LAMINAR NATURAL CONVECTION IN SQUARE ENCLOSURE UNDER AN EXTERNALLY EVANESCENT MAGNETIC FIELD

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ABSTRACT This paper numerically investigates the effect of an externally evanescent magnetic field on flow patterns and heat transfer of fluid in a square cavity. The horizontal walls of the enclosure are assumed to be insulated while the vertical walls are kept isothermal. A control volume finite element method is used to solve the conservation equations at Prandtl number of 0.71. The effect of constant Hartman number on Nusselt number was studied. Validation tests with existing data demonstrate the aptitude of the present method to produce accurate results. The effects of magnetic field inclination angle from 0° to 90° on streamlines distributions are shown for different values of Hartman number. For Grashof number equal to 10^5, the values of relaxation time of the magnetic field are chosen, so that the Lorentz force acts only in the transient state of Nusselt number in natural convection. The Nusselt number was calculated for different values of the inverse relaxation time varying from 0 to +∞. The magnitude and the number of oscillations of the Nusselt number were observed. It has been found that no oscillation was seen at relaxation time equal to 20.

NOMENCLATURE

a	thermal diffusivity (m^2s^{-1})
B_0	initial magnetic field (T)
B	magnetic field ($B = B_0 e^{-\gamma t}$)
E	electrical force (N)
g	acceleration due to gravity (m^2s^{-2})
Gr	thermal Grashof number
Ha_0	initial Hartmann number ($Ha_0 = B_0 L\sqrt{\sigma/\mu}$)
Ha	Hartmann number ($Ha = Ha_0\, exp(-n\tau)$)
J	current density ($A\ m^{-2}$)
k	conductivity ($J\ m^{-1}s^{-1}K^{-1}$)
L	cavity length (m)
n	inverse relaxation time (s)
p	pressure (Nm^{-2})
P	dimensionless pressure
Pr	Prandtl number
q	heat flux
t	time (s)
T	temperature (K)
T_0	bulk temperature ($T_0 = (T_h + T_c)/2$)
ΔT	temperature difference ($\Delta T = T_h - T_c$)
w	velocity
W	dimensionless velocity

u,v velocity components in x,y directions (ms^{-1})
U,V dimensionless velocity, components in x,y directions
x, y Cartesian coordinates (m)
X, Y dimensionless Cartesian coordinates

Greek symbols
α magnetic field inclination angle (°)
β coefficient of thermal expansion (K^{-1})
θ dimensionless temperature
μ dynamic viscosity (kg $m^{-1}s^{-1}$)
σ electrical conductivity of the fluid
χ relaxation time (s^{-1})
ν cinematic viscosity ($m^{-2}s^{-1}$)
τ dimensionless time
Ω system volume

Subscripts
a dimensionless
c cold wall
h hot wall

1. INTRODUCTION

The heat transfer and the magnetic field coupled by the natural convection in a fluid in square or rectangular enclosures have been paid considerable attention in the recent years. Growing applications in material industry, geophysics, and engineering processes are using an external magnetic field as control drive since the force of Lorentz removes the currents of convection by reducing speeds.

The study of Garandet et al. [1] provides an analytical solution to the governing equations of magneto-hydro-dynamics to be used to model the effect of a transverse magnetic field on natural convection in a two-dimensional cavity. Rudraiah et al. [2] used modified Alternating Direction Implicit (ADI) fine difference scheme to solve the vorticity-stream function formulation of natural convection inside a rectangular enclosure in the presence of a magnetic field. The numerical results showed that the magnetic field suppresses the rate of convective heat. Al Chaar et al. [3] numerically studied two-dimensional natural convection in a shallow cavity heated from below in the presence of inclined magnetic field. The numerical results show that magnetic field reduces the heat transfer and inhibit the onset of the convection current. Furthermore, the convection modes inside the cavity are found to depend strongly upon both the strength and orientation of the magnetic field. A horizontal magnetic field is found to be the most effective in suppressing the convective flow. Al Najem et al. [4] determined the flow and temperature fields under a transverse magnetic field in a tilted square enclosure. Results show that the magnetic field on convection currents and heat transfer is more significant for low inclination angles and high Grashof numbers. Toshio et al. [5] studied and modelled the magnetizing force for convection of gas with a temperature gradient. This investigation shows that the convection of fluid can be explained by the repulsion of hot fluid from the hot wall to weak magnetic field and the attraction of cold fluid to the strong magnetic field. Hassain et al. [6] numerically investigated the effect of surface tension on unsteady natural convection flow of an electrically-conducting fluid in a rectangular enclosure under an externally imposed magnetic field with internal heat generation. The results show that a change of direction of the external magnetic field force from horizontal to vertical leads to decreases in the flow rates in both the primary and secondary cells and causes an increase in the effect of the thermo-capillary force. Cem Ece et al. [7] studied a laminar natural convection flow in the presence of a magnetic field in an inclined rectangular cavity. The results show that the flow characteristics, and therefore convection heat transfer, strongly depend on the strength and direction of magnetic field, the aspect ratio and the inclination of the enclosure. The local Nusselt number increases considerably with Grashof number since the circulation becomes stronger and the magnetic field significantly reduces the local Nusselt number by suppressing the convection currents. In many magnetohydrodynamics studied cases, the magnetic field is generally kept constant. In this paper, our paramount objective is to study the magnetic field effect in transient state of natural convection, which imposes the use of evanescent magnetic field. Our study is principally focalised on the effect of decreasing magnetic field on heat transfer and flow patterns, in transient state of natural convection, which has not been tackled yet. The effect of magnetic field in stationary state is investigated as a particular case.

2. MATHEMATICAL FORMULATION

Imposed magnetic field acting on Newtonian fluid enclosed in a confined differential heated square cavity is considered in this study (figure 1).

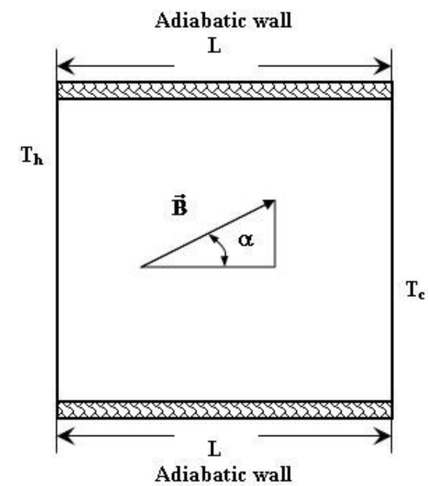

Figure1. Schematic confined diagram of the problem under consideration.

The fluid is modelled as a Boussinesq incompressible fluid, whose properties are described by its kinematic viscosity ν, thermal diffusivity a and thermal volumetric expansion coefficient β_T. The orientation of the magnetic field forms an angle α with horizontal axis.
The electric current vector J is defined by:

$$J = \sigma\left(E + W \times B\right) \tag{1}$$

The electric force per unit charge (E) is negligible compared to the magnetic force per unit charge $\left(W \times B\right)$ as it is given in Woods [8].

Under the above assumptions, the conservation equations for mass, momentum and energy in a two-nondimensional form are written as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$\frac{\partial U}{\partial t}+div\left(U.W-gradU\right)=-\frac{\partial P}{\partial x}+Ha^{2}\left(V.sin\,\alpha.cos\,\alpha-U.sin^{2}\,\alpha\right) \tag{3}$$

$$\frac{\partial V}{\partial t}+div\left(V.W-gradV\right)=-\frac{\partial P}{\partial y}+\left[Gr_{T}.\theta\right]+Ha^{2}\left(U.cos\,\alpha.sin\,\alpha-V.cos^{2}\,\alpha\right) \tag{4}$$

$$\frac{\partial \theta}{\partial t}+div\left(\theta.W-\frac{1}{Pr}grad\theta\right)=0 \tag{5}$$

Where the dimensionless variables are defined by:

$$X=\frac{x}{L};\ Y=\frac{y}{L};\ U=\frac{uL}{a};\ V=\frac{vL}{a};\ \theta=\frac{T-T_{0}}{T_{h}-T_{c}};\ P=\frac{pL^{2}}{\rho a^{2}};$$
$$Gr_{T}=\frac{g\beta_{T}\Delta TL^{3}}{\nu^{2}};\ \tau=\frac{at}{L^{2}};\ Ha^{2}=\frac{B^{2}L^{2}\sigma}{\mu} \tag{6}$$

The boundary conditions are:
$U=V=0$ for all walls ; $\theta=0.5$ on plane $X=0$ and $\theta=-0.5$ on plane $X=1$
$\frac{\partial \theta}{\partial Y}=0$ on planes $Y=1$ and $Y=0$

The initial conditions are:
At $\tau=0$; $U=V=P=0$ and $\theta=0.5-X$ for whole space.

The average Nusselt number is expressed as:

$$Nu=\int_{0}^{1}\left|\frac{\partial \theta}{\partial y}\right|dx \tag{7}$$

3. NUMERICAL PROCEDURE

A modified version of the Control Volume Finite-Element Method (CVFEM) of Saabas and Baliga [9] is adapted to the standard staggered grid in which pressure and velocity components are stored at different points. The SIMPLER algorithm was applied to resolve the pressure-velocity coupling in conjunction with an Alternating Direction Implicit (ADI) scheme for performing the time evolution. The dimensionless average Nusselt number for the entire cavity *Nu* is easily obtained by equation (7) from the known temperature and velocity fields at any instant τ given by solving equations (2) through to (5). The shape function describing the variation of the dependant variable ψ (= U, V, θ) is needed to calculate the flux across the control-volume faces. We have followed Saabas and Baliga [9] in assuming linear and exponential variations respectively when the dependant variable ψ is calculated in the convective term of the conservation equations. More details and discussions about CVFEM are available in such works as Prakash [10], Hookey [11], Elkaim et al. [12] and Saabas

Saabas and Baliga [9]. The numerical code used here is described and validated in details in Abbassi et al. [13].

4. RESULTS AND DISCUSSION

The present study is restricted to the non-reactive fluids with Prandlt number equal to *0.71*. The Grashof number is ranging between 10^3 and 10^5. Our objective is mainly focused on the effect of the magnetic field within the transient state of natural convection, and more precisely on the fluctuation of the Nusselt number for high Grashof numbers, without disturbing the stationary state. Then, we need an evanescent magnetic field, which can be expressed as:

$$B = B_0 e^{-\gamma t} \quad \left(\gamma \in IR^+\right) \tag{8}$$

Using equation (6), the Hartmann number is a decreasing function versus time, and can be written as:

$$Ha = Ha_0 e^{-\left(\frac{\gamma L^2}{a}\right)\tau} \tag{9}$$

The parameter γ was selected so that the inverse of the magnetic field relaxation time $n = \left(\frac{\gamma L^2}{a}\right)$ takes prime numbers. Therefore, the Hartman number can be written as:

$$Ha = Ha_0 e^{-n\tau} \tag{10}$$

It is important to note that for $n = 0$, the magnetic field takes constant value and can therefore disturb the stationary state. The values of parameter n are chosen, so that the magnetic field acts only in the transient state of natural convection for Grashof number equal to 10^5. This can be illustrated by figure 2 which reveals the variation of the Nusselt and Hartmann numbers versus time ($Gr = 10^5$). It can be concluded from figure 2 that for $n \geq 20$ the magnetic field affects only the transient regime. For validation purposes, let us begin with the case of $n = 0$ (constant magnetic field). In this case, the Hartmann number, the inclination angle of magnetic field, and the Grashof number are ranging from *0* to *100*, *0* to *90°*, and 10^3 to 10^5 respectively. Figure 3 shows the variation of the Nusselt number versus the Hartmann number at zero inclination angle of magnetic field. Good agreement with the work of Al-Najem et al. [4] is noticed in this figure since heat transfer decreases by increasing the Hartmann number. Therefore, the magnetic field seems to suppress convection and to retard fluid motion via the Lorentz force.

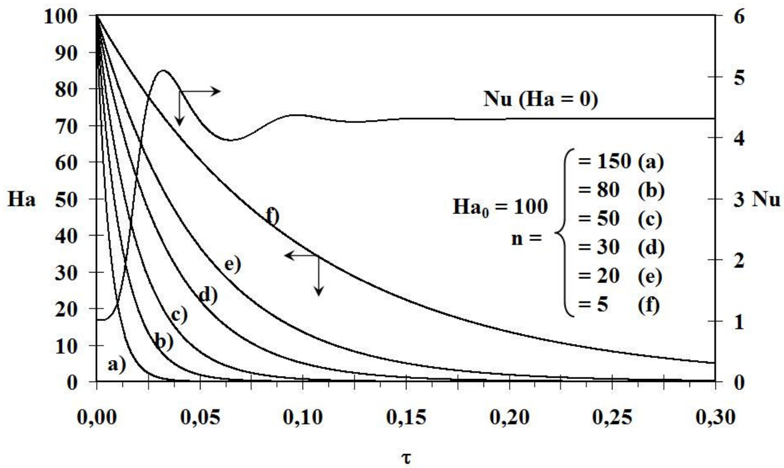

Figure 2. Hartman number for different constants n and Nusselt number distribution for $Ha = 0$, $Gr = 10^5$ and $\alpha = 0°$ all in function of dimensionless time.

Furthermore, for constant magnetic field *(n = 0)*, the influence of Hartmann number on the flow patterns was investigated. We display in figure 4 the evolution of streamlines with the Hartmann number and the inclination angle of magnetic field (for brevity the Grashof number is equal to 10^4). It can be pointed out from figure 4 that, for small value of Hartmann number *(Ha = 10)*, the flow patterns seem to be similar when the magnetic field angle inclination increases from *0°* to *90°*. However, for high value of Hartmann number, the flow patterns change noticeably, and one can observe an elongation of the eddy and a clockwise rotation of its axis from the vertical to the horizontal, when increasing the inclination angle of the magnetic field from *0°* to *90°*. As can be noticed in figure 4 also, for constant values of magnetic field inclination angle, the increase of Hartmann number tends to slow down the movement of the fluid. Again, the single eddy is found to be elongating (vertically for *0°* and horizontally for *90°*).

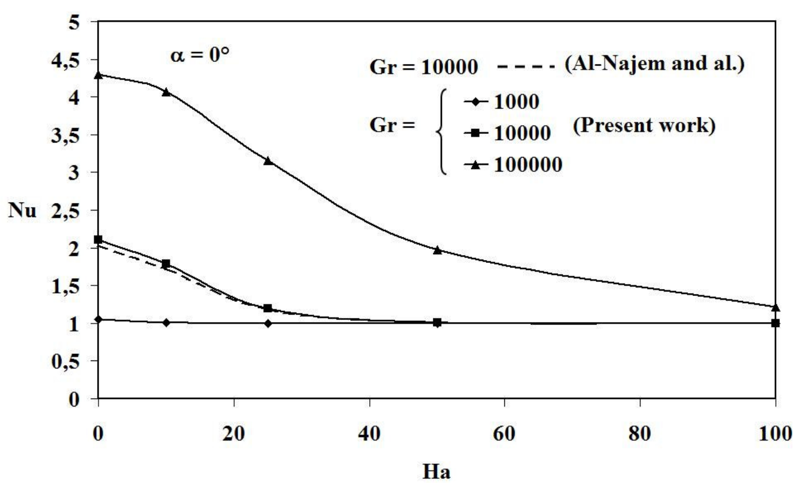

Figure 3. Nusselt number distribution as function of Ha ($0 \le Ha \le 100$) for $\alpha = 0°$.

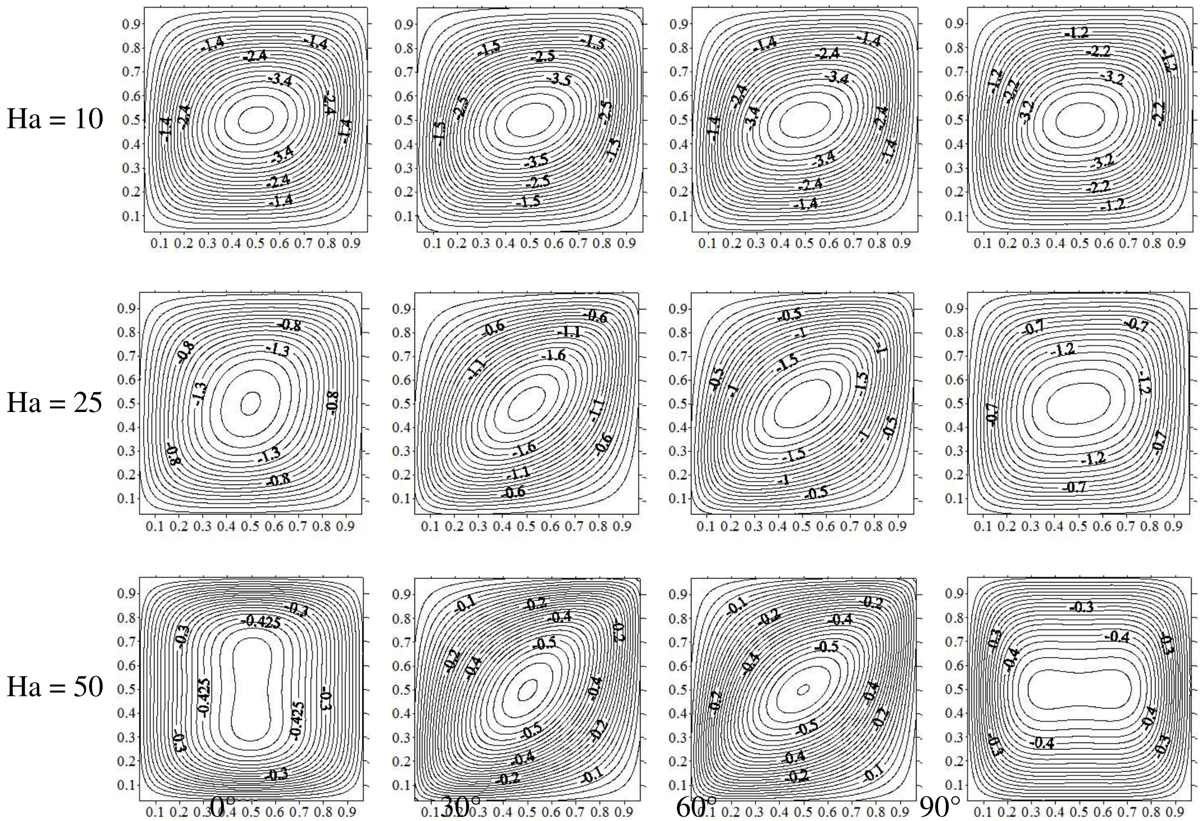

Figure 4. Stream lines for Ha *($10 \le Ha \le 50$)* and α *($0 \le \alpha \le 90°$)* at $Gr = 10^4$.

On the other hand, for high value of Hartmann number *(Ha = 50)*, the increase of magnetic field inclination angle tends to rotate the single eddy axis clockwise. In fact, the magnetic field applied in the XY-plane induces an electric current in the Z-direction, which generates a magnetic force (Lorentz force) in the XY-plane. The direction and the magnitude of the Lorentz force are at the origin of the eddy elongation and of its central axis rotation. Let us consider the two particular cases of the magnetic field inclination angle *0°* and *90°*. At magnetic field inclination angle *0°* (i.e. the magnetic field is parallel to the X-direction), the Lorentz force acts along the Y-direction and its magnitude is proportional to the Y-component of the velocity vector, then becomes maximal when the velocity vector is vertical, causing the elongation of the eddy. As the Hartmann number increases, the Lorentz force effect increases. Consequently, the elongation of the eddy becomes more significant and its axis approaches the vertical. Furthermore, the Lorentz force is opposed in direction to the Y-component of the velocity vector, which brings about the retardation effect. At magnetic field inclination angle *90°*, the magnetic field is vertical and the Lorentz force acts along the Y-direction and induces a horizontal elongation of the eddy. In this case, its magnitude is proportional to the X-component of the velocity vector. Also, its direction is opposed to the X-component of the velocity vector, which then reduces the strength of circulation inside the cavity. We investigate now the case of non constant magnetic fields $(n \neq 0)$. The evolution of the Nusselt number with the Hartmann number at the onset of natural convection are illustrated in figure 5 (the X-axis is given in logarithmic scale), for Grashof and initial Hartmann numbers equal to 10^5 and *100*, respectively.

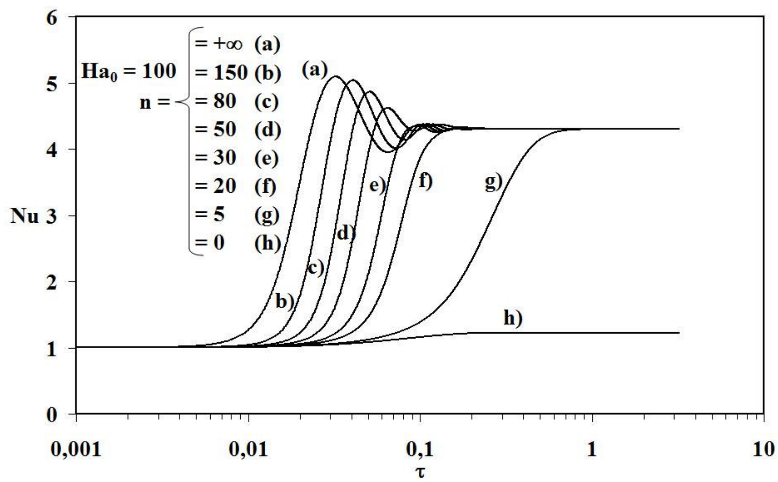

Figure 5. Nusselt number distribution as function of logarithmic coordinate dimensionless time for different values of n at $Ha_0 = 100$, $Gr = 10^5$ and $\alpha = 0°$.

Oscillations of the Nusselt number can be observed in figure 5. These fluctuations and magnitude of oscillations are important for lower values of relaxation time and diminish as the relaxation time increases (n decreases). At lower value of relaxation time, fluctuations of the Nusselt number indicate that the flow exhibits oscillatory behaviour. At the very beginning of the transient state, heat transfer is mainly due to heat conduction since the Nusselt number is equal to unity. The isotherms are nearly parallel to the active walls generating a horizontal temperature gradient. The streamlines are those of a single spiral with its center being at the center of the cavity. As time proceeds, the isotherms are gradually deformed by convection generating a vertical temperature gradient while the horizontal temperature gradient diminishes at the center of the cavity becoming locally negative, which causes an elongation of the central streamline and the development of a second spiral in the core. The transition from a single to a double configuration may induce generation of internal waves in the temperature fields which can be at the origin of the oscillations of the whole cavity. The current result is consistent with the findings of Ivey [14] and Schladow [15] who demonstrated the existence of transient oscillations in enclosures consisting of two isothermal vertical walls and two adiabatic horizontal walls. Ivey [14] claimed the transient oscillations occurred because of an internal hydraulic jump with an increase of the horizontal intrusion layers. These oscillations were found to disappear as the interior is set in motion and stratifing in temperature, increasing the thickness of the intrusion and flooding the hydraulic jump. Transient oscillations consisting of two distinct boundary layer instabilities and a whole cavity oscillation were observed by Schladow [15]. The whole cavity oscillations were attributed to the horizontal pressure gradient established by changes in the intrusion temperature field. The magnitude and the number of oscillations of the Nusselt number decrease by increasing the relaxation time (decreasing n). This is due to the fact that the transition from a single to a double configuration is made gradually, so inducing a decrease of the amplitude of the generated internal thermal waves in the cavity. At critical relaxation time ($\chi = 1/20$), the instability of the Nusselt number becomes insignificant and therefore the internal thermal waves disappear in the cavity. It is important to note that, for all used values of relaxation time (except for the case $n = 0$), the Nusselt number reaches the same constant value. This is due to the fact that the magnetic field disappears in the stationary state. Also, one can notice in figure 5 that the stage of the pure conduction regime increases with increasing relaxation time of the magnetic field. This can be explained in the following way: for constant dimensionless time, the Hartmann number increases with increasing relaxation time, inducing an increase of the Lorentz force. This retards the appearance of the convection regime and decreases its stage before reaching the steady state.

5. CONCLUSION

Imposed evanescent magnetic field acting on Newtonian Boussinesq incompressible fluid enclosed in heated square cavity was studied in the present paper. The values of relaxation time of the magnetic field are chosen so that the magnetic field acts only in the transient state of natural convection. The validation of numerical results was presented at constant magnetic field. The results about the evanescent magnetic field can be summarized as follows:

1. The fluctuations and magnitude of oscillations of the Nusselt number are important for lower values of relaxation time and diminish as the relaxation time increases (n decreases).
2. Fluctuations of the Nusselt number were observed at lower value of relaxation time, indicating that the flow exhibits oscillatory behavior.
3. The magnitude and the number of oscillations of the Nusselt number decrease when the relaxation time increases.
4. At critical relaxation time $(\chi = 1/20)$, the instability of the Nusselt number becomes insignificant and therefore the internal thermal and viscous waves disappear in the cavity.
5. The stage of the pure conduction regime increases with increasing relaxation time of the magnetic field.

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