NATURAL CONVECTION IN A FINNED RAYLEIGH-BENARD CUBICAL ENCLOSURE

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ABSTRACT

The papers deals with a numerical 3D study of natural convection in a finned Rayleigh-Benard (RB) cubical enclosure. A single fin with a thickness of 10% of the cavity side (and a height of 50%) is placed vertically on the bottom hot wall at T_H. The working fluid is air with Prandtl number Pr = 0.71 and the Rayleigh number (*Ra*) varies from 10^3 to 10^5. The solid-to-fluid thermal conductivity ratio (R_k) was fixed at $R_k = 7000$, corresponding to a metal of high conductivity. The top wall is at the temperature $T_C < T_H$ and the remaining four surfaces are insulated. Inside the RB enclosure, the flow structure and the temperature distribution are presented in terms of mean velocity vector plots and isotherm plots. The effects of the Rayleigh number on the mean heat transfer rate through the cold wall are presented and discussed. A correlation between the averaged Nusselt number through the top wall and *Ra* is proposed.

INTRODUCTION

Investigations on heat transfer and fluid dynamics in fluid-filled enclosures with fins mounted on the active walls have expanded during the last years. The majority of works report simulations of steady, 2D flow and temperature fields. The interest is to control and/or optimize the heat transfer rate in the cavity by the addition of fins.

Within the framework of 2D investigations, one can cite the publications of Frederick [1989], Tasnim and Collins [2004] and Arquis and Rady [2005]. For more realistic 3D numerical investigations less works are encountered in the literature. However, due to the recent developments of computer technologies, the works by Frederick [2007] and Frederick and Moraga [2007] are representative.

In the present numerical investigation, a 3D "in-house" code was used to study the flow structure and heat transfer in a finned cubical enclosure. The physical model and coordinates are shown in figure1. A rectangular, conducting fin is centrally affixed to the hot wall at the temperature T_H, whereas the cold wall at the temperature T_C. The four remaining walls of the enclosure are kept insulated. The parameters of the problem are the Rayleigh number *Ra*, the Prandtl number *Pr*, the dimensionless fin length and width (*s/H* and *e/H*), and the thermal conductivity ratio, $R_k = k_s / k_f$.

In order to reduce the parameter combinations, we choose to vary only the Rayleigh number in the interval $10^3 \le Ra \le 10^5$. The thermal conductivity ratio is fixed at $R_k = 7000$ (metallic solid of high thermal conductivity). Six different values of *Ra* are considered, i.e., *Ra*=10^3, 10^4, 2.5×10^4, 5×10^4, 7.5×10^4 and 10^5. The length and width ratios are kept constant, i.e., *s/H*=0.5 and *e/H*=0.1.

GOVERNING EQUATIONS

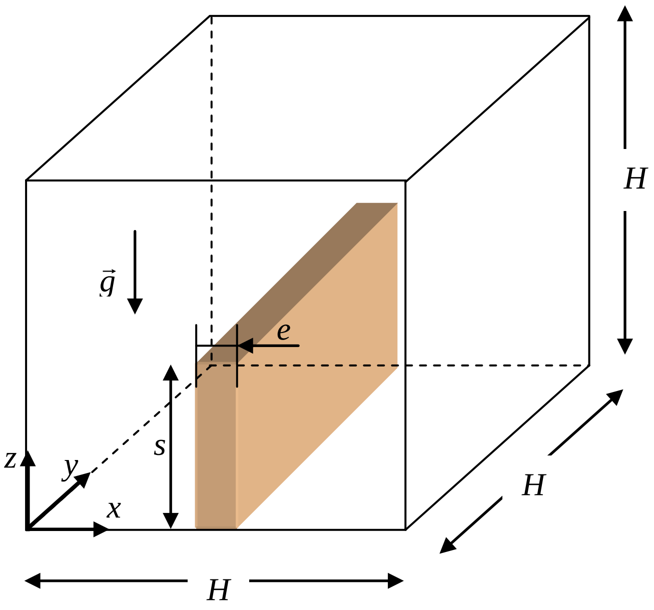

Figure 1. Physical model and coordinates

The governing equations of continuity (1), momentum (2)-(3) and energy (4) for unsteady laminar flow in Cartesian coordinates take the following dimensionless form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+\frac{\partial (u\;\;)}{\partial x}+\frac{\partial (vu)}{\partial y}+\frac{\partial (wu)}{\partial z}=-\frac{\partial p}{\partial x}+Pr\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \qquad (2)$$

$$\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+\frac{\partial (vv)}{\partial y}+\frac{\partial (wv)}{\partial z}=-\frac{\partial p}{\partial y}+Pr\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \qquad (3)$$

$$\frac{\partial w}{\partial t}+\frac{\partial (u\;\;)}{\partial x}+\frac{\partial (vw)}{\partial y}+\frac{\partial (ww)}{\partial z}=-\frac{\partial p}{\partial z}+Pr\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+RaPr\theta \qquad (4)$$

$$\frac{\partial \theta}{\partial t}+\frac{\partial (u\theta)}{\partial x}+\frac{\partial (v\theta)}{\partial y}+\frac{\partial (w\theta)}{\partial z}=\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right) \qquad (5)$$

Within the fin, the dimensionless energy equation is:

$$\frac{\partial \theta}{\partial t}+\frac{\partial (u\theta)}{\partial x}+\frac{\partial (v\theta)}{\partial y}+\frac{\partial (w\theta)}{\partial y}=R_k\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right). \qquad (6)$$

Eqs. (1)-(6) where obtained using H, α, ρ and $(T_H - T_C)$ as reference quantities. The non dimensional temperature is defined in terms of the wall temperature difference and a reference temperature as:

$$\theta=\frac{T-T_r}{T_H-T_C} \quad \text{and} \quad T_r=\frac{1}{2}\left(T_H+T_C\right)$$

T_H is the temperature of the hot wall, and T_C is that of the cold wall. The Rayleigh number and Prandtl number are, respectively:

$Ra = \dfrac{g\beta(T_H - T_C)H^3}{\alpha\nu}$ and $Pr = \dfrac{\nu}{\alpha}$ where α is the thermal diffusivity and ν the kinematic viscosity.

The enclosure boundary conditions consist of no-slip and no penetration walls, i.e., u=v=w=0 on all walls. The thermal boundary conditions on the bottom and top walls are:

$\theta_{z=0} = \theta_H = +\dfrac{1}{2}$ and $\theta_{z=1} = \theta_C = -\dfrac{1}{2}$. The remaining walls are adiabatic.

Zero velocities (u=v=w=0) are specified on the fin faces. The base and tip fin are located at $z = 0$ and $z = s/H$, respectively, while the left and right fin sides are located at $x = \dfrac{1}{2} - \dfrac{e}{2H}$ and $x = \dfrac{1}{2} + \dfrac{e}{2H}$. The fin base is at $\theta = \dfrac{1}{2}$ and the thermal boundary conditions for the other fin faces are:

$$\left(\frac{\partial\theta}{\partial z}\right)_f = R_k\left(\frac{\partial\theta}{\partial z}\right)_s \text{ at } z = s/H$$

$$\left(\frac{\partial\theta}{\partial x}\right)_f = R_k\left(\frac{\partial\theta}{\partial x}\right)_s \text{ at } x = \frac{1}{2} - \frac{e}{2H} \text{ and } x = \frac{1}{2} + \frac{e}{2H}.$$

Here, the subscripts f and s indicate that the gradients are evaluated at the fluid and at the solid side of the interface, respectively.

NUMERICAL METHOD

The unsteady Navier-Stokes and energy equations are discretized using staggered, non-uniform control volumes. A projection method is used to couple the momentum and continuity equations. A finite-volume method is used to discretize the Navier–Stokes and energy equations. The discretized momentum and energy equations are resolved using the red and black successive over relaxation method RBSOR. The Poisson pressure correction equation is solved using a full multigrid method as suggested by Ben Cheikh et al. [2007].

Table 1
Comparison between the present results and those of Frederick [2007]

Ra	$\langle Nu \rangle$		
	Present work	Frederick [2007]	Difference
10^4	2.6791	2.6360	1.64 %
10^5	5.7901	5.7997	-0.17 %
10^6	11.5348	11.4806	0.48 %

The computer code was first validated with a test problem related to a work of Frederick [2007]. It consists of a differentially heated cube with a horizontal fin situated on the right hot wall. The left wall is cold while the four remaining surfaces are insulated. Table 1 lists our numerical values along with those of Frederick [2007] relatively to the overall heat transfer (Nusselt number) through the cold wall for $10^4 \le Ra \le 10^6$ and $R_k = 7000$ and good agreement is evident.

RESULTS AND DISCUSSION

In this paragraph, the Prandtl number is equal to 0.71, and the Rayleigh number varies from 10^3 to 10^5. The dimensionless time step varies from $\Delta t = 5\times10^{-6}$ to $\Delta t = 5\times10^{-7}$ depending on the Rayleigh number. The grid size is 64×48×64 with non uniform spaces near the active walls and the boundaries of the fin. Steady state was considered as achieved according to the following criterion:

$$\sum_{i,j,k} \frac{\left|X_{i,j,k}^{n+1} - X_{i,j,k}^{n}\right|}{\left|X_{i,j,k}^{n}\right|} \le 10^{-5}. \qquad (7)$$

Here, X represents the variable u, v, w or θ, the superscript n refers to the iteration number and (i,j,k) refers to the space coordinates.
Figure 2 represents the vector fields in the mid plane x-z for the different considered Rayleigh numbers.

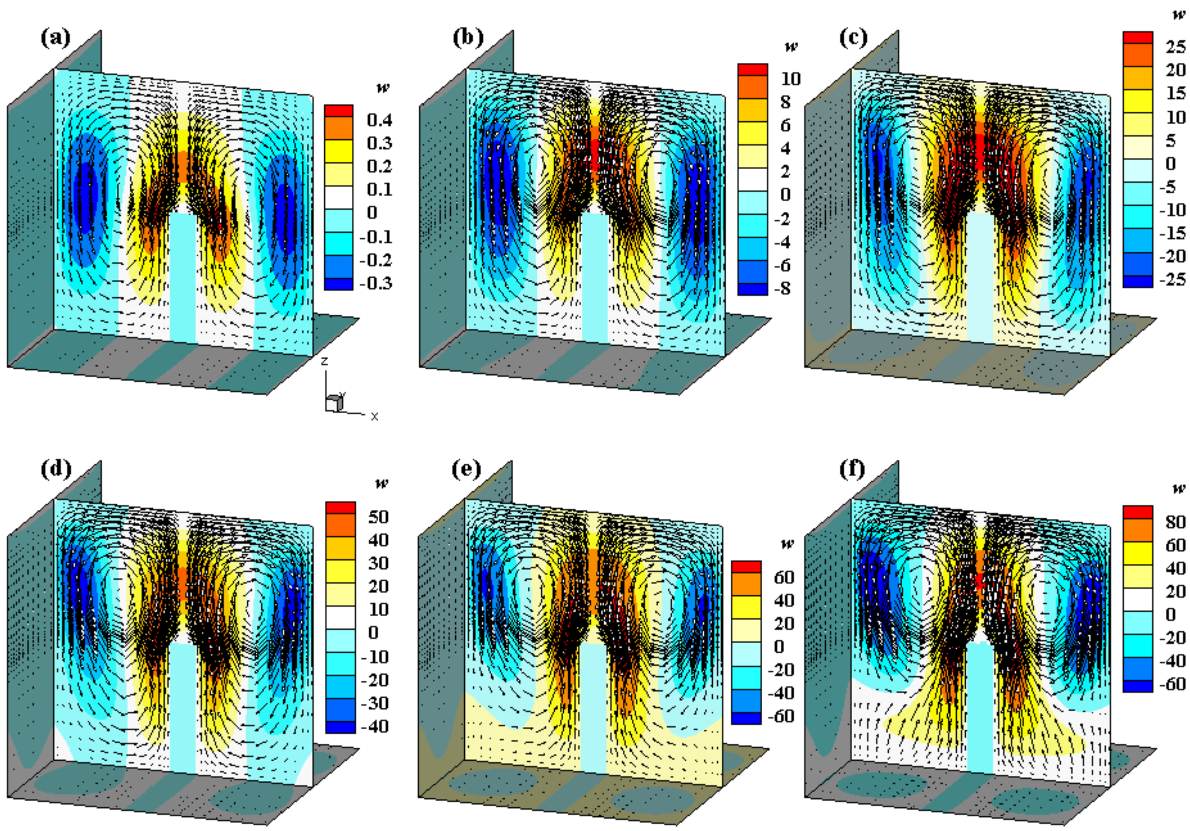

Fig. 2. Vector fields in the mid plane x-z for (a) : Ra=10^3, (b) : Ra=10^4, (c) : Ra=2.5×10^4 , (d) : Ra=5×10^4 , (e) : Ra=7.5×10^4 and (f) : Ra=10^5.

It is observed that in the range $10^3 \le Ra \le 10^5$, the flow is symmetric to both mid-planes x=0.5 and y=0.5 and dominated by two rolling cells situated on the right and the left of the fin. When the Rayleigh number increases, the intensity of the rotating cells elevates and natural convection becomes dominating. It is also noted that high vertical velocities are situated near the top edges of the fin and close to the lateral walls parallel to the fin. The temperature distribution illustrated by Figure 3 shows that for $Ra=10^3$, the heat transfer is mostly conductive. For $Ra \geq 10^4$, the flow is dominated by natural convection mechanism and a large amount of heat is dragged from the lateral sides of the fin up to the cold wall due to the sweeping rolls situated there.

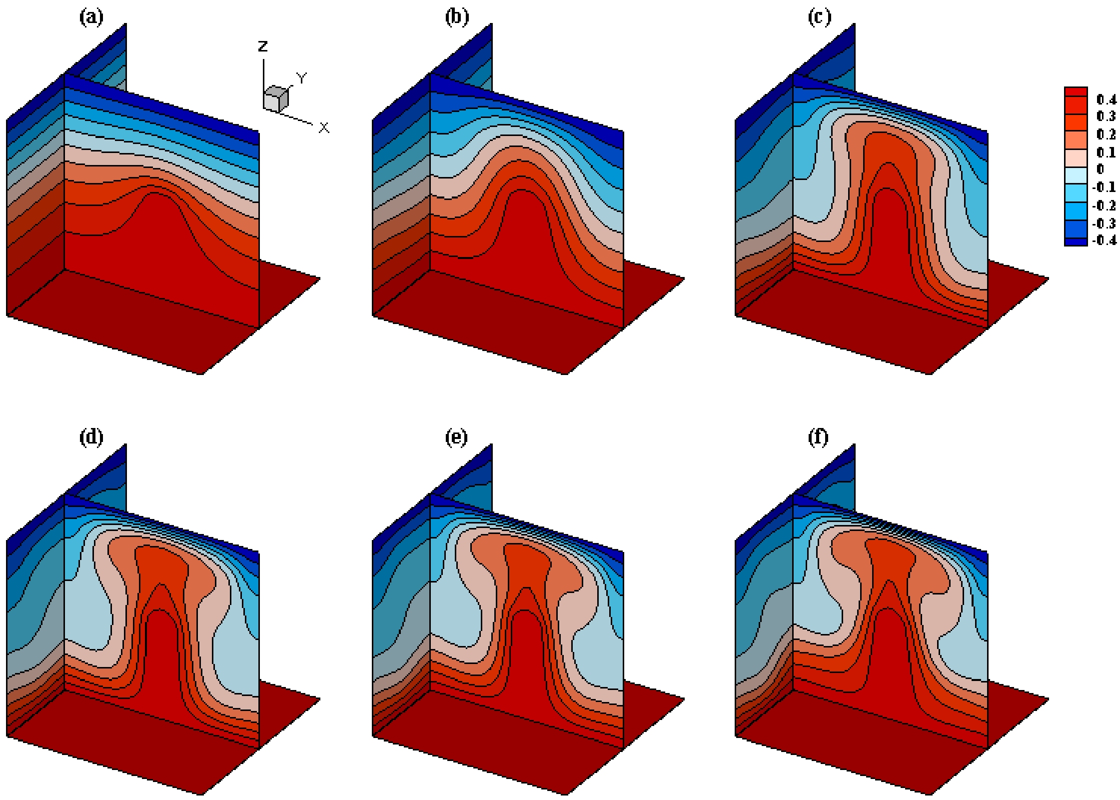

Fig. 3. Isotherm plots for (a) : $Ra=10^3$, (b) : $Ra=10^4$, (c) : $Ra=2.5\times10^4$, (d) : $Ra=5\times10^4$, (e) : $Ra=7.5\times10^4$ and (f) : $Ra=10^5$. ($k_r=7000$).

It is easier to study the heat transfer rate through the cold wall instead the hot wall. To accomplish this, we define the local and mean Nusselt numbers by their respective expressions:

$$Nu = -\left.\frac{\partial \theta}{\partial z}\right|_{z=1} \quad \text{and} \quad \langle Nu \rangle = \int_0^1 \int_0^1 Nu \, \mathrm{d}x\mathrm{d}y.$$

The local Nusselt number distribution is reported in Fig. 4. It is observed that maximum values of Nu are localized at the coordinates ($x=0.5;y=0.5$) for $Ra=10^3$, $Ra=10^4$, $Ra=2.5\times10^4$ and $Ra=10^5$. However, when $Ra=5\times10^4$ and $Ra=7.5\times10^4$, two locations where the local heat transfer is maximum are observed, i.e., ($x=0.5;y=0.256$) , ($x=0.5;y=0.744$) and ($x=0.5;y=0.209$), ($x=0.5;y=0.791$), respectively.

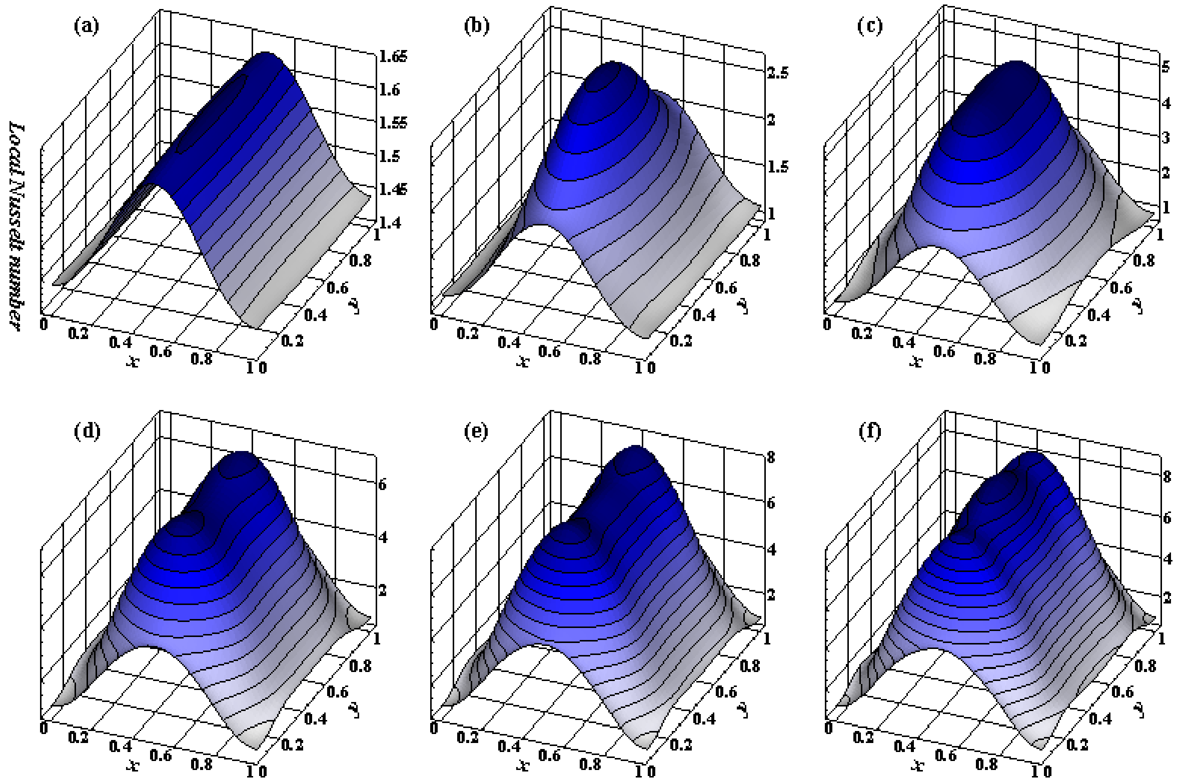

Fig. 4. Local Nusselt number for (a) : Ra=10^3, (b) : Ra=10^4, (c) : Ra=2.5×10^4 , (d) : Ra=5×10^4 , (e) : Ra=7.5×10^4 and (f) : Ra=10^5.

As far as the mean heat transfer rate is concerned, the averaged Nusselt number is plotted in Fig. 5 versus the Rayleigh number.

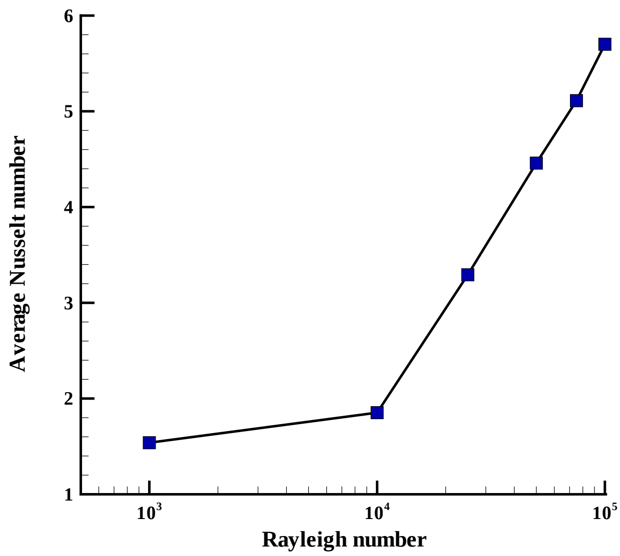

Figure 5. Averaged Nusselt number versus Rayleigh number

As expected, the heat transfer rate increases with the Rayleigh number. For Ra=10^3, $\langle Nu \rangle$ is close to unity, which means that the heat transfer mechanism is almost due to conduction. For $Ra \geq 10^4$, (in the interval where convection predominates) a correlation between $\langle Nu \rangle$ and Ra has been constructed. This correlation is given by the following expression:

$$\langle Nu \rangle = 3.72 \times \log Ra - 13.04$$

CONCLUSION

In this work, a numerical three dimensional investigation was carried on to study natural convection in a finned cubical enclosure of Rayleigh-Benard type. The numerical method is based on the finite volume technique and a multigrid acceleration. The results showed that the flow is specially dominated by two contra rotating cells for which the intensity increases with *Ra*. High values of kinetic energy are localized near the top of the fin and the lateral walls. For specific values of *Ra*, two locations indicative of maximum local heat transfer through the top wall are observed. For $10^3 \leq Ra \leq 10^5$, a correlation between the averaged Nusselt number and the Rayleigh number has been determined.

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