ENTROPY GENERATION IN NATURAL CONVECTION UNDER AN EVANESCENT MAGNETIC FIELD

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ABSTRACT We numerically study the effect of an externally-evanescent magnetic field on total entropy generation in conducting and non-reactive fluid enclosed in a square cavity. The horizontal walls of the enclosure are assumed to be insulated while the vertical walls are kept isothermal. A control volume finite element method is used to solve the conservation equations at Prandtl number of 0.71. The values of relaxation time of the magnetic field are chosen, so that the Lorentz force acts only in the transient state of entropy generation in natural convection. The total entropy generation was calculated for fixed value of irreversibility distribution ratio, different relaxation time varying from 0 to 1/5 and Grashof number equal to 10^5.

NOMENCLATURE

a	thermal diffusivity (m^2s^{-1})
B_0	initial magnetic field (T)
B	magnetic field (T)
E	electrical force (N)
F_m	magnetic force (N)
g	acceleration due to gravity (m^2s^{-2})
Gr	thermal Grashof number
Ha_0	initial Hartman number ($Ha_0 = B_0 L\sqrt{\sigma/\mu}$)
Ha	Hartman number
J	current density (A m^{-2})
k	conductivity (J $m^{-1}s^{-1}K^{-1}$)
L	cavity length (m)
p	pressure (Nm^{-2})
P	dimensionless pressure
Pr	Prandtl number
q	heat flux
t	time (s)
T	temperature (K)
T_0	bulk temperature ($T_0 = \left(T_h + T_c\right)/2$)
ΔT	temperature difference ($\Delta T = T_h - T_c$)
w	velocity (m/s)
W	dimensionless velocity
u,v	velocity components in x,y directions (ms^{-1})
U,V	dimensionless velocity, components in x,y directions
x, y	Cartesian coordinates (m)
X, Y	dimensionless Cartesian coordinates

Greek symbols

α magnetic field inclination angle (°)
β coefficient of thermal expansion (K^{-1})
θ dimensionless temperature
μ dynamic viscosity (kg $m^{-1}s^{-1}$)
φ electrostatic potential
σ dimensionless entropy generation
σ_e electrical conductivity of the fluid
ξ relaxation time (s^{-1})
ν cinematic viscosity ($m^{-2}s^{-1}$)
τ dimensionless time
χ_i irreversibility distribution ratio (i =1, 2)
Ω system volume

Subscripts

a dimensionless
c cold wall
h hot wall

1. INTRODUCTION

In recent years, many studies are focalized on the characteristics of transfer phenomena and their optimum that can be attained by minimizing entropy generation. Different sources of irreversibility are responsible for entropy generation, for example heat and /or mass transfer, viscous dissipation effect, magnetic field effect…etc. The literature on the subject is well-reviewed by Bejan and his coworkers (see Bejan [1-3] and Bejan et al. [4]). Similarly, Arpaci and his coworkers also partook in the discussion (see Arpaci [5], Arpaci and Selamet [6], Arpaci and Selamet [7], Arpaci and Selamet [8], Arpaci [9], Arpaci and Esmaeeli [10] and Arpaci [11]). Further, for a mixed convective flow, recent works cited in Mahmoud et al. [12]. A detailed analysis of the entropy generation in a vertical non-porous channel with transverse hydromagnetic effect is given. Mahmud and Fraser [13] investigated first law and second law aspects of fluid flow and heat transfer inside a vertical porous channel with a transverse magnetic field. Haddad et al. [14] conducted a study on entropy generation through a single rectangular microchannel with constant heat flux boundary condition. They numerically modelled microchannel in order to find velocity and temperature distributions along the coolant flow and channel width directions.
Al-Odat et al. [15] investigated the magnetic field effect on local entropy generation in a steady two-dimensional laminar forced-convection flow past a horizontal plate.
In all these contributions, the magnetic field is generally kept constant. In the present magnetohydrodynamic investigation our primary objective is to study the magnetic field effect on the total entropy generation evolution in transient natural convection, without disturbing the steady one. This imposes the employment of evanescent magnetic field, which has not been tried yet.

2. MATHEMATICAL FORMULATION

In figure 1, we are considering a problem of a Newtonian fluid enclosed in heated square cavity, acted upon by an imposed evanescent magnetic field.

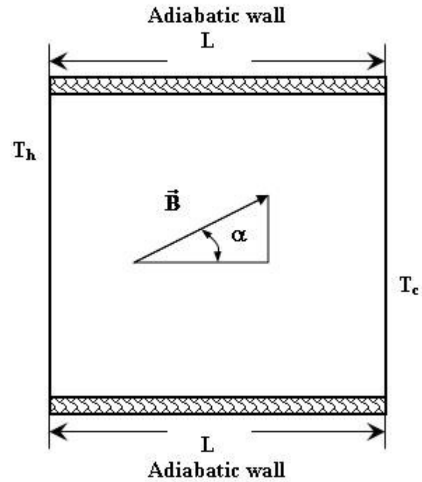

Fig. 1: Schematic confined diagram of the problem under consideration.

This fluid is modelling as a Boussinesq incompressible fluid, whose properties are described by its kinematics viscosity (ν), thermal diffusivity (a) and thermal volumetric expansion coefficient (β_T). The induced magnetic field is negligible in comparison to the imposed field. The electric field must be irrotational $\left(\nabla\times E=0\right)$ and can be written as: $E=-\nabla\varphi$, where φ is the electrostatic potential. The divergence of electric field leads to the following expression: $\nabla J=-\sigma_e\nabla^2\varphi+\sigma_e\nabla\cdot\left(W\times B\right)=0$, as it is given by Davidson [16]. We assume that there is no imposed electric field, therefore the electrostatic potential is equal to zero and the electric force is negligible compared to the magnetic force $\left(W\times B\right)$ as it is given in Woods [17]. Consequently, the electric field is reduced to $J=\sigma_e\left(W\times B\right)$ and the magnetic force can be written as:

$$F_m=J\times B=\sigma_e\left(V\times B\right)\times B \tag{1}$$

Under the above assumptions, the conservation equations for mass, momentum and energy in a two-non-dimensional form are as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$\frac{\partial U}{\partial t}+div\left(U.W-gradU\right)=-\frac{\partial P}{\partial x}+Ha^2\left(V.sin\,\alpha.cos\,\alpha-U.sin^2\,\alpha\right) \tag{3}$$

$$\frac{\partial V}{\partial t}+div\left(V.W-gradV\right)=-\frac{\partial P}{\partial y}+\left[Gr_T.\theta\right]+Ha^2\left(U.cos\,\alpha.sin\,\alpha-V.cos^2\,\alpha\right) \tag{4}$$

$$\frac{\partial\theta}{\partial t}+div\left(\theta.W-\frac{1}{Pr}grad\theta\right)=0 \tag{5}$$

Where the dimensionless variables are defined by:

$$X=\frac{x}{L};\ Y=\frac{y}{L};\ U=\frac{uL}{a};\ V=\frac{vL}{a};\ \theta=\frac{T-T_0}{T_h-T_c};\ P=\frac{pL^2}{\rho a^2};$$
$$Gr_T=\frac{g\beta_T \Delta T L^3}{\nu^2};\ \tau=\frac{at}{L^2};\ Ha^2=\frac{B^2L^2\sigma}{\mu} \tag{6}$$

The boundary conditions are:
$U=V=0$ for all walls ; $\theta=0.5$ on plane $X=0$ and $\theta=-0.5$ on plane $X=1$
$\frac{\partial\theta}{\partial Y}=0$ on planes $Y=1$ and $Y=0$

The initial conditions are:
At $\tau=0$; $U=V=P=0$ and $\theta=0.5-X$ for whole space.

3. ENTROPY GENERATION

The irreversibility involved in the system due to momentum and energy transport between the vertical walls of the enclosure sets the fluid in non-equilibrium state and induces a continuous entropy production in the system. In two-dimensional coordinates system and using the dimensionless variables listed in equation (6), the dimensional local entropy generation (Woods [17]) can be simplified and made dimensionless as:

$$\sigma_{l,a}=\sigma_{l,a,H}+\sigma_{l,a,F}+\sigma_{l,a,M} \tag{7}$$

with :

$$\sigma_{l,a,H}=\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2 \tag{8}$$

$$\sigma_{l,a,F}=\chi_1\left[2\left(\frac{\partial U}{\partial X}\right)^2+2\left(\frac{\partial V}{\partial Y}\right)^2+\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)^2\right] \tag{9}$$

$$\sigma_{l,a,M}=\chi_2\left[U.\sin\alpha-V.\cos\alpha\right]^2 \tag{10}$$

In the equation (7), the first term represents the entropy generation due to heat transfer, the second is due to viscous dissipation and the third term is due to magnetic field. Dimensionless terms denoted χ_i (i=1,2) are irreversibility distribution ratios given by:

$$\chi_1=\frac{\mu T_0}{k}\left(\frac{a}{L(\Delta T)}\right)^2 \text{ and } \chi_2=\chi_1 Ha^2 \tag{11}$$

The dimensionless total entropy generation over the system volume is defined by integrating Equation (7):

$$\sigma_t=\int_\Omega \sigma_{l,a}\,d\Omega \tag{12}$$

4. NUMERICAL PROCEDURE

A modified version of the Control Volume Finite-Element Method (CVFEM) of Saabas and Baliga [18] is adapted to the standard staggered grid in which pressure and velocity components are stored at different points. The SIMPLER algorithm was applied to resolve the pressure-velocity coupling in conjunction with an Alternating Direction Implicit (ADI) scheme for performing the time evolution. From the known temperature and velocity fields at any instant τ given by solving equations (2 – 5), the local entropy generation $\sigma_{l,a}$ is evaluated at any node of the domain by Eq. (7). The dimensionless total entropy generation for the entire cavity σ_t is easily obtained by equation (12). The shape function describing the variation of the dependant variable ψ (= U, V, θ) is needed to calculate the flux across the control-volume faces. We have followed Saabas and Baliga [18] in assuming linear and exponential variations respectively when the dependant variable ψ is calculated in the diffusive and in the convective terms of the conservation equations. More details and discussions about CVFEM are available in the works of Prakash [19], Hookey [20], Elkaim et al. [21], Saabas and Baliga [18]...etc. The numerical code used here is described and validated in detail in Abbassi et al. [22] and Magherbi et al. [23].

5. RESULTS AND DISCUSSION

All industrial systems induce entropy generation and, thus, destroy system available work and reduce its efficiency. We are also convinced that a part of total entropy generation is at the origin of gradual deterioration of mechanical system (principally at the starting operation). Numerous investigations were carried out with an aim of optimizing entropy generation. In several real cases, we believe that optimizing entropy generation in the stationary state can often change the desired product characteristics, which is generally disagreeable. This leads us to optimise the entropy generation only in transient state by applying an evanescent magnetic field (Equation (13)) on the fluid enclosed in a square cavity. This can make profit of available work, minimize the potential damage at the beginning of the operation system and additionally maintain the characteristics of the end product.

$$B = B_0 e^{-\gamma t} \quad \left(\gamma \in IR^+\right) \tag{13}$$

Using Equation (6), the Hartman number is a decreasing function versus time and can be written as:

$$Ha = Ha_0 e^{-\left(\frac{\gamma L^2}{a}\right)\tau} \tag{14}$$

The parameter γ was selected so the inverse of the magnetic field relaxation time $1/\xi = \left(\gamma L^2/a\right)$ takes prime numbers. Therefore, the Hartman number can be written as:

$$Ha = Ha_0 e^{-\frac{\tau}{\xi}} \tag{15}$$

In this study, the Prandlt and the Grashof numbers are equal to 0.71 and 10^5, respectively. The magnetic field inclination angle is equal to zero. The initial Hartman number (Ha_0) is equal to 100. The irreversibility distribution ratio χ_1 is kept constant and equal to 10^{-3}. It is important to note that, for higher values of relaxation time ξ, the magnetic field takes practically constant value and can therefore disturb the stationary state. The values of parameter ξ are chosen so that the magnetic field acts only in the transient state of natural convection. This is illustrated in figure 2, which shows the variation of the entropy generation and Hartman number versus time. It can be concluded from figure 2, that the dimensionless total entropy generation increases suddenly and takes maximum value ($\sigma_{t,Max}$) at the very beginning of natural convection. Thereafter, it decreases with oscillatory behaviour to finally reach a constant value in the steady state. Fluctuations of the total entropy generation at high Grashof numbers indicate that the flow exhibits oscillatory behavior which depends on the boundary conditions. This result is consistent with the finding of Ivey [24], Schladow [25] and magherbi et al. [23]. These latter explained, from a thermodynamic viewpoint that, for higher Grashof numbers, the steady state is relatively far from the equilibrium state. Therefore, a rotation around the steady state is possible and the system is in the case of a spiral approach towards this state corresponding to an oscillation of the total entropy generation. Consequently, the system evolves in the non-linear branch of irreversible phenomena, since the Prigogine's theorem of minimum entropy production is unproven.

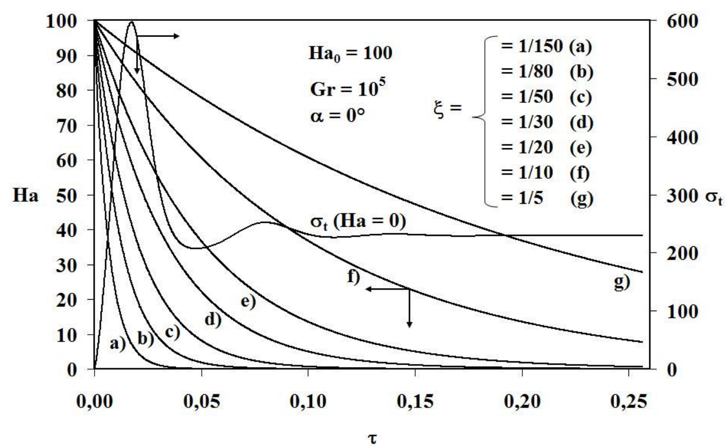

Fig. 2: Hartman number for different relaxation times ξ and entropy production distribution for $\chi_1 = 10^{-3}$ $Ha = 0$, $Gr = 10^5$ and $\alpha = 0°$ all in function of dimensionless time.

The effect of Hartmann number on entropy generation fluctuations for relatively high values of Grashof number is illustrated in figure 3 (the X-axis is given in logarithmic scale). From this figure, it is clear that the value reached by entropy generation at the steady state of natural convection is the same for all considered relaxation time. This result is very logical since the magnetic field takes zero value at the steady state. Therefore, at steady state and for relatively high values of Grashof number (10^5), the flow structure is the same as that obtained in the absence of magnetic field, which consists in a double spiral configuration and viscous boundary layers in close proximity to the active walls. Similar observations regarding the evolution of the dimensionless total entropy generation to the ones given in figure 2 can be conducted from figure 3, except the influence of the relaxation time on the magnitude and the number of oscillations of total entropy generation. As seen in figure 3, total entropy generation begins to increase at the very start of the transient state due to the initial conditions of the fluid temperature and velocity. Insignificant influence of the relaxation time (except for zero magnetic field) on the dimensionless total entropy generation was seen for dimensionless time $\tau \leq 0.002$ since curves of entropy generation practically coincide. This is because the entropy generation at the very beginning of transient state is mainly due to the relatively important Hartman number and, therefore, magnetic irreversibly. As time proceeds, a bifurcation of the total entropy generation depending on the values of the relaxation time was observed. From the bifurcation point, as shown in figure 3, the maximum of total entropy generation decreases with the increase of the relaxation time. Also, the number and the amplitude of oscillations of the total entropy generation decrease. This can be explained by the fact that the internal thermal and viscous waves, resulting from the transition from one to double configuration in the whole cavity, are gradually suppressed by the Lorentz force when the relaxation time increases.

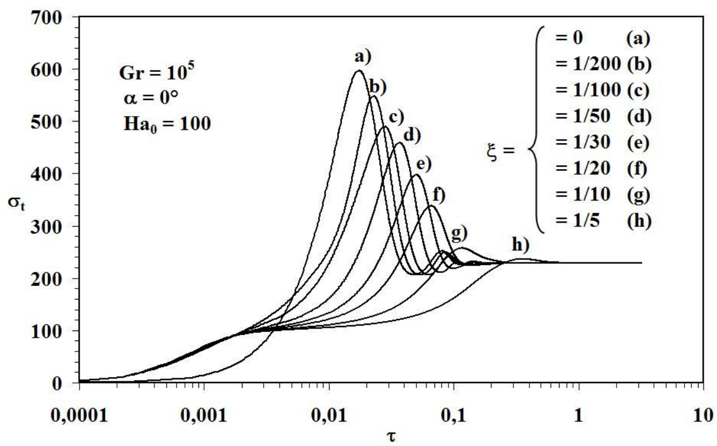

Fig. 3: Entropy production distribution as function of logarithmic coordinate dimensionless time for different values of ξ at $\chi_1 = 10^{-3}$, $Ha_0 = 100$, $Gr = 10^5$ and $\alpha = 0°$.

Notice that no oscillation can be observed at critical relaxation time equal or superior to *1/10*. Accordingly, the total entropy generation tends towards asymptotically the steady state. This is due to the adequate value of the Hartmann number throughout the transient regime and precisely when the transition to the double configuration occurs. So, the Lorentz force is always able to eliminate internal thermal and viscous waves. From a thermodynamic viewpoint, for relaxation time $\xi \geq 1/10$, the asymptotic behavior of the total entropy generation in function of time shows that the system returns directly towards the steady state.

6. CONCLUSION

Imposed evanescent magnetic field acting on Newtonian Boussinesq incompressible fluid enclosed in heated square cavity was investigated in this study. The values of relaxation time of the magnetic field are chosen so that the magnetic field acts only in the transient state of natural convection. The total entropy generation was calculated only in transient state by applying the considered magnetic field with the aim of keeping constant the desired product characteristics. Results show that the dimensionless total entropy generation increases abruptly and takes maximum value at the very beginning of natural convection. Thereafter, it decreases with oscillatory behavior and finally reaches a constant value in the steady state. The effect of Hartmann number on entropy generation fluctuations for relatively high values of Grashof number was also studied. Results show that the influence of the relaxation time (except for zero magnetic field) on irreversibility is insignificant for dimensionless time $\tau \leq 0.002$, corresponding to a bifurcation point of total entropy generation. Results also show that, from the bifurcation point, the number and the amplitude of oscillations of the total entropy generation decrease when the relaxation time is increased. It was remarked that no oscillation can be observed at critical relaxation time equal to *1/10* corresponding to an asymptotic behaviour of the total entropy generation towards the steady state. The Prigogine's theorem of minimum entropy production is therefore verified.

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