PREDICTION OF EQUILIBRIUM STATES OF KINEMATIC AND THERMAL FIELDS IN HOMOGENEOUS TURBULENCE SUBMITTED TO THE ROTATION

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ABSTRACT In this work, effects of rotation on the evolution of kinematic and thermal fields in homogeneous sheared turbulence are investigated using second order closure modeling. The Launder-Reece-Rodi models, the Speziale-Sarkar-Gatski model and the Shih-Lumley models are retained for pressure-strain correlation and pressure-temperature correlation. Whereas classic models are retained for time evolution equations of kinematic and thermal dissipation rates. The fourth order Runge-Kutta method is used to resolve three non linear differential systems obtained after modeling. The numerical integration is carried out separately for several values of the dimensionless rotation number R equal to 0, 0.25 and 0.5. The obtained results are compared to the recent results of Direct Numerical Simulations of G.Brethouwer. The results have confirmed the asymptotic equilibrium behaviors of kinematic and thermal dimensionless parameters. Furthermore they have shown that rotation affects not only kinematic field but also thermal field. The coupling between the Speziale-Sarkar-Gatski model and the Launder-Reece-Rodi model is of a big contribution on the prediction of kinematic and thermal fields.

NOMENCLATURE

b_{ij}	components of the anisotropic tensor
K	turbulent kinetic energy, $m^2 s^{-2}$
p	fluctuation of pressure, Nm^{-2}
R	non dimensional rotation number $R = \Omega / S$
S	mean shear, s^{-1}
G	mean temperature gradient, $°Cm^{-1}$
	$G = \dfrac{d\,\overline{T}}{d\,x_2}$
t	time, s
$\overline{T}$	mean temperature, °C

u_i fluctuation of the velocity, ms^{-1}

$u_i' = (\overline{u_i^2})^{1/2}\ ms^{-1}$

$\overline{u_i u_j}$ Components of Reynolds tensor, $m^2 s^{-2}$

$U_{i,j}$ mean velocity gradient, s^{-1}

$\overline{u_i \theta}$ turbulent thermal flux,°Cms^{-1}

δ_{ij} Kronecker Symbol

Ω Rotation rate, s^{-1}

θ fluctuation of the temperature, °C

$\overline{\theta^2}$ variance of the fluctuation of the température, °C^2

$\theta' = (\overline{\theta^2})^{1/2}$, °C

$\tau = St$ non dimensional time

INTRODUCION

The study of the turbulence using second order models remains one of the most important approaches used to describe and to analyse turbulent flows especially homogeneous sheared turbulent flows submitted to the rotation effects. Rotation takes important place in various physics and industrial applications like oceanic current, atmospheric boundary, mechanical and thermal systems like turbomachinary. In this work, the Launder-Reece-Rodi (LRR) models and the Shih-Lumley (SL) models are retained for the kinematic and thermal fields of an homogeneous sheared turbulence submitted to rotation. However the Speziale-Sarkar-Gatski (SSG) model is retained only for the kinematic field, since it has not yet been extended to the thermal field. It is coupled with the Launder-Reece-Rodi for the thermal field.
We start in the following paragraph by presenting the general equations which describe the considered flow. Numerical integration of non dimensional equations describing the flow, presentation and discussion of results make the object of paragraph three. Finally, general conclusion is presented at the last paragraph.

GENERAL EQUATIONS

Double Correlations Equations We recall in this paragraph time evolution equations of the components $\overline{u_i u_j}$ of Reynolds stress tensor, the component $\overline{\theta u_i}$ of turbulent thermal flux, turbulent kinetic energy K and variance of the fluctuation of temperature $\overline{\theta^2}$. These equations are written as part of a conventional second order modeling on the following forms:

$$\frac{d}{dt}\overline{u_i u_j} = P_{ij} + \phi_{ij} - \varepsilon_{ij} \tag{1}$$

$$\frac{d}{dt}\overline{\theta u_i} = P_{i\theta} + \phi_{i\theta} - \varepsilon_{i\theta} \tag{2}$$

$$\frac{d}{dt}K = P - \varepsilon \tag{3}$$

$$\frac{d}{dt}\overline{\theta^2} = P_\theta - \varepsilon_\theta \tag{4}$$

In these equations, terms denoted by P are terms of production due to mean kinematic and thermal gradients:

$$P_{ij} = -\overline{u_j u_k}\frac{\partial \overline{U_i}}{\partial x_k} - \overline{u_i u_k}\frac{\partial \overline{U_j}}{\partial x_k} - 2(\varepsilon_{imk}\overline{u_j u_k} + \varepsilon_{jmk}\overline{u_i u_k})\Omega_m \tag{5}$$

$$P_{i\theta} = -\overline{u_i u_k}\frac{\partial \overline{T}}{\partial x_k} - (S\,\delta_{i1}\delta_{k2} + 2\,\varepsilon_{imk}\Omega_m)\overline{\theta u_k} \tag{6}$$

$$P = -\overline{u_i u_k}\frac{\partial \overline{U_i}}{\partial x_k} - 2\varepsilon_{imk}\Omega_m\overline{u_i u_k} \tag{7}$$

$$P_\theta = -2\overline{u_i\theta}\frac{\partial \overline{T}}{\partial x_i} \tag{8}$$

The terms denoted by ε are terms of dissipation due to molecular effects:

$$\varepsilon_{ij} = 2\nu\,\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} \tag{9}$$

$$\varepsilon_{i\theta} = (\alpha + \nu)\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial \theta}{\partial x_k}} \tag{10}$$

$$\varepsilon = \nu\,\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k}} \tag{11}$$

$$\varepsilon_\theta = 2\alpha \overline{\frac{\partial \theta}{\partial x_k}\frac{\partial \theta}{\partial x_k}} \tag{12}$$

Finally, ϕ_{ij} and $\phi_{i\theta}$ are respectively terms of pressure-strain correlation and pressure-temperature correlation:

$$\phi_{ij} = -(\overline{u_j \frac{\partial p}{\partial x_i} + u_i \frac{\partial p}{\partial x_j}}) \tag{13}$$

$$\phi_{i\theta} = -\overline{\theta \frac{\partial p}{\partial x_i}} \tag{14}$$

Non-Dimensional Equations It is essential here to note, that second order modeling concerns terms noted ϕ and the time evolution equations of dissipation rates ε and ε_θ. The non linear systems are casted in non dimensional forms when dimensionless parameters are introduced. These parameters are the components b_{ij} of the anisotropic tensor b, the dimensionless shear number $\frac{\varepsilon}{KS}$ for the kinematic field and the parameters $\rho_1 = \frac{\overline{u_1\theta}}{\overline{u_2\theta}}$, $\rho_2 = \frac{\overline{u_1\theta}}{u_1'\theta'}$, $\rho_3 = \frac{\theta'/G}{q'/S}$ for the thermal field. In the goal of a simplified form of this paper only the system modeled by the Launder-Reece-Rodi is presented here.

$$\frac{db_{11}}{d\tau} = -2(1-2R)b_{12} + C_2(1-\frac{2}{3}C_2)b_{12} - \frac{1}{3}\frac{\varepsilon}{kS} + 2b_{12}(b_{11}+\frac{1}{3}) + \frac{\varepsilon}{kS}(b_{11}+\frac{1}{3}) \tag{15}$$

$$\frac{db_{22}}{d\tau} = -4Rb_{12} + C_2(1-\frac{2}{3}C_2)b_{12} - \frac{1}{3}\frac{\varepsilon}{kS} + 2b_{12}(b_{22}+\frac{1}{3}) + \frac{\varepsilon}{kS}(b_{22}+\frac{1}{3}) \tag{16}$$

$$\frac{db_{12}}{d\tau} = -(1-2R)(b_{22}+\frac{1}{3}) - 2R(b_{11}+\frac{1}{3}) + C_2 b_{12} + \frac{2}{3}C_2^2 b_{12} + 2b_{12}^2 + \frac{\varepsilon}{kS}b_{12} \tag{17}$$

$$\frac{d}{d\tau}(\frac{\varepsilon}{kS}) = ((1-C_{\varepsilon 2})\frac{\varepsilon}{kS} + 2(1-C_{\varepsilon 1})b_{12})\frac{\varepsilon}{kS} \tag{18}$$

$$\frac{d\rho_1}{d\tau} = -\frac{1}{\rho_3}\frac{b_{12}}{(b_{11}+\frac{1}{3})^{\frac{1}{2}}} - C_1\frac{\varepsilon}{kS}\rho_1 - \frac{\rho_1}{\rho_2}(1.8-3R) + \rho_1(1-2R)\frac{b_{12}}{b_{11}+\frac{1}{3}} + \frac{C_1}{4}\rho_1\frac{\varepsilon}{kS}\frac{b_{11}}{b_{11}+\frac{1}{3}}$$

$$+\frac{C_2}{4}(1-\frac{2}{3}C_2)\rho_1\frac{b_{12}}{b_{11}+\frac{1}{3}} + \frac{1}{6}\rho_1\frac{\varepsilon}{kS}\frac{1}{b_{11}+\frac{1}{3}} + \frac{\rho_1^2}{\rho_2\rho_3} + \frac{1}{2}\rho_1\frac{\varepsilon}{kS}r_c \tag{19}$$

$$\frac{d\rho_2}{d\tau} = -\frac{\rho_2}{\rho_1\rho_3}\frac{b_{12}}{(b_{11}+\frac{1}{3})^{\frac{1}{2}}} + 3R - 1.8 + (R-0.2)\rho_2^2 - \frac{\rho_2^2}{\rho_1\rho_3}\frac{b_{22}+\frac{1}{3}}{(b_{11}+\frac{1}{3})^{\frac{1}{2}}} \tag{20}$$

$$\frac{d\rho_3}{d\tau} = -\frac{\rho_1}{\rho_2}(b_{11}+\frac{1}{3})^{\frac{1}{2}} - \frac{1}{2}\rho_3\frac{\varepsilon}{kS}r_c + \rho_3 b_{12} + \frac{1}{2}\rho_3\frac{\varepsilon}{kS} \tag{21}$$

NUMERICAL INTEGRATION AND RESULTS

The non linear differential equations are submitted to the initial conditions of the Direct Numerical Simulations (DNS) of G.Brethouwer. A fourth order Runge-Kutta method is used to integrate the mentioned systems and is advanced to long time evolution for the different values 0, 0.25 and 0.5 of the rotation number R. The model 1 is attributed to the LRR model, the coupling (SSG+LRR) will be called model 2 and the SL model is called model 3. The numerical results have shown a tendency of dimensionless parameters to an asymptotic equilibrium states. On the following tables equilibrium values of these parameters are presented.

Table 1

Equilibrium values of dimensionless parameters predicted by the model 1

	b_{11}	b_{22}	b_{12}	ε / KS	ρ_1	ρ_2	ρ_3
R=0	0.192	-0.095	-0.185	0.182	-0.996	0.393	1.56
R=0.25	-0.059	0.155	-0.207	0.206	0.297	0.217	1.85
R=0.5	…	…	…	0	…	…	3.8

Table 2

Equilibrium values of dimensionless parameters predicted by the model 2

	b_{11}	b_{22}	b_{12}	ε / KS	ρ_1	ρ_2	ρ_3
R=0	0.220	-0.147	-0.164	0.167	0.298	1.12	2.44
R=0.25	0.537	-0.201	-0.365	0.387	0.621	-1.46	2.27
R=0.5	1	-0.230	-0.689	0.730	∞	-1.8	∞

Table 3

Equilibrium values of dimensionless parameters predicted by the model 3

	b_{11}	b_{22}	b_{12}	ε / KS	ρ_1	ρ_2	ρ_3
R=0	0.114	-0.116	-0.121	0.144	-0.865	0.616	0.834
R=0.25	-0.051	0.048	-0.159	0.189	-0.233	0.303	0.937
R=0.5	-0.123	0.120	0	0	…	…	…

Equilibrium values have been obtained for the most dimensionless parameters. An exception is observed for the value R=0.5 in table 1. A significant improvement is observed on both tables 2 and 3 for the models 2 and 3. These findings have been explained that the models 2 and 3 are submitted to the strict conditions of the realizability, whereas the model 1 have been submitted only to the known relations of kinematic constraints. The components b_{ij} of the anisotropic tensor is affected by the rotation, especially, a significant growth is obtained for the principal component b_{12} of anisotropy when the dimensionless rotation numbers grows from 0. to 0.5. Furthermore, the thermal field is affected by rotation. The turbulent flux rate is concerned by the most important influence of rotation.

Now the evolutions of kinematic and thermal non dimensional rates are discussed. Then the behavior of the kinetic turbulent energy is studied.
Figure 1 shows the time evolutions of the component b_{11} of anisotropic tensor in the case R=0.25. On this figure, a perfect agreement between the model 1 and the model 2 is observed in one hand, in the other hand, the agreement of each of three models results with DNS results of G.Brethouwer is clearly noted.
Figure 2 shows the time evolutions of the component b_{22} of anisotropic tensor which are compared with results of DNS of G.Brethouwer. On this figure we see clearly that the model 1 indicates the same qualitative changes as DNS results of G.Brethouwer moving from R=0 to R=0.25.
Figure 3 shows the time evolutions of the thermal dimensionless parameter $\rho_3 = \dfrac{\theta' / G}{q' / S}$. We observe in this figure a qualitative agreement between the results given by both the model 1 and model 3 with the

DNS results of G.Brethouwer. Furthermore, the most important agreement is observed on the evolution predicted by the model 3.
Figure 4 shows the evolutions of the dimensionless shear number ε / KS with non dimensional time St. these evolutions exhibit clearly the eventual equilibrium asymptotic behavior in various cases of dimensionless rotating number which shows growth moving from R=0 to R=0.5. While it leads to a decay moving from R=-0.75 to R=0.
The evolutions of the kinetic energy K in non dimensional time St according to the model 2 is shown in figure 5. All six cases have an initial phase of turbulent kinetic energy due to the isotropic initial conditions. The non rotating case with R=0 develops shear production of turbulence at about St=5 and eventually shows an exponential growth of the turbulent kinetic energy with different cases of rotation, the time evolutions of the turbulent kinetic energy exhibits the same behavior. All configurations show the strongest eventual growth of The kinetic energy K.

CONCLUSION

In this work we have studied the kinematic and thermal fields of homogeneous sheared turbulence submitted to the rotation using three second order closure models. The numerical integration conducted by the fourth order Runge-kutta method and advanced to long time evolution has shown that the predictions of retained second order models confirm the existence of asymptotic equilibrium states. The principal results of this work are the following:
The model 1 has predicted equilibrium states but the values are three to four times greater than the DNS values of G.Brethouwer. Whereas the model 3 has shown better predictions than the model 1 compared to DNS values. The model 2 has shown the best prediction of equilibrium states. Furthermore, this model has been of great contribution and has improved the asymptotic values predicted by model 1. As for the kinetic energy, it shows the exponential growth with the non dimensional time. Finally, we have noted that the rotation has the influence not only on the kinematic field of homogeneous turbulence but also on the thermal field.

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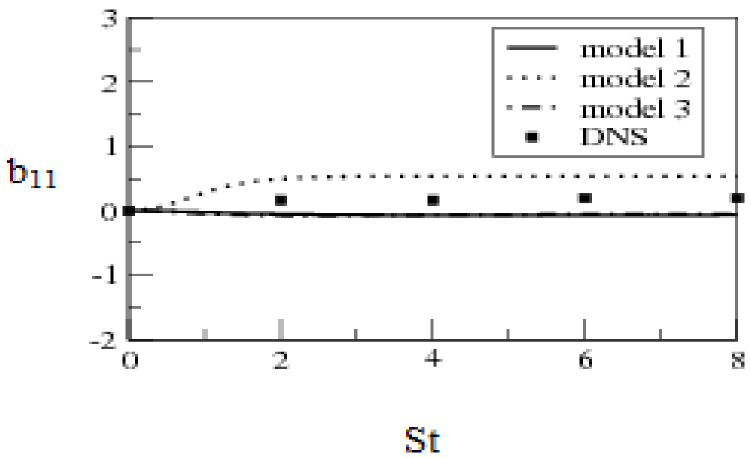

Figure 1: Time evolutions of the component b_{11} of the anisotropic tensor for the case R=0.25

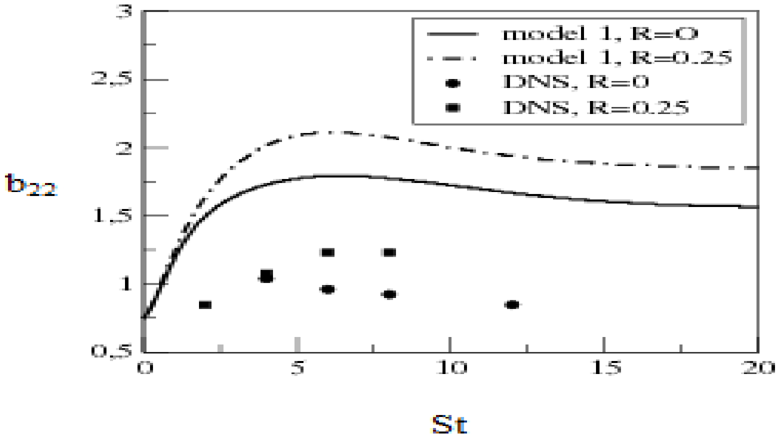

Figure 2: Time evolutions of the component b_{22} of the anisotropic tensor for the cases R=0 and R=0.25

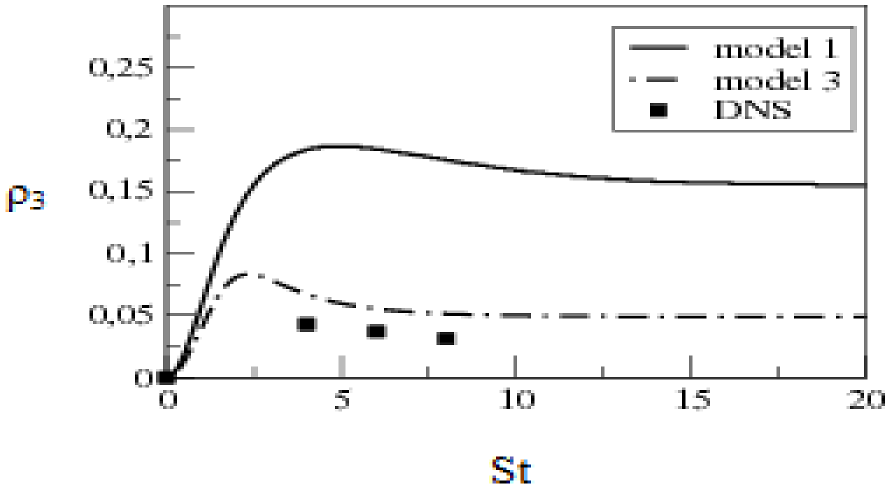

Figure 3 : Time evolutions of the dimensionless parameter $\rho_3 = \dfrac{\theta' / G}{q' / S}$

for the case R=0 .25

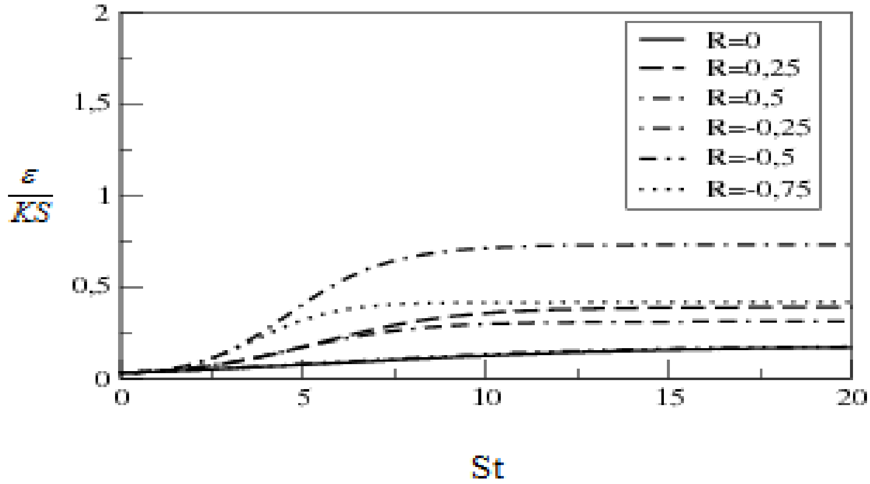

Figure 4: Time evolutions of the dimensionless rate $\frac{\varepsilon}{KS}$ for various cases of non dimensional rotation number

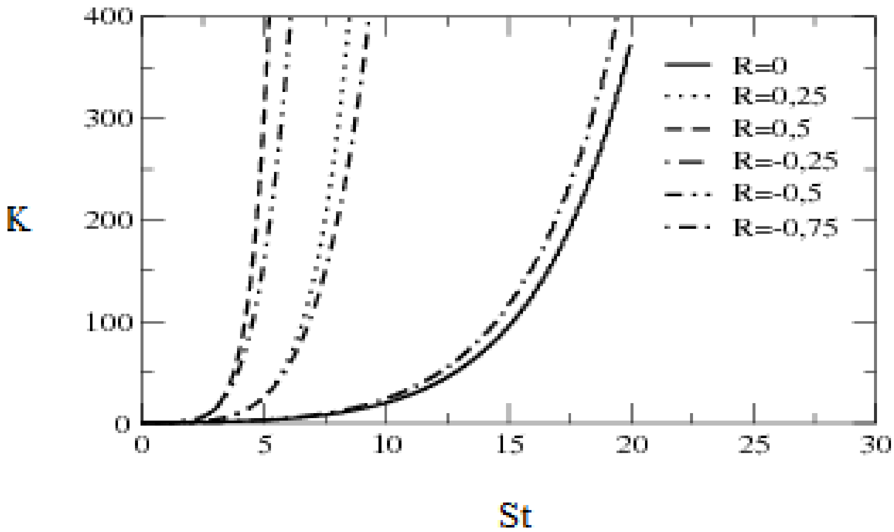

Figure 5: Time evolutions of the turbulent kinetic energy for various Cases of on dimensional rotation number