Can Inhomogeneities Accelerates the Universe?

M. R. Martins and M. F. A. da Silva

Departamento de Física Teórica,

Universidade do Estado do Rio de Janeiro,

Rua São Francisco Xavier 524*, Maracanã,* 20550 − 013

Rio de Janeiro, RJ, Brazil

Anzhong Wang

Departamento de Física Teórica,

Universidade do Estado do Rio de Janeiro,

Rua São Francisco Xavier 524*, Maracanã,* 20550 − 013

Rio de Janeiro, RJ, Brazil and Baylor University - USA

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Abstract

Recently Moffat Showed that the Szafron inhomogeneous cosmological solution can lead to an accelerating inflationary period without a large initial vacuum energy and with $\Lambda = 0$ in the early universe near the Planck time, combined with an accelerating universe at late times, in the non-linear regime with galaxy structure and voids, without need to introduce negative pressures or a cosmological constant. Here we present an inhomogeneous and anisotropic cosmological model, in 2+1 gravity, which satisfy all the energy conditions, although it generates an accelerated universe. We work in a 2+1 gravity scenario in order to simplify the equations system and to allow us to find analytical and simple solutions. Our propose is basically to improve our understanding on the role of inhomogeneities on the acceleration of the universe.

1 Solutions of the field equations

The general metric is given by

$$ds^2 = e^{2\phi}dt^2 - e^{2\psi}dr^2 - r^2S^2d\theta^2, \tag{1}$$

The non-null components of the Einstein‘s tensor are

$$G_{tt} = \frac{e^{-2\psi}}{rS}\{e^{2\phi}\psi_{,r}rS_{,r} + e^{2\phi}\psi_{,r}S - e^{2\phi}rS_{,rr} - 2e^{2\phi}S_{,r} + e^{2\psi}\psi_{,t}rS_{,t}\}, \tag{2}$$

$$G_{tr} = \frac{1}{rS}\{\phi_{,r} rS_{,t} + \psi_{,t} rS_{,r} + \psi_{,t} S - rS_{,tr} - S_{,t}\}, \tag{3}$$

$$G_{rr} = \frac{e^{-2\phi}}{rs}\{e^{2\phi}\phi_{,r} rS_{,r} + e^{2\phi}\phi_{,r} S + e^{2\psi}\phi_{,t} rS_{,t} - e^{2\psi} rS_{,tt}\}, \tag{4}$$

$$\begin{aligned} G_{\theta\theta} &= r^2 S^2\{e^{-2\psi}\phi_{,r}^2 - e^{-2\psi}\phi_{,r}\psi_{,r} + e^{-2\psi}\phi_{,rr} + \\ &\quad (e^{-2\phi}\phi_{,t} - e^{2\phi}\psi_{,t})\psi_{,t} - e^{-2\phi}\phi_{,tt}\}. \end{aligned} \tag{5}$$

In order to study these solutions with self-similarity of the second kind we introduce two new adimensional variables, χ and τ, through of the relations

$$\chi = \ln(z) = \ln\left[\frac{r}{(t)^{\frac{1}{\alpha}}}\right], \tag{6}$$

$$\tau = -\ln(t), \tag{7}$$

where α is an adimensional constant.

Using these transformations, and imposing that the metric must to depend only on the variable χ, we find

$$G_{tt} = -\frac{1}{\alpha^2 r^2 S e^{2\psi}}\{\alpha^2 e^{2\phi}[S_{,\chi\chi} + S_{,\chi} - \psi_{,\chi}(S_{,\chi} + S)] - \frac{r^2}{t^2}\psi_{,\chi} S_{,\chi} e^{2\psi}\}, \tag{8}$$

$$G_{tr} = \frac{1}{\alpha t r S}\{S_{,\chi\chi} - \psi_{,\chi}(S_{,\chi} + S) - S_{,\chi}(\phi_{,\chi} - 1)\}, \tag{9}$$

$$G_{rr} = \frac{1}{\alpha^2 r^2 S e^{2\phi}}\{\alpha^2 e^{2\phi}[\phi_{,\chi}(S_{,\chi}+S)] - \frac{r^2 e^{2\psi}}{t^2}(S_{,\chi\chi} - S_{,\chi}\phi_{,\chi}) - \frac{\alpha r^2}{t^2} e^{2\psi} S_{,\chi}\}, \tag{10}$$

$$\begin{aligned} G_{\theta\theta} &= \frac{S^2}{\alpha^2}\{\alpha^2 e^{-2\psi}[\phi_{,\chi\chi} + \phi_{,\chi}(\phi_{,\chi} - \psi_{,\chi} - 1)] - \\ &\quad \frac{r^2 e^{-2\phi}}{t^2}[\psi_{,\chi\chi} - \psi_{,\chi}(\phi_{,\chi} - \psi_{,\chi} - \alpha)]\}. \end{aligned} \tag{11}$$

The momentum-energy tensor is given by

$$T_{\mu\nu} = \rho u_\mu u_\nu + p_r r_\mu r_\nu + p_\theta \theta_\mu \theta_\nu, \tag{12}$$

where ρ is the energy density, p_r and p_θ are the radial and the tangencial pressures, respectively, with u_μ, r_μ and θ_μ being given by

$$u_\mu = l e^{\phi(\chi)}\delta^t_\mu, \qquad r_\mu = l e^{\psi(\chi)}\delta^r_\mu, \qquad \theta_\mu = l r S(\chi)\delta^\theta_\mu, \tag{13}$$

where we adopted the comoving frame.

Substituting (??) - (??) and (??) into the field equations given bellow

$$G_{\mu\nu} = KT_{\mu\nu}, \tag{14}$$

we have

$$\rho = -\frac{1}{l^2K}\{\frac{1}{r^2}e^{-2\psi}[y_{,\chi} + (y+1)(y-\psi_{,\chi})] - \frac{1}{\alpha^2t^2}e^{-2\phi}\psi_{,\chi}y\}, \tag{15}$$

$$y_{,\chi} = y\phi_{,\chi} + (y+1)(\psi_{,\chi} - y), \tag{16}$$

$$p_r = \frac{1}{l^2K}\{\frac{1}{r^2}e^{-2\psi}\phi_{,\chi}(y+1) - \frac{1}{\alpha^2t^2}[y_\chi + y(y-\phi_\chi+\alpha)]\}, \tag{17}$$

$$p_\theta = \frac{1}{l^2K}\{\frac{e^{-2\psi}}{r^2}[\phi_{,\chi\chi}+\phi_{,\chi}(\phi_{,\chi}-\psi_{,\chi}-1)] - \frac{e^{-2\phi}}{\alpha^2t^2}[\psi_{,\chi\chi}-\psi_{,\chi}(\phi_\chi-\psi_{,\chi}-\alpha)]\}. \tag{18}$$

Note that in the above equations we make

$$y(\chi) = \frac{S_{,\chi}}{S}. \tag{19}$$

Then we have 4 equations and 6 functions to determine, that are ϕ,ψ, S, ρ, p_r and p_θ. Thus, we consider an additional equation, which is $p_r = 0$, in order to solve the system. Doing the coordinates transformation (??), the state equation furnishes 2 more equations, that are

$$e^{-2\psi}\phi_\chi(y+1) = 0, \tag{20}$$

$$e^{-2\phi}[y_{,\chi} + y(y-\phi_\chi+\alpha)] = 0, \tag{21}$$

For equation (??) there are 2 possible solutions

$$(i) y + 1 = 0 \tag{22}$$

$$(ii) \phi_{,\chi} = 0 \tag{23}$$

From (??) we have

$$y = -1. \tag{24}$$

Putting (??) into (??) we find

$$S(\chi) = S_0e^{-\chi}, \tag{25}$$

Substituting (??) into (??) we obtain a differential equation for $\phi(\chi)$

$$\phi_{,\chi} = \alpha - 1, \tag{26}$$

which admites the solution

$$\phi(\chi) = (\alpha - 1)\chi + \phi_0. \tag{27}$$

On the other hand, substituting (??) into (??) we obtain

$$\phi_{,\chi} = 0, \tag{28}$$

implying, from (??), that $\alpha = 1$, which characterizes a first kind self-similar solution, with the function ψ undetermined. As here we are interested only on second kind self-similarity ($\alpha \neq 0$ and $\alpha \neq 1$), we discard this solution.

From solution (??), we have

$$\phi(\chi) = \phi_0. \tag{29}$$

Now, substituting (??) into (??) we get a non linear differential equation for $y(\chi)$ given by

$$y_{,\chi} = y(y + \alpha), \tag{30}$$

which has the solution

$$y(\chi) = \frac{\alpha}{[c\alpha e^{\alpha\chi} - 1]}. \tag{31}$$

This result allows two different solutions. If we choose $c \neq 0$, we have collapse forming black hole or naked singularity [?]. For $c = 0$, y reduces to $y = -\alpha$.

2 Caso 1: $c = 0$

In this case we need to return to the equation (??) in order to reobtain the expression for the function S. Then, from equation (??) we have

$$y(\chi) = -\alpha, \tag{32}$$

which implies

$$S(\chi) = S_0 e^{-\alpha\chi} \tag{33}$$

and

$$y(\chi) = -\alpha\chi + \Psi_0. \tag{34}$$

The energy density is given by

$$\rho = \frac{1}{Kl^2 e^{2\phi_0} t^2}. \tag{35}$$

The metric for this solution can be written as

$$ds^2 = e^{2\phi_0} dt^2 - \frac{r^{2\alpha}}{t^2}[\Psi_0^2 dr^2 + r^2 d\theta^2]. \tag{36}$$

The condition $R_g > 0$ imposes that $S_0 > 0$. From the expression to ρ we can identify a singularity on $t = 0$, which corresponds to a $R_g = \infty$, as a result of an accelerated expansion, as can see in the following

$$\dot{R}_g = \frac{S_0 r^{(1-\alpha)}}{t^2} > 0, \tag{37}$$

$$\ddot{R}_g = \frac{2S_0 r^{(1-\alpha)}}{(t)^3} > 0. \tag{38}$$

We have an accelerated cosmological model, an accelerated cosmological model which begins in a initial singularity ($t = 0$), with all the energy conditions satisfied.

3 Conclusion

We obtain all the possible solutions of the Einstein's equations for a anisotropic and spherically symmetric, self-similar of the second kind fluid in a space-time (2+1)-dimensional. We need to introduce a state equation, $p_r = 0$, in order to solve the problem and showed that the unique solution in this case represents a dust fluid. The global properties are studied, always considered the energy conditions. They reveled that for $\alpha < 1$, $S_0 < 0$ and $\alpha c > 0$ the collapse results in a black hole formation. It is interesting to investigate if the solution can represent a critical solution for the gravitational collapse. Besides, choosing $c = 0$ and $S_0 > 0$, we have an accelerated cosmological model which begins in a initial singularity ($t = 0$), with all the energy conditions satisfied.

4 References

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