PRECISION EXPERIMENTS IN MUON PHYSICS

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Abstract: Why is the muon so important in particle physics? - We now have particle accelerators which can produce high-intensity and high-quality muon beams. Further improvements are on the way. - The muon does not have strong interactions. It is therefore an ideal particle to explore the field of electroweak interactions. - The properties of the muon: mass, lifetime, magnetic moment, have already been measured to great precision and future improvements are still possible. - The decay modes of the muon are an important source of information: 1) A precise study of the main decay mode gives access to the "Michel parameters" which have definite values in the Standard Model. 2) Especially interesting are the decays which are forbidden in the Standard Model. Presently, very stringent upper limits have been obtained and further progress is expected. These upper limits can be used to constrain theories of electroweak interactions beyond the Standard Model.

1 Introduction

The talk will start with some historical considerations. Then, the present experimental situation will be reviewed, in connection with the verification of the Standard-Model predictions. Finally, new experiments will be able to improve the sensitivity to the so-called "rare decay modes of the muon" and possibly discover some effects which will point to physics "beyond the Standard Model". In writing this report the author has used a lot of information taken from the review paper of Y. Kuno and Y. Okada[1], and from the Particle Data Group report[2].

2 Historical notes

See the contribution by Zichichi[3] at the International Nuclear Physics Conference 2007, Tokyo, Japan.

The story starts with the Yukawa proposal[4]: nucleons (protons, neutrons) should interact via the exchange of a new (then unknown) particle. Since the nuclear force has a short range (of the order of femtometers) the particle in question must have a large mass (of the order of 100 MeV); it is therefore called a "meson" (it has an intermediate mass between an electron and a nucleon).

Shortly after Yukawa's proposal a particle with such an intermediate mass was found in the cosmic radiation[5]. It was called "the μ meson".

But is it really the Yukawa meson, the mediator of the strong interaction between nucleons?

This particle does not interact strongly with matter [6]. Therefore it cannot be the Yukawa meson. But what is it? Something unexpected. Isidor Rabi asked "Who ordered that?"

Early experiments showed that the μ^+ meson does not decay according to:

$$\mu^+ \to e^+ \gamma$$

The first search for this mode was performed at Chalk River, Canada, by Hincks and Pontecorvo[7].

3 Positively (μ^+) and negatively charged (μ^-)muons

In matter, positive and negative muons behave differently.

The μ^+ feels the repulsive force from the positively charged atomic nucleus, but it can attract a negative electron and form with it a bound state μ^+e^- called "muonium". This is the lightest "atom" existing in nature. But the lifetime of muonium is limited because the muon will decay (its lifetime is not much different from the lifetime of the free muon).

A negative muon feels the attraction of atomic nuclei and will possibly form a muonic atom. The muon is bound in an atomic orbit; but, since the muon has a larger mass compared to the electron (about 200 times larger) the muonic orbits are about 200 times smaller than the electronic orbits. Then what happens:

1. The μ^- can decay as if it were free, but the lifetime of this bound muon is different from the lifetime of a free muon. This will happen mainly for muonic atoms formed with light nuclei.

2. The μ^- can be captured by the atomic nucleus and undergo the transition (induced by the weak interaction):
$$\mu^-(A, Z) \rightarrow \nu_\mu(A, Z-1)$$
This process in called "muon capture". It is the most frequent disappearance mode for $Z > 10$ since the probability for the capture process goes like Z^4.

4 Ordinary muon decay

It was soon realized that the μ meson had nothing to do with the nuclear interaction, something which justified a change of name (the "muon"). Experiments showed that its decay is a three-body decay, with the emission of an electron (positron) and neutral particles which are supposed to be neutrinos.
$$\mu^\pm \rightarrow e^\pm \nu\nu$$
The shape of the positron (or electron) energy spectrum was first calculated by Louis Michel[8], who introduced a parameter called ρ. The shape of the spectrum is well reproduced with the value $\rho = 3/4$.

After the "Lee and Yang" proposal (non-conservation of parity in weak interactions) and the experiments which followed (including polarisation effects), muon decay data had to be described by four parameters: ρ, η, ξ and δ[8,9,10,11,1]. These four parameters are now called the "Michel parameters". In the Standard Model they have definite values:
$$\rho = 3/4 \quad \eta = 0 \quad \xi = 1 \quad \delta = 3/4$$
The observables of muon decay are given by the following expression:
$$\frac{d^2\Gamma(\mu^+ \rightarrow e^+\nu\overline{\nu})}{dx\ d\cos\theta_e} = \frac{m_\mu}{4\pi^3 W_{e\mu}^4} G_F^2 \sqrt{x^2 - x_0^2} \Big(F_{IS}(x) \pm P_\mu \cos\theta_e F_{AS}(x) \Big) \Big(1 + \vec{P}_e(x, \theta_e) \cdot \vec{\zeta} \Big)$$
with the following definitions:
$$W_{e\mu} = (m_\mu^2 + m_e^2)/(2m_\mu) \quad x = E_e/W_{e\mu} \quad x_0 = m_e/W_{e\mu}$$
$\vec{P}_e(x, \theta_e)$ is the polarization vector of the positron. θ_e is the angle between the electron (or positron) momentum and the muon polarization vector $\vec{P}_\mu$. The vector $\vec{\zeta}$ is the direction along which the $e^\pm$ polarization is measured.

The ± sign corresponds to $\mu^{\pm}$ muons.

The functions F_{IS} and F_{IS} are given by:

$$F_{IS} = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$F_{AS} = \frac{1}{3}\xi\sqrt{x^2 - x_0^2}\Big[1 - x + \frac{2}{3}\delta\Big(4x - 3 + (\sqrt{1 - x_0^2} - 1)\Big)\Big]$$

x runs from $x_0 = 9.7 \times 10^{-3}$ to 1.

Some complications arise from the presence of radiative corrections (the muon is no longer a point particle, it is surrounded by a cloud of virtual particles: virtual photons, virtual electron-positron pairs, etc...) These radiative corrections have been the object of many detailed calculations. It is necessary to take these corrections into account when performing precision experiments.

Many experiments have been devoted to the determination of the Michel parameters, as attempts to observe some “physics beyond the Standard Model”.

The most recent one (TWIST = TRIUMF Weak Interaction Test) is being performed at TRIUMF, the Canadian Meson Factory, located in Vancouver. The challenge is to measure the Michel parameters in a single experiment.

The experiment uses positively charged muons and selects the so-called “surface muons”, i.e. muons which are produced from pions decaying at the surface of the target. These muons are fully longitudinally polarized (it is possible to neglect the neutrino mass). They leave the target without losing energy and they are guided by a magnetic system to the center of a large detector, where they decay. The positron tracks are detected by a system of sensitive chambers so that their trajectories can be reconstructed and the positron momenta determined. More information (with pictures) can be found on the TWIST web site http://twist.triumf.ca/.

The analysis of the TWIST data is not fully completed yet but several papers have already been published. Essentially, all the results obtained are consistent with the Standard Model but improved upper limits have been obtained on several parameters:

4.1 The parameter ρ

The TWIST experiment has produced the following result[12]

$$\rho = 0.75080 \pm 0.00044 \text{ (stat.)} \ \pm 0.00093 \text{ (syst.)} \ \pm 0.00023$$

where the last uncertainty represents the dependence of ρ on the Michel parameter η. This result sets new limits on the $W_L - W_R$ mixing angle in left-right symmetric models.

4.2 The parameter η

The influence of the parameter η on the positron spectrum is restricted to the lower end of the positron energy spectrum, therefore the TWIST experiment cannot compete with the PSI experiment which measured the perpendicular (in the decay plane) polarization of the positron in muon decay. Reference [2] gives the following result:

$$\eta = -0.007 \pm 0.013$$

which is consistent with zero, the Standard Model value.

4.3 The parameter $P_\mu\xi$

Due to the difficulty of knowing with sufficient precision the polarisation P_μ of the muon at the point of decay, it is more convenient to publish the product $P_\mu\xi$, which is still a good test of the Standard Model. The published result[13] is:

$$P_\mu\xi = 1.0003 \pm 0.0006(stat.) \pm 0.0038(syst.)$$

This result agrees with previous measurements but is over a factor of 2 more precise. It also agrees with the Standard Model prediction for $P_\mu\xi$ and leads to restrictions on left-right symmetric models.

4.4 The parameter δ

The TWIST experiment has published[14]:

$$\delta = 0.74964 \pm 0.00066(stat.) \pm 0.00112(syst.)$$

This result is in agreement with the Standard Model prediction of 0.75.

4.5 About the tensor interaction

The question of a possible tensor interaction (not existing in the Standard Model) has been a long-standing problem since an experiment done in Protvino (Russia) on the $\pi^- \rightarrow e^-\nu\gamma$ showed a deficit of events in a certain kinematic region which could not be explained by the Standard Model. A study of the $\pi^+ \rightarrow e^+\nu\gamma$ at PSI has not produced convincing results. Several papers by Chizhov[15] deal with the problem of the tensor interaction in muon decay.

4.6 Other muon decay modes

The decay:

$$\mu^+ \rightarrow e^+\overline{\nu}_\mu\nu_e$$

is the main mode of the muon decay (close to 100%). The other decay modes are[2]:

$$\mu^+ \rightarrow e^+\overline{\nu}_\mu\nu_e\gamma \quad (1.4 \pm 0.4)\%$$

This decay is the most dangerous background source for the study of the $\mu^+ \rightarrow e^+\gamma$ decay.

$$\mu^+ \rightarrow e^+\overline{\nu}_\mu\nu_e e^+e^- \quad (3.4 \pm 0.4) \times 10^{-5}$$

5 Forbidden muon decays

In the Standard Model there is a conservation law for the leptonic numbers L_e and L_μ which are defined as:

	e^-, ν_e	e^+, $\overline{\nu}_e$	μ^-, ν_μ	μ^+, $\overline{\nu}_\mu$
L_e	+1	−1	0	0
L_μ	0	0	+1	−1

The muon decays which do no respect this rule are called “lepton-flavour violating”.

5.1 The $\mu^+ \rightarrow e^+\gamma$ decay

Many efforts have been devoted to the possible observation of this lepton-flavor-violating decay, especially at the meson factories: LAMPF (USA), PSI (Switzerland), TRIUMF (Canada). The best upper limit has been reached at LAMPF:

$$B.R.(\mu^+ \rightarrow e^+\gamma) \leq 3.2 \times 10^{-11}$$

The most ambitious experiment, in preparation at PSI, aims at the $10^{-13} - 10^{-14}$ level.

Comment on neutrino mixing: in principle, neutrino mixing (which has been observed) can induce the $\mu^+ \rightarrow e^+\gamma$ decay, but, due to the smallness of the neutrino masses in comparison to the mass of the W boson the effect would be too small to be detected.

5.2 The $\mu^+ \rightarrow e^+\gamma\gamma$ decay

This decay has been studied at the meson factories. The result is an upper limit:

$$B.R.(\mu^+ \rightarrow e^+\gamma\gamma) \leq 7.2 \times 10^{-11}$$

5.3 The $\mu^+ \rightarrow e^+e^-e^+$ decay

Studied at PSI with the SINDRUM detector.

$$B.R.(\mu^+ \rightarrow e^+e^-e^+) \leq 1.0 \times 10^{-12}$$

5.4 Muon-to-electron conversion in a nucleus

It is the process:

$$\mu^-(A,Z) \rightarrow e^-(A,Z)$$

One is looking for a ground-state to ground state transition. The electron is monoenergetic with an energy much above the end point of the $\mu^+ \rightarrow e^+\nu_e\overline{\nu}_\mu$, a big advantage for the signal-to-background ratio.

The first search was conducted many years ago by Lagarrigue and Peyrou[16], using cosmic rays.

The experimental results are expressed as the ratio:

$$\frac{\mu^-(A,Z) \rightarrow e^-(A,Z)}{\mu^-(A,Z) \rightarrow \nu_\mu(A,Z-1)}$$

Among the best upper limits.

$$\mu^- Ti \rightarrow e^- Ti \quad 4.6 \times 10^{-12} \quad \text{TRIUMF}$$

$$\mu^- Pb \rightarrow e^- Pb \quad 4.6 \times 10^{-11} \quad \text{PSI}$$

5.5 Muon-to-positron conversion in a nucleus

It is the process:

$$\mu^-(A,Z) \rightarrow e^+(A,Z-2)$$

In this case the positron has a wide energy distribution. This process relates two nuclei which differ by two units in the atomic number. It has some similarity with neutrinoless double-beta decay:

$$(A, Z) \rightarrow e^- e^- (A, Z + 2)$$

which has been and is still being actively studied because its observation would prove the Majorana character of the neutrino (neutrino and antineutrino are the same particle).

See [2] for the upper limits, which are of the order of $10^{-11} - 10^{-10}$.

5.6 Muonium to antimuonium conversion

It is the process:

$$\mu^+ e^- \rightarrow \mu^- e^+$$

An incident positive muon combines with an atomic electron to form a muonic atom. The muonium to antimuonium conversion would signal itself by the emission of a positron.

For the interpretation of experimental data the effective Lagrangian associated with the muonium to antimuonium conversion is written as[2]:

$$L = \frac{1}{\sqrt{2}} G_C \left[\overline{\psi}_\mu \gamma_\lambda (1 - \gamma_5) \psi_e \right] \left[\overline{\psi}_\mu \gamma_\lambda (1 - \gamma_5) \psi_e \right] + h.c.$$

The result is expressed in terms of the ratio $R_g = G_C / G_F$, where G_F is the Fermi constant:

$$R_g < 0.0030$$

6 Muon capture by the atomic nucleus

The basic weak interaction is of the $V - A$ type. This is true for purely leptonic processes, but the situation is different for processes involving strongly-interacting particles (hadrons). One could expect that the vector and axial vector coupling constant are modified and that new couplings appear. One very interesting fact is that the vector coupling constant g_V is not modified (not renormalized) in the presence of the strong interaction. This has led to the "Conserved Vector Current Theory" of Feynman and Gell-Mann, which has been confirmed by many experiments, opening the door to the Standard Model which unifies weak and electromagnetic interactions.

But the same situation does not appear for the axial-vector interaction, and the coupling constant g_A is renormalized in the strong interaction sector. Instead on the interaction $V - A$ (for instance in muon decay) we observe something like $V - 1.2A$ in neutron decay.

In a more general way, in addition to the modification of the coupling constant g_A there are new interactions which appear in the weak-interaction Hamiltonian: scalar (S), weak magnetism (M), tensor (T), and pseudoscalar (P):

$$V^\mu = G_V(q^2) \gamma^\mu + \frac{1}{2m} G_M(q^2) \sigma^{\mu\nu} q_\nu + \frac{1}{2m} G_S(q^2) q^\mu$$

$$A^\mu = G_A(q^2) \gamma^\mu \gamma^5 + \frac{1}{2m} G_P(q^2) \gamma^5 q^\mu + \frac{1}{2m} G_T(q^2) \sigma^{\mu\nu} q_\nu$$

The existence of weak magnetism has been confirmed by many experiments, starting with C.S. Wu. Scalar and tensor interactions have not been established with certainty. The size of the pseudoscalar interaction has been predicted and confirmed in several experiments. One particular process is particularly sensitive to the pseudoscalar interaction: radiative muon capture on the proton.

$$\mu^- P \rightarrow \nu_\mu N \gamma$$

Experimentally, the source of protons is a hydrogen target. Liquid hydrogen is a convenient choice but the chemistry of muons in liquid hydrogen is complicated and not enough understood. An experiment using hydrogen gas has been performed at PSI (MUCAP)[17]. The result is a value of the induced pseudoscalar coupling constant $g_p(q^2 = -0.88m_\mu^2) = 7.3 \pm 1.1$ which is in agreement with calculations based on approximate chiral symmetry of QCD (Quantum Chromo Dynamics).

7 Static properties of the muon

7.1 Muon lifetime

The muon lifetime is very important for the determination of the fundamental constant (Fermi coupling constant) G_F. The relation is[1] :

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[F\left(\frac{m_e^2}{m_\mu^2}\right) + 4\eta\frac{m_e}{m_\mu}G\left(\frac{m_e^2}{m_\mu^2}\right) - \frac{32}{3}\left(\rho - \frac{3}{4}\right)\frac{m_e^2}{m_\mu^2}\left(1 - \frac{m_e^4}{m_\mu^4}\right)\right]\times$$

$$\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2}\right)\left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

The functions F and G have the following expressions:

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 lnx$$

$$G(x) = 1 + 9x - 9x^2 - x^3 + 6x(1 + x)lnx$$

$\alpha(m_\mu)$ is the so-called "fine structure constant", not exactly a constant, evaluated at the muon mass.

This expression is quite general. If the Standard Model is assumed the expression simplifies:

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2}\right)\left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

7.2 Muon mass

The muon mass is presently known with a spectacular accuracy[2]. The best measurements of the muon mass are based on the muonic atoms and the value of the muon mass is best expressed in terms of atomic mass units:

$$m_\mu = 0.1134289264 \pm 0.0000000030 \quad \text{a.m.u.}$$

The expression in MeV is less precise because of a larger uncertainty in the conversion factor a.m.u to MeV.

$$m_\mu = 105.6583692 \pm 0.0000094 \text{ MeV}$$

The muon mass is an important number which enters in the expression used to get the Fermi constant G_F.

7.3 Muon magnetic moment

For a classical particle of spin 1/2 the magnetic moment would be given by:

$$\vec{\mu} = g\frac{e\hbar}{2mc}\vec{S}$$

with a gyromagnetic ratio $g = 2$.

However, a physical particle is surrounded by virtual photons, virtual particle-antiparticle pairs and the gyromagnetic factor g is different from 2 by a small amount. This quantity $(g-2)$ is extremely important because it provides a very sensitive test of quantum electrodynamics and could shed some light on physics beyond the Standard Model.

Recently, an experimental result obtained at Brookhaven National Laboratory disagreed with the theoretical prediction [18].

$$a_\mu(Expt) = 11659208.0 \times 10^{-10}$$

$$\Delta a_\mu(Expt - SM) = (22.4 \pm 10 \text{ to } 26.1 \pm 9.4) \times 10^{-10}$$

The announcement started a very intense theoretical activity on possible manifestations of physics “beyond the Standard Model”. There have been some doubts about the theoretical calculation [19]. After correction, a small discrepancy remains. This situation justifies further experimental effort.

7.4 Muon electric dipole moment

The electric dipole moment is a physical quantity which violates parity and time-reversal invariance, and further efforts are deployed to improve the accuracy on this important quantity. The present value is[2]:

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e-cm}$$

8 References

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