TRANSIENT THERMAL AND VISCOUS IRREVERSIBILITIES THROUGH A POISEUILLE-BENARD CHANNEL IN PRESENCE OF A MAGNETIC FIELD

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ABSTRACT The influence of an external magnetic field on transient thermal and viscous irreversibilities in Poiseuille-Benard channel for the laminar flow of an incompressible electrically conducting fluid is numerically investigated by solving the mass, momentum and energy conservation equations. Numerical simulation was performed for Reynolds number (Re) equal to 10 and an irreversibility distribution ratio (ϕ) equal to 0.1. The Stuart number (N) that describes the magnetic field effect, ranges between 0 and 10. Rayleigh and Prandtl numbers are fixed respectively at 10^4 and 0.71.

INTRODUCTION

Fluid flow inside a channel with circular cross-section or made of two parallel plates is of great interest in thermal engineering as they appear in many industrial applications. Flows of an electrically conducting fluid are encountered in many industrial applications, such as, purification of molten materials from non-metallic inclusions, liquid metal, plasma, geothermal energy extractions and in many other applications. Oreper and Szekely [1983] showed that the magnetic field suppresses the natural convection currents and the magnetic field strength is one of the most important factors for crystal formation. Natural convection of low Prandtl number fluid in the presence of a magnetic field was numerically studied by Ozone and Maruo [1987], they obtained correlations for the Nusselt number in terms of Rayleigh, Prandtl and Hartman numbers. Rudraiah et al. [1995] numerically investigated the effect of a transverse magnetic field on natural convection flow inside a rectangular enclosure with isothermal vertical walls and adiabatic horizontal walls and found out that a circulating flow is formed with a relatively weak magnetic field and that the convection is suppressed and the rate of convective heat transfer is decreased when the magnetic field strength increases. Wang and Chen [2005] studied the effect of magnetic field on mixed convection boundary layer flow in inclined wavy plates. They showed that the action of the magnetic field tends to accelerate the flow near the leading edge and to decelerate it far downstream of the leading edge.

Thermal gradients and viscous effects are responsible of entropy generation in any thermal device, and the performance of the system is enhanced by minimizing these effects via the study of parameters of the problem under consideration [Bejan 1996]. Mahmud and Fraser [2004] studied entropy generation in a fluid saturated porous cavity for laminar magnetohydrodynamic natural convection, where the magnetic force is assumed acting along the direction of the gravity force. It was found that increasing Hartmann number (i.e. magnetic force), tends to retard the fluid motion, both average Nusselt and dimensionless entropy generation numbers decrease with increasing Hartmann number and approach a limiting value (asymptotic value). For a mixed convection flow, Mahmud et al. [2003] gave a detailed analysis on the nature of entropy generation and on the irreversibility sources inside a non porous vertical channel submitted to a transversal magnetic field.

Abbassi et al. [2003] studied numerically entropy generation in Poiseuille-Benard channel flow. They showed that entropy generation is largely higher near the channel walls than that in the central flow.

PROBLEM FORMULATION

Two-dimensional mixed convection flow of an electrically conducting fluid in Poiseuille-Benard channel of length L and height H, submitted to a uniform magnetic field normal to the flow and a vertical thermal gradient is simulated as illustrated in Figure 1. Temperature of incoming stream is supposed linear from T_H (hot temperature) at the bottom of the channel wall to T_C (cold temperature) at the upper wall. Normal component of the velocity is assumed to be zero, and a fully developed parabolic profile for the axial velocity component is deployed. The aspect ratio is fixed at L/H = 5. This aspect ratio value was found sufficient for the numerical study of Poiseuille-Benard channel flow as it was indicated by Nicolas et al. [1997].

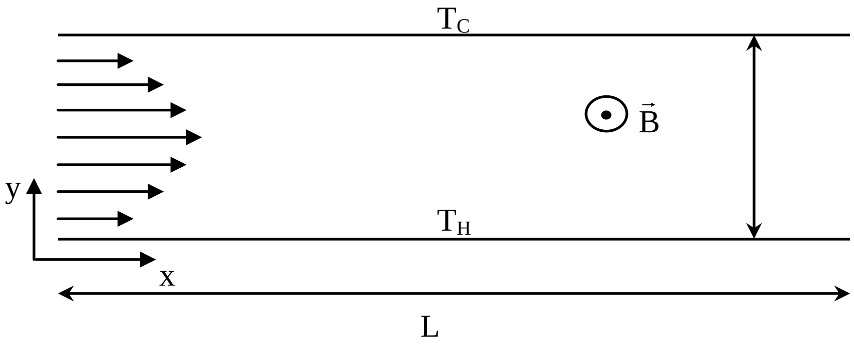


Figure 1: Poiseuille-Benard channel configuration in presence of an external magnetic field normal to the flow.

Boundary conditions expressed in dimensionless form are as follows:

$$
\begin{aligned}
&0 \le x \le L/H,\ y = 0:\ u = v = 0,\ \theta = 1\\
&0 \le x \le L/H,\ y = 1:\ u = v = 0,\ \theta = 0\\
&x = 0,\ 0 \le y \le 1:\ u = 6y\,(1 - y),\ v = 0,\ \theta = 1 - y\\
&x = L/H,\ 0 \le y \le 1:\ \frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial x} = 0,\ \int_0^1 u\,dy = 1
\end{aligned}
\qquad (1)
$$

φ is the dimensionless dependent variable (φ = u, v, θ) and $\vec{V}$ is the dimensionless velocity vector. θ is the dimensionless temperature and u and v are the dimensionless velocity vector components.

GOVERNING EQUATIONS

With reference to a Newtonian incompressible fluid of constant physical properties, continuity, change of linear momentum and energy equations are written in dimensionless form as follows:

$$\nabla.\vec{V} = 0 \qquad (2)$$

$$\frac{\partial u}{\partial \tau} + \nabla.\vec{J}_u = - \frac{\partial P}{\partial x} - N\,u \qquad (3)$$

$$\frac{\partial v}{\partial \tau} + \nabla.\vec{J}_v = - \frac{\partial P}{\partial y} + \frac{Ra}{Pe\,Re}\theta - N\,v \qquad (4)$$

$$\frac{\partial \theta}{\partial \tau} + \nabla.\vec{J}_\theta = 0 \qquad (5)$$

where:

$$\vec{J}_u = u\vec{V} - \frac{1}{Re}\vec{\nabla}u \qquad (6)$$

$$\vec{J}_v = v\vec{V} - \frac{1}{Re}\vec{\nabla}v \qquad (7)$$

$$\vec{J}_\theta = \theta\,\vec{V} - \frac{1}{Re\,Pr}\vec{\nabla}\theta \qquad (8)$$

N, Ra, Pe, Re and θ are respectively the Stuart, the Rayleigh, the Peclet and the Reynolds numbers and the dimensionless temperature. These dimensionless variables are defined as follows:

$$N = \frac{\sigma H B^2}{\rho u_0}, Ra = \frac{g\beta\Delta T H^2}{\nu u_0}, Re = \frac{H u_0}{\nu}, Pe = Re.Pr, \theta = \frac{T - T_C}{T_H - T_C} \qquad (9)$$

σ is the electrical conductivity of the fluid. Pr (= μ C_p/λ) is the Prandtl number. The Stuart number N, is the criterion of the relative importance of the magnetic force (Lorentz force) to the inertia force. ρ, u_0, β, ΔT (= T_H - T_C), μ, C_P, ν and λ are respectively the mass density, a characteristic velocity, thermal expansion factor, temperature difference, dynamic viscosity, specific heat, kinematic viscosity and thermal conductivity of the fluid. B is the external magnetic field.

ENTROPY GENERATION

Irreversible nature of heat transfer and viscous effects cause continuous generation of entropy in the fluid. Entropy generation is then due to non-equilibrium flow imposed by boundary conditions through the channel walls. Since entropy generation results from the heat transfer and the fluid friction, known scalar fields of temperature and velocity components provide the calculation of the degraded energy expressed by entropy generation. The rate of entropy generation in its dimensional form is given by:

$$\dot{S}_{gen} = \frac{\lambda}{T^2}\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{\mu}{T}\left(2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right) \qquad (10)$$

First term of the right hand side of equation (10) represents local irreversibility due to thermal gradients and the second is due to fluid friction. Under these conditions, dimensionless local entropy generation due to thermal gradients and that due to velocity gradients are given by:

$$s_t = \left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2 \quad (11)$$

$$s_v = \phi \left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right] \quad (12)$$

ϕ is the irreversibility distribution ratio given by:

$$\phi = \frac{\mu T_0}{\lambda} \left(\frac{u_0}{\Delta T}\right)^2 \quad (13)$$

T_0 ($= (T_C + T_H)/2$) is the mean temperature. Calculations of irreversibilities due to thermal (S_t) and velocity (S_v) gradients are then obtained by integrating equations (11) and (12) over the entire domain. Effect of the magnetic field on entropy generation is presented by Stuart number which is obtained in velocity as well as temperature equations.

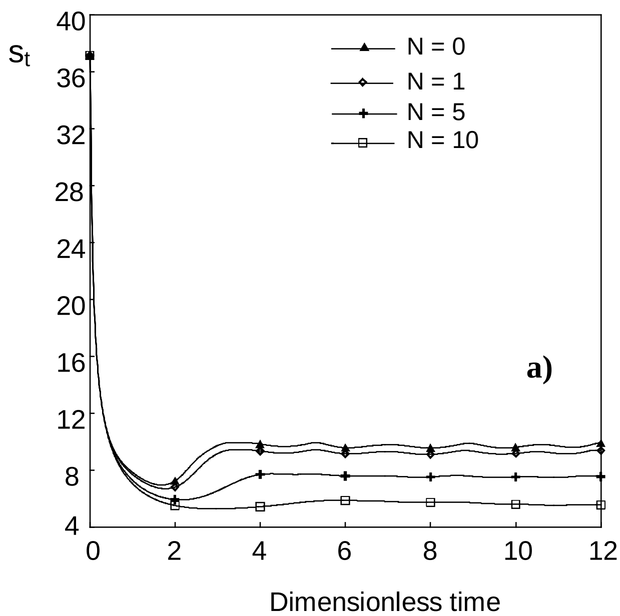
NUMERICAL METHOD

The present study is based on a modified version of control volume finite element method (CVFEM) of Saabas and Baliga [1994], adapted to the staggered grids in which pressure and velocity components are stored at different points. SIMPLER algorithm was applied to resolve the pressure-velocity coupling in conjunction with an alternating direction implicit scheme (ADI) for performing the time evolution. The used numerical code was described and validated in details in Abbassi et al. [2001a, 2001b]. From the known temperature and velocity scalar fields at any time τ given by solving equations (3)-(5). In this study, the use of a uniform grid of 101x26 nodal points with a step time $\Delta\tau = 10^{-3}$ was found sufficiently enough to achieve the imposed global and local convergences criteria given by:

$$\nabla . \vec{V} \leq 10^{-4}, \ \max \left| \frac{\varphi^{\tau+\Delta\tau} - \varphi^{\tau}}{\varphi^{\tau+\Delta\tau}} \right| \leq 10^{-5} \quad (10)$$

THERMAL AND VISCOUS IRREVERSIBILITIES IN PRESENCE OF A MAGNETIC FIELD

As a complementary study of Abbassi et al. [2003], this paper is focused on studying the effect of an external imposed magnetic field normal to the flow on entropy generation. For Reynolds number, irreversibility distribution ratio and Rayleigh number values equal respectively to 10, 0.1 and 10^4, Figure 2.a shows that transient irreversibility due to thermal gradients passes from a maximum value at the very beginning of time, towards an oscillatory behaviour for lower Stuart number values ($0 \leq N \leq 1$) and towards an asymptotic behavior for higher Stuart numbers (N ≥ 5). The increase of magnetic effect induces the decrease of thermal irreversibility. Figure 2.b shows that viscous irreversibility passes from a minimum value at the very beginning of the transient state towards an oscillatory behaviour for lower values of Stuart number and towards an asymptotic behaviour for higher values of Stuart number (N ≥ 10). Magnitude of viscous and thermal irreversibilities is considerably decreased as time proceeds for higher Stuart number values. As a consequence, application of an external magnetic field induces considerable decrease of energy loss expressed by entropy generation. It's important to notice that from N = 10, the oscillatory behavior vanishes and irreversibility magnitude are practically constant over time for both thermal and viscous irreversibilities. This is proved by the plot of streamlines (Figure 3) and isothermal charts (Figure 4), in which one can distinguish the considerable changes in flow structure at low Stuart number. In fact, on increasing the Stuart number from N = 5 to N = 10, one can easily observe the destruction of the cells near to the bottom wall (just at the inlet channel) and consequently thermal and velocity gradients are absent in this region as illustrated in Figures 3 and 4. This means that on increasing values of Stuart number which expresses the magnetic force, deceleration of the electrically conducting fluid flow is obtained and convective cells disappeared from the inlet flow channel. At the beginning, cells of smaller amplitudes are squeezed, and then destroyed. In the same way, as the Stuart number increases inner cells undergo the same effect. Finally the Von Karman street disappears for higher magnetic force, inducing formation of streamlines parallel to the upper cooled and lower heated walls of the channel.



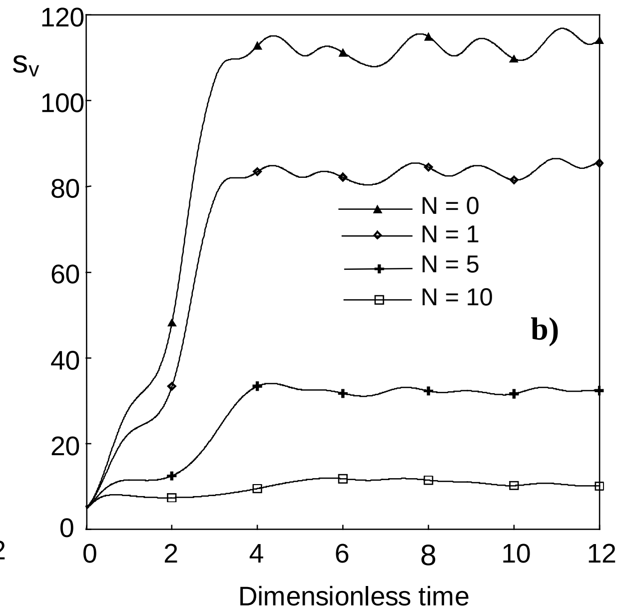
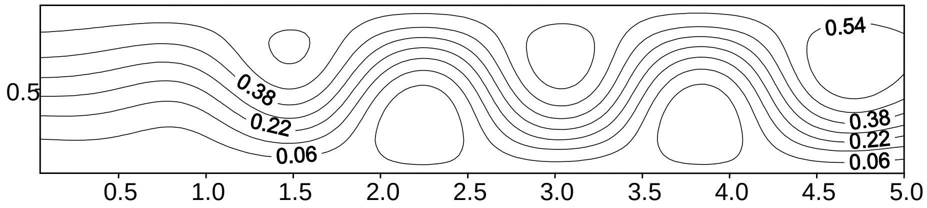
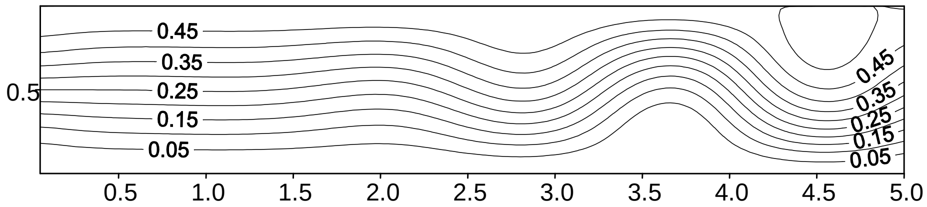


Figure 2: Transient entropy generation versus time for different Stuart number values at Re = 10, Ra = 10^4 and $\phi = 0.1$: a) thermal irreversibility, b) viscous irreversibility.

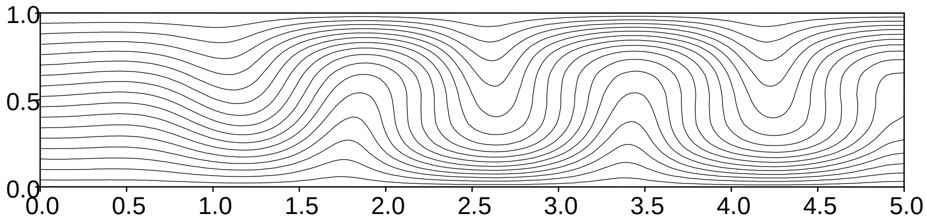


N = 5

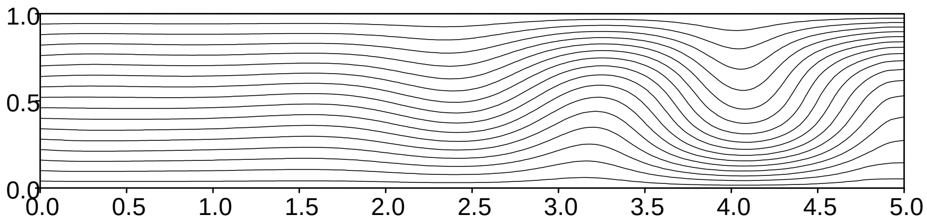


N = 10

Figure 3: Streamlines for Re = 10, Ra = 10^4 and $\phi = 0.1$: N = 5, N = 10



N = 5



N = 10

Figure 4: Isothermal charts for Re = 10, Ra = 10^4 and $\phi = 0.1$: N = 5, N = 10.

CONCLUSION

Transient thermal and viscous irreversibilities in presence of a magnetic field versus time are numerically studied. The magnetic field causes an important decrease in irreversibility magnitude and we pass from the non linear branch of thermodynamics of irreversible process characterized by an oscillatory behavior towards an asymptotic state which characterizes the linear branch of thermodynamics of irreversible process.

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