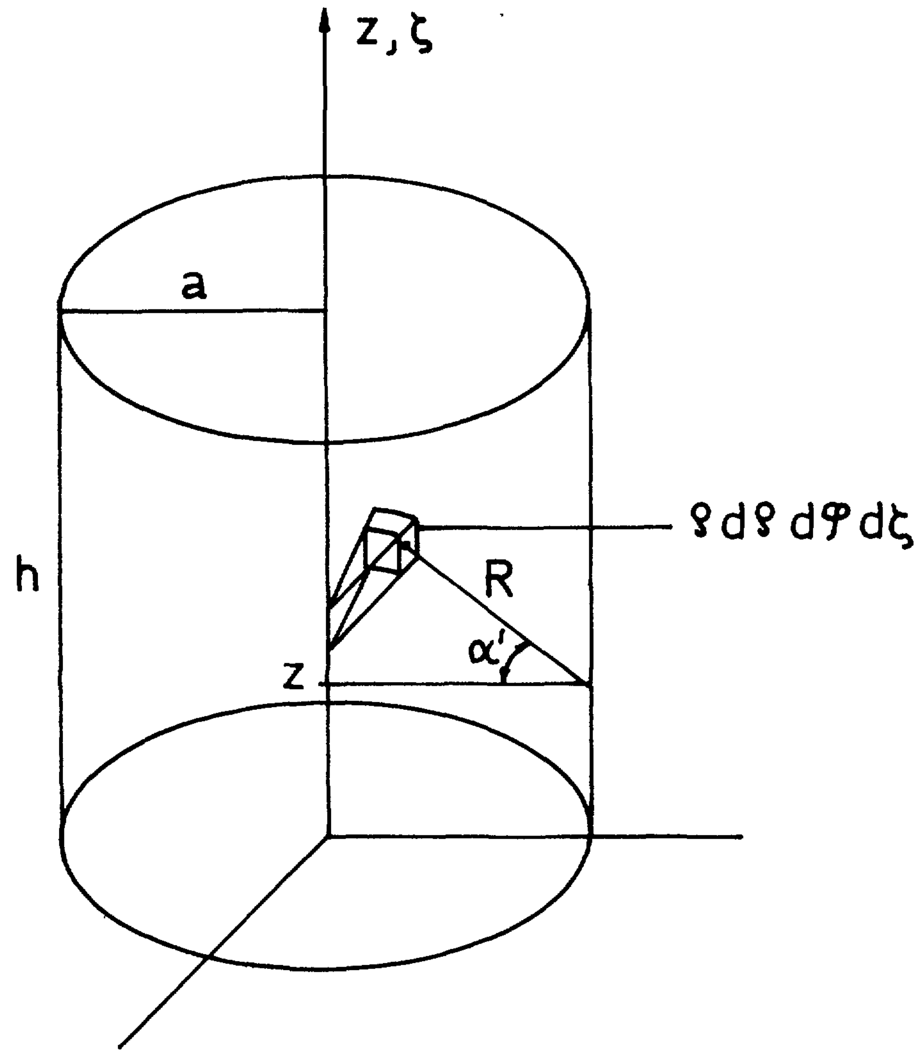

Figure 4

Notations for the calculation of escape through the curved surface of a cylinder

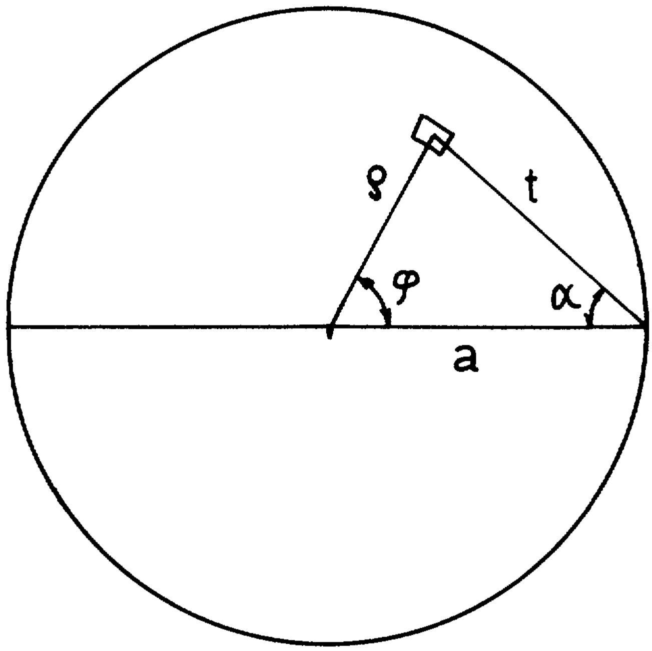

Figure 5

Change of variables (equation 20)

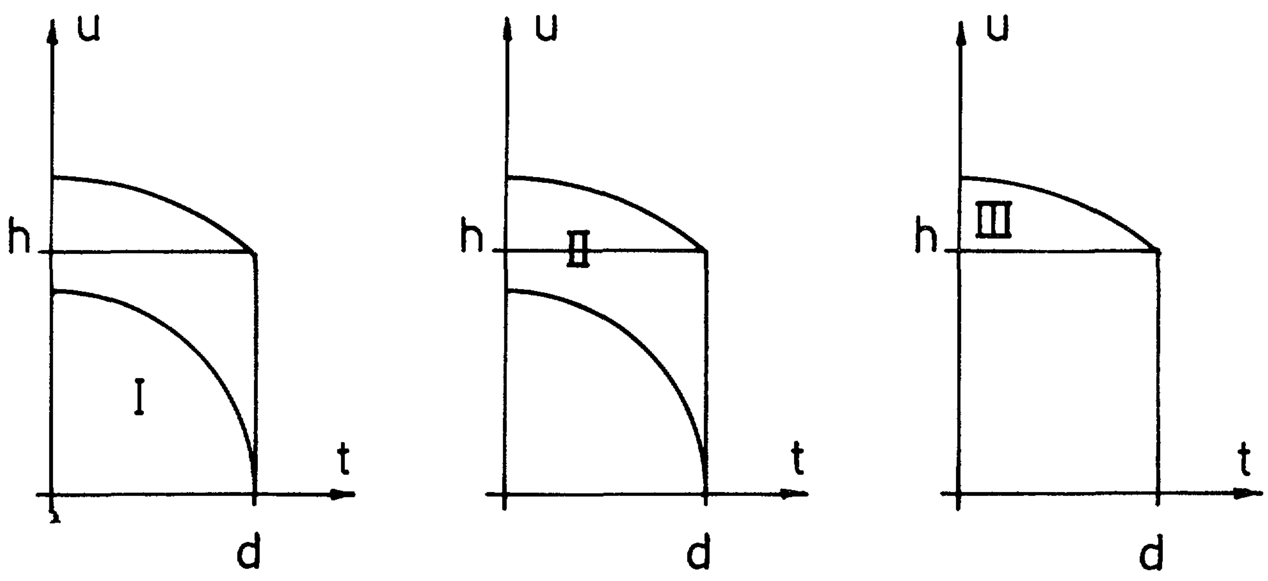

Figure 6

Regions of integration (equation 31)

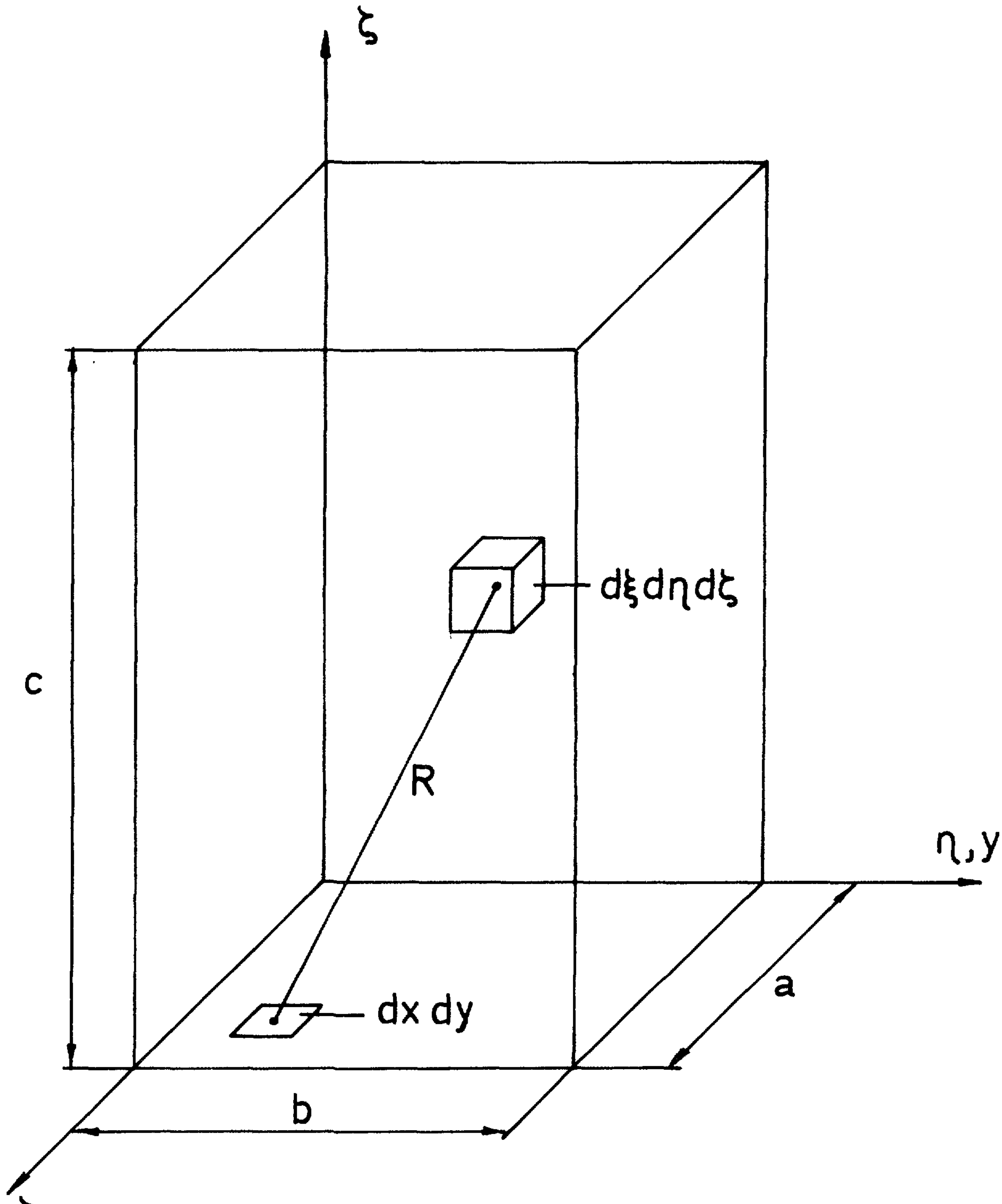

Figure 7

Notations for the calculation of escape through the bottom surface of a cuboid

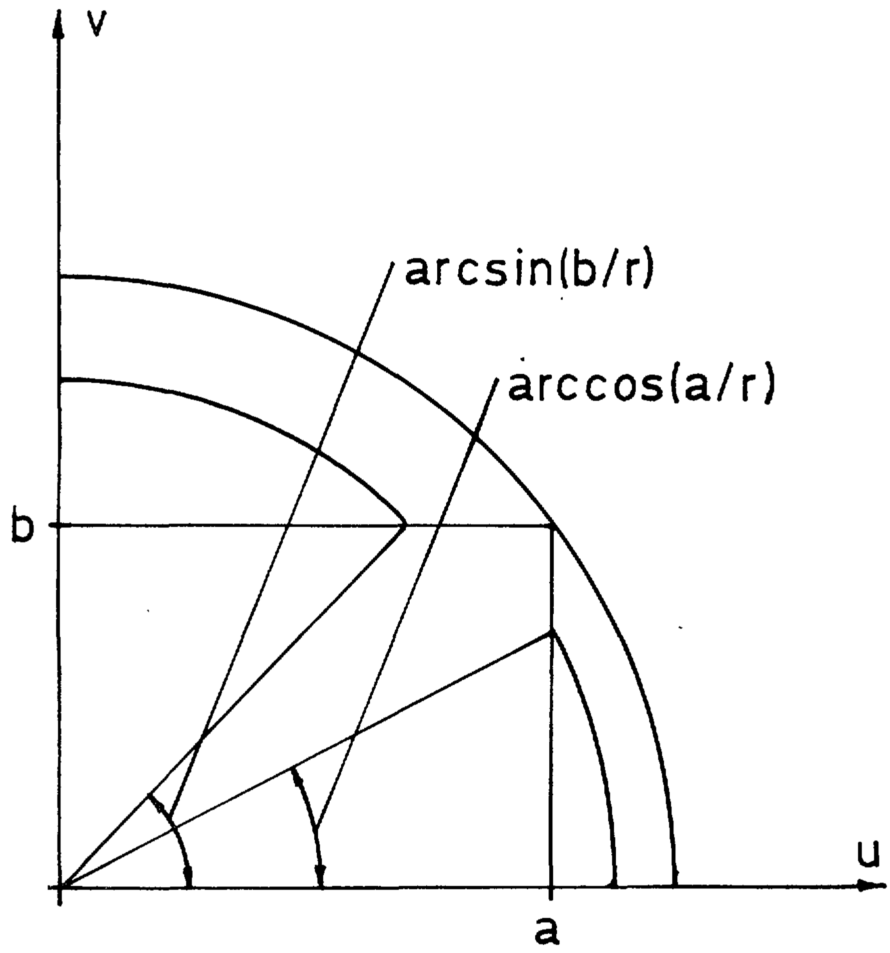

Figure 8

Change to polar coordinates (equation 49)

LIST OF PUBLISHED AE-REPORTS

1–200. (See the back cover earlier reports.)

201. Heat transfer analogies. By A. Bhattacharyya. 1965. 55 p. Sw. cr. 8:–.
202. A study of the "384" KeV complex gamma emission from plutonium-239. By R. S. Forsyth and N. Ronqvist. 1965. 14 p. Sw. cr. 8:–.
203. A scintillometer assembly for geological survey. By E. Dissing and O. Landström. 1965. 16 p. Sw. cr. 8:–.
204. Neutron-activation analysis of natural water applied to hydrogeology. By O. Landström and C. G. Wenner. 1965. 28 p. Sw. cr. 8:–.
205. Systematics of absolute gamma ray transition probabilities in deformed odd-A nuclei. By S. G. Malmskog. 1965. 60 p. Sw. cr. 8:–.
206. Radiation induced removal of stacking faults in quenched aluminium. By U. Bergenlid. 1965. 11 p. Sw. cr. 8:–.
207. Experimental studies on assemblies 1 and 2 of the fast reactor FR0. Part 2. By E. Hellstrand, T. Andersson, B. Brunfelter, J. Kockum, S-O. Londen and L. I. Tirén. 1965. 50 p. Sw. cr. 8:–.
208. Measurement of the neutron slowing-down time distribution at 1.46 eV and its space dependence in water. By E. Möller. 1965. 29 p. Sw. cr. 8:–.
209. Incompressible steady flow with tensor conductivity leaving a transverse magnetic field. By E. A. Witalis. 1965. 17 p. Sw. cr. 8:–.
210. Methods for the determination of currents and fields in steady two-dimensional MHD flow with tensor conductivity. By E. A. Witalis. 1965. 13 p. Sw. cr. 8:–.
211. Report on the personnel dosimetry at AB Atomenergi during 1964. By K. A. Edvardsson. 1966. 15 p. Sw. cr. 8:–.
212. Central reactivity measurements on assemblies 1 and 3 of the fast reactor FR0. By S-O. Londen. 1966. 58 p. Sw. cr. 8:–.
213. Low temperature irradiation applied to neutron activation analysis of mercury in human whole blood. By D. Brune. 1966. 7 p. Sw. cr. 8:–.
214. Characteristics of linear MHD generators with one or a few loads. By E. A. Witalis. 1966. 16 p. Sw. cr. 8:–.
215. An automated anion-exchange method for the selective sorption of five groups of trace elements in neutron-irradiated biological material. By K. Samsahl. 1966. 14 p. Sw. cr. 8:–.
216. Measurement of the time dependence of neutron slowing-down and thermalization in heavy water. By E. Möller. 1966. 34 p. Sw. cr. 8:–.
217. Electrodeposition of actinide and lanthanide elements. By N-E. Bärring. 1966. 21 p. Sw. cr. 8:–.
218. Measurement of the electrical conductivity of He^3 plasma induced by neutron irradiation. By J. Braun and K. Nygaard. 1966. 37 p. Sw. cr. 8:–.
219. Phytoplankton from Lake Magelungen, Central Sweden 1960–1963. By T. Willén. 1966. 44 p. Sw. cr. 8:–.
220. Measured and predicted neutron flux distributions in a material surrounding av cylindrical duct. By J. Nilsson and R. Sandlin. 1966. 37 p. Sw. cr. 8:–.
221. Swedish work on brittle-fracture problems in nuclear reactor pressure vessels. By M. Grounes. 1966 34 p. Sw. cr. 8:–.
222. Total cross-sections of U, UO_2 and ThO_2 for thermal and subthermal neutrons. By S. F. Beshai. 1966. 14 p. Sw. cr. 8:–.
223. Neutron scattering in hydrogenous moderators, studied by the time dependent reaction rate method. By L. G. Larsson, E, Möller and S. N. Purohit. 1966. 26 p. Sw. cr. 8:–.
224. Calcium and strontium in Swedish waters and fish, and accumulation of strontium-90. By P-O. Agnedal. 1966. 34 p. Sw. cr. 8:–.
225. The radioactive waste management at Studsvik. By R. Hedlund and A. Lindskog. 1966. 14 p. Sw cr. 8:–.
226. Theoretical time dependent thermal neutron spectra and reaction rates in H_2O and D_2O. S. N. Purohit. 1966. 62 p. Sw. cr. 8:–.
227. Integral transport theory in one-dimensional geometries. By I. Carlvik. 1966. 65 p. Sw. cr. 8:–.
228. Integral parameters of the generalized frequency spectra of moderators. By S. N. Purohit. 1966. 27 p. Sw. cr. 8:–.
229. Reaction rate distributions and ratios in FR0 assemblies 1, 2 and 3. By T. L. Andersson. 1966. 50 p. Sw. cr. 8:–.
230. Different activation techniques for the study of epithermal spectra, applied to heavy water lattices of varying fuel-to-moderator ratio. By E. K. Sokolowski. 1966. 34 p. Sw. cr. 8:–.
231. Calibration of the failed-fuel-element detection systems in the Ågesta reactor. By O. Strindehag. 1966. 52 p. Sw. cr. 8:–.
232. Progress report 1965. Nuclear chemistry. Ed. by G. Carleson. 1966. 26 p. Sw. cr. 8:–.
233. A summary report on assembly 3 of FR0. By T. L. Andersson, B. Brunfelter, P. F. Cecchi, E. Hellstrand, J. Kockum, S-O. Londen and L. I. Tirén. 1966. 34 p. Sw. cr. 8:–.
234. Recipient capacity of Tvären, a Baltic Bay. By P.-O. Agnedal and S. O. W. Bergström. 1966. 21 p. Sw. cr. 8:–.
235. Optimal linear filters for pulse height measurements in the presence of noise. By K. Nygaard. 1966. 16 p. Sw. cr. 8:–.
236. DETEC, a subprogram for simulation of the fast-neutron detection process in a hydro-carbonous plastic scintillator. By B. Gustafsson and O. Aspelund. 1966. 26 p. Sw. cr. 8:–.
237. Microanalys of fluorine contamination and its depth distribution in zircaloy by the use of a charged particle nuclear reaction. By E. Möller and N. Starfelt. 1966. 15 p. Sw. cr. 8:–.
238. Void measurements in the regions of sub-cooled and low-quality boiling. P. 1. By S. Z. Rouhani. 1966. 47 p. Sw. cr. 8:–.
239. Void measurements in the regions of sub-cooled and low-quality boiling. P. 2. By S. Z. Rouhani. 1966. 60 p. Sw. cr. 8:–.
240. Possible odd parity in ^{125}Xe. By L. Broman and S. G. Malmskog. 1966. 10 p. Sw. cr. 8:–.
241. Burn-up determination by high resolution gamma spectrometry: spectra from slightly-irradiated uranium and plutonium between 400–830 keV. By R. S. Forsyth and N. Ronqvist. 1966. 22 p. Sw. cr. 8:–.
242. Half life measurements in ^{155}Gd. By S. G. Malmskog. 1966. 10 p. Sw. cr. 8:–.
243. On shear stress distributions for flow in smooth or partially rough annuli. By B. Kjellström and S. Hedberg. 1966. 66 p. Sw. cr. 8:–.
244. Physics experiments at the Ågesta power station. By G. Apelqvist, P.-Å. Bliselius, P. E. Blomberg, E. Jonsson and F. Åkerhielm. 1966. 30 p. Sw. cr. 8:–.
245. Intercrystalline stress corrosion cracking of inconel 600 inspection tubes in the Ågesta reactor. By B. Grönwall, L. Ljungberg, W. Hübner and W. Stuart. 1966. 26 p. Sw. cr. 8:–.
246. Operating experience at the Ågesta nuclear power station. By S. Sandström. 1966. 113 p. Sw. cr. 8:–.
247. Neutron-activation analysis of biological material with high radiation levels. By K. Samsahl. 1966. 15 p. Sw. cr. 8:–.
248. One-group perturbation theory applied to measurements with void. By R. Persson. 1966. 19 p. Sw. cr. 8:–.
249. *Optimal linear filters. 2. Pulse time measurements in the presence of noise.* By K. Nygaard. 1966. 9 p. Sw. cr. 8:–.
250. The interaction between control rods as estimated by second-order one-group perturbation theory. By R. Persson. 1966. 42 p. Sw. cr. 8:–.
251. Absolute transition probabilities from the 453.1 keV level in 183W. By S. G. Malmskog. 1966. 12 p. Sw. cr. 8:–.
252. Nomogram for determining shield thickness for point and line sources of gamma rays. By C. Jönemalm and K. Malén. 1966. 33 p. Sw. cr. 8:–.
253. Report on the personnel dosimetry at AB Atomenergi during 1965. By K. A. Edwardsson. 1966. 13 p. Sw. cr. 8:–.
254 Buckling measurements up to 250°C on lattices of Ågesta clusters and on D_2O alone in the pressurized exponential assembly TZ. By R. Persson, A. J. W. Andersson and C.-E. Wikdahl. 1966. 56 p. Sw. cr. 8:–.
255 Decontamination experiments on intact pig skin contaminated with beta-gamma-emitting nuclides. By K. A. Edwardsson, S. Hagsgård and Å. Swensson. 1966. 35 p. Sw. cr. 8:–.
256. Perturbation method of analysis applied to substitution measurements of buckling. By R. Persson. 1966. 57 p. Sw. cr. 8:–.
257. The Dancoff correction in square and hexagonal lattices. By I. Carlvik. 1966. 35 p. Sw. cr. 8:–.
258. Hall effect influence on a highly conducting fluid. By E. A. Witalis. 1966. 13 p. Sw. cr. 8:–.
259. Analysis of the quasi-elastic scattering of neutrons in hydrogenous liquids. By S. N. Purohit. 1966. 26 p. Sw. cr. 8:–.
260. High temperature tensile properties of unirradiated and neutron irradiated 20Cr–35Ni austenitic steel By R B Roy and B Solly. 1966. 25 p. Sw. cr. 8:–.
261. On the attenuation of neutrons and photons in a duct filled with a helical plug. By E. Aalto and Å. Krell 1966. 24 p. Sw. cr. 8:–.
262. Design and analysis of the power control system of the fast zero energy reactor FR-O. By N. J. H. Schuch. 1966. 70 p. Sw. cr. 8:–.
263. Possible deformed states in ^{115}In and ^{117}In. By A. Bäcklin, B. Fogelberg and S. G. Malmskog. 1967. 39 p. Sw. cr. 10:–.
264. Decay of the 16.3 min. ^{182}Ta isomer. By M. Höjeberg and S. G. Malmskog. 1967. 13 p. Sw. cr. 10:–.
265. Decay properties of ^{147}Nd. By A. Bäcklin and S. G. Malmskog. 1967. 15 p. Sw. cr. 10: –.
266. The half life of the 53 keV level in ^{197}Pt. By S. G. Malmskog. 1967. 10 p. Sw. cr. 10:–.
267. Burn-up determination by hight resolution gamma spectrometry: Axial and diametral scanning experiments. By R. S. Forsyth, W. H. Blackadder and N. Ronqvist. 1967. 18 p. Sw. cr. 10:–.
268. On the properties of the $s_{1/2} \longrightarrow d_{3/2}$ transition in ^{199}Au. By A. Bäcklin and S. G. Malmskog. 1967. 23 p. Sw. cr. 10:–.
269. Experimental equipment for physics studies in the Ågesta reactor. By G. Bernander, P. E. Blomberg and P.-O. Dubois. 1967. 35 p. Sw. cr. 10:–.
270. An optical model study of neutrons elastically scattered by iron, nickel, cobalt, copper, and indium in the energy region 1.5 to 7.0 MeV. By B. Holmqvist and T. Wiedling. 1967. 20 p. Sw. cr. 10:–.
271. Improvement of reactor fuel element heat transfer by surface roughness. By B. Kjellström and A. E. Larsson. 1967. 94 p. Sw. cr. 10:–.
272. Burn-up determination by high resolution gamma spectormetry Fission product migration studies. By R. S. Forsyth, W. H. Blackadder and N. Ronqvist. 1967. 19 p. Sw. cr. 10:–.
273. Monoenergetic critical parameters and decay constants for small spheres and thin slabs. By I. Carlvik. 24 p. Sw. cr. 10:–.
274. Scattering of neutrons by an anharmonic crystal. By T. Högberg, L. Bohlin and I. Ebbsjö. 1967. 38 p. Sw. cr. 10:–.
275. The $I\Delta KI=1$, E1 transitions in odd-A isotopes of Tb and Eu. By S. G. Malmskog, A. Marelius and S. Wahlborn. 1967. 24 p. Sw. cr. 10:–.
276. A burnout correlation for flow of boiling water in vertical rod bundles. By Kurt M. Becker. 1967. 102 p. Sw. cr. 10:–.
277. Epithermal and thermal spectrum indices in heavy water lattices. By E. K. Sokolowski and A. Jonsson. 1967. 44 p. Sw. cr. 10:–.
278. On the $d_{5/2} \leftrightarrow g_{7/2}$ transitions in odd mass Pm nuclei. By A. Bäcklin and S. G. Malmskog. 1967. 14 p. Sw. cr. 10:–.
279. Calculations of neutron flux distributions by means of integral transport methods. By I. Carlvik. 1967. 94 p. Sw. cr. 10:–.
280. On the magnetic properties of the K=1 rotational band in ^{188}Re. By S. G. Malmskog and M. Höjeberg. 1967. 18 p. Sw. cr. 10:–.
281. Collision probabilities for finite cylinders and cuboids. By I. Carlvik. 1967. 28 p. Sw. cr. 10:–.

Förteckning över publicerade AES-rapporter

1. Analys medelst gamma-spektrometri. Av D. Brune. 1961. 10 s. Kr 6:–.
2. Bestrålningsförändringar och neutronatmosfär i reaktortrycktankar – några synpunkter. Av M. Grounes. 1962. 33 s. Kr 6:–.
3. Studium av sträckgränsen i mjukt stål. Av G. Östberg och R. Attermo 1963. 17 s. Kr 6:–.
4. Teknisk upphandling inom reaktorområdet. Av Erik Jonson. 1963. 64 s. Kr 8:–.
5. Ågesta Kraftvärmeverk. Sammanställning av tekniska data, beskrivningar m. m. för reaktordelen. Av B. Lilliehöök. 1964. 336 s. Kr 15:–.
6. Atomdagen 1965. Sammanställning av föredrag och diskussioner. Av S Sandström. 1966. 321 s. Kr 15:–.

Additional copies available at the library of AB Atomenergi, Studsvik, Nyköping, Sweden. Micronegatives of the reports are obtainable through Filmprodukter, Gamla landsvägen 4, Ektorp, Sweden.

EOS-tryckerierna, Stockholm 1967