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La leptogénèse savoureuse

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Leptogenesis

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Abstract

Leptogenesis is a class of scenarios where the baryon asymmetry of the Universe is produced from a lepton asymmetry generated in the decays of a heavy sterile neutrino. We explain the motivation for leptogenesis. We review the basic mechanism, and describe subclasses of models. We then focus on recent developments in the understanding of leptogenesis: finite temperature effects, spectator processes, and in particular the significance of flavour physics.

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1 The Baryon Asymmetry of the Universe

1.1 Observations

Observations indicate that the number of baryons (protons and neutrons) in the Universe is unequal to the number of antibaryons (antiprotons and antineutrons). To the best of our understanding, all the structures that we see in the Universe – stars, galaxies, and clusters – consist of matter (baryons and electrons) and there is no antimatter (antibaryons and positrons) in appreciable quantities. Since various considerations suggest that the Universe has started from a state with equal numbers of baryons and antibaryons, the observed baryon asymmetry must have been generated dynamically, a scenario that is known by the name of *baryogenesis*.

One may wonder why we think that the baryon asymmetry has been dynamically generated, rather than being an initial condition. There are at least two reasons for that. First, if a baryon asymmetry had been an initial condition, it would have been a highly fine-tuned one. For every 6,000,000 antiquarks, there should have been 6,000,001 quarks. Such a fine-tuned condition seems very implausible. Second, and perhaps more important, we have excellent reasons, based on observed features of the cosmic microwave background radiation, to think that inflation took place during the history of the Universe. Any primordial baryon asymmetry would have been exponentially diluted away by the required amount of inflation.

The baryon asymmetry of the Universe poses a puzzle in particle physics. The Standard Model (SM) of particle interactions contains all the ingredients that are necessary to dynamically generate such an asymmetry in an initially baryon-symmetric Universe. Yet, it fails to explain an asymmetry as large as the one observed (see *e.g.* [1]). New physics is called for. The new physics must [2], first, distinguish matter from antimatter in a more pronounced way than do the weak interactions of the SM. Second, it should depart from thermal equilibrium during the history of the Universe.

The baryon asymmetry of the Universe can be defined in two equivalent ways:

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \Big|_0 = (6.21 \pm 0.16) \times 10^{-10}, \quad (1.1)$$

$$Y_{\Delta B} \equiv \frac{n_B - n_{\bar{B}}}{s} \Big|_0 = (8.75 \pm 0.23) \times 10^{-11} \quad (1.2)$$

where n_B , $n_{\bar{B}}$, n_γ and s are the number densities of, respectively, baryons, antibaryons, photons and entropy, a subscript 0 implies “at present time”, and the numerical value is from combined microwave background and large scale structure data (WMAP 5 year data, Baryon Acoustic Oscillations and Type Ia Supernovae) [3]. It is convenient to calculate $Y_{\Delta B}$, the baryon asymmetry relative to the entropy density s , because $s = g_*(2\pi^2/45)T^3$ is conserved during the expansion of the Universe (g_* is the number of degrees of freedom in the plasma, and T is the temperature; see the discussion and definitions below eqn (13.6)). The two definitions (1.1) and (1.2) are related through $Y_{\Delta B} = (n_{\gamma 0}/s_0)\eta \simeq \eta/7.04$. A third, related way to express the asymmetry is in terms of the baryonic fraction of the critical energy density,

$$\Omega_B \equiv \rho_B/\rho_{\text{crit}}. \quad (1.3)$$

The relation to η is given by

$$\eta = 2.74 \times 10^{-8} \Omega_B h^2, \quad (1.4)$$

where $h \equiv H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1} = 0.701 \pm 0.013$ [4] is the present Hubble parameter.

The value of baryon asymmetry of the Universe is inferred from observations in two independent ways. The first way is via big bang nucleosynthesis [5–7]. This chapter in cosmology predicts the abundances of the light elements, D, ^3He , ^4He , and ^7Li . These predictions depend on essentially a single parameter which is η . The abundances of D and ^3He are very sensitive to η . The reason is that they are crucial in the synthesis of ^4He via the two body reactions $\text{D}(p, \gamma)^3\text{He}$ and $^3\text{He}(\text{D}, p)^4\text{He}$. The rate of these reactions is proportional to the number densities of the incoming nuclei which, in turn, depend on η : $n(\text{D}) \propto \eta$ and $n(^3\text{He}) \propto \eta^2$. Thus, the larger η , the later these ^4He -producing processes will stop (that is, become slower than the expansion rate of the Universe), and consequently the smaller the freeze-out abundances

of D and of ^3He will be. The abundance of ^4He is less sensitive to η . Larger values of η mean that the relative number of photons, and in particular photons with energy higher than the binding energies of the relevant nuclei, is smaller, and thus the abundances of D, ^3He and ^3H build up earlier. Consequently, ^4He synthesis starts earlier, with a larger neutron-to-proton ratio, which results in a higher ^4He abundance. The dependence of the ^7Li -abundance on η is more complicated, as two production processes with opposite η -dependencies play a role.

The primordial abundances of the four light elements can be inferred from various observations. The fact that there is a range of η which is consistent with all four abundances gives a strong support to the standard hot big bang cosmology. This range is given (at 95% CL) by [5]

$$4.7 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}, \quad 0.017 \leq \Omega_B h^2 \leq 0.024. \quad (1.5)$$

The second way to determine Ω_B is from measurements of the cosmic microwave background (CMB) anisotropies (for a pedagogical review, see [8, 9]). The CMB spectrum corresponds to an excellent approximation to a blackbody radiation with a nearly constant temperature T . The basic observable is the temperature fluctuation $\Theta(\hat{\mathbf{n}}) = \Delta T/T$ ($\hat{\mathbf{n}}$ denotes the direction in the sky). The analysis is simplest in Fourier space, where we denote the wavenumber by k .

The crucial time for the CMB is that of recombination, when the temperature dropped low enough that protons and electrons could form neutral hydrogen. This happened at redshift $z_{\text{rec}} \sim 1000$. Before this time, the cosmological plasma can be described, to a good approximation, as a photon-baryon fluid. The main features of the CMB follow from the basic equations of fluid mechanics applied to perfect photon-baryon fluid, neglecting dynamical effects of gravity and the baryons:

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0, \quad c_s \equiv \sqrt{\dot{p}/\dot{\rho}} = \sqrt{1/3}, \quad (1.6)$$

where c_s is the sound speed in the dynamically baryon-free fluid (ρ and p are the photon energy density and pressure). These features in the anisotropy spectrum are: the existence of peaks and troughs, the spacing between adjacent peaks, and the location of the first peak. The modifications due to gravity and baryons can be understood from adding their effects to eqn (1.6),

$$\ddot{\Theta} + c_s^2 k^2 \Theta = F, \quad c_s = \frac{1}{\sqrt{3(1 + 3\rho_B/4\rho_\gamma)}}, \quad (1.7)$$

where F is the forcing term due to gravity, and ρ_B is the baryon energy density. The physical effect of the baryons is to provide extra gravity which enhances the compression into potential wells. The consequence is enhancement of the compressional phases which translates into enhancement of the odd peaks in the spectrum. Thus, a measurement of the odd/even peak disparity constrains the baryon energy density. A fit to the most recent observations (WMAP5 data only, assuming a Λ CDM model with a scale-free power spectrum for the primordial density fluctuations) gives (at 2σ) [4]

$$0.02149 \leq \Omega_B h^2 \leq 0.02397. \quad (1.8)$$

The impressive consistency between the nucleosynthesis (1.5) and CMB (1.8) constraints on the baryon density of the Universe is another triumph of the hot big-bang cosmology. A consistent theory of baryogenesis should explain $n_B \approx 10^{-10} s$.

1.2 Ingredients and Mechanisms

The three ingredients required to dynamically generate a baryon asymmetry were given by Sakharov in Ref. [2]:

1. Baryon number violation: This condition is required in order to evolve from an initial state with $Y_{\Delta B} = 0$ to a state with $Y_{\Delta B} \neq 0$.

2. C and CP violation: If either C or CP were conserved, then processes involving baryons would proceed at precisely the same rate as the C- or CP-conjugate processes involving antibaryons, with the overall effects that no baryon asymmetry is generated.
3. Out of equilibrium dynamics: In chemical equilibrium, there are no asymmetries in quantum numbers that are not conserved (such as B , by the first condition).

These ingredients are all present in the Standard Model. However, no SM mechanism generating a large enough baryon asymmetry has been found.

1. Baryon number is violated in the Standard Model, and the resulting baryon number violating processes are fast in the early Universe [10]. The violation is due to the triangle anomaly, and leads to processes that involve nine left-handed quarks (three of each generation) and three left-handed leptons (one from each generation). A selection rule is obeyed,⁴

$$\Delta B = \Delta L = \pm 3. \quad (1.9)$$

At zero temperature, the amplitude of the baryon number violating processes is proportional to $e^{-8\pi^2/g^2}$ [11], which is too small to have any observable effect. At high temperatures, however, these transitions become unsuppressed [10].

2. The weak interactions of the SM violate C maximally and violate CP via the Kobayashi-Maskawa mechanism [12]. This CP violation can be parameterized by the Jarlskog invariant [13] which, when appropriately normalized, is of order 10^{-20} . Since there are practically no kinematic enhancement factors in the thermal bath [14–16], it is impossible to generate $Y_{\Delta B} \sim 10^{-10}$ with such a small amount of CP violation. Consequently, baryogenesis implies that there must exist new sources of CP violation, beyond the Kobayashi-Maskawa phase of the Standard Model.
3. Within the Standard Model, departure from thermal equilibrium occurs at the electroweak phase transition [17, 18]. Here, the non-equilibrium condition is provided by the interactions of particles with the bubble wall, as it sweeps through the plasma. The experimental lower bound on the Higgs mass implies, however, that this transition is not strongly first order, as required for successful baryogenesis. Thus, a different kind of departure from thermal equilibrium is required from new physics or, alternatively, a modification to the electroweak phase transition.

This shows that baryogenesis requires new physics that extends the SM in at least two ways: It must introduce new sources of CP violation and it must either provide a departure from thermal equilibrium in addition to the electroweak phase transition (EWPT) or modify the EWPT itself. Some possible new physics mechanisms for baryogenesis are the following:

GUT baryogenesis [19–28] generates the baryon asymmetry in the out-of-equilibrium decays of heavy bosons in Grand Unified Theories (GUTs). The Boltzmann Equations (BE) for the bosons and the baryon asymmetry are studied, for instance, in [29–31]. BE are also required for leptogenesis, and in this review, we will follow closely the analysis of Kolb and Wolfram [31]. The GUT baryogenesis scenario has difficulties with the non-observation of proton decay, which puts a lower bound on the mass of the decaying boson, and therefore on the reheat temperature after inflation⁵. Simple inflation models do not give such a high reheat temperature, which in addition, might regenerate unwanted relics. Furthermore, in the simplest GUTs, $B + L$ is violated but $B - L$ is not. Consequently, the $B + L$ violating SM sphalerons, which are in equilibrium at $T \lesssim 10^{12}$ GeV, would destroy this asymmetry⁶.

Leptogenesis was invented by Fukugita and Yanagida in Ref. [35]. New particles – singlet neutrinos – are introduced via the seesaw mechanism [36–40]. Their Yukawa couplings provide the necessary new

⁴This selection rule implies that the sphaleron processes do not mediate proton decay.

⁵Bosons with $m > T_{reheat}$ could nonetheless be produced during “preheating” [32, 33], which is a stage between the end of inflation and the filling of the Universe with a thermal bath.

⁶A solution to this problem [34], in the seesaw model, could be to have fact L violation (due to the righthanded neutrinos) at $T > 10^{12}$ GeV. This would destroy the L component of the asymmetry, and the remaining B component would survive the sphalerons.

source of CP violation. The rate of these Yukawa interactions can be slow enough (that is slower than H , the expansion rate of the Universe, at the time that the asymmetry is generated) that departure from thermal equilibrium occurs. Lepton number violation comes from the Majorana masses of these new particles, and the Standard Model sphaleron processes still play a crucial role in partially converting the lepton asymmetry into a baryon asymmetry [41]. This review focuses on the simplest and theoretically best motivated realization of leptogenesis: thermal leptogenesis with hierarchical singlet neutrinos.

Electroweak baryogenesis [17, 42, 43] is the name for a class of models where the departure from thermal equilibrium is provided by the electroweak phase transition. In principle, the SM belongs to this class, but the phase transition is not strongly first order [44] and the CP violation is too small [14, 15]. Thus, viable models of electroweak baryogenesis need a modification of the scalar potential such that the nature of the EWPT changes, and new sources of CP violation. One example [45] is the 2HDM (two Higgs doublet model), where the Higgs potential has more parameters and, unlike the SM potential, violates CP. Another interesting example is the MSSM (minimal supersymmetric SM), where a light stop modifies the Higgs potential in the required way [46, 47] and where there are new, flavour-diagonal, CP violating phases. Electroweak baryogenesis and, in particular, MSSM baryogenesis, might soon be subject to experimental tests at the CERN LHC.

The Affleck-Dine mechanism [48, 49]. The asymmetry arises in a classical scalar field, which later decays to particles. In a SUSY model, this field could be some combination of squark, Higgs and slepton fields. The field starts with a large expectation value, and rolls towards the origin in its scalar potential. At the initial large distances from the origin, there can be contributions to the potential from baryon or lepton number violating interactions (mediated, for instance, by heavy particles). These impart a net asymmetry to the rolling field. This generic mechanism could produce an asymmetry in any combination of B and L .

Other, more exotic scenarios, are described in Ref. [1].

1.3 Thermal leptogenesis: advantages and alternatives

There are many reasons to think that the Standard Model is only a low energy effective theory, and that there is new physics at a higher energy scale. Among these reasons, one can list the experimental evidence for neutrino masses, the fine-tuning problem of the Higgs mass, the dark matter puzzle, and gauge coupling unification.

It would be a particularly interesting situation if a new physics model, motivated by one of the reasons mentioned above, would also provide a viable mechanism for baryogenesis. Leptogenesis [35] is such a scenario, because it is almost unavoidable when one invokes the seesaw mechanism [36–40] to account for the neutrino masses and to explain their unique lightness. Indeed, the seesaw mechanism requires that lepton number is violated, provides in general new CP violating phases in the neutrino Yukawa interactions, and for a large part of the parameter space predicts that new, heavy singlet neutrinos decay out of equilibrium. Thus, all three Sakharov conditions are naturally fulfilled in this scenario. The question of whether leptogenesis is the source of the observed baryon asymmetry is then, within the seesaw framework, a quantitative one.

In the context of the seesaw extension of the SM, there are several ways to produce a baryon asymmetry. They all have in common the introduction of singlet neutrinos N_i with masses M_i that are usually ⁷ heavier than the electroweak breaking scale, $M_i \gg v_u$. They may differ, however, in the cosmological scenario, and in the values of the seesaw parameters. (The number of seesaw parameters is much larger than the number of measured, light neutrino parameters.) A popular possibility, which this review focuses on, is “thermal leptogenesis” with hierarchical masses, $M_1 \ll M_{j>1}$ [35]. The N_1 particles are produced by scattering in the thermal bath, so that their number density can be calculated from the seesaw parameters and the reheat temperature of the Universe.

Thermal leptogenesis has been studied in detail by many people, and there have been numerous clear and pedagogical reviews. Early analyses, focusing on hierarchical singlet neutrinos, include [51–54] (see the thesis [55] for details, in particular of supersymmetric leptogenesis with superfields). The importance

⁷An exception is the ν MSSM [50].

of including the wave function renormalisation of the decaying singlet, in calculating the CP asymmetry, was recognised in [56]. Various reviews [57, 58] were written at this stage, a pedagogical presentation that introduces BE, and discusses models and supersymmetry can be found in [58]. More detailed calculations, but which do not include flavour effects, are presented in [59, 60]: thermal effects were included in [60], and [59] gives many useful analytic approximations. The importance of flavour effects was emphasized in [61] for resonant leptogenesis with degenerate N_i , and in [62–64] for hierarchical singlets. An earlier “flavoured” analysis is [65], and flavoured BE are presented in [66]. The aim of this review is to pedagogically introduce flavour effects, which can change the final baryon asymmetry by factors of few or more. Some previous reviews of flavour effects can be found in the TASI lectures of [67], and in the conference proceedings [68–70]. They are also mentioned in Refs. [71, 72].

A potential drawback of thermal leptogenesis, for hierarchical masses M_i ’s, is the lower bound on M_1 [73, 74] (discussed in Section 5.2), which gives a lower bound on the reheat temperature. This bound might be in conflict with an upper bound on the reheat temperature that applies in supersymmetric models with a “gravitino problem” [75–81]. The lower bound on M_1 can be avoided with quasi-degenerate M_i ’s [56, 61, 82–85], where the CP violation can be enhanced in $N_i - N_j$ mixing. This scenario is discussed in Section 10.2.

Other leptogenesis scenarios, some of which work at lower reheat temperatures, include the following:

- “Soft leptogenesis” [86–88], which can work even in a one-generation SUSY seesaw. Here the source of both lepton number violation and CP violation is a set of soft supersymmetry breaking terms.
- The Affleck Dine mechanism [48, 49, 89] where the scalar condensate carries lepton number. For a detailed discussion, see, for instance, [89].
- The N_i ’s could be produced non-thermally, for instance in inflaton decay [90, 91], or in preheating [92, 93].
- The singlet sneutrino \tilde{N} could be the inflaton, which then produces a lepton asymmetry in its decays [94].
- Models with a (*e.g.* flavour) symmetry that breaks below the leptogenesis scale, and involve additional heavy states that carry lepton number [95].

1.4 Reading this review

This review contains chapters at different levels of detail, which might not all be equally interesting to all readers. Analytic formulae, which can be used to estimate the baryon asymmetry, can be found in Appendix 16. Appendix 12 is a dictionary of the notation used in this review. A reader interested in a qualitative understanding and wishing to avoid Boltzmann Equations (BE), can read Section 2, perhaps Section 3, Sections 4, 5.1 and 5.2, and browse Sections 9 and 10.

For a more quantitative understanding of thermal leptogenesis, some acquaintance with the BE is required. In addition to the Sections listed above, readers may wish to review preliminary definitions in Appendix 12, read the introduction to simple BE of Sections 6.1 and 6.2, and browse Sections 7 and 8, and Appendix 14.

Aficionados of leptogenesis may also be interested in Section 6.3, in which more complete BE are derived, and Appendix 15 that describes the toy model which motivates our claims about flavour effects.

The layout of the review is the following: In Section 2 we briefly review the seesaw mechanism and describe various useful parameterizations thereof. Section 3 introduces the non-perturbative $B + L$ violating interactions which are a crucial ingredient for baryogenesis through leptogenesis. Section 4 gives an overview of leptogenesis, using rough estimates to motivate the qualitative behaviour. Flavour dependent quantities (CP asymmetries and efficiencies) are introduced already in this section: flavour blind equations in section 4 would be inapplicable later in the review. The calculation of the CP asymmetry and bounds thereon are presented in Section 5. Subsections 5.3 and 5.4 discuss the implications of CPT and unitarity, relevant to a reader who wishes to derive the BE. Simple BE that describe interactions

mediated by the neutrino Yukawa coupling are derived in Section 6.2, using definitions from Section 6.1. (Formulae for the number densities and definitions of the rates, calculated at zero temperature, can be found in Appendix 13.) In Section 6.3 the most relevant scattering interactions, and a large set of $\mathcal{O}(1)$ effects, are included. The next three sections refine the analysis by implementing various ingredients that are omitted in the basic calculation. Section 7 is an overview of finite temperature effects. Section 8 analyzes the impact of spectator processes. (These are often described by chemical equilibrium conditions, which are reviewed in Appendix 14.) Section 9 is devoted to a detailed discussion of flavour effects. In section 10, we present some variations on the simplest framework: non-hierarchical singlets (subsection 10.2), soft leptogenesis (subsection 10.3), Dirac leptogenesis (subsection 10.4), leptogenesis with scalar triplets (subsection 10.5), and with fermion triplets (subsection 10.6). Our conclusions and some prospects for future developments are summarized in Section 11. We collect all the symbols that are used in this review into a table in Appendix 12, where we also give the equation number where they are defined. The basic ingredients and consequences of kinetic equilibrium are reviewed in Appendix 13, while those of chemical equilibrium are reviewed in Appendix 14. The BE used in the main text are not covariant under transformations of the lepton flavour basis. Appendix 15 discusses, via a toy model, covariant equations for a “density matrix”, and explains how a flavour basis for the BE is singled out by the dynamics. Approximate solutions to simple BE are given in Appendix 16.

2 Seesaw Models

Measurements of fluxes of solar, atmospheric, reactor and accelerator neutrinos give clear evidence that (at least two of) the observed weakly interacting neutrinos have small masses. Specifically, two neutrino mass-squared differences have been established,

$$\Delta m_{21}^2 = (7.9 \pm 0.3) \times 10^{-5} \text{ eV}^2 \equiv m_{\text{sol}}^2, \quad |\Delta m_{32}^2| = (2.6 \pm 0.2) \times 10^{-3} \text{ eV}^2 \equiv m_{\text{atm}}^2. \quad (2.1)$$

If these are ‘‘Majorana’’ masses, then they violate lepton number and correspond to a mass matrix of the following form:

$$\mathcal{L}_{m_\nu} = \frac{1}{2} \bar{\nu}^c_\alpha [m]_{\alpha\beta} \nu_\beta + \text{h.c.} \quad (2.2)$$

The mass terms $[m]_{\alpha\beta}$ break the $SU(2)_L$ gauge symmetry as a triplet. Consequently, they cannot come from renormalizable Yukawa couplings with the Standard Model Higgs doublet. (This stands in contrast to the charged fermion masses which break $SU(2)_L$ as doublets and are generated by the renormalizable Yukawa couplings.) Instead, they are likely to arise from the dimension five operator $(\ell_\alpha \phi)(\ell_\beta \phi)$. Seesaw models are full high energy models which induce this effective low energy operator at tree level. Seesaw models are attractive because they naturally reproduce the small masses of the doublet neutrinos. Explicitly, if the exchanged particle has a mass M which, by assumption, is much heavier than the electroweak breaking scale v_u , then the light neutrino mass scale is v_u^2/M , which is much smaller than v_u , the charged fermion mass scale.

There are three types of seesaw models, which differ by the properties of the exchanged heavy particles:

- **Type I:** $SU(3) \times SU(2) \times U(1)$ -singlet fermions;
- **Type II:** $SU(2)$ -triplet scalars;
- **Type III:** $SU(2)$ -triplet fermions.

We now describe these three types of seesaw models in more detail, with particular emphasis on the type I seesaw, which is well motivated by various extensions of the SM. We also comment on the supersymmetric seesaw framework.

2.1 Singlet fermions (Type I)

In the type I seesaw [36, 37, 39, 40], two or three singlet fermions N_i (sometimes referred to as ‘‘right-handed neutrinos’’) are added to the Standard Model. They are assumed to have large Majorana masses. The leptonic part of the Lagrangian can be written in the mass basis of the charged leptons and of the singlet fermions as follows:⁸

$$\mathcal{L} = +[\mathbf{h}]_\beta^* (\bar{\ell}_\beta \phi^{c*}) e_{R\beta} - [\lambda]_{\alpha k}^* (\bar{\ell}_\alpha \phi^*) N_k - \frac{1}{2} \bar{N}_j M_j N_j^c + \text{h.c.}, \quad (2.3)$$

where ϕ is the Higgs field, with vev $v_u \simeq 174$ GeV, and the parentheses indicate anti-symmetric $SU(2)$ contraction:

$$(\bar{\ell}_\alpha \phi^*) = (\bar{\nu}_{L\alpha}, \bar{e}_{L\alpha}) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ \phi^{*0} \end{pmatrix}. \quad (2.4)$$

Twenty-one parameters are required to fully determine the Lagrangian of eqn (2.3). To see this, notice that it is always possible to choose a basis for the N_i 's where the mass matrix M is diagonal $M = D_M$, with three positive and real eigenvalues. Similarly, one can choose a basis for the ℓ_α and $e_{R\beta}$ such that the charged lepton Yukawa matrix \mathbf{h} is diagonal and real (in particular, $\mathbf{h}\mathbf{h}^\dagger = D_{\mathbf{h}}^2$) giving other three parameters. Then, in this basis, the neutrino Yukawa matrix λ is a generic complex matrix, from which

⁸The Yukawa indices are ordered left-right, and the definition of λ is aimed to reproduce the Lagrangian that corresponds to the superpotential of eqn (2.21).

three phases can be removed by phase redefinitions on the ℓ_α , leaving 9 moduli and 6 phases as physical parameters. Therefore there are in total 21 real parameters for the lepton sector. See [96] for a more elegant counting, in particular of the phases.

If the effective mass matrix m of the light neutrinos is normalized as in eqn (2.2) then, in the charged lepton mass basis, it is given by

$$[m]_{\alpha\beta} = [\lambda]_{\alpha k} M_k^{-1} [\lambda]_{\beta k} v_u^2. \quad (2.5)$$

One has to take care of using a consistent set of definitions of the mass and the Higgs vev [97]. In particular, note the factor of 1/2 in eqn (2.2) and our use of $v_u = 174$ GeV, both of which are important in the relation (2.5). These choices are also important for the proof of eqn (4.8), so $v_u = 174$ GeV should be used in the definition of \tilde{m} and m_* of eqn (4.7).

The leptonic mixing matrix U is extracted by diagonalizing $[m]$:

$$[m] = U^* D_m U^\dagger, \quad (2.6)$$

where $D_m = \text{diag}\{m_1, m_2, m_3\}$. The matrix U is 3×3 and unitary and therefore depends, in general, on six phases and three mixing angles. Three of the phases can be removed by redefining the phases of the charged lepton doublet fields. Then, the matrix U can be conveniently parametrized as

$$U = \hat{U} \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta}) \quad , \quad (2.7)$$

where α and β are termed ‘‘Majorana’’ phases (if the neutrinos were Dirac particles, these two phases could be removed by redefining the phases of the neutrino fields), and \hat{U} can be parametrized in a fashion similar to the CKM matrix:

$$\hat{U} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \quad , \quad (2.8)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. There are various other possible phase conventions for U (see *e.g.* the first appendix of [98] for a list and translation rules).

In the leptonic sector of the SM augmented with the Majorana neutrino mass matrix of eqn (2.5), there are twelve physical parameters: The 3 charged lepton masses m_e, m_μ, m_τ , the 3 neutrinos masses m_1, m_2, m_3 , and the 3 angles and 3 phases of the mixing matrix U . Seven of these parameters are measured ($m_e, m_\mu, m_\tau, \Delta m_{21}^2, |\Delta m_{32}^2|, s_{12}, s_{23}$). There is an upper bound on the mixing angle s_{13} . The mass of the lightest neutrino and three phases of U are unknown.

In addition, there are 9 unknown parameters in the high scale theory. They are, however, relevant to leptogenesis.

2.1.1 Parametrizing the seesaw

The usual ‘‘**top-down parametrization**’’ of the theory, which applies at energy scales $\Lambda \gtrsim M_i$, is given in eqn (2.3). To relate various parametrizations of the seesaw, it is useful to diagonalize λ , which can be done with a bi-unitary transformation:

$$\lambda = V_L^\dagger D_\lambda V_R \quad (2.9)$$

Thus in the top-down approach, the lepton sector can be described by the nine eigenvalues of D_M, D_λ and $D_{\mathbf{h}}$, and the six angles and six phases of V_L and V_R . In this parametrization, the inputs are masses and coupling constants of the propagating particles at energies Λ , so it makes ‘‘physical’’ sense.

Alternatively, the (type I) seesaw Lagrangian of eqn (2.3) can be described with inputs from the left-handed sector [99]. This is referred to as a ‘‘**bottom-up parametrization**’’, because the left-handed ($SU(2)_L$ -doublet) particles have masses \lesssim the weak scale. $D_{\mathbf{h}}, U$ and D_m , can be taken as a subset of the inputs. To identify the remainder, consider the ℓ basis where m is diagonal, so as to emphasize the parallel between this parametrization and the previous one (this is similar to the N basis being chosen

to diagonalize M). If one knows $\lambda\lambda^\dagger \equiv W_L^\dagger D_\lambda^2 W_L$ in the D_m basis, then the N masses and mixing angles can be calculated:

$$M^{-1} = D_\lambda^{-1} W_L D_m W_L^T D_\lambda^{-1} = V_R D_M^{-1} V_R^T \quad (2.10)$$

In this parametrization, there are three possible basis choices for the ℓ vector space: the charged lepton mass eigenstate basis (D_h), the neutrino mass eigenstate basis (D_m), and the basis where the λ is diagonal. The first two choices are physical, that is, U rotates between these two bases. D_h , D_m and U contain the 12 possibly measurable parameters of the SM seesaw. The remaining 9 parameters can be taken to be D_λ and V_L (or $W = V_L U^*$). If supersymmetry (SUSY) is realized in nature, these parameters may contribute to the Renormalization Group (RG) running of the slepton mass matrix [100].

The **Casas-Ibarra parametrization** [101] is very convenient for calculations. It uses the three diagonal matrices D_M , D_m and D_{hh^\dagger} , the unitary matrix U defined in eqn (2.6) and a complex orthogonal matrix R . In the mass basis for the charged leptons and for the singlet fermions, which is used in the Lagrangian of eqn (2.3), R is given by

$$R = D_m^{-1/2} U^T \lambda D_M^{-1/2} v_u. \quad (2.11)$$

R can be written as $R = \hat{R} \text{diag}\{\pm 1, \pm 1, \pm 1\}$ where the ± 1 are related to the CP parities of the N_i , and \hat{R} is an orthogonal complex matrix

$$\hat{R} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{bmatrix}, \quad (2.12)$$

where $c_{ij} = \cos z_{ij}$, $s_{ij} = \sin z_{ij}$, with z_{ij} complex angles.

An alternative parametrization [102], that can be useful for quasi-degenerate N or ν masses, is given by

$$R = O \times \exp\{iA\}, \quad (2.13)$$

where O is a real orthogonal matrix, and A a real anti-symmetric matrix.

In summary, the lepton sector of the seesaw extension of the SM can be parametrized with D_h , the real eigenvalues of two more matrices, and the transformations among the bases where the matrices are diagonal. The matrices-to-be-diagonalized can be chosen in various ways:

1. “top-down” – input the N sector: D_M , $D_{\lambda^\dagger\lambda}$, and V_R and V_L .
2. “bottom-up” – input the ν_L sector: D_m , $D_{\lambda\lambda^\dagger}$, and V_L and U .
3. “intermediate” – the Casas-Ibarra parametrization: D_M , D_m , and U and R .

2.1.2 The two right-handed neutrino model

The minimal seesaw models that are viable have only two N_i 's [103]. Such models, known as “two right-handed neutrino” (2RHN) models, have a strong predictive power. The 2RHN model can be thought of as the limit of the three generation model, where N_3 decouples from the theory because it is either very heavy or has very small couplings, that is, $(|\lambda_{3\alpha}|^2 M_3^{-1}) / (|\lambda_{i\alpha}|^2 M_i^{-1}) \rightarrow 0$ for $i = 1, 2$. In the 2RHN models, M is a 2×2 matrix, and λ is a 3×2 matrix. It was originally introduced and studied with a particular texture [103, 104], and later studied in general (see *e.g.* [105] for a review). Here we follow the parametrization of the general model used in [106].

The Lagrangian of the lepton sector is of the form of eqn (2.3), but with the sum over the singlet indices restricted to two generations ($\alpha, \beta = e, \mu, \tau$; $j, k = 1, 2$). The model has fourteen independent parameters, which can be classified as follows: 5 masses ($m_e, m_\mu, m_\tau, M_1, M_2$), and 9 real parameters in λ (three phases can be removed from the six complex elements of λ by phase redefinitions on the doublets).

In this model, the lightest neutrino is massless, $m_1 = 0$, and so its associated phase vanishes, $\alpha = 0$. This situation implies that there are ten parameters that can be determined by low energy physics, instead of the usual twelve.

In the mass eigenstate bases of charged leptons and singlet fermions, the 3×2 matrix λ can be written, analogously to eqn (2.9), as follows:

$$[V_L]^\dagger \begin{bmatrix} 0 & 0 \\ \lambda_2 & 0 \\ 0 & \lambda_3 \end{bmatrix} [V_R] \quad (2.14)$$

where $[V_R]$ is a 2×2 unitary matrix with one angle and one phase, $[V_L]$ is a 3×3 matrix which can be written as U^*W^\dagger , with 4 of the six off-diagonal elements of W vanishing.

As mentioned above, the main prediction of this model is that one of the doublet neutrinos is massless. This excludes the possibility of three quasi-degenerate light neutrinos, and leaves only two other options:

1. Normal hierarchy, with $m_1 = 0$, $m_2 = m_{\text{sol}}$, $m_3 = m_{\text{atm}}$.
2. Inverted hierarchy, with $m_3 = 0$, $m_{1,2} \approx m_{\text{atm}}$, $m_2 - m_1 = m_{\text{sol}}^2/(2m_{\text{atm}})$.

2.2 Triplet scalars (Type II)

One can generate neutrino masses by the tree level exchange of $SU(2)$ -triplet scalars [40,107–110]. These $SU(2)$ -triplets should be color-singlets and carry hypercharge $Y = +1$ (in the normalization where the lepton doublets have $Y = -1/2$). In the minimal model, there is a single such scalar, which we denote by T . The relevant new terms in the Lagrangian are

$$\mathcal{L}_T = -M_T^2|T|^2 + \frac{1}{2}([\lambda_L]_{\alpha\beta} \ell_\alpha \ell_\beta T + M_T \lambda_\phi \phi \phi T^* + \text{h.c.}). \quad (2.15)$$

Here, M_T is a real mass parameter, λ_L is a symmetric 3×3 matrix of dimensionless, complex Yukawa couplings, and λ_ϕ is a dimensionless complex coupling.

The exchange of scalar triplets generates an effective dimension-5 operator $\ell \ell \phi \phi$ which, in turn, leads to neutrino masses. The triplet contribution to the neutrino masses, m_{II} , is

$$[m_{\text{II}}]_{\alpha\beta} = [\lambda_L]_{\alpha\beta} \frac{\lambda_\phi v_u^2}{M_T}. \quad (2.16)$$

This model for neutrino masses has eleven parameters beyond those of the Standard Model: 8 real and 3 imaginary ones. Of these, 6 + 3 can in principle be determined from the light neutrino parameters, while 2 (M_T and $|\lambda_\phi|$) are related to the full high energy theory.

The model involves lepton number violation because the co-existence of λ_L and λ_ϕ does not allow a consistent way of assigning a lepton charge to T , and new sources of CP violation via the phases in λ_L and λ_ϕ .

The supersymmetric triplet model [111] must include, for anomaly cancellation, two triplet superfields, T and \bar{T} , of opposite hypercharge. Only one of these couples to the leptons. The relevant superpotential terms are

$$W_T = M_T T \bar{T} + \frac{1}{\sqrt{2}}([\lambda_L]_{\alpha\beta} L_\alpha T L_\beta + \lambda_{H_d} H_d T H_d + \lambda_{H_u} H_u \bar{T} H_u), \quad (2.17)$$

leading to

$$[m_{\text{II}}]_{\alpha\beta} = [\lambda_L]_{\alpha\beta} \frac{\lambda_{H_u} v_u^2}{M_T}. \quad (2.18)$$

2.3 Triplet fermions (Type III)

One can generate neutrino masses by the tree level exchange of $SU(2)$ -triplet fermions T_i^a [112–114] (i denotes a heavy mass eigenstate while a is an $SU(2)$ index). These $SU(2)$ -triplets should be color-singlets and carry hypercharge 0. The relevant Lagrangian terms have a form that is similar to the singlet-fermion case (2.3), but the contractions of the $SU(2)$ indices is different, so we here show it explicitly:

$$\mathcal{L}_{T^a} = [\lambda_T]_{\alpha k} \tau_{\rho\sigma}^a \ell_\alpha^\rho \phi^\sigma T_k^a - \frac{1}{2} M_i T_i^a T_i^a + \text{h.c.}. \quad (2.19)$$

Here, M_i are real mass parameters, while λ_T is a 3×3 matrix of dimensionless, complex Yukawa couplings.

The exchange of fermion triplets generates an effective dimension-5 operator $\ell\ell\phi\phi$ which, in turn, leads to neutrino masses. The triplet contribution to the neutrino masses, m_{III} , is

$$[m_{\text{III}}]_{\alpha\beta} = [\lambda_T]_{\alpha k} M_k^{-1} \lambda_{\beta k} v_u^2. \quad (2.20)$$

As in the standard seesaw model, this model for neutrino masses has eighteen parameters beyond those of the Standard Model: 12 real and 6 imaginary ones.

The model involves lepton number violation because the co-existence of λ_T and M_k does not allow a consistent way of assigning a lepton charge to T_k , and new sources of CP violation via the phases in λ_T .

2.4 Supersymmetry and singlet fermions

One of the motivations of supersymmetry (SUSY) is to cancel quadratically divergent contributions to the Higgs mass. In the seesaw extension of the SM, a large mass scale M_i is present. This results in an additional set of corrections of $O(\lambda^2 M^2)$.⁹ Cancelling the contributions from the large seesaw scale motivates the supersymmetric version of the model.

The superpotential for the leptonic sector of the type I seesaw is

$$W = \frac{1}{2} N_i^c M_i N_i^c + (L_\alpha H_u) [\lambda]_{\alpha i} N_i^c - (L_\alpha H_d) [\mathbf{h}]_\alpha E_\alpha^c, \quad (2.21)$$

where L_α and E_α^c are respectively the $SU(2)$ doublets and singlets lepton superfields, and H_u and H_d are the Higgs superfields. (To resemble the SM notation, the scalar components of the Higgs superfields will be denoted as ϕ_u and ϕ_d .) The $SU(2)$ contractions in parentheses are anti-symmetric, as in eqn (2.4). Both Higgs bosons, ϕ_u and ϕ_d , have vacuum expectation values (vevs): $\langle \phi_i \rangle \equiv v_i$. Their ratio is defined as

$$\tan \beta \equiv \frac{v_u}{v_d}. \quad (2.22)$$

When including ‘‘flavour effects’’ in supersymmetric leptogenesis, the value of $\tan \beta$ is relevant, because the Yukawa coupling $\mathbf{h}_{e,\mu,\tau} \propto m_{e,\mu,\tau} / \cos \beta$.

To agree with experimental constraints (while keeping supersymmetry as a solution to the m_H^2 -fine-tuning problem), it is important to add soft SUSY-breaking terms:

$$\frac{1}{2} [\tilde{m}_L^2]_{\alpha\beta} \tilde{L}_\alpha^* \tilde{L}_\beta + \dots + \frac{1}{2} \tilde{N}_i^c [BM]_{ij} \tilde{N}_j^c + [A\lambda]_{\alpha i} (\tilde{L}_\alpha \phi_u) \tilde{N}_i^c + [A\mathbf{h}]_{\beta\alpha} (\tilde{L}_\alpha \phi_d) \tilde{E}_\alpha^c + \text{h.c.}, \quad (2.23)$$

where the ... represent soft masses-squared for all the scalars. In the thermal leptogenesis scenario that this review concentrates on, the soft SUSY breaking terms give $O(m_{\text{SUSY}}^2/M^2)$ corrections, and can be neglected. In other mechanisms, such as soft leptogenesis [86–88] and Affleck-Dine leptogenesis [48, 49, 89], the soft parameters play a central role.

The interesting feature of the SUSY seesaw, is that the neutrino Yukawa couplings may contribute to the RG running of the soft slepton mass matrix, and induce flavour-changing mass-squared terms. Consider, for example, the Type I seesaw, with universal soft masses at some high scale Λ : $[\tilde{m}_L^2]_{\alpha\beta} = m_0^2 \delta_{\alpha\beta}$, $[A\lambda]_{\alpha i} = A_0 [\lambda]_{\alpha i}$. Then at the electroweak scale, in the flavour basis, the RG contributions to the off-diagonal elements of the mass-squared matrix can be estimated at leading log as follows:

$$[\tilde{m}_L^2]_{\alpha\beta} = -\frac{3m_0^2 + A_0^2}{16\pi^2} \sum_i \lambda_{\alpha i} \log \frac{M_i^2}{\Lambda^2} \lambda_{\beta i}^* \quad (2.24)$$

In general, the soft mass matrix is unknown. One can argue, however, that the flavour-changing mass-squared matrix elements (off-diagonal in the flavour basis) are at least of order eqn (2.24), because we do

⁹The requirement of no excessive fine tuning in the cancellation of these contributions has been used to set the bound $M_i \lesssim 10^7$ GeV [115].

not expect fine-tuned cancellations among different contributions. The flavour off-diagonal terms induce processes like $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ [100], at rates of order [116]

$$\frac{\Gamma(\ell_\alpha \rightarrow \ell_\beta\gamma)}{\Gamma(\ell_\alpha \rightarrow \ell_\beta\nu\bar{\nu})} = \frac{\alpha^3}{G_F^2} \frac{|[\tilde{m}_L^2]_{\alpha\beta}|^2}{m_{\text{susy}}^8} (1 + \tan^2 \beta), \quad (2.25)$$

where m_{susy} is a typical slepton mass. For reasonable values of the unknown parameters, one obtains predictions [101, 116–118] that are in the range of current or near-future experiments [119–121].

The neutrino Yukawa couplings make real and imaginary contributions to the soft parameters. These phases give contributions to lepton electric dipole moments [122–124] which are orders of magnitude below current bounds, but possibly accessible to future experiments.

The seesaw contribution to RG running of the slepton masses is relevant to leptogenesis, because the slepton mass matrix could be an additional low energy footprint of the seesaw model. As discussed in Section 2.1, it is possible to reconstruct the Yukawa matrix λ and the masses M_i , from the light neutrino mass matrix m_ν of eqn (2.5), and the Yukawa combination $\lambda\lambda^\dagger$ that enters the RG equations. So it is interesting to study correlations between low energy observables and a large enough baryon asymmetry at low enough T_{reheat} . Early (unflavoured) studies can be found in refs. [125, 126], and in many other works: [127] (degenerate N_i), [118] (hierarchical N_i), [102] (degenerate light neutrinos), and [128] (type II). Recent flavoured analyses can be found in refs. [129, 130] (hierarchical N_i) and [131, 132] (degenerate N_i).

3 Anomalous $B + L$ Violation

The aim of this section is to give a qualitative introduction¹⁰ to the non-perturbative baryon number violating interactions that play a crucial role in leptogenesis. A similar discussion can be found in Ref. [67], while more details and references can be found, for instance, in Section 2 of Ref. [17], and in Refs. [133–136].

From a theoretical perspective, the baryon number B and the three lepton flavour numbers L_α are conserved in the renormalisable Lagrangian of the Standard Model. Furthermore, experimentally, the proton has not been observed to decay: $\tau_p \gtrsim 10^{33}$ years [137,138]. (For a review of proton decay, see [139].) However, due to the chiral anomaly, there are non-perturbative gauge field configurations [11, 140, 141] which can act as sources for $B + L_e + L_\mu + L_\tau$. (Note that $B - L_e - L_\mu - L_\tau$ is conserved.) In the early Universe, at temperatures above the electroweak phase transition (EWPT), such configurations occur frequently [10, 22, 142], and lead to rapid $B + L$ violation. These configurations are commonly referred to as “sphalerons” [143–145].

3.1 The chiral anomaly

For a pedagogical introduction to the chiral anomaly, see for instance Ref. [133].

Consider the Lagrangian for a massless Dirac fermion ψ with $U(1)$ gauge interactions:

$$\mathcal{L} = \bar{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}. \quad (3.1)$$

It is invariant under the local symmetry:

$$\psi(x) \rightarrow e^{i\theta(x)}\psi(x), \quad A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\theta(x). \quad (3.2)$$

It is also invariant under a global “chiral” symmetry:

$$\psi(x) \rightarrow e^{i\gamma_5\phi}\psi(x). \quad (3.3)$$

The associated current,

$$j_5^\mu = \bar{\psi}\gamma_5\gamma^\mu\psi, \quad (3.4)$$

is conserved at tree level, but not in the quantum theory. This can be related to the regularization of loops—renormalization introduces a scale, and the scale breaks the chiral symmetry, as would a fermion mass (see, for instance, chapter 13 of Ref. [133]). Indeed, at one loop, one finds

$$\partial_\mu j_5^\mu = \frac{1}{16\pi^2}\tilde{F}_{\mu\nu}F^{\mu\nu} = \frac{\epsilon^{\rho\sigma\mu\nu}}{16\pi^2}F^{\rho\sigma}F^{\mu\nu}. \quad (3.5)$$

The right-hand side can be written as a total divergence involving gauge fields, and is related to their topology: it counts the “winding number”, or Chern-Simons number, of the field configuration. (An instructive 1+1 dimensional model, where the topology is easy to visualize, can be found in Ref. [136].) In four dimensions, the space-time integral of the right-hand side of eqn (3.5) vanishes for an Abelian gauge field, but can be non-zero for non-Abelian fields.

In the context of leptogenesis, we are looking for an anomaly in the $B + L$ current. Within the four-dimensional SM, it arises due to the $SU(2)$ gauge interactions, which are chiral and non-Abelian. We neglect other interactions in the following (see Ref. [17] for a discussion of the effects of Yukawa and $SU(3)_C \times U(1)_Y$ interactions). The fermions that are relevant to our discussion are the three generations of quark and lepton doublets: $\{\psi_L^i\} = \{q_L^{\alpha,\beta}, \ell_L^\alpha\}$, where α, β are generation indices, a, b are colour indices, and A, B are $SU(2)$ indices. The Lagrangian terms for the $SU(2)$ gauge interactions read

$$\mathcal{L} = \sum_i \bar{\psi}_L^i \gamma^\mu (\partial_\mu - i\frac{g}{2}\sigma^A W_\mu^A) \psi_L^i. \quad (3.6)$$

¹⁰This Section is based on a lecture given by V. Rubakov at the Lake Louise Winter Institute, 2008.

It has twelve global $U(1)$ symmetries (one for each field):

$$\psi_L^i(x) \rightarrow e^{i\beta} \psi(x)_L^i \quad . \quad (3.7)$$

The chiral currents associated to these transformations,

$$j_\mu^i = \overline{\psi}_L^i \gamma_\mu \psi_L^i, \quad (3.8)$$

are conserved at tree level, but are “anomalous” in the quantum theory:

$$\partial^\mu j_\mu^i = \frac{1}{64\pi^2} F_{\mu\nu}^A \tilde{F}^{\mu\nu A}. \quad (3.9)$$

Let us define $Q^i(t) = \int j_0^i d^3x$, $\Delta Q^i = Q^i(+\infty) - Q^i(-\infty)$, and let us suppose for the moment that there exist field configurations such that

$$\Delta Q^i = \frac{1}{64\pi^2} \int d^4x F_{\mu\nu}^A \tilde{F}^{\mu\nu A} \quad (3.10)$$

is a non-zero integer. This implies that fermions will be created, even though there is no perturbative interaction in the Lagrangian that generates them. One way [146] to understand “where they come from” is to think in the Dirac sea picture, and place the chiral fermions $\{\psi_L^i\}$ in an external gauge field for which the right hand side of eqn (3.10) is non-zero. In the ground state at $t \rightarrow -\infty$, all the negative energy states are filled, and all the positive energy states are empty. As the fermions are massless, there is no mass gap at $E = 0$. At any given t , one can solve for the eigenvalues of the fermion Hamiltonian. See, for instance, Ref. [134] for a discussion. One finds that the levels move as a function of t : negative energy states from the sea acquire positive energy, and empty positive energy states could become empty sea states. In the case of the chiral $SU(2)$ of the SM, one finds that, for each species of doublet, what was a filled left-handed state in the sea at $t \rightarrow -\infty$, becomes a particle at $t \rightarrow +\infty$. See figure 3.1. This “level-crossing” occurs for each type of fermion, so the gauge field configuration centered at $t = 0$ in figure 3.1, is a source for nine quarks and three leptons.

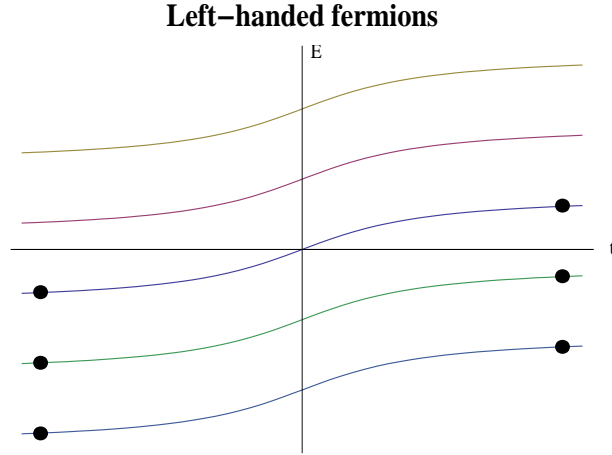


Figure 3.1: Evolution with time of the energy eigenstates of chiral fermions in a gauge field background with $\tilde{F}F \neq 0$.

3.2 $B + L$ violating rates

At zero temperature, gauge field configurations that give non-zero $\int d^4x \tilde{F}F$ correspond to tunneling configurations, and are called instantons [147] (for reviews, see *e.g.* Refs. [135, 136]). They change fermion

number by an integer N , so the instanton action is large:

$$\left| \frac{1}{4g^2} \int d^4x F_{\mu\nu}^A F^{\mu\nu A} \right| \geq \left| \frac{1}{4g^2} \int d^4x F_{\mu\nu}^A \tilde{F}^{\mu\nu A} \right| \geq \frac{64\pi^2 N}{4g^2} .$$

The first inequality follows from the Schwartz inequality (see [136]). Consequently, the associated rate is highly suppressed,

$$\Gamma \propto e^{-(\text{instanton Action})} \sim e^{-4\pi/\alpha_W} ,$$

and the mediated $B + L$ violation is unobservably small. Moreover, the instantons do not threaten the stability of the proton [11], because an instanton acts as a source for three leptons (one from each generation), and nine quarks (all colours and generations), so it induces $\Delta B = \Delta L = 3$ processes. Notice that the three quantum numbers $B/3 - L_\alpha$ are not anomalous, so they are conserved in the SM.

If the ground state of the gauge fields is pictured as a periodic potential, with minima labeled by integers, then the instantons correspond to vacuum fluctuations that tunnel between minima. With this analogy, one can imagine that at finite temperature, a thermal fluctuation of the field could climb *over* the barrier. The sphaleron [143–145] is such a configuration, in the presence of the Higgs vacuum expectation value. The $B + L$ violating rate mediated by sphalerons is Boltzmann suppressed:

$$\Gamma_{\text{sph}} \propto e^{-E_{\text{sph}}/T} ,$$

where $E_{\text{sph}} = 2Bm_W/\alpha_W$ is the height of the barrier at $T = 0$, and $1.5 \lesssim B \lesssim 2.75$ is a parameter that depends on the Higgs mass.

For leptogenesis, we are interested in the $B + L$ violating rate at temperatures far above the EWPT. The large $B + L$ violating gauge field configurations occur frequently at $T \gg m_W$ [148–152]. The rate can be estimated as (see [153] for a recent discussion)

$$\Gamma_{B+L} \simeq 250 \alpha_W^5 T . \quad (3.11)$$

This implies that, for temperatures below 10^{12} GeV and above the EWPT, $B + L$ violating rates are in equilibrium [153, 154].

4 A Toy Model of Thermal Leptogenesis

In this section, the baryon asymmetry produced by thermal leptogenesis is *estimated*. The goal is to provide a basic understanding and useful formulae, while avoiding, at this stage, the details. The CP asymmetry in the singlet fermion decay is introduced as a parameter, and discussed in more detail in Section 5. The dynamics is estimated by dimensional analysis; Boltzmann equations appear in Section 6.

In this section, we make the following simplifying assumptions:

1. The lepton asymmetry is produced in a single flavour α ;
2. The N -masses are hierarchical: $M_1 \sim 10^9 \text{ GeV} \ll M_2, M_3$. (The kinematics is simpler in an effective theory of a propagating N_1 and effective dimension-five operators induced by N_2 and N_3).
3. Thermal production of N_1 and negligible production of N_2, N_3 .

When any of these three assumptions does not hold, there are interesting modifications of the simplest scenario that we describe in this section. The effects of flavour are discussed in section 9, the consequences of non-hierarchical M_i 's (and the possible effects of $N_{2,3}$) are summarised in section 10.2, and other leptogenesis mechanisms were mentioned in section 1.3.

The basic idea is the following. Scattering processes produce a population of N_1 's at temperatures $T \sim M_1$. Then this N_1 population decays away because, when the temperature drops below M_1 , the equilibrium number density is exponentially suppressed $\propto e^{-M_1/T}$. If the N_1 interactions are CP violating, asymmetries in all the lepton flavours can be produced. If the relevant interactions are out-of-equilibrium, the asymmetries may survive. They can then be reprocessed into a baryon asymmetry by the SM $B + L$ violating processes that have been discussed in Section 3.

A unique feature of thermal leptogenesis, which distinguishes it from other “out-of-equilibrium decay” scenarios of baryogenesis, is that the same coupling constant controls the production and later disappearance of the population of N 's. As demonstrated in the seminal works [53, 155], a sufficient number density of N 's can be produced via their Yukawa coupling λ . The CP asymmetry in the processes that produce the N population is closely related to the CP asymmetry in the N decays, which wipe out the N population. In particular, in our toy model of hierarchical N_i 's, the CP asymmetry in the scattering interactions by which the N_1 population is produced is equal in magnitude but opposite in sign to the CP asymmetry in N_1 decays. At first sight, this suggests that the final lepton asymmetry is zero.¹¹ A non-zero asymmetry survives, however, because the initial anti-asymmetry made with the N population is depleted by scattering, decays, and inverse decays. This depletion is called **washout**, and is critical to thermal leptogenesis. The importance of flavour effects in leptogenesis is a consequence of the crucial role played by washout: the initial state of washout interactions contains a lepton, so it is important to know which leptons are distinguishable.

Our aim here is to estimate the asymmetry-to-entropy ratio by considering the Sakharov conditions. Each condition gives a suppression factor. The baryon asymmetry can be approximated as

$$Y_{\Delta B} \simeq \frac{135\zeta(3)}{4\pi^4 g_*} \sum_{\alpha} \epsilon_{\alpha\alpha} \times \eta_{\alpha} \times C. \quad (4.1)$$

The first factor is the equilibrium N_1 number density divided by the entropy density at $T \gg M_1$, of $\mathcal{O}(4 \times 10^{-3})$ when the number of relativistic degrees of freedom g_* is taken $\simeq 106$, as in the SM. An equilibrium number density of N_1 's is the maximum that can arise via thermal N_1 production. It is produced when $\lambda_{\alpha 1}$ is large. A smaller N_1 density is parameterized in η_{α} . As concerns the other factors in (4.1), we note the following:

¹¹Notice that the potential cancellation is between the CP asymmetry in processes with N and ℓ_{α} in the final state, such as $X \rightarrow N\ell_{\alpha}$ scattering, and the asymmetry in processes with N in the initial state and ℓ_{α} in the final state, such as $N \rightarrow \phi\ell_{\alpha}$. Only processes with ℓ_{α} in the final state can generate an asymmetry. In particular, there is no cancellation between the asymmetry produced in decays and inverse decays. It is intuitive, and straightforward to verify (see Section 6), that interactions with the lepton in the initial state can wash-out the asymmetry, but not produce it.

1. $\epsilon_{\alpha\alpha}$ is the CP asymmetry in N_1 decay. For every $1/\epsilon_{\alpha\alpha}$ N_1 decays, there is one more ℓ_α than there are $\bar{\ell}_\alpha$'s.
2. η_α is the efficiency factor. Inverse decays, other “washout” processes, and inefficiency in N_1 production, reduce the asymmetry by $0 < \eta_\alpha < 1$. In particular, $\eta_\alpha = 0$ is the limit of N_1 interactions in perfect equilibrium, so no asymmetry is created.
3. C describes further reduction of the asymmetry due to fast processes which redistribute the asymmetry that was produced in lepton doublets among other particle species. These include gauge, third generation Yukawa, and $B + L$ violating non-perturbative processes. As we discuss in sections 8 and 9.3, C is a matrix in flavour space, but for simplicity we approximate it here as a single number.

Formulae for $\epsilon_{\alpha\alpha}$ and η_α can be located in this review, by consulting the table in section 12. Our aim is now to estimate $\epsilon_{\alpha\alpha}$, η_α and C .

4.1 CP violation ($\epsilon_{\alpha\alpha}$)

To produce a net asymmetry in lepton flavour α , the N_1 must have L_α -violating interactions (see section 4.3), and different decay rates to final states with particles or anti-particles. The asymmetry in lepton flavour α produced in the decay of N_1 is defined by

$$\epsilon_{\alpha\alpha} \equiv \frac{\Gamma(N_1 \rightarrow \phi\ell_\alpha) - \Gamma(N_1 \rightarrow \bar{\phi}\bar{\ell}_\alpha)}{\Gamma(N_1 \rightarrow \phi\ell) + \Gamma(N_1 \rightarrow \bar{\phi}\bar{\ell})} \quad (4.2)$$

where $\bar{\ell}$ denotes the anti-particle of ℓ . N_1 is a Majorana fermion, so $\overline{N_1} = N_1$ ¹². The asymmetry $\epsilon_{\alpha\alpha}$ is normalized to the total decay rate, so that the Boltzmann Equations are linear in flavour space. When we include additional lepton generations, we find that the CP asymmetry is a diagonal element of a matrix, so we give it a double flavour index already.

By definition, $|\epsilon_{\alpha\alpha}| \leq 1$. Usually, it is much smaller than that. It is a function of the parameters of the Lagrangian (2.3). This dependence is evaluated in section 5. The requirement that it is large enough to account for the observed baryon asymmetry (roughly speaking, $|\epsilon_{\alpha\alpha}| > 10^{-7}$) gives interesting constraints on these parameters.

4.2 Out-of-equilibrium dynamics (η_α)

The non-equilibrium which is necessary for thermal leptogenesis is provided by the expansion of the Universe: interaction rates which are of order, or slower, than the Hubble expansion rate H are not fast enough to equilibrate particle distributions. Interactions can be classified as much faster than H , much slower, or of the same order. For the purposes of making analytic estimates (and writing Boltzmann codes), it is convenient to have a single scale problem. The timescale of leptogenesis is H^{-1} , so we neglect interactions that are much slower than H . Interactions whose rates are faster than H are resummed into thermal masses, and impose chemical and kinetic equilibrium conditions on the distributions of particles whose interactions are fast.

For the initial conditions, we assume that after inflation the Universe reheats to a thermal bath at temperature T_{reheat} which is composed of particles with gauge interactions. A thermal number density of N_1 ($n_{N_1} \simeq n_\gamma$) is produced if $T_{\text{reheat}} \gtrsim M_1/5$ [59, 60], and if the production timescale for N_1 's, $1/\Gamma_{\text{prod}}$, is shorter than the age of the Universe $\sim 1/H$. The N_1 can be produced by inverse decays $\phi\ell_\alpha \rightarrow N_1$ and, most effectively, by $2 \rightarrow 2$ scatterings involving the top quark or electroweak gauge bosons. We here neglect the gauge interactions (although $g_2 > h_t$ at 10^{10} GeV) because it is simpler and formally consistent, and because the $O(\alpha)$ corrections do not give important new effects. N_1 can be produced by

¹²In supersymmetry, N_1 represents the chiral superfield, so one may wish to distinguish N_1 from $\overline{N_1}$. In that case, the $\epsilon_{\alpha\alpha}$ arise in corrections to a “D-term”, and can be defined by replacing $\Gamma(N_1 \rightarrow \bar{\phi}\bar{\ell}_\alpha) \rightarrow \Gamma(\overline{N_1} \rightarrow \bar{\phi}\bar{\ell}_\alpha)$ in eqn 4.2.

s or t channel exchange of a Higgs : $q_L t_R \rightarrow \phi \rightarrow \ell_\alpha N$ or $\ell_\alpha t_R \rightarrow \phi \rightarrow q_L N$. So the production rate can be estimated (by dimensional analysis in zero temperature field theory) as

$$\Gamma_{\text{prod}} \sim \sum_{\alpha} \frac{h_t^2 |\lambda_{\alpha 1}|^2}{4\pi} T. \quad (4.3)$$

If $\Gamma_{\text{prod}} > H$ then, since $h_t \sim 1$, the N_1 total decay is also “in equilibrium”:

$$\Gamma_D > H(T = M_1), \quad (4.4)$$

where

$$\Gamma_D = \sum_{\alpha} \Gamma_{\alpha\alpha} = \sum_{\alpha} \Gamma(N_1 \rightarrow \phi \ell_\alpha, \bar{\phi} \bar{\ell}_\alpha) = \frac{[\lambda^\dagger \lambda]_{11} M_1}{8\pi}, \quad (4.5)$$

and

$$H(T = M_1) = 1.66 g_*^{1/2} \frac{T^2}{m_{\text{pl}}} \Big|_{T=M_1}. \quad (4.6)$$

Here g_* is the number of relativistic degrees of freedom in the thermal bath (see eqn 13.8). Within the SM, $g_* = 106.75$,

It is useful to introduce two dimensionful parameters [155] \tilde{m} and m_* , which are of the order of the light neutrino masses, and which represent, respectively, the decay rate Γ_D and expansion rate $H(T = M_1)$:

$$\begin{aligned} \tilde{m} &\equiv \sum \tilde{m}_{\alpha\alpha} \equiv \sum_{\alpha} \frac{\lambda_{\alpha 1}^* \lambda_{\alpha 1} v_u^2}{M_1} = 8\pi \frac{v_u^2}{M_1^2} \Gamma_D, \\ m_* &\equiv 8\pi \frac{v_u^2}{M_1^2} H|_{T=M_1} \simeq 1.1 \times 10^{-3} \text{ eV}. \end{aligned} \quad (4.7)$$

It can be shown [156] that

$$\tilde{m} > m_{\text{min}}, \quad (4.8)$$

where m_{min} is the smallest light neutrino mass (this is relevant for degenerate light neutrino masses), and that “usually” $\tilde{m} \gtrsim m_{\text{sol}}$ [157] (see also [158]). The $\Gamma_D > H$ condition reads, in the language of \tilde{m} and m_* , simply as

$$\tilde{m} > m_*. \quad (4.9)$$

If indeed $\tilde{m} \gtrsim m_{\text{sol}}$, then this condition is satisfied. This range of parameters is referred to as “strong washout”. The converse case, $\tilde{m} < m_*$, is referred to as “weak washout”.

In the **strong washout scenario**, at $T \sim M_1$, a thermal number density of N_1 is obtained ($n_{N_1} \sim n_\gamma$), and the total lepton asymmetry $Y_L \simeq 0$ (any asymmetry made with the N_1 is washed out). As the temperature drops and the N_1 ’s start to decay, the inverse decays $\ell_\alpha \phi \rightarrow N_1$, which can wash out the asymmetry, may initially be fast compared to H . Suppose that this is indeed the case for flavour α . Then the asymmetry in lepton flavour α will survive once the partial inverse decays from flavour α are “out of equilibrium”:

$$\Gamma_{ID}(\phi \ell_\alpha \rightarrow N_1) \simeq \frac{1}{2} \Gamma_{\alpha\alpha} e^{-M_1/T} < H = 1.66 \sqrt{g_*} \frac{T^2}{m_{\text{pl}}} \quad (4.10)$$

where the partial decay rate $\Gamma_{\alpha\alpha}$ is defined in eqn (4.5), and $\Gamma_{ID} \simeq e^{-M_1/T} \Gamma_D$. At temperature T_α where eqn (4.10) is satisfied, the remaining N_1 density is Boltzmann suppressed, $\propto e^{-M_1/T_\alpha}$. Below T_α , the N_1 ’s decay “out of equilibrium”, and contribute to the lepton flavour asymmetry. So the efficiency factor η_α for flavour α can be estimated as

$$\eta_\alpha \simeq \frac{n_{N_1}(T_\alpha)}{n_{N_1}(T \gg M_1)} \simeq e^{-M_1/T_\alpha} \simeq \frac{m_*}{\tilde{m}_{\alpha\alpha}} \quad (\tilde{m} > m_*, \tilde{m}_{\alpha\alpha} > m_*), \quad (4.11)$$

where $m_*/\tilde{m}_{\alpha\alpha} = H(T = M_1)/\Gamma_{\alpha\alpha}$. This approximation applies for $\tilde{m}_{\alpha\alpha} > m_* \simeq 10^{-3} \text{ eV}$.

Now consider an **intermediate** case, where $\tilde{m} > m_*$ (strong washout), but $\tilde{m}_{\alpha\alpha} < m_*$. In this case, the N_1 number density reaches its equilibrium value, because it has large couplings to other flavours, but the coupling $\lambda_{\alpha 1}$ to the flavour we are interested in is small. The (anti-)asymmetry in flavour α is of order $-\epsilon_{\alpha\alpha} n_\gamma$. As the population of N_1 decays at $T \lesssim M_1$, a lepton asymmetry $\sim \epsilon_{\alpha\alpha} n_\gamma$ is produced. Consequently, at lowest order in $\tilde{m}_{\alpha\alpha}$, the lepton asymmetry vanishes.¹³ A small part of the asymmetry (in flavour α) made during N_1 production is, however, washed out before N_1 decay. This fraction can be estimated as $\sim -(\tilde{m}_{\alpha\alpha}/m_*)\epsilon_{\alpha\alpha} n_\gamma$, yielding an efficiency factor¹⁴

$$\eta_\alpha \sim \frac{\tilde{m}_{\alpha\alpha}}{m_*} \quad (\tilde{m} > m_*, \tilde{m}_{\alpha\alpha} < m_*). \quad (4.12)$$

In the **weak washout scenario**, not only $\tilde{m}_{\alpha\alpha} < m_*$, as above, but also the total decay rate is small, $\tilde{m} < m_*$. In this case, the N_1 number density does not reach the equilibrium number density $\sim n_\gamma$. Production is most efficient at $T \sim M_1$, when the age of the Universe satisfies $2\tau_U = 1/H$, so $n_{N_1} \sim \Gamma_{\text{prod}} \tau_U n_\gamma \sim (\tilde{m}/m_*) n_\gamma$. The cancellation, at lowest order, of the lepton anti-asymmetry and asymmetry is as in the intermediate case above, so that the efficiency factor can be estimated as

$$\eta_\alpha \sim \frac{\tilde{m}_{\alpha\alpha} \tilde{m}}{m_*^2} \quad (\tilde{m} < m_*, \tilde{m}_{\alpha\alpha} < m_*). \quad (4.13)$$

4.3 Lepton and $B + L$ violation (C)

The interactions of N_1 violate L because lepton number cannot be consistently assigned to N_1 in the presence of λ and M . If $L(N_1) = 1$, then $\lambda_{\alpha 1}$ respects L but M_1 violates it by two units. If $L(N_1) = 0$, then M_1 respects L but $\lambda_{\alpha 1}$ violates it by one unit. The N_1 decay, which depends on both M_1 and $\lambda_{\alpha 1}$, does not conserve L . The heavy mass eigenstate is its own anti-particle, so it can decay to both $\ell\phi$ and $\bar{\ell}\phi^*$. If there is an asymmetry in the rates, a net lepton asymmetry will be produced.

The baryon number violation is provided by $B + L$ changing SM non-perturbative processes [17] (see Section 3). Their approximate rate is given in eq. (3.11) and is faster than the Hubble expansion H in the thermal plasma for $T \lesssim 10^{12}$ GeV. The asymmetry in lepton flavour α , produced in the N_1 decay, contributes to the density of $B/3 - L_\alpha$, which is conserved by the SM interactions. In equilibrium, this excess of $B - L$ implies (for the SM) a baryon excess [41]:

$$Y_{\Delta B} \simeq \frac{12}{37} \sum_\alpha Y_{\Delta_\alpha} \quad (4.14)$$

where Y_{Δ_α} is the asymmetry in $B/3 - L_\alpha$, divided by the entropy density. The value of $C = 12/37$ applies in the SM (see eqn 14.17). In the MSSM, it is $10/31$ (see eqn 14.18).

So far, the focus has been on the neutrino Yukawa interactions, which produce an asymmetry in the lepton doublets ℓ_α . The SM interactions, which redistribute the asymmetries among other particles, are included in Section 8. As discussed in section 9.3, these interactions usually give $O(1)$ effects, which are parameterized with the A -matrix [65] that is derived in section 14. One effect that can be explained already at this stage is the following. When the charged lepton Yukawa coupling h_α is in chemical equilibrium, that is, when interaction rates such as $\Gamma(\text{gauge boson} + e_R \leftrightarrow \ell + \phi)$ are fast compared to H , the lepton asymmetry in flavour α is shared between $e_{R\alpha}$ and ℓ_α . But only the part that remains in ℓ_α is washed out by the neutrino Yukawa interactions, so there is a mild reduction in washout.

¹³This cancellation is discussed in more detail in ref. [59], when they consider production by inverse decays. This cancellation is absent in their later discussion when production by scattering is included, because CP violation in scattering was neglected.

¹⁴This estimate assumes the momentum distribution $f(p)$ is thermal (see Appendix 13). This is a usual assumption in leptogenesis, where Boltzmann Equations for the total number densities are solved. Differences that arise when the BE are solved mode by mode were studied in [159].

4.4 Putting it all together

An estimate for the baryon asymmetry can be obtained from eqn (4.1), with the prefactor C from eqn (4.14):

$$Y_{\Delta B} \sim 10^{-3} \epsilon_{\alpha\alpha} \eta_{\alpha},$$

where the CP asymmetry $\epsilon_{\alpha\alpha}$ is taken from eqn (5.9) or (5.13), and the efficiency factor is taken from eqn (4.11), (4.12) or (4.13). To obtain more accurate estimates, as can be found in Appendix 16, the dynamics should be calculated via the Boltzmann Equations, introduced in Section 6.

We can anticipate the flavour issues, which are discussed in Section 9, by supposing that there are CP asymmetries in all flavours. Then, in the strong washout case for all flavours, we obtain

$$Y_{\Delta B} \sim 10^{-3} \sum_{\alpha} \epsilon_{\alpha\alpha} \eta_{\alpha} \sim 10^{-3} m_* \sum_{\alpha} \frac{\epsilon_{\alpha\alpha}}{\tilde{m}_{\alpha\alpha}} \quad (\text{flavoured, strong washout}) \quad (4.15)$$

where the flavours summed over are presumably the charged lepton mass eigenstates. Alternatively, one might choose $\ell_{\alpha} = \hat{\ell}_{N_1}$, the direction in flavour space into which N_1 decays (see eqn 5.11). Then ϵ_{11} is the total CP asymmetry ϵ and $\Gamma(N \rightarrow \phi \ell_1)$ is the total decay rate Γ_D . One obtains

$$Y_{\Delta B} \sim 10^{-3} m_* \frac{\epsilon}{\tilde{m}} \quad (\text{single flavour, strong washout}), \quad (4.16)$$

which is simpler but different. In Appendix 15.4 (see also Section 9) we discuss why and when the charged lepton mass basis is the relevant one.

5 CP Violation

In section 5.1, we review how to calculate a CP asymmetry in N_1 decays, including the vertex and wavefunction [160] contributions. In Section 5.2, we calculate $\epsilon_{\alpha\alpha}$ for hierarchical N_i , and give the formulae for less hierarchical N_i , as calculated in [56]. It was shown in [73,74], that for hierarchical N_i , there is an upper bound on the CP asymmetry proportional to M_1 . Section 5.2 contains a derivation of this bound, which gives a strong constraint on models, and a discussion of various loopholes. Sections 5.3–5.4 discuss general constraints from S-matrix unitarity and CPT invariance, which have implications for the generation of a cosmological asymmetry from decays. In particular, we explain a subtlety in the analysis: the contribution of an on-shell N_1 to scattering rates should be subtracted, as it is already included in the decays and inverse decays.

5.1 CP violation in N_1 decays

The CP asymmetry in lepton flavour α , produced in the decay of N_1 , is defined in eqn (4.2):

$$\epsilon_{\alpha\alpha} \equiv \frac{\Gamma(N_1 \rightarrow \phi \ell_\alpha) - \Gamma(N_1 \rightarrow \bar{\phi} \bar{\ell}_\alpha)}{\Gamma(N_1 \rightarrow \phi \ell) + \Gamma(N_1 \rightarrow \bar{\phi} \bar{\ell})} \quad (5.1)$$

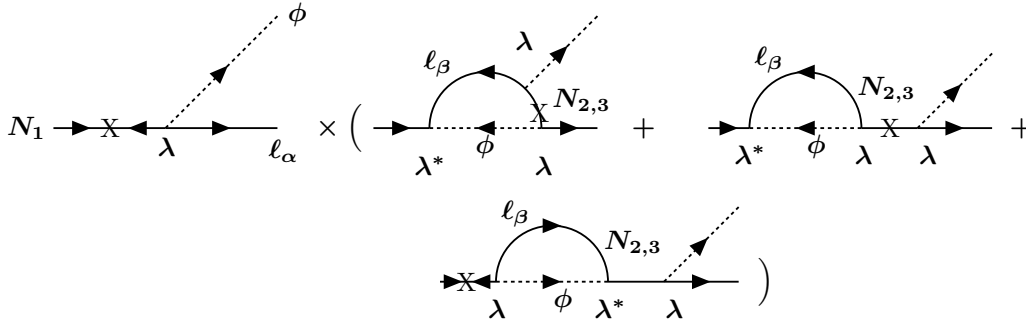


Figure 5.1: The diagrams contributing to the CP asymmetry $\epsilon_{\alpha\alpha}$. The flavour of the internal lepton ℓ_β is summed. The internal ℓ_β and Higgs ϕ are on-shell. The X represents a Majorana mass insertion. Line direction is “left-handedness”, assigning to scalars the handedness of their SUSY partners. The loop diagrams on the first line are lepton flavour and lepton number violating. The last diagram is lepton flavour changing but “lepton number conserving”, in the sense that it makes no contribution to the total CP asymmetry ϵ . It is suppressed by an additional factor $M_1/M_{2,3}$ [see eqn (5.13)].

The CP asymmetry $\epsilon_{\alpha\alpha}$ arises from the interference of tree-level (subscript 0) and one-loop (subscript 1) amplitudes. This is discussed further in section 5.3. As noted in [160], it is important to include all the one loop diagrams, including the wavefunction corrections. The tree and loop matrix elements can each be separated into a coupling constant part c and an amplitude part \mathcal{A} :

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 = c_0 \mathcal{A}_0 + c_1 \mathcal{A}_1. \quad (5.2)$$

For instance, in the tree level decay of figure (5.1),

$$c_0 = \lambda_{\alpha 1}^* \quad \mathcal{A}_0(N \rightarrow \phi^\dagger \bar{\ell}_\alpha) = \bar{u}_{\ell_\alpha} P_R u_N \quad . \quad (5.3)$$

The matrix element for the CP conjugate process is

$$\bar{\mathcal{M}} = c_0^* \bar{\mathcal{A}}_0 + c_1^* \bar{\mathcal{A}}_1, \quad (5.4)$$

where¹⁵ $|\overline{\mathcal{A}}_i|^2 = |\mathcal{A}_i|^2$. Thus the CP asymmetry can be written

$$\begin{aligned}\epsilon_{\alpha\alpha} &= \frac{\int |c_0\mathcal{A}_0 + c_1\mathcal{A}_1|^2 \tilde{\delta} d\Pi_{\ell,\phi} - \int |c_0^*\mathcal{A}_0 + c_1^*\mathcal{A}_1|^2 \tilde{\delta} d\Pi_{\ell,\phi}}{2 \sum_{\beta} \int |c_0\mathcal{A}_0|^2 \tilde{\delta} d\Pi_{\ell,\phi}} \\ &= \frac{\text{Im}\{c_0c_1^*\} 2 \int \text{Im}\{\mathcal{A}_0\mathcal{A}_1^*\} \tilde{\delta} d\Pi_{\ell,\phi}}{\sum_{\alpha} |c_0|^2 \int |\mathcal{A}_0|^2 \tilde{\delta} d\Pi_{\ell,\phi}},\end{aligned}\quad (5.5)$$

where

$$\tilde{\delta} = (2\pi)^4 \delta^4(P_i - P_f), \quad d\Pi_{\ell,\phi} = d\Pi_{\ell} d\Pi_{\phi} = \frac{d^3 p_{\phi}}{2E_{\phi}(2\pi)^3} \frac{d^3 p_{\ell}}{2E_{\ell}(2\pi)^3}, \quad (5.6)$$

and P_i, P_f are, respectively, the incoming four-momentum (in this case P_N) and the outgoing four-momentum (in this case $p_{\phi} + p_{\ell}$). The loop amplitude has an imaginary part when there are branch cuts corresponding to intermediate on-shell particles (see Cutkosky Rules in [161], or eqn (5.19)), which can arise in the loops of figure 5.1 when the ϕ and ℓ_{β} are on-shell:

$$2\text{Im}\{\mathcal{A}_0\mathcal{A}_1^*\} = \mathcal{A}_0(N \rightarrow \phi\ell_{\alpha}) \sum_{\beta} \int \mathcal{A}_0^*(N \rightarrow \bar{\ell}'_{\beta}\bar{\phi}') \tilde{\delta}' d\Pi_{\ell'_{\beta},\phi'} \mathcal{A}_0^*(\bar{\ell}'_{\beta}\bar{\phi}' \rightarrow \phi\ell_{\alpha}). \quad (5.7)$$

Here ϕ' and ℓ'_{β} are the (assumed massless) intermediate on-shell particles, and $d\Pi_{\ell'_{\beta},\phi'}$ is the integration over their phase space.

5.2 $\epsilon_{\alpha\alpha}$ and the lower bound on M_1

In the limit $M_2, M_3 \gg M_1$, the effects of N_2, N_3 can be represented by an effective dimension-5 operator. In the diagram of 5.1, this corresponds to shrinking the heavy propagator in the loop to a point. For calculating $\epsilon_{\alpha\alpha}$, the Feynman rule for the dimension-5 operator can be taken $\propto [m]/v_u^2$. (There is a contribution to $[m]$ from N_1 exchange, which is not present in the dimension-5 operator that is obtained by integrating out N_2 and N_3 . But the N_1 -mediated part of $[m]$ makes no contribution to the imaginary part for $\epsilon_{\alpha\alpha}$.) Then, we obtain for the relevant coupling constants

$$c_0 = \lambda_{\alpha 1}^* \quad c_1 = 3 \sum_{\beta} \lambda_{\beta 1} [m^*]_{\beta\alpha} / v^2.$$

The factor of three comes from careful book-keeping of weak $SU(2)_L$ indices; The dimension-5 operator is

$$\frac{[m]_{\alpha\beta}}{2v_u^2} (\nu_L^{\alpha} \phi_0 - e_L^{\alpha} \phi^+) (\nu_L^{\beta} \phi_0 - e_L^{\beta} \phi^+) + \text{h.c.} \quad (5.8)$$

This leads to a Feynman rule $2(\delta_{\rho}^{\alpha} \delta_{\sigma}^{\beta} + \delta_{\rho}^{\beta} \delta_{\sigma}^{\alpha}) [m]_{\alpha\beta} / (2v_u^2)$ for the vertex $\nu_L^{\rho} \phi_0 \nu_L^{\sigma} \phi_0$ or $e_L^{\rho} \phi^+ e_L^{\sigma} \phi^+$, but to a Feynman rule $-[m]_{\rho\sigma} / v_u^2$ for $\nu_L^{\rho} \phi_0 e_L^{\sigma} \phi^+$ or $\nu_L^{\sigma} \phi_0 e_L^{\rho} \phi^+$. Summing over all possible lepton/Higgs combinations in the loop gives the factor of three. It can also be seen in the theory with propagating $N_{2,3}$: The charged and the neutral components of the intermediate ϕ' and ℓ'_{β} contribute in the N_1 wavefunction correction, giving a factor of 2, but only the charged or the neutral ϕ' and ℓ'_{β} appear in the vertex correction.

To obtain the amplitude ratio, in the case of hierarchical N 's, we take $\mathcal{A}_0^*(\bar{\ell}'_{\beta}\bar{\phi}' \rightarrow \phi\ell_{\alpha}) = \bar{v}_{\ell}^{\alpha} P_L u_{\ell}^{\beta}$, and after spin sums we obtain:¹⁶

$$\epsilon_{\alpha\alpha} = \frac{3M_1}{16\pi v_u^2 [\lambda^{\dagger} \lambda]_{11}} \text{Im} \{ [\lambda]_{\alpha 1} [m^* \lambda]_{\alpha 1} \} \quad (5.9)$$

¹⁵In the CP conjugate amplitude, $\overline{\mathcal{A}}$, the u_{ℓ} spinors are replaced by v_{ℓ} spinors. Since, however, $\bar{u}_{\ell} u_{\ell} = \not{p} = \bar{v}_{\ell} v_{\ell}$, the $|\text{magnitude}|^2$ is the same.

¹⁶The various 2's for the initial and final state averages and sums are discussed around eqn (13.18). They cancel in the ratio and can be ignored here.

where there is no sum on α in this equation.

The upper bound

$$|\epsilon_{\alpha\alpha}| \leq \frac{3M_1 m_{\max}}{16\pi v_u^2} \sqrt{B_{\phi\ell_\alpha}^{N_1} + B_{\phi\bar{\ell}_\alpha}^{N_1}} \quad (5.10)$$

where $B_{xy}^{N_1} \equiv \Gamma(N_1 \rightarrow xy)/\Gamma_D$ and m_{\max} is the largest light neutrino mass, can be derived by defining the unit vector

$$\hat{\ell}_{N_1} = \frac{\lambda_{\alpha 1}}{\sqrt{\sum_\beta |\lambda_{\beta 1}|^2}} \quad (5.11)$$

and using $|m \cdot \hat{\ell}_{N_1}^*| \leq |m_{\max} \hat{\ell}_{N_1}|$.

The upper bound on $|\epsilon_{\alpha\alpha}|$ can be used to obtain a lower bound on M_1 and the reheat temperature of the Universe for thermal leptogenesis with hierarchical N_i . The first calculations [73, 74] used the total asymmetry $^{17} \epsilon = \sum_\alpha \epsilon_{\alpha\alpha}$; the differences are discussed around eqn (5.17). The estimate for the baryon asymmetry, eqn (4.1), combined with eqn (4.14), will match the observed asymmetry of eqn (1.2) for $\sum_\alpha \epsilon_{\alpha\alpha} \eta_\alpha \sim 10^{-7}$. The efficiency factor, $0 < \eta_\alpha < 1$, is usually $\lesssim 0.1$ (see section 6), which implies

$$\epsilon_{\alpha\alpha} \gtrsim 10^{-6} \quad .$$

Taking $m_{\max} = m_{\text{atm}}$ in the upper bound of eqn (5.10), we find

$$M_1 \gtrsim 10^9 \text{ GeV}. \quad (5.12)$$

A more precise bound can be obtained numerically. The CP asymmetries $\epsilon_{\alpha\alpha}$ can be smaller when η_α is larger, that is, when there is less washout, which occurs when $\Gamma_D \sim H(T = M_1)$. For this range of parameters, analytic approximations are not so reliable (see the plots of section 9.4.3).

The bound of eqn (5.12) is restrictive, so it is worth repeating the list of assumptions that lead to it and to enumerate some mechanisms that evade it:

1. The bound applies for non-degenerate heavy neutrinos. The CP asymmetry can be much larger for quasi-degenerate N_i , with $M_1 - M_2 \sim \Gamma_D$ (see section 10.2.2).
2. The bound actually applies only for strongly hierarchical heavy neutrinos. To prove the upper bound on ϵ , for arbitrary λ compatible with the observed $[m]$, requires $M_{2,3} > 100M_1$ [162, 163]. For a milder hierarchy, it is possible to tune λ such that a dimension-seven operator gives a contribution to ϵ that can be as high as $M_1^3/(M_2^2 M_3)$ and thus possibly exceed the bound [164], but no contribution to $[m]$. If such tuning is neglected, the bound is “usually” good for $M_{2,3} > 10M_1$.¹⁸
3. The bound can be evaded by adding particles and interactions [165]. Some of the possibilities are the following:

- In multi-Higgs models, the vev(s) of the scalars that appear in the light neutrino mass matrix may be unknown. The CP asymmetries $\epsilon_{\alpha\alpha}$ increase as these vevs become smaller.

For instance, writing $v_u = 174 \sin \beta$ GeV in a model with two Higgs doublets, the bound becomes $M_1 \gtrsim \sin^2 \beta \times 10^9$ GeV, which can be significantly weaker if $\sin \beta \ll 1$ [166]. It is interesting to note, in this regard, that this is not the case in the supersymmetric Standard Model,¹⁹ in spite of its being a two-Higgs model. The reason is that in this framework $\tan \beta > 1$. If there were extra Higgs doublets, or a non-analytic term $LH_d^* N$ with large $\tan \beta$, then thermal leptogenesis could be successful for lower M_1 and T_{reheat} values.

¹⁷Notice that [74] used ϵ for the MSSM, which is twice as big, see eqn (5.15). The bound given in [74] therefore had 8π rather than 16π in the denominator.

¹⁸This can be seen from [163], where the procedure of scanning of the parameter space is described. There is no similar information in [162], where contrary claims are made.

¹⁹It is desirable, within the supersymmetric framework, to avoid the bound (5.12) because it may be in conflict with upper bounds on T_{reheat} from the gravitino problem.

In “inverse seesaw” models [167], which contain additional singlet fermions and scalars, the light neutrino masses are proportional to the unknown vev(s) of the additional scalar(s). For sufficiently small vev(s), hierarchical leptogenesis can work down to the TeV scale [168–170].

- A minimal extension that works down to the TeV, is to add a singlet [171].
 - Including a fourth lepton generation allows thermal leptogenesis at $T \sim \text{TeV}$ [172–174].
 - Consider the case that the number of heavy singlet neutrinos is larger than three. Their contribution to the effective operator $(\ell\phi)(\ell\phi)$ has no effect on the bounds on the flavoured asymmetries $\epsilon_{\alpha\alpha}$. In contrast, the contribution of many singlets in weak washout can enhance the final baryon asymmetry, allowing thermal leptogenesis at T_{reheat} that is a factor ~ 30 lower than in the case of three singlet neutrinos [175]. In the case where leptogenesis is unflavoured, extra singlets weaken the upper bound on $|\epsilon|$ from $3M_1(m_3 - m_1)/(16\pi v^2)$ to $3M_1 m_3/(16\pi v^2)$ [175, 176].
4. For degenerate light neutrinos, $m_{\text{max}} > m_{\text{atm}}$, so the individual flavour asymmetries can be larger. However, the efficiency factor is smaller, so for $m_{\text{max}} \lesssim \text{eV}$ (a conservative interpretation of the cosmological bound [177–179]) the lower bound on M_1 is similar to eqn (5.12) [62, 180, 181].

One can go beyond the effective theory and incorporate the $N_{2,3}$ states as dynamical degrees of freedom. For a not-too degenerate N_i spectrum, $M_i - M_j \gg \Gamma_D$ (the case of $M_i - M_j \sim \Gamma_D$ is discussed in Section 10.2.2), one obtains

$$\begin{aligned} \epsilon_{\alpha\alpha} &= \frac{1}{(8\pi)} \frac{1}{[\lambda^\dagger \lambda]_{11}} \sum_j \text{Im} \{ (\lambda_{\alpha 1}^*) (\lambda^\dagger \lambda)_{1j} \lambda_{\alpha j} \} g(x_j) \\ &\quad + \frac{1}{(8\pi)} \frac{1}{[\lambda^\dagger \lambda]_{11}} \sum_j \text{Im} \{ (\lambda_{\alpha 1}^*) (\lambda^\dagger \lambda)_{j1} \lambda_{\alpha j} \} \frac{1}{1 - x_j}, \end{aligned} \quad (5.13)$$

where

$$x_j \equiv M_j^2 / M_1^2,$$

and, within the SM [56],

$$g(x) = \sqrt{x} \left[\frac{1}{1-x} + 1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right] \xrightarrow{x \gg 1} -\frac{3}{2\sqrt{x}} - \frac{5}{6x^{3/2}} + \dots \quad (5.14)$$

In the MSSM, N_1 decays to a slepton + Higgsino, as well as to lepton + Higgs. The sum of the asymmetries to leptons and to sleptons is about twice larger than the SM asymmetry [56]:

$$g(x) = -\sqrt{x} \left(\frac{2}{x-1} + \ln [1 + 1/x] \right) \xrightarrow{x \gg 1} -\frac{3}{\sqrt{x}} - \frac{3}{2x^{3/2}} + \dots \quad (5.15)$$

The first line of eqn (5.13) corresponds to the diagrams on the first row of figure 5.1 while the second line [56, 63, 66, 95] corresponds to the diagram of the second row. This contribution violates the single lepton flavours but conserves the total lepton number, and thus it vanishes when summed over α :

$$\epsilon \equiv \sum_{\alpha} \epsilon_{\alpha\alpha} = \frac{1}{(8\pi)} \frac{1}{[\lambda^\dagger \lambda]_{11}} \sum_j \text{Im} \{ [(\lambda^\dagger \lambda)_{1j}]^2 \} g(x_j) . \quad (5.16)$$

As discussed in Sections 5.2 and 9.4.3, the upper bound on the flavoured CP asymmetries $\epsilon_{\alpha\alpha}$ can be used to obtain a lower bound on the reheat temperature. Here we discuss the upper bound on the total CP asymmetry $\epsilon = \sum_{\alpha} \epsilon_{\alpha\alpha}$ of eqn (5.16) [73, 74]:

$$|\epsilon| < \frac{3}{16\pi} \frac{(m_{\text{max}} - m_{\text{min}}) M_1}{v_u^2} \times \beta(\tilde{m}, m_{\text{max}}, m_{\text{min}}) \quad (5.17)$$

where m_{\max} (m_{\min}) is the largest (smallest) light neutrino mass, and $\beta \sim 1$ can be found in [162].

The interesting feature of the bound (5.17), is that it *decreases* for degenerate light neutrinos. This was used to obtain an upper bound on the light neutrino mass scale from unflavoured leptogenesis [157, 182] (discussed in Section 9.4.1), and explains the interest in the form of the function β . However, the maximum CP asymmetry in a given flavour is unsuppressed for degenerate light neutrinos [62], so flavoured leptogenesis can be tuned to work for degenerate light neutrinos, as discussed in Section 9.4.1.

5.3 Implications of CPT and unitarity for CP violation in decays

S-matrix unitarity and CPT invariance give useful constraints on CP violation (see *e.g.* [24, 31, 183]). This section is a brief review of some relevant results for CP violation in decays. CP transforms a particle ℓ_α into its antiparticle which we represent as $\bar{\ell}_\alpha$.

Useful relations, between matrix elements and their CP conjugates, can be obtained from the unitarity of the S-matrix $\mathbf{S} = \mathbf{1} + i\mathbf{T}$:

$$\mathbf{1} = \mathbf{S}\mathbf{S}^\dagger = (\mathbf{1} + i\mathbf{T})(\mathbf{1} - i\mathbf{T}^\dagger) \quad (5.18)$$

which implies that $i\mathbf{T}_{ab} = i\mathbf{T}_{ba}^* - [\mathbf{T}\mathbf{T}^\dagger]_{ab}$. Assuming that the transition matrix \mathbf{T} can be perturbatively expanded in some coupling constant λ , it follows from

$$|\mathbf{T}_{ab}|^2 - |\mathbf{T}_{ba}|^2 = -2 \operatorname{Im} \{ [\mathbf{T}\mathbf{T}^\dagger]_{ab} \mathbf{T}_{ba}^* \} + |[\mathbf{T}\mathbf{T}^\dagger]_{ab}|^2 \quad (5.19)$$

that CP violation in a tree process, such as N_j decay, can first arise in the loop corrections. Notice that the unstable N_1 is being treated as an asymptotic state (the unitary S-matrix is defined between asymptotic states); this approximation requires some care, as discussed around eqn (5.24).

CPT, which should be a symmetry of Quantum Field Theories, implies

$$|\mathcal{M}(a \rightarrow b)|^2 = |\mathcal{M}(\bar{b} \rightarrow \bar{a})|^2 \quad (5.20)$$

where $i\mathcal{M}(a \rightarrow b)(2\pi)^4 \delta^4(\sum_i^n p_i - \sum_f^m q_f)$ is the $i\mathbf{T}_{ba}$ matrix element from an initial state of particles $\{a_1(p_1), \dots, a_n(p_n)\}$ to a final state of particles $\{b_1(q_1), \dots, b_m(q_m)\}$. In particular, for a Majorana fermion N_1 , which is its own antiparticle,

$$|\mathcal{M}(N \rightarrow \ell_\alpha \phi)|^2 = |\mathcal{M}(\bar{\phi} \bar{\ell}_\alpha \rightarrow N)|^2 \quad (5.21)$$

Following many textbooks (for instance, section 3.6 of [183]), one can show from unitarity and CPT that the total decay rate of a particle X and its antiparticle \bar{X} are the same. The unitarity condition $\sum_{\{b\}} \langle X | \mathbf{S} | b \rangle \langle b | \mathbf{S}^\dagger | X \rangle = 1$ implies

$$\sum_{\{b\}} |\mathcal{M}(X \rightarrow b)|^2 = \sum_{\{b\}} |\mathcal{M}(b \rightarrow X)|^2, \quad (5.22)$$

where the sum is over all accessible states b . Combined with the CPT condition of eqn (5.20) (with $a = X$), one obtains, as anticipated,

$$\sum_{\{b\}} |\mathcal{M}(X \rightarrow b)|^2 = \sum_{\{b\}} |\mathcal{M}(\bar{X} \rightarrow b)|^2. \quad (5.23)$$

(Notice that $\{\bar{b}\} = \{b\}$.) It is nonetheless possible to have a CP asymmetry in a partial decay rate. In the case of N_1 , which decays to $\phi \ell_\alpha$ and $\bar{\phi} \bar{\ell}_\alpha$, the asymmetry of eqn (5.1) can be non-zero.

N_1 can be approximated as an asymptotic state, for unitarity purposes, if its lifetime is long compared to the S-matrix timescale. This timescale can be identified as $1/\sqrt{s_{\text{kin}}}$ (where s_{kin} is a Lorentz invariant measure of the center of mass energy of the process, for instance the Mandelstam variable s for $2 \rightarrow 2$ scattering), because, in calculating (for instance) a decay rate, one squares the S-matrix element, using

$$\left| \delta^4 \left(\sum_i^n p_i - \sum_f^m q_f \right) \right|^2 = \delta^4 \left(\sum_i^n p_i - \sum_f^m q_f \right) V \tau, \quad (5.24)$$

where V and τ are the volume of the box and the time interval in which the interaction takes place. For finite τ , there must be an uncertainty in the energy conservation δ -function of order $1/\tau$, so the $\delta(E)$ makes sense for $\sqrt{s_{\text{kin}}} \gg 1/\tau$. Consequently, unitarity is satisfied for matrix elements with N_1 in the initial or final state when $\Gamma_D \ll \sqrt{s_{\text{kin}}} \sim M_1$ (the narrow width approximation).

One must take care, however, to subtract from scattering rates into true asymptotic states the contribution of on-shell N_1 's (to avoid double counting). This is usually done in the narrow width approximation (see *e.g.* ref. [60] for a clear discussion). Then one can check the result by verifying the CPT and unitarity constraints. Here we do the converse: use CPT and unitarity to guess what should be subtracted (see section 3.8 of [183] for a more complete analysis).

To see how this works, consider the process $\phi\ell_\alpha \rightarrow \text{anything}$, at $O(\lambda^4)$ in the rate. The possible final states are N_1 , with one-loop corrections, and $\bar{\phi}\bar{\ell}_\beta$ or $\phi\ell_\beta$, at tree level. Then one can make the following three observations:

1. Unitarity and CPT imply (see eqn (5.23) with the initial state $X = \phi\ell_\alpha$) that there can be no CP asymmetry in this total rate:

$$|\mathcal{M}(\phi\ell_\alpha \rightarrow \text{anything})|^2 = |\mathcal{M}(\bar{\phi}\bar{\ell}_\alpha \rightarrow \text{anything})|^2. \quad (5.25)$$

2. For leptogenesis to work, we need $\epsilon_{\alpha\alpha} \neq 0$ which, by CPT, implies that there is a CP asymmetry in the partial rates to N_1 :

$$|\mathcal{M}(\bar{\phi}\bar{\ell}_\alpha \rightarrow N_1)|^2 - |\mathcal{M}(\phi\ell_\alpha \rightarrow N_1)|^2 \neq 0. \quad (5.26)$$

3. From the unitarity constraint (5.19), there should be no CP asymmetry to cancel (5.26) in the tree-level scatterings.

The apparent contradiction arises because on-shell s -channel exchange of N_1 is included in the scattering, so we have counted it twice. This on-shell part, also referred to as ‘‘Real Intermediate State’’, should therefore be subtracted from the scattering:

$$\begin{aligned} |\mathcal{M}(\phi\ell_\alpha \rightarrow \text{anything})|^2 &= |\mathcal{M}(\phi\ell_\alpha \rightarrow N)|^2 + \sum_{\beta} \left(|\mathcal{M}(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2 - |\mathcal{M}^{\text{os}}(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2 \right. \\ &\quad \left. + |\mathcal{M}(\phi\ell_\alpha \rightarrow \phi\ell_\beta)|^2 - |\mathcal{M}^{\text{os}}(\phi\ell_\alpha \rightarrow \phi\ell_\beta)|^2 \right), \end{aligned} \quad (5.27)$$

where \mathcal{M}^{os} stands for the on-shell contribution to the amplitude. It is simple to check that the asymmetry (5.25) vanishes as required if the subtracted matrix element squared, denoted by \mathcal{M}' ,

$$|\mathcal{M}'(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2 \equiv |\mathcal{M}(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2 - |\mathcal{M}^{\text{os}}(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2, \quad (5.28)$$

is taken as follows:

$$|\mathcal{M}'(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2 = |\mathcal{M}(\phi\ell_\alpha \rightarrow \bar{\phi}\bar{\ell}_\beta)|^2 - |\mathcal{M}(\phi\ell_\alpha \rightarrow N)|^2 B_{\bar{\phi}\bar{\ell}_\beta}^{N_1}. \quad (5.29)$$

where $B_{\bar{\phi}\bar{\ell}_\beta}^{N_1}$ is the branching ratio for $N \rightarrow \bar{\phi}\bar{\ell}_\beta$. This is the subtracted matrix-element-squared one obtains in the narrow width approximation. It will be useful for writing Boltzmann Equations for the lepton asymmetry.

5.4 CP violation in scattering

Scattering processes are relevant for the production of the N_1 population, because decay and inverse decay rates are suppressed by a time dilation factor $\propto M_1/T$. The $N_1 = \bar{N}_1$ particles can be produced by s -channel ϕ -exchange in $qt^c \rightarrow N\ell_\alpha$ and $\bar{q}\bar{t}^c \rightarrow N\bar{\ell}_\alpha$, and by t -channel ϕ -exchange in $q\bar{\ell}_\alpha \rightarrow N\bar{t}^c$, $t^c\bar{\ell}_\alpha \rightarrow N\bar{q}$, $\ell_\alpha\bar{t}^c \rightarrow Nq$ and $\bar{q}\ell_\alpha \rightarrow Nt^c$.

In this section, we explicitly calculate the CP asymmetry in scattering processes, for the case of hierarchical N_j , and show that it is the same as in decays and inverse decays [64]. This result was found in refs. [61, 85, 184] for the case of resonant leptogenesis. To introduce CP violation in scattering into the Boltzmann Equations, one must correctly include all processes of order $h_t^2 \lambda^4$ with the on-shell intermediate state N_{1s} subtracted out [185]. This is done in section 6.3, following the analysis of [185].

For simplicity, we work at zero temperature, in the limit of hierarchical singlet fermions. This means we follow the framework of subsection 5.2, that is, we calculate in the effective field theory with particle content of the SM $+N_1$, where the effects of the heavier N_2 and N_3 appear in the dimension-five operator of eqn (5.8).

We define the CP asymmetries in $\Delta L = 1$ scattering (mediated by s - and t -channel Higgs boson exchange) as

$$\hat{\epsilon}_{\alpha\alpha}^s = \frac{\sigma(t^c q \rightarrow N \ell_\alpha) - \bar{\sigma}(\bar{q} \bar{t}^c \rightarrow N \bar{\ell}_\alpha)}{\sigma + \bar{\sigma}}, \quad (5.30)$$

$$\begin{aligned} \hat{\epsilon}_{\alpha\alpha}^t &= \frac{\sigma(q N \rightarrow \bar{t}^c \ell_\alpha) - \bar{\sigma}(\bar{q} N \rightarrow t^c \bar{\ell}_\alpha)}{\sigma + \bar{\sigma}} \\ &= \frac{\sigma(q \bar{\ell}_\alpha \rightarrow \bar{t}^c N) - \bar{\sigma}(\bar{q} \ell_\alpha \rightarrow t^c N)}{\sigma + \bar{\sigma}}, \end{aligned} \quad (5.31)$$

where the cross-sections in the denominator are summed over flavour. The initial state density factors cancel in the ratio, so the cross-sections σ , $\bar{\sigma}$ can be replaced by the matrix elements squared $|\mathcal{M}|^2$, integrated over final state phase space $\int d\Pi$. Separating the tree and loop matrix elements into a coupling constant part c and an amplitude part \mathcal{A} , as in eqn (5.2), the CP asymmetry can be written as in eqn (5.5). The loop amplitude has an imaginary part when there are branch cuts corresponding to intermediate on-shell particles, which can arise here in a bubble on the N line at the $N\phi_\alpha$ vertex, *e.g.* for s -channel Higgs exchange:

$$\text{Im}\{\mathcal{A}_0(t^c q \rightarrow N \ell_\alpha) \mathcal{A}_1^*(t^c q \rightarrow N \ell_\alpha)\} = \mathcal{A}_0(t^c q \rightarrow N \ell_\alpha) \int \mathcal{A}_0^*(t^c q \rightarrow \ell_\alpha \ell'_\beta \phi') d\Pi' \mathcal{A}_0^*(\ell'_\beta \phi' \rightarrow N). \quad (5.32)$$

Here ϕ' and ℓ'_β are the (assumed massless) intermediate on-shell particles, and $d\Pi'$ is the integration over their phase space.

In the scattering process, $c_0 = h_t \lambda_{1\alpha}^*$ and $c_1 = 3h_t \lambda_{1\beta} [m^*]_{\beta\alpha} / v^2$, where h_t is the top Yukawa coupling. The complex coupling constant combination in the scattering processes is clearly the same as in $\epsilon_{\alpha\alpha}$ for decays discussed in section 5.1. To obtain the amplitude ratio [the second ratio in eqn (5.5)], we take, for instance, $\mathcal{A}_0(N \rightarrow \bar{\phi} \bar{\ell}^\alpha) = \bar{u}_\ell P_L u_N$. After performing straightforward spin sums, we find that it is the same for scattering and for N decay, so

$$\hat{\epsilon}_{\alpha\alpha}^s = \hat{\epsilon}_{\alpha\alpha}^t = \epsilon_{\alpha\alpha}. \quad (5.33)$$

CPT and unitarity are realized in the scattering process ($qt^c \rightarrow N \ell_\alpha$) in a similar way to inverse decays. They should hold order by order in perturbation theory, so we work at order $\lambda^2 \lambda_\alpha^2 h_t^2$, and define

$$|\mathcal{M}(qt^c \rightarrow N \ell_\alpha)|^2 = |\mathcal{M}_s|^2 (1 + \epsilon_{\alpha\alpha}), \quad (5.34)$$

where $|\mathcal{M}_s|^2 \propto \lambda_\alpha^2 h_t^2$, and $|\mathcal{M}_s|^2 \epsilon_{\alpha\alpha} \propto \lambda^2 \lambda_\alpha^2 h_t^2$. At order $\lambda^2 \lambda_\alpha^2 h_t^2$, we should also include various $2 \rightarrow 3$ tree diagrams without N_1 in the final state. Following the inverse decay discussion, one can write

$$\begin{aligned} |\mathcal{M}(qt^c \rightarrow X \ell_\alpha)|^2 &= |\mathcal{M}(qt^c \rightarrow N \ell_\alpha)|^2 + \sum_\beta [|\mathcal{M}(qt^c \rightarrow \ell_\beta \phi_\alpha)|^2 - |\mathcal{M}^{\text{os}}(qt^c \rightarrow \ell_\beta \phi_\alpha)|^2] \\ &\quad + \sum_\beta [|\mathcal{M}(qt^c \rightarrow \bar{\ell}_\beta \bar{\phi}_\alpha)|^2 - |\mathcal{M}^{\text{os}}(qt^c \rightarrow \bar{\ell}_\beta \bar{\phi}_\alpha)|^2] \\ &= |\mathcal{M}_s|^2 (1 + \epsilon_{\alpha\alpha}) + |\mathcal{M}(qt^c \rightarrow \ell \phi_\alpha)|^2 - |\mathcal{M}_s|^2 (1 + \epsilon_{\alpha\alpha}) \frac{(1 + \epsilon)}{2} \end{aligned}$$

$$\begin{aligned}
& + |\mathcal{M}(qt^c \rightarrow \bar{\ell}\bar{\phi}\ell_\alpha)|^2 - |\mathcal{M}_s|^2(1 + \epsilon_{\alpha\alpha})\frac{(1 - \epsilon)}{2} \\
= & \sum_{\beta} [|\mathcal{M}(qt^c \rightarrow \ell_\beta\phi\ell_\alpha)|^2 + |\mathcal{M}(qt^c \rightarrow \bar{\ell}_\beta\bar{\phi}\ell_\alpha)|^2] , \tag{5.35}
\end{aligned}$$

where, in the narrow width approximation,

$$|\mathcal{M}^{\text{os}}(qt^c \rightarrow \bar{\ell}_\beta\bar{\phi}\ell_\alpha)|^2 = |\mathcal{M}(qt^c \rightarrow N\ell_\alpha)|^2 \times B_{\phi\ell_\beta}^N \tag{5.36}$$

In eqn (5.35), the CP asymmetry $\epsilon_{\alpha\alpha}$ has disappeared in the final result, so if we repeat the calculation for the CP conjugate initial state, $\bar{q}\bar{t}^c$, we should obtain the same result, verifying that a CP asymmetry in $qt^c \rightarrow N\ell_\alpha$ is consistent with CPT and unitarity. Furthermore, the final result of eqn (5.35) is reassuring, because the unstable state N has disappeared. There is no CP violation in the total rate for $qt^c \rightarrow$ asymptotic (stable) final states, but CP violation in the partial rate to the unstable N is possible. This can be relevant to the final value of the baryon symmetry when some of the lepton flavours are weakly washed out.

6 Boltzmann Equations

The lepton (and baryon) asymmetry produced via leptogenesis, can be computed by solving the Boltzmann equations (BE). These describe the out-of-equilibrium dynamics of the processes involving the heavy singlet fermions. The aim of this chapter is to derive the basic Boltzmann equations, restricted to the non-supersymmetric framework, and to the case where the processes that generate a lepton asymmetry involve just the lightest singlet fermion N_1 . Modifications from supersymmetry (see *e.g.* [186]) are described in Section 10.1. Possible contributions from heavier singlet neutrinos $N_{2,3}$ [65, 187, 188] are considered in Section 10.2.1.

To derive the BE one has to consider a large set of processes, as well as abundances and density asymmetries of many types of particles, and the use of notations in extended form can result in rather cumbersome equations. Thus, we start in Section 6.1 by introducing some compact notation. To make the navigation through the details in the following subsections easier, we also present the general structure of the final equations.

Simple Boltzmann equations, taking into account decays, inverse decays, and $2 \leftrightarrow 2$ scatterings mediated by N_1 exchange, are derived in Section 6.2 and are given in eqns (6.16) and (6.28). In Section 6.3 we include scattering processes with N_1 on an external leg, and we also discuss $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ processes, since the CP asymmetries of their off-shell parts must be taken into account for consistency. The corresponding BE for the evolution of the N_1 density is given in eqn (6.51), and for the relevant flavour asymmetry in eqns (6.53), (6.54) and (6.56).

We emphasize that to reach quantitatively accurate results, one has to take into account (i) some relevant interactions that do not involve the Majorana neutrinos, and (ii) flavour effects. These tasks are taken in the following sections.

6.1 Notation

In this subsection we introduce our notation. A brief introduction to particle number densities and rates in the early Universe can be found on Appendix 13. We denote the thermally averaged rate for an initial state A to go into the final state B , summed over initial and final spin and gauge degrees of freedom, as (see eqn 13.14)

$$\gamma_B^A \equiv \gamma(A \rightarrow B). \quad (6.1)$$

The difference between the rates of CP-conjugate processes is written as

$$\Delta\gamma_B^A \equiv \gamma_B^A - \gamma_{\bar{B}}^{\bar{A}}. \quad (6.2)$$

We denote by n_a the number density for the particle a , by n_a^{eq} its equilibrium density, and by s the entropy density (see Appendix 13). We define:

$$Y_a \equiv \frac{n_a}{s}, \quad y_a \equiv \frac{Y_a}{Y_a^{\text{eq}}}, \quad \Delta y_a \equiv y_a - y_{\bar{a}}. \quad (6.3)$$

Thus, we write all particle densities (Y_a) normalized to the entropy density. To simplify the expressions, we rescale the densities Y_a by the equilibrium density of the corresponding particle ($Y_a^{\text{eq}} = n_a^{\text{eq}}/s$). We denote the asymmetries of the rescaled densities by Δy_a .

The difference between a process and its time reversed, weighted by the densities of the initial state particles, is defined as

$$[A \leftrightarrow B] \equiv \left(\prod_{i=1}^n y_{a_i} \right) \gamma_B^A - \left(\prod_{j=1}^m y_{b_j} \right) \gamma_A^B, \quad (6.4)$$

where the state A contains the particles a_1, \dots, a_n while the state B contains the particles b_1, \dots, b_m . We consider only processes in which at most one intermediate state heavy neutrino N_1 can be on-shell. In these cases, a primed notation $\gamma_B^{\prime A}$ (and $[A \leftrightarrow B]'$) refers to rates with the resonant intermediate state

(RIS) subtracted. In other words, for the process $A \rightarrow B$, we distinguish the on-shell piece $\gamma_B^{\text{os}A}$ from the off-shell piece $\gamma_B^{\prime A}$:

$$\gamma_B^{\prime A} \equiv \gamma_B^A - \gamma_B^{\text{os}A}. \quad (6.5)$$

In the simple case where only $2 \leftrightarrow 2$ scatterings are considered, the on-shell part is just

$$\gamma_B^{\text{os}A} \equiv \gamma_{N_1}^A B_B^{N_1}, \quad (6.6)$$

where $B_B^{N_1}$ is the branching ratio for N_1 decays into the final state B . To include processes of higher order in the couplings, eqn (6.6) needs to be generalized [185].

We introduce from the start a set of BE that allow a proper treatment of flavour effects. To do that, we write down the BE for the evolution of the density of the heavy singlet fermions N and of the asymmetry for a generic lepton flavour α . Below $T \sim 10^{12}$ GeV ($T \sim 10^9$ GeV), reactions mediated by the τ (μ) Yukawa couplings become faster than the Universe expansion rate, possibly resolving the flavour composition of these lepton doublets (see Appendix 15.4). As discussed in Appendix 15, if the charged lepton Yukawa interactions are fast compared to both H and Γ_{ID} , the equations of motion for the lepton asymmetry reduce to the BE in the flavour basis. Assuming that the flavour basis does not change during leptogenesis, we can work with simple projections onto flavour of all the relevant quantities. However, we still adopt a double-index notation for most of the flavour-dependent quantities, as a reminder that they correspond to diagonal elements of matrices in flavour space. For example, we denote the density of leptons of flavour α by

$$Y_L^{\alpha\alpha} \equiv Y_{\ell_\alpha} + Y_{e_\alpha}, \quad (6.7)$$

where Y_{ℓ_α} is the density of the two gauge degrees of freedom in ℓ_α . The inclusion of the density Y_{e_α} for the right-handed charged lepton e_α is required when (some of) the L -conserving charged lepton Yukawa interactions become fast compared to the Universe expansion rate, since in this case they transfer part of the asymmetry to the right handed degrees of freedom (see section 8).

We define the asymmetry in the densities of leptons and antileptons of flavour α as

$$Y_{\Delta L_\alpha} \equiv Y_L^{\alpha\alpha} - Y_{\bar{L}}^{\alpha\alpha}. \quad (6.8)$$

When the rates of charged lepton Yukawa interactions are negligible, the lepton asymmetry is stored only in the lepton doublets, and one simply has $Y_{\Delta L_\alpha} = Y_{\ell_\alpha} - Y_{\bar{\ell}_\alpha}$.

As we explain below, the following asymmetries are particularly useful in the context of leptogenesis:

$$Y_{\Delta_\alpha} \equiv \frac{Y_{\Delta B}}{3} - Y_{\Delta L_\alpha}, \quad (6.9)$$

where $Y_{\Delta B}$ is the baryon asymmetry to entropy ratio. For time derivative, we use

$$\dot{Y} \equiv \frac{sH_1}{z} \frac{dY}{dz}, \quad (6.10)$$

where

$$z \equiv M_1/T, \quad H_1 \equiv H(T = M_1). \quad (6.11)$$

We split the contributions to the evolution equation for $Y_L^{\alpha\alpha}$ into three parts:

$$\dot{Y}_L^{\alpha\alpha} = \left(\dot{Y}_L^{\alpha\alpha}\right)_I + \left(\dot{Y}_L^{\alpha\alpha}\right)_{II} + \left(\dot{Y}_L^{\alpha\alpha}\right)_{\text{sphal}}. \quad (6.12)$$

1. $\left(\dot{Y}_L^{\alpha\alpha}\right)_I$ includes contributions of $\mathcal{O}(\lambda^2)$ and of $\mathcal{O}(\lambda^4)$. It is evaluated in section 6.2.
2. $\left(\dot{Y}_L^{\alpha\alpha}\right)_{II}$ includes contributions of $\mathcal{O}(\lambda^2 h_t^2, \lambda^4 h_t^2)$ and $\mathcal{O}(\lambda^2 g^2, \lambda^4 g^2)$, with g a generic gauge coupling constant. It is evaluated in section 6.3.
3. $\left(\dot{Y}_L^{\alpha\alpha}\right)_{\text{sphal}}$ represents the change in the lepton densities due to electroweak sphalerons.

As concerns the sphaleron effects, although their precise rates are hard to estimate, it is known that below $T \simeq 10^{12}$ GeV they are a source of rapid baryon number violation, but they leave $B - L$ unchanged. More precisely, sphalerons generate the same change in the baryon and lepton number of each generation,

$$(\dot{Y}_{\Delta L_\alpha})_{\text{sphal}} = \frac{1}{3}(\dot{Y}_{\Delta B})_{\text{sphal}}, \quad (6.13)$$

leaving unchanged the charge densities of eqn (6.9). Hence, it is convenient to write an equation directly for these quantities. By subtracting from eqn (6.12) the analogous equation for the density of antileptons $\dot{Y}_L^{\alpha\alpha}$ and by subtracting again the result from the equation that describes the evolution of the baryon asymmetry, $(\dot{Y}_{\Delta B})/3 = (\dot{Y}_{\Delta B})_{\text{sphal}}/3$, one obtains the evolution equations

$$\dot{Y}_{\Delta_\alpha} = -\left(\dot{Y}_{\Delta L_\alpha}\right)_I - \left(\dot{Y}_{\Delta L_\alpha}\right)_{II} \quad (6.14)$$

that do not depend on the sphaleron rates.

The heavy fermions N_1 are treated as quasi-stable particles on the time scale of the Universe expansion. This is justified by the fact that leptogenesis requires that the N_1 lifetime is of the order of the expansion time H_1^{-1} . The final baryon asymmetry depends on the density of the neutrinos N_1 as a function of time, so a Boltzmann equation for Y_{N_1} is needed. It is convenient to split also this equation into two parts:

$$\dot{Y}_{N_1} = \left(\dot{Y}_{N_1}\right)_I + \left(\dot{Y}_{N_1}\right)_{II}. \quad (6.15)$$

The term $(\dot{Y}_{N_1})_I$ includes the contributions of terms up to $\mathcal{O}(\lambda^2)$ and is evaluated in section 6.2. The term $(\dot{Y}_{N_1})_{II}$ includes the contributions up to $\mathcal{O}(\lambda^2 h_i^2)$ or $\mathcal{O}(\lambda^2 g^2)$ and is evaluated in section 6.3.

6.2 The $\mathcal{O}(\lambda^2)$ and $\mathcal{O}(\lambda^4)$ terms

This section aims to obtain the basic BE which depend only on the neutrino Yukawa coupling λ , including only the terms $(\dot{Y}_{\Delta L_\alpha})_I$ of eqn (6.14) and $(\dot{Y}_{N_1})_I$ of eqn (6.15). The SM gauge interactions are assumed to be fast, which ensures kinetic equilibrium for the particle distributions. All other SM interactions, including sphalerons, are neglected. The processes of N_1 decay and inverse decay and two-to-two scattering mediated by the N_i 's are included. However, the latter is important mainly to subtract real intermediate states, but its effects are small in the temperature range $T < 10^{12}$ GeV and in a first approximation can be neglected (see the appendix of [62] for a brief discussion, and [59] for a detailed one).

The Boltzmann equation for Y_{N_1} , including only decays and inverse decays, is given by

$$\begin{aligned} \left(\dot{Y}_{N_1}\right)_I &= \sum_{\beta} \left\{ [\ell_{\beta}\phi \leftrightarrow N_1] + [\bar{\ell}_{\beta}\bar{\phi} \leftrightarrow N_1] \right\} \\ &= - \sum_{\beta} \left[y_{N_1} \left(\gamma_{\phi\ell_{\beta}}^{N_1} + \gamma_{\bar{\phi}\bar{\ell}_{\beta}}^{N_1} \right) - y_{\phi} y_{\ell_{\beta}} \gamma_{N_1}^{\phi\ell_{\beta}} - y_{\bar{\phi}} y_{\bar{\ell}_{\beta}} \gamma_{N_1}^{\bar{\phi}\bar{\ell}_{\beta}} \right] \simeq - (y_{N_1} - 1) \gamma_{N_1 \rightarrow 2}, \end{aligned} \quad (6.16)$$

where in the last expression we have approximated the ϕ and ℓ_{β} number densities with their equilibrium densities, neglecting small corrections $\propto (\gamma_{N_1}^{\bar{\phi}\bar{\ell}_{\beta}} - \gamma_{N_1}^{\phi\ell_{\beta}})(y_{\bar{\ell}_{\beta}} - y_{\ell_{\beta}})$ that are second order in the small quantity $\epsilon_{\beta\beta}$. We use

$$\gamma_{N_1 \rightarrow 2} = \sum_{\beta} (\gamma_{\ell_{\beta}\phi}^{N_1} + \gamma_{\bar{\ell}_{\beta}\bar{\phi}}^{N_1}) \quad (6.17)$$

for the thermally averaged two body N_1 decay rate.

To obtain the BE for the lepton asymmetry we should work to order λ^4 , because $\epsilon_{\alpha\alpha} = \mathcal{O}(\lambda^4)$. At this order, the doublet leptons participate in the following three types of processes:

- (i) $1 \leftrightarrow 2$ processes: the (CP violating) decays $N_1 \rightarrow \ell_{\alpha}\phi$, and inverse decays $\ell_{\alpha}\phi \rightarrow N_1$;

- (ii) $2 \leftrightarrow 2$ scatterings mediated by s -channel N_1 exchange: $\ell_\alpha \phi \leftrightarrow \ell_\beta \phi$ with $\beta \neq \alpha$, and $\ell_\alpha \phi \leftrightarrow \bar{\ell}_\beta \bar{\phi}$;
- (iii) $2 \leftrightarrow 2$ scatterings mediated by t - and u -channel N_1 exchange: $\bar{\phi} \bar{\phi} \leftrightarrow \ell_\alpha \ell_\beta$ and $\phi \bar{\phi} \leftrightarrow \ell_\alpha \bar{\ell}_\beta$. (Neglecting provisionally thermal effects, no RIS can appear in this case, and hence there are no on-shell contributions to be subtracted.)

Accordingly, at this order the evolution equation for the density of the lepton flavour α reads

$$\left(\dot{Y}_L^{\alpha\alpha}\right)_I = \left(\dot{Y}_L^{\alpha\alpha}\right)_{1\leftrightarrow 2} + \left(\dot{Y}_L^{\alpha\alpha}\right)_{2\leftrightarrow 2}^{N_s} + \left(\dot{Y}_L^{\alpha\alpha}\right)_{2\leftrightarrow 2}^{N_t}, \quad (6.18)$$

where

$$\left(\dot{Y}_L^{\alpha\alpha}\right)_{1\leftrightarrow 2} = [N_1 \leftrightarrow \ell_\alpha \phi], \quad (6.19)$$

$$\left(\dot{Y}_L^{\alpha\alpha}\right)_{2\leftrightarrow 2}^{N_s} = \sum_{\beta} [\bar{\ell}_\beta \bar{\phi} \leftrightarrow \ell_\alpha \phi]' + \sum_{\beta \neq \alpha} [\ell_\beta \phi \leftrightarrow \ell_\alpha \phi]', \quad (6.20)$$

$$\left(\dot{Y}_L^{\alpha\alpha}\right)_{2\leftrightarrow 2}^{N_t} = \sum_{\beta} \{[\phi \bar{\phi} \leftrightarrow \ell_\alpha \bar{\ell}_\beta] + (1 + \delta_{\alpha\beta})[\bar{\phi} \bar{\phi} \leftrightarrow \ell_\alpha \ell_\beta]\}. \quad (6.21)$$

We first focus on (i) of eqn (6.19) and (ii) of eqn (6.20) that give rise to the source term for the asymmetry. The processes (iii) of eqn (6.21) contribute only to the washout (at $\mathcal{O}(\lambda^4)$) and will be added at the end of the section. As discussed in section 5.3, some care is required in combining (i) and (ii): In the $2 \leftrightarrow 2$ scattering, the contribution from on-shell s -channel N_1 -exchange is already accounted for by the decays and inverse decays. This contribution should be removed, to avoid double-counting, by using a subtracted $|\mathcal{M}|^2$ (see eqn 5.29), as indicated by the primed notation in eqn. (6.20). That is, we include only the off-shell part of the scatterings rate density

$$\gamma'_{\ell_\beta \phi}{}^{\ell_\alpha \phi} = \gamma_{\ell_\beta \phi}{}^{\ell_\alpha \phi} - \gamma_{N_1}^{\ell_\alpha \phi} B_{\ell_\beta \phi}^{N_1} \quad (6.22)$$

that has a CP asymmetry of the same order as the CP asymmetries of decays and inverse decays. The scatterings $\ell_\alpha \phi \leftrightarrow \ell_\beta \phi$ (with $\beta \neq \alpha$) are treated in a similar way.

The equations for the comoving number densities of the lepton doublet, and of the anti-lepton doublet, are therefore

$$\left(\dot{Y}_{\ell_\alpha}\right)_I = [N_1 \leftrightarrow \ell_\alpha \phi] + \sum_{\beta} [\bar{\ell}_\beta \bar{\phi} \leftrightarrow \ell_\alpha \phi]' + \sum_{\beta} [\ell_\beta \phi \leftrightarrow \ell_\alpha \phi]', \quad (6.23)$$

$$\left(\dot{Y}_{\bar{\ell}_\alpha}\right)_I = [N_1 \leftrightarrow \bar{\ell}_\alpha \bar{\phi}] + \sum_{\beta} [\ell_\beta \phi \leftrightarrow \bar{\ell}_\alpha \bar{\phi}]' + \sum_{\beta} [\bar{\ell}_\beta \bar{\phi} \leftrightarrow \bar{\ell}_\alpha \bar{\phi}]'. \quad (6.24)$$

Note that the sum in the last two terms of the two equations has been extended to include the contributions from $\beta = \alpha$ which cancel in the difference between the process and its time reversed. Eqns (6.23) and (6.24) can be written more explicitly as

$$\left(\dot{Y}_{\ell_\alpha}\right)_I = y_{N_1} \gamma_{\ell_\alpha \phi}^{N_1} + \sum_{\beta} \left(y_\phi y_{\ell_\beta} \gamma'_{\ell_\alpha \phi}{}^{\ell_\beta \phi} + y_{\bar{\phi}} y_{\bar{\ell}_\beta} \gamma'_{\ell_\alpha \phi}{}^{\bar{\ell}_\beta \bar{\phi}} \right) - y_\phi y_{\ell_\alpha} \sum_{\beta} \left(\gamma_{\ell_\beta \phi}{}^{\ell_\alpha \phi} + \gamma_{\bar{\ell}_\beta \bar{\phi}}{}^{\ell_\alpha \phi} \right), \quad (6.25)$$

$$\left(\dot{Y}_{\bar{\ell}_\alpha}\right)_I = y_{N_1} \gamma_{\bar{\ell}_\alpha \bar{\phi}}^{N_1} + \sum_{\beta} \left(y_{\bar{\phi}} y_{\bar{\ell}_\beta} \gamma'_{\bar{\ell}_\alpha \bar{\phi}}{}^{\bar{\ell}_\beta \bar{\phi}} + y_\phi y_{\ell_\beta} \gamma'_{\bar{\ell}_\alpha \bar{\phi}}{}^{\ell_\beta \phi} \right) - y_{\bar{\phi}} y_{\bar{\ell}_\alpha} \sum_{\beta} \left(\gamma_{\ell_\beta \phi}{}^{\bar{\ell}_\alpha \bar{\phi}} + \gamma_{\bar{\ell}_\beta \bar{\phi}}{}^{\bar{\ell}_\alpha \bar{\phi}} \right). \quad (6.26)$$

Using eqns (6.5) and (6.6) and CPT, we rewrite eqn (6.25) as follows:

$$\begin{aligned} \left(\dot{Y}_{\ell_\alpha}\right)_I &= \gamma_{\ell_\alpha \phi}^{N_1} \left(y_{N_1} - \sum_{\beta} \left[y_\phi y_{\ell_\beta} B_{\ell_\beta \phi}^{N_1} + y_{\bar{\phi}} y_{\bar{\ell}_\beta} B_{\bar{\ell}_\beta \bar{\phi}}^{N_1} \right] \right) - y_\phi y_{\ell_\alpha} \gamma_{N_1}^{\ell_\alpha \phi} \left[1 - \sum_{\beta} (B_{\ell_\beta \phi}^{N_1} + B_{\bar{\ell}_\beta \bar{\phi}}^{N_1}) \right] \\ &- y_\phi y_{\ell_\alpha} \sum_{\beta} \left[\gamma_{\ell_\beta \phi}{}^{\ell_\alpha \phi} + \gamma_{\bar{\ell}_\beta \bar{\phi}}{}^{\ell_\alpha \phi} \right] + \sum_{\beta} \left[y_\phi y_{\ell_\beta} \gamma_{\ell_\alpha \phi}{}^{\ell_\beta \phi} + y_{\bar{\phi}} y_{\bar{\ell}_\beta} \gamma_{\ell_\alpha \phi}{}^{\bar{\ell}_\beta \bar{\phi}} \right], \end{aligned} \quad (6.27)$$

and similarly for the analogous terms in eqn. (6.26). Some comments are in order with regard to eqn (6.27):

1. At the order in λ we are working in this section $\sum_{\beta} (B_{\ell_{\beta}\phi}^{N_1} + B_{\bar{\ell}_{\beta}\bar{\phi}}^{N_1}) = 1$. Consequently, the second term in the first line vanishes.
2. As concerns the second line of eqn. (6.27), the first term is proportional to $\gamma(\ell_{\alpha}\phi \rightarrow \text{anything})$, while the second term is the sum of $\gamma(\text{anything} \rightarrow \phi\ell_{\alpha})$ and terms that are second order in the asymmetry. We know from eqn (5.23) that there can be no CP asymmetry in differences of the form $\gamma(X \leftrightarrow \text{anything}) - \gamma(\bar{X} \leftrightarrow \text{anything})$. Consequently, these terms do not contribute to the CP asymmetry.
3. The source term ($\propto \epsilon_{\alpha\alpha}$) in the BE arises from the first term of (6.27) and from its analog in $Y_{\bar{\ell}_{\alpha}}$. We can set the sum of the bracketed branching ratios to 1, because corrections to this approximation are second order in the asymmetry. We thus obtain the correct behaviour: no asymmetry can be generated in thermal equilibrium ($y_{N_1} = 1$).
4. The last term in eqn. (6.27) does not contribute to the washout of $y_{\ell_{\alpha}}$. Nevertheless this term should not be dropped since, as it will become clear in section 9.3, it does induce washouts for the charge density $Y_{\Delta_{\alpha}}$ in eqn. (6.9).

By subtracting (6.26) from (6.25), and including the contribution of the t and u -channel processes (iii) of eqn.(6.21), we obtain the complete BE at $\mathcal{O}(\lambda^4)$:

$$\begin{aligned} \left(\dot{Y}_{\Delta L_{\alpha}}\right)_I &= (y_{N_1} - 1)\Delta\gamma_{\ell_{\alpha}\phi}^{N_1} - (\Delta y_{\ell_{\alpha}} + \Delta y_{\phi}) \sum_{\beta} \left(\gamma_{\ell_{\beta}\phi}^{\ell_{\alpha}\phi} + \gamma_{\bar{\ell}_{\beta}\bar{\phi}}^{\ell_{\alpha}\phi}\right) \\ &+ \sum_{\beta} (\Delta y_{\ell_{\beta}} + \Delta y_{\phi}) \left(\gamma_{\ell_{\alpha}\phi}^{\ell_{\beta}\phi} - \gamma_{\bar{\ell}_{\alpha}\bar{\phi}}^{\ell_{\beta}\phi}\right) + \left(\dot{Y}_{\Delta L_{\alpha}}\right)_{2\leftrightarrow 2}^{w, N_t}. \end{aligned} \quad (6.28)$$

The term

$$\left(\dot{Y}_{\Delta L_{\alpha}}\right)_{2\leftrightarrow 2}^{w, N_t} = - \sum_{\beta} \left[(1 + \delta_{\alpha\beta})(\Delta y_{\ell_{\alpha}} + \Delta y_{\ell_{\beta}} + 2\Delta y_{\phi}) \gamma_{\bar{\phi}\bar{\phi}}^{\ell_{\alpha}\ell_{\beta}} + (\Delta y_{\ell_{\alpha}} - \Delta y_{\ell_{\beta}}) \gamma_{\bar{\phi}\bar{\phi}}^{\ell_{\alpha}\bar{\ell}_{\beta}} \right] \quad (6.29)$$

is straightforwardly obtained by subtracting from eqn (6.21) the analogous equation for $\bar{\ell}_{\alpha}$. In eqns (6.28) and (6.29) we approximate the washout rates with their tree level values, and we linearize in the asymmetry densities Δy .

Note that there are no resonant contributions to the washout from the second line of eqn (6.28) since the on-shell parts contained in $\gamma_{\ell_{\alpha}\phi}^{\ell_{\beta}\phi}$ and $\gamma_{\bar{\ell}_{\alpha}\bar{\phi}}^{\ell_{\beta}\phi}$ cancel in the difference. The washout term in the first line of eqn. (6.28) can be written as the sum of resonant ($\mathcal{O}(\lambda^2)$) and non-resonant ($\mathcal{O}(\lambda^4)$) parts. By dropping all the subleading non-resonant terms, we obtain an approximate expression, valid at $\mathcal{O}(\lambda^2)$:

$$\begin{aligned} \left(\dot{Y}_{\Delta L_{\alpha}}\right)_I &\simeq (y_{N_1} - 1)\Delta\gamma_{\ell_{\alpha}\phi}^{N_1} - (\Delta y_{\ell_{\alpha}} + \Delta y_{\phi})\gamma_{\ell_{\alpha}\phi}^{N_1} \\ &\simeq \left[(y_{N_1} - 1)\epsilon_{\alpha\alpha} - \frac{1}{2}(\Delta y_{\ell_{\alpha}} + \Delta y_{\phi})\frac{\tilde{m}_{\alpha\alpha}}{\tilde{m}} \right] \gamma_{N_1 \rightarrow 2}, \end{aligned} \quad (6.30)$$

where $\epsilon_{\alpha\alpha}$ is defined in eqn (4.2), $m_{\alpha\alpha}/\tilde{m} = \Gamma(N_1 \rightarrow \ell_{\alpha}\phi, \bar{\ell}_{\alpha}\bar{\phi})/\Gamma_D$, and $\gamma_{N_1 \rightarrow 2}$ is defined in eqn (6.17). Using eqns (6.10), (13.2) and (13.18), and provisionally neglecting the contribution to the washout of the Higgs asymmetry Δy_{ϕ} (see section 8), eqn. (6.30) can be written in a more explicit form:

$$\frac{d}{dz} (Y_{\Delta L_{\alpha}})_I = \frac{z}{H_1} \left((Y_{N_1} - Y_{N_1}^{\text{eq}}) \frac{K_1(z)}{K_2(z)} \epsilon_{\alpha\alpha} - \frac{g_{N_1}}{4g_{\ell}} z^2 K_1(z) Y_{\Delta L_{\alpha}} \frac{\tilde{m}_{\alpha\alpha}}{\tilde{m}} \right) \Gamma_D. \quad (6.31)$$

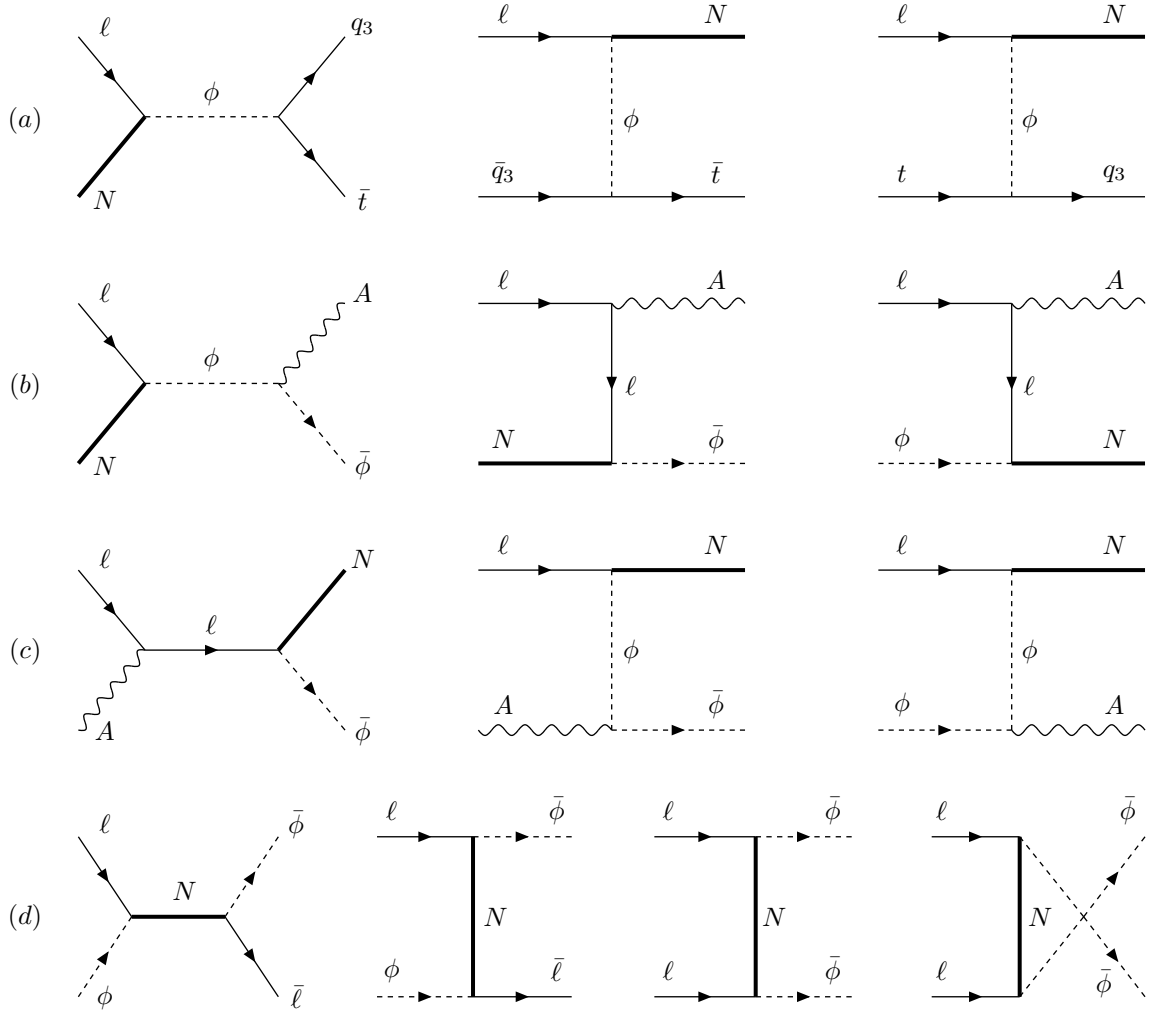


Figure 6.1: Diagrams for various $2 \leftrightarrow 2$ scattering processes: (a) scatterings with the top-quarks, (b), (c) scatterings with the gauge bosons ($A = B, W_i$ with $i = 1, 2, 3$), (d) $\Delta L = 2$ scatterings mediated by N_1 .

6.3 The $\mathcal{O}(h_t^2 \lambda^2)$ and $\mathcal{O}(h_t^2 \lambda^4)$ terms

In this section, we include processes involving the top Yukawa coupling h_t . Processes involving gauge bosons can be included in a similar way and we add them in our final expressions.

We denote the left-handed third-generation quark doublet by q_3 , and the $SU(2)$ -singlet top by t . The inclusion of $1 \leftrightarrow 3$ decays and inverse decays such as $N_1 \leftrightarrow \ell_\alpha \bar{q}_3 t$, and of $N_1 \ell_\alpha \leftrightarrow q_3 \bar{t}$ scatterings mediated by Higgs exchange, follows lines analogous to those presented in the previous section. For the $\mathcal{O}(h_t^2 \lambda^2)$ contributions to the evolution of the N_1 density, we obtain:

$$\left(\dot{Y}_{N_1}\right)_{II} = -(y_{N_1} - 1) [\gamma_{N \rightarrow 3} + \gamma_{\text{top}}^{2 \leftrightarrow 2}]. \quad (6.32)$$

Here,

$$\gamma_{N \rightarrow 3} \equiv \sum_{\beta} (\gamma_{\ell_{\beta} \bar{q}_3 t}^{N_1} + \gamma_{\bar{\ell}_{\beta} q_3 \bar{t}}^{N_1}), \quad (6.33)$$

is the contribution from decays into three-body final states, while

$$\gamma_{\text{top}}^{2\leftrightarrow 2} = \sum_{\beta} \left(\gamma_{q_3\bar{t}}^{N_1\ell_\beta} + \gamma_{\bar{q}_3t}^{N_1\bar{\ell}_\beta} + \gamma_{\ell_\beta t}^{N_1q_3} + \gamma_{\bar{\ell}_\beta\bar{t}}^{N_1\bar{q}_3} + \gamma_{\bar{\ell}_\beta q_3}^{N_1t} + \gamma_{\ell_\beta\bar{q}_3}^{N_1\bar{t}} \right), \quad (6.34)$$

is the contribution from Higgs mediated scatterings: the first two terms correspond to s -channel Higgs exchange, while the other four (that are all equal at leading order) correspond to t - and u -channel Higgs exchange (see fig 6.1(a)).

Regarding the evolution of the lepton asymmetries, the derivation of the BE is more subtle. Once we include the CP violating asymmetries in $1 \leftrightarrow 3$ (inverse) decays, like $N_1 \leftrightarrow \ell_\alpha \bar{q}_3 t$, and in $2 \leftrightarrow 2$ scatterings, like $N_1 \ell_\alpha \leftrightarrow q_3 \bar{t}$, we must include also the asymmetries of various off-shell $2 \leftrightarrow 3$ scatterings, which contribute to the source term at the same order in the couplings. Accordingly, we write the term $(\dot{Y}_L^{\alpha\alpha})_{II}$ of eqn (6.12) as follows:

$$\left(\dot{Y}_L^{\alpha\alpha} \right)_{II} = \left(\dot{Y}_L^{\alpha\alpha} \right)_{2\leftrightarrow 3} + \left(\dot{Y}_L^{\alpha\alpha} \right)_{2\leftrightarrow 3}^{\text{sub}} + \left(\dot{Y}_L^{\alpha\alpha} \right)_{2\leftrightarrow 3}^{N_t}, \quad (6.35)$$

where

$$\left(\dot{Y}_L^{\alpha\alpha} \right)_{2\leftrightarrow 3} = [N_1 \leftrightarrow \ell_\alpha \bar{q}_3 t] + [q_3 \bar{t} \leftrightarrow N_1 \ell_\alpha] + [N_1 \bar{t} \leftrightarrow \bar{q}_3 \ell_\alpha] + [N_1 q_3 \leftrightarrow t \ell_\alpha]; \quad (6.36)$$

$$\begin{aligned} \left(\dot{Y}_L^{\alpha\alpha} \right)_{2\leftrightarrow 3}^{\text{sub}} &= \sum_{\beta \neq \alpha} \{ [\ell_\beta \phi \leftrightarrow \bar{q}_3 t \ell_\alpha]' + [\ell_\beta \phi q_3 \leftrightarrow t \ell_\alpha]' + [\ell_\beta \phi \bar{t} \leftrightarrow \bar{q}_3 \ell_\alpha]' \\ &\quad + [\ell_\beta \bar{q}_3 t \leftrightarrow \ell_\alpha \phi]' + [\ell_\beta \bar{q}_3 \leftrightarrow \ell_\alpha \phi \bar{t}]' + [\ell_\beta t \leftrightarrow \ell_\alpha \phi q_3]' \} \\ &+ \sum_{\beta} \{ [\bar{\ell}_\beta \bar{\phi} \leftrightarrow \ell_\alpha \bar{q}_3 t]' + [\bar{\ell}_\beta \bar{\phi} q_3 \leftrightarrow t \ell_\alpha]' + [\bar{\ell}_\beta \bar{\phi} \bar{t} \leftrightarrow \bar{q}_3 \ell_\alpha]' + [\bar{\ell}_\beta q_3 \bar{t} \leftrightarrow \ell_\alpha \phi]' \\ &\quad + [\bar{\ell}_\beta q_3 \leftrightarrow \ell_\alpha \phi \bar{t}]' + [\bar{\ell}_\beta \bar{t} \leftrightarrow \ell_\alpha \phi \bar{q}_3]' + [\bar{q}_3 t \leftrightarrow \ell_\alpha \phi \bar{\ell}_\beta]' + [q_3 \bar{t} \leftrightarrow \ell_\alpha \phi \bar{\ell}_\beta]' \} \\ &+ \sum_{\beta} (1 + \delta_{\alpha\beta}) [q_3 \bar{t} \leftrightarrow \ell_\alpha \phi \ell_\beta]'; \end{aligned} \quad (6.37)$$

$$\begin{aligned} \left(\dot{Y}_L^{\alpha\alpha} \right)_{2\leftrightarrow 3}^{N_t} &= \sum_{\beta \neq \alpha} \{ [\ell_\beta q_3 \bar{t} \leftrightarrow \ell_\alpha \bar{\phi}] + [\ell_\beta \bar{\phi} \leftrightarrow \ell_\alpha q_3 \bar{t}] + [\ell_\beta \bar{\phi} t \leftrightarrow \ell_\alpha q_3] + [\ell_\beta \bar{\phi} \bar{q}_3 \leftrightarrow \ell_\alpha \bar{t}] \} \\ &+ \sum_{\beta} \{ [q_3 \bar{t} \bar{\phi} \leftrightarrow \bar{\ell}_\beta \ell_\alpha] + [\bar{q}_3 t \bar{\phi} \leftrightarrow \bar{\ell}_\beta \ell_\alpha] + [\bar{q}_3 \bar{\phi} \leftrightarrow \bar{\ell}_\beta \ell_\alpha \bar{t}] + [t \bar{\phi} \leftrightarrow \bar{\ell}_\beta \ell_\alpha \bar{q}_3] \} \\ &+ \sum_{\beta} (1 + \delta_{\alpha\beta}) \{ [\bar{t} \bar{\phi} \leftrightarrow \ell_\alpha \bar{q}_3 \ell_\beta] + [q_3 \bar{t} \bar{\phi} \leftrightarrow \ell_\alpha \ell_\beta] + [q_3 \bar{\phi} \leftrightarrow \ell_\alpha t \ell_\beta] \}. \end{aligned} \quad (6.38)$$

As in the previous section, the asymmetries in the off-shell $2 \leftrightarrow 3$ rates in eqn (6.37) can be estimated by relating them to the asymmetries of the corresponding on-shell parts. However, for $2 \leftrightarrow 3$ scatterings, the definition of the on-shell part is more subtle, because after a real N_1 is produced in a collision, it has a certain probability to scatter before decaying. Consider, for example, the process $\ell_\beta \phi q_3 \rightarrow t \ell_\alpha$. The contribution to this process from the exchange of an on-shell N_1 corresponds to the production process $\ell_\beta \phi \rightarrow N_1$, followed by the scattering $N_1 + q_3 \rightarrow t \ell_\alpha$ that is mediated by a Higgs in the t channel. Processes of this kind can generally be written as $AX \rightarrow Y$ where A denotes a possible state to which N_1 can decay. The corresponding on-shell rate is then

$$\gamma_Y^{\text{os}AX} = \gamma_{N_1}^A P_Y^{N_1 X}, \quad (6.39)$$

where $P_Y^{N_1 X}$ is the probability that N_1 scatters with X to produce Y . Processes in which the on-shell N_1 can disappear only by decaying (as for example $\ell_\beta \phi \leftrightarrow \ell_\alpha \bar{q}_3 t$ or $\ell_\beta t \leftrightarrow \ell_\alpha \phi q_3$) can generally be written as $A \rightarrow B$ or as $X \rightarrow BY$, where both A and B denote possible final states for N_1 decays. The corresponding

on-shell rates are

$$\gamma_B^{\text{os}A} = \gamma_{N_1}^A P_B^{N_1}, \quad (6.40)$$

$$\gamma_{BY}^{\text{os}X} = \gamma_{N_1 Y}^X P_B^{N_1}. \quad (6.41)$$

Note that because of the fact that in the dense plasma N_1 can be scattered inelastically before decaying, as described by eqn (6.39), the quantities $P_B^{N_1}$ in eqns (6.40) and (6.41) differ from the usual notion of branching ratios at zero temperature. In particular, scattering rates should also be included in normalizing properly the decay probabilities. The quantities P_b^a then denote the general probabilities that N_1 contained in state a ends up producing a state b . In the case under discussion, we have, for example,

$$\begin{aligned} P_{\ell_\alpha \phi}^{N_1} &= \frac{\gamma_{\ell_\alpha \phi}^{N_1}}{\gamma_{\text{all}}}, & P_{\ell_\alpha \bar{q}_3 t}^{N_1} &= \frac{\gamma_{\ell_\alpha \bar{q}_3 t}^{N_1}}{\gamma_{\text{all}}}, \\ P_{q_3 t}^{N_1 \ell_\alpha} &= \frac{\gamma_{q_3 t}^{N_1 \ell_\alpha}}{\gamma_{\text{all}}}, & P_{t \ell_\alpha}^{N_1 q_3} &= \frac{\gamma_{t \ell_\alpha}^{N_1 q_3}}{\gamma_{\text{all}}}, & P_{\bar{q}_3 \ell_\alpha}^{N_1 \bar{t}} &= \frac{\gamma_{\bar{q}_3 \ell_\alpha}^{N_1 \bar{t}}}{\gamma_{\text{all}}}, \end{aligned} \quad (6.42)$$

with similar definitions for the probabilities of the CP conjugate processes. The probabilities are normalized in terms of the sum of all the rates:

$$\gamma_{\text{all}} = \sum_{\beta} (\gamma_{\ell_\alpha \phi}^{N_1} + \gamma_{\bar{\ell}_\alpha \phi}^{N_1} + \gamma_{\ell_\alpha \bar{q}_3 t}^{N_1} + \gamma_{\ell_\alpha q_3 \bar{t}}^{N_1} + \gamma_{q_3 \bar{t}}^{N_1 \ell_\alpha} + \gamma_{\bar{q}_3 t}^{N_1 \bar{\ell}_\alpha} + \gamma_{\bar{\ell}_\alpha \bar{t}}^{N_1 \bar{q}_3} + \gamma_{\ell_\alpha t}^{N_1 q_3} + \gamma_{\ell_\alpha \bar{q}_3}^{N_1 \bar{t}} + \gamma_{\bar{\ell}_\alpha q_3}^{N_1 t}). \quad (6.43)$$

To the order in the Yukawa couplings that we are considering, the unitarity condition for the sum of the branching ratios of N_1 into all possible final states, $\sum_Y B_Y^{N_1} = 1$, is then generalized to $\sum_{X,Y} P_Y^{N_1 X} = 1$. In other words, the probabilities for all the possible ways through which N_1 can disappear add up to unity.

To include the new sources of CP asymmetries, we now need to subtract from eqns (6.36,6.37) the analogous equations for $Y_{\bar{L}}^{\alpha\alpha}$. For the source term, we obtain:

$$\left(\dot{Y}_{\Delta L_\alpha}\right)_{II}^s = \left(\dot{Y}_{\Delta L_\alpha}\right)_{2\leftrightarrow 3}^s + \left(\dot{Y}_{\Delta L_\alpha}\right)_{2\leftrightarrow 3}^{s, \text{sub}}, \quad (6.44)$$

where we neglect the CP asymmetries of the $2 \leftrightarrow 3$ processes with N_1 exchanged in the t -channel eqn. (6.38) that are of higher order in the couplings. For the first term in the r.h.s. of eqn (6.44) we have

$$\left(\dot{Y}_{\Delta L_\alpha}\right)_{2\leftrightarrow 3}^s = (y_{N_1} + 1) \left[\Delta\gamma_{\ell_\alpha \bar{q}_3 t}^{N_1} + \Delta\gamma_{\ell_\alpha \bar{q}_3}^{N_1 \bar{t}} + \Delta\gamma_{\ell_\alpha t}^{N_1 q_3} - \Delta\gamma_{q_3 \bar{t}}^{N_1 \ell_\alpha} \right]. \quad (6.45)$$

Eliminating the subtracted rates by writing their CP asymmetries as minus the CP asymmetries of the on-shell rates, and keeping terms up to $\mathcal{O}(\lambda^4 h_t^2)$, we obtain for the second term in eqn (6.44)

$$\left(\dot{Y}_{\Delta L_\alpha}\right)_{2\leftrightarrow 3}^{s, \text{sub}} = -2 \Delta\gamma_{\ell_\alpha \phi}^{N_1} \left[1 - \sum_{\beta} \left(P_{\ell_\beta \phi}^{N_1} + P_{\bar{\ell}_\beta \phi}^{N_1} \right) \right] - 2 \left[\Delta\gamma_{\ell_\alpha \bar{q}_3 t}^{N_1} + \Delta\gamma_{\ell_\alpha \bar{q}_3}^{N_1 \bar{t}} + \Delta\gamma_{\ell_\alpha t}^{N_1 q_3} - \Delta\gamma_{q_3 \bar{t}}^{N_1 \ell_\alpha} \right]. \quad (6.46)$$

Note that, at $\mathcal{O}(\lambda^2 h_t^2)$, the sum of the branching ratios in eqn (6.27) of the previous section is not unity, so the first term in that equation does not have the correct thermodynamic behaviour $\propto (y_{N_1} - 1)$, and the second term does not vanish. By keeping track carefully of the relevant higher order terms, the source term arising from the difference between (6.27) and the analog equation for $Y_{\bar{L}_\alpha}$ reads

$$\left(\dot{Y}_{\Delta L_i}\right)_I^s = \left(y_{N_1} + 1 - 2 \sum_{\beta} \left[P_{\ell_\beta \phi}^{N_1} + P_{\bar{\ell}_\beta \phi}^{N_1} \right] \right) \Delta\gamma_{\ell_\alpha \phi}^{N_1}, \quad (6.47)$$

where $\sum_{\beta} (P_{\ell_{\beta}\phi}^{N_1} + P_{\bar{\ell}_{\beta}\bar{\phi}}^{N_1}) < 1$. However, the first term in eqn (6.46) combines with eqn (6.47) to yield for the source term involving $\Delta\gamma_{\ell_{\alpha}\phi}^{N_1}$ the correct behavior $\propto (y_{N_1} - 1)$. Summing up eqns (6.45), (6.46) and (6.47), we obtain the final expression for the source term that holds at $\mathcal{O}(\lambda^4 h_t^2)$:

$$\left(\dot{Y}_{\Delta L_{\alpha}}\right)_{I+II}^s = (y_{N_1} - 1) \left[\Delta\gamma_{\ell_{\alpha}\phi}^{N_1} + \Delta\gamma_{\ell_{\alpha}\bar{q}_3 t}^{N_1} + \Delta\gamma_{\ell_{\alpha}\bar{q}_3}^{N_1\bar{t}} + \Delta\gamma_{\ell_{\alpha}t}^{N_1 q_3} - \Delta\gamma_{q_3\bar{t}}^{N_1\ell_{\alpha}} \right]. \quad (6.48)$$

Regarding the washouts, we neglect the contributions from eqns (6.37) and (6.38) since they both involve non-resonant $2 \leftrightarrow 2$ scatterings that are of higher order in the couplings. We retain only the $\mathcal{O}(\lambda^2 h_t^2)$ contributions of eqn (6.36). Subtracting from eqn (6.36) the analogous equations for $\bar{\ell}_{\alpha}$, we obtain the relevant washout term:

$$\left(\dot{Y}_{\Delta L_{\alpha}}\right)_{II}^w \simeq \left(\dot{Y}_{\Delta L_{\alpha}}\right)_{2\leftrightarrow 2}^{1\leftrightarrow 3} \quad (6.49)$$

where

$$\begin{aligned} \left(\dot{Y}_{\Delta L_{\alpha}}\right)_{2\leftrightarrow 2}^{1\leftrightarrow 3} = & \left[(\Delta y_{q_3} - \Delta y_t - \Delta y_{\ell_{\alpha}}) \gamma_{\ell_{\alpha}\bar{q}_3 t}^{N_1} + (\Delta y_{q_3} - \Delta y_t - y_{N_1} \Delta y_{\ell_{\alpha}}) \gamma_{N_1\ell_{\alpha}}^{q_3\bar{t}} \right. \\ & \left. + (\Delta y_{q_3} - y_{N_1} \Delta y_t - \Delta y_{\ell_{\alpha}}) \gamma_{\bar{q}_3\ell_{\alpha}}^{N_1\bar{t}} + (y_{N_1} \Delta y_{q_3} - \Delta y_t - \Delta y_{\ell_{\alpha}}) \gamma_{t\ell_{\alpha}}^{N_1 q_3} \right]. \end{aligned} \quad (6.50)$$

Note that while the contributions from the RIS-subtracted $2 \leftrightarrow 3$ scatterings in eqn (6.37) must be taken into account to obtain the correct form of the source term, neglecting them in the washouts, as we do in eqn (6.50), does not have a significant effect on the numerical results.

The inclusion of N_1 decays into three body final states is qualitatively required if we want to take into account all processes of the same order in the couplings, and to incorporate consistently $2 \leftrightarrow 3$ scatterings where the on-shell piece involves a $1 \rightarrow 3$ decay (for example, $\ell_{\beta}\phi \rightarrow N_1 \rightarrow \ell_{\alpha}\bar{q}_3 t$). The quantitative impact is, however, rather small: while for zero temperature this decay has a large enhancement related to small momentum-values for the Higgs propagating off-shell, for the relevant temperatures, the finite value of the thermal mass prevents this enhancement, and the decay rate is below 6% of the two-body decay rate [185]. We thus neglect the washout term of $3 \rightarrow 1$ inverse decays, and the contribution of the three-body decay CP asymmetry $\Delta\gamma_{\ell_{\alpha}\bar{q}_3 t}^{N_1}$ to the source term in eqn (6.48).

Following the same procedure outlined above, it is possible to include in the BE other relevant processes, such as those involving the gauge bosons [60, 85, 189]. With all the subdominant terms neglected and with the effects of the gauge bosons included, the simplified expression of the BE for the evolution of Y_{N_1} reads:

$$\dot{Y}_{N_1} = -(y_{N_1} - 1) [\gamma_{N \rightarrow 2} + \gamma_{\text{top}}^{2 \leftrightarrow 2} + \gamma_A^{2 \leftrightarrow 2}], \quad (6.51)$$

where $A = W_i$ or B for $SU(2)$ and $U(1)$ bosons respectively. The term involving the gauge bosons is

$$\gamma_A^{2 \leftrightarrow 2} = \sum_{\beta} \left(\gamma_{A\bar{\phi}}^{N_1\ell_{\beta}} + \gamma_{A\phi}^{N_1\bar{\ell}_{\beta}} + \gamma_{A\bar{\ell}_{\beta}}^{N_1\bar{\phi}} + \gamma_{A\bar{\ell}_{\beta}}^{N_1\phi} + \gamma_{\ell_{\beta}\phi}^{N_1 A} + \gamma_{\bar{\ell}_{\beta}\bar{\phi}}^{N_1 A} \right). \quad (6.52)$$

where a sum over the gauge boson degrees of freedom in all the rate densities is understood. In eqn (6.51) we neglect three-body decays involving the gauge bosons like $\gamma_{A\phi\ell_{\alpha}}^{N_1}$ that are suppressed by phase space factors. We also neglect the contributions to the washouts from gauge bosons $2 \leftrightarrow 3$ processes.

We can finally write the simplified evolution equation for the charge-densities $Y_{\Delta_{\alpha}}$ [defined in eqn (6.9)] in terms of the source and the washout terms:

$$\dot{Y}_{\Delta_{\alpha}} = \left(\dot{Y}_{\Delta_{\alpha}}\right)^s + \left(\dot{Y}_{\Delta_{\alpha}}\right)^w. \quad (6.53)$$

The source term is given by

$$\begin{aligned} \left(\dot{Y}_{\Delta_{\alpha}}\right)^s &= -(y_{N_1} - 1) \left[\Delta\gamma_{\ell_{\alpha}\phi}^{N_1} + \Delta\gamma_{\ell_{\alpha}\bar{q}_3}^{N_1\bar{t}} + \Delta\gamma_{\ell_{\alpha}t}^{N_1 q_3} - \Delta\gamma_{q_3\bar{t}}^{N_1\ell_{\alpha}} - \Delta\gamma_{A\bar{\phi}}^{N_1\ell_{\alpha}} + \Delta\gamma_{A\bar{\ell}_{\alpha}}^{N_1\bar{\phi}} + \Delta\gamma_{\ell_{\alpha}\phi}^{N_1 A} \right] \\ &\simeq -(y_{N_1} - 1) [\gamma_{N \rightarrow 2} + \gamma_{\text{top}}^{2 \leftrightarrow 2} + \gamma_A^{2 \leftrightarrow 2}] \epsilon_{\alpha\alpha}. \end{aligned} \quad (6.54)$$

In the second line we use the approximate equality between the scatterings and the decay asymmetries that was discussed in section 5.4, for example,

$$\frac{\Delta\gamma_{N_1\ell_\alpha}^{q_3\bar{t}}}{\gamma_{N_1\ell_\alpha}^{q_3\bar{t}}} \simeq \frac{\Delta\gamma_{\ell_\alpha\phi}^{N_1}}{\gamma_{\ell_\alpha\phi}^{N_1}} = \epsilon_{\alpha\alpha}. \quad (6.55)$$

The washout term is given by

$$\begin{aligned} \left(\dot{Y}_{\Delta_\alpha}\right)^w &= \sum_\beta \left[(\Delta y_{\ell_\alpha} + \Delta y_\phi) \left(\gamma_{\bar{\ell}_\beta\bar{\phi}}^{\ell_\alpha\phi} + \gamma_{\ell_\beta\phi}^{\ell_\alpha\phi} \right) + (\Delta y_{\ell_\beta} + \Delta y_\phi) \left(\gamma_{\bar{\ell}_\beta\bar{\phi}}^{\ell_\alpha\phi} - \gamma_{\ell_\beta\phi}^{\ell_\alpha\phi} \right) \right] \\ &+ \sum_\beta \left[(1 + \delta_{\alpha\beta}) (\Delta y_{\ell_\alpha} + \Delta y_{\ell_\beta} + 2\Delta y_\phi) \gamma_{\bar{\phi}\bar{\phi}}^{\ell_\alpha\ell_\beta} + (\Delta y_{\ell_\alpha} - \Delta y_{\ell_\beta}) \gamma_{\bar{\phi}\bar{\phi}}^{\ell_\alpha\bar{\ell}_\beta} \right] \\ &+ (y_{N_1} \Delta y_{\ell_\alpha} - \Delta y_{q_3} + \Delta y_t) \gamma_{q_3\bar{t}}^{N_1\ell_\alpha} + [2\Delta y_{\ell_\alpha} - (y_{N_1} + 1)(\Delta y_{q_3} - \Delta y_t)] \gamma_{t\ell_\alpha}^{N_1q_3} \\ &- (y_{N_1} \Delta y_{\ell_\alpha} + \Delta y_\phi) \gamma_{A\bar{\phi}}^{N_1\ell_\alpha} + (y_{N_1} \Delta y_\phi + \Delta y_{\ell_\alpha}) \gamma_{A\bar{\ell}_\alpha}^{N_1\phi} + (\Delta y_\phi + \Delta y_{\ell_\alpha}) \gamma_{\phi\ell_\alpha}^{N_1A}. \end{aligned} \quad (6.56)$$

In the third line we use the equality of the t - and u -channels top-quark scatterings to set $\gamma_{\bar{q}_3\bar{\ell}_\alpha}^{N_1\bar{t}} = \gamma_{t\ell_\alpha}^{N_1q_3}$. Confronting eqn (6.54) with eqn (6.51), we learn that the BE for the evolution of the Δ_α charge density can be written as follows:

$$\dot{Y}_{\Delta_\alpha} = \dot{Y}_{N_1} \epsilon_{\alpha\alpha} + \left(\dot{Y}_{\Delta_\alpha}\right)^w. \quad (6.57)$$

Two comments are in order:

1. The washout term of eqn (6.56) depends on the density asymmetries of various particle types which evolve with time. In principle, we need to know their time evolution in order to solve eqn (6.57). However, as discussed in Sections 8.2 and 9.3, and, in more detail, in Appendix 14, the chemical equilibrium constraints from fast SM reactions always allow one to express all the relevant density asymmetries Δy_a in terms of the Y_{Δ_β} . Doing that, one obtains a closed system of differential equations involving only the flavoured charge densities.
2. Eqn (6.57) shows that if washouts were neglected (and the value of $\epsilon_{\alpha\alpha}$ assumed independent of the temperature; see Section 7), the final value of Y_{Δ_α} would be simply proportional to the initial value of Y_{N_1} . Therefore, in case that the λ -interactions are the only source of populating the N_1 degree of freedom, the final asymmetry would vanish if not for the presence of the washouts.

7 Thermal Effects

At the high temperatures at which leptogenesis occurs, the light particles involved in the processes relevant for the generation of an initial lepton number asymmetry are in equilibrium with the hot plasma. The thermal effects give corrections to several ingredients in the analysis: (i) coupling constants, (ii) particle propagators (leptons, quarks, gauge bosons and the Higgs) and (iii) CP-violating asymmetries. These three effects are discussed in turn in the following three subsections. A dedicated study of thermal corrections to leptogenesis processes with a discussion of the leading numerical effects can be found in [60].

7.1 Coupling constants

A detailed study of gauge and Yukawa couplings renormalization in a thermal plasma can be found in [190]. In practice, it is a very good approximation to use the zero-temperature renormalization group equations for the top-quark Yukawa coupling and for the gauge couplings, with a renormalization scale $\Lambda \sim 2\pi T$ [60]. The value $\Lambda > T$ is related to the fact that the average energy of the colliding particles in the plasma is larger than the temperature.

The renormalization effects for the neutrino couplings are also well known [191, 192]. In the non-supersymmetric case, to a good approximation these effects can be described by a simple rescaling of the low energy neutrino mass matrix $m(\mu) = r \cdot m$, where $1.2 \lesssim r \lesssim 1.3$ for $10^8 \text{ GeV} \lesssim \mu \lesssim 10^{16} \text{ GeV}$ [60]. Therefore, RG effects on neutrino couplings can be accounted for by increasing the values of the neutrino mass parameters (for example, \tilde{m}) as measured at low energy by $\approx 20\% - 30\%$ (depending on the leptogenesis scale). In the supersymmetric case one expects a milder enhancement, but uncertainties related with the precise value of the top-Yukawa coupling can be rather large (see fig.3 in ref. [60]).

7.2 Decays and scatterings

In the thermal plasma, any particle with sizeable couplings to the background states acquires a thermal mass that, modulo the renormalization of the relevant couplings, is proportional to the plasma temperature. Consequently, decay and scattering rates are modified. The diagrams corresponding to the relevant leptogenesis processes for which these corrections should be estimated are given in fig. 5.1 ($1 \leftrightarrow 2$ processes) and in fig. 6.1 (for the $2 \leftrightarrow 2$ scatterings).

Thermal corrections to particle masses have been thoroughly studied in both the standard model and the supersymmetric standard model [193–198]. The singlet neutrinos have no gauge interactions, their Yukawa couplings are generally small and, during the relevant era, their bare mass is of the order of the temperature or larger. Consequently, to a good approximation, corrections to their masses can be neglected. We thus need to account for the thermal masses of only the lepton doublets, the third generation quarks, the Higgs and the gauge bosons (and, in the supersymmetric case, also their superpartners). Explicit expressions for the thermal masses that enter the relevant leptogenesis processes are collected in appendix B of [60]. For the following qualitative discussion, it is enough to keep in mind that, within the leptogenesis temperature range, $m_\phi(T) \gtrsim m_{q,t}(T) \gg m_\ell(T)$. The most important effects relate to four classes of leptogenesis processes:

(i) *Decays and inverse decays.* Since thermal corrections to the Higgs mass are particularly large ($m_\phi(T) \approx 0.4T$), decays and inverse decays become kinematically forbidden in the temperature range in which $m_\phi(T) - m_\ell(T) < M_{N_1} < m_\phi(T) + m_\ell(T)$. For lower temperatures, the usual processes $N_1 \leftrightarrow \ell\phi$ can occur. For higher temperatures, the Higgs is heavy enough that it can decay: $\phi \leftrightarrow \ell N_1$. A rough estimate of the kinematically forbidden region yields $2 \lesssim T/M_1 \lesssim 5$. The important point is that these corrections are effective only at $T > M_1$. In the parameter region $\tilde{m} > 10^{-3} \text{ eV}$, that is favored by the measurements of the neutrino mass-squared differences, the N_1 number density and its L -violating reactions attain thermal equilibrium at $T \approx M_1$ and erase quite efficiently any memory of the specific conditions at higher temperatures. Consequently, in the strong washout regime, these thermal corrections have practically no effect on the final value of the baryon asymmetry.

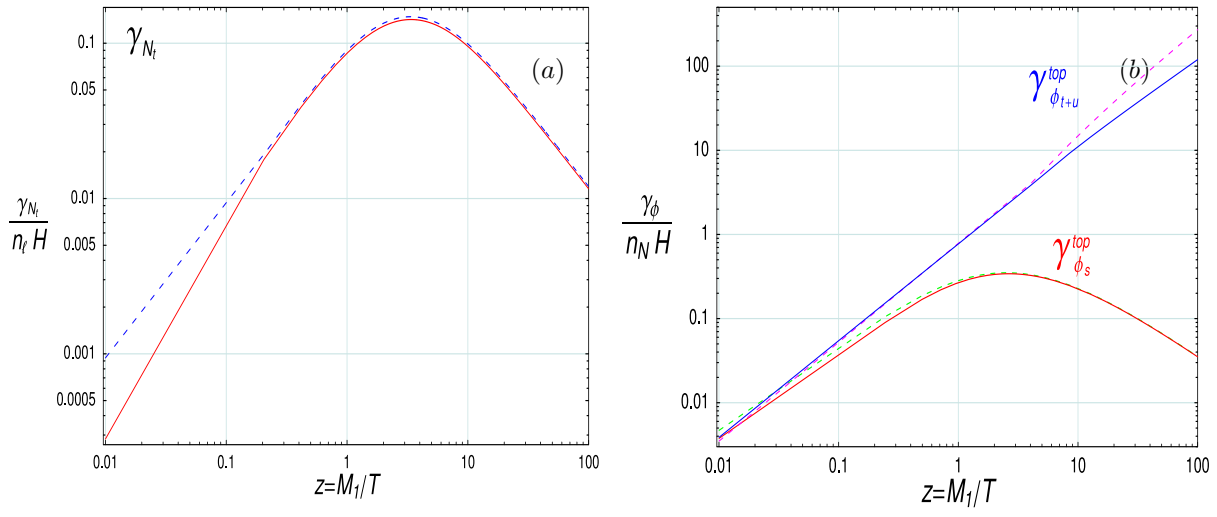


Figure 7.1: Comparison between scattering rates in the Standard Model with (solid lines) and without (dashed lines) thermal corrections. We use $\tilde{m} = 0.06$ eV and $M_1 = 10^{10}$ GeV. (a) γ_{N_1} : the rate density of the t -channel N_1 -exchange scattering $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$, normalized to $n_\ell H$. (b) $\gamma_{\phi_s}^{\text{top}}$ and $\gamma_{\phi_{t+u}}^{\text{top}}$: the rate densities of the Higgs exchange scatterings in, respectively, the s -channel ($q_3 \bar{t} \leftrightarrow \ell N$) and the t - and u -channels ($q_3 N \leftrightarrow \ell t$ and $\bar{t} N \leftrightarrow \ell \bar{q}_3$), normalized to $n_{N_1} H$. (Figures adapted from ref. [199].)

(ii) $\Delta L = 2$ scatterings. A comparison of the scattering rates for $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$ with and without thermal corrections is given in fig. 7.1(a) (adapted from [199]). The reaction densities are computed for $\tilde{m} = 0.06$ eV and $M_1 = 10^{10}$ GeV and are plotted as a function of decreasing temperature (increasing $z = M_1/T$). It is apparent that for this process thermal effects are sizeable only in the high temperature range $z < 1$. For $\ell\bar{\phi} \leftrightarrow \bar{\ell}\phi$ scatterings, a new resonant contribution can appear at high temperatures due to the fact that N_1 can go on-shell when exchanged in the u -channel [60]. As regards the off-shell contributions to this process, they are affected by thermal corrections in a way similar to $\ell\bar{\phi} \leftrightarrow \bar{\ell}\phi$, that is mainly at $M_1/T < 1$. We conclude that, for the $\Delta L = 2$ rates, thermal corrections are sizeable only at high temperatures. In the theoretically preferred regime, $\tilde{m} > m_*$, the related effects can be neglected.

(iii) $\Delta L = 1$ scatterings with top-quarks. Comparisons between the thermally-corrected and -uncorrected rates of the $\Delta L = 1$ top-quark scattering $\gamma_{\phi_s}^{\text{top}} \equiv \gamma(q_3 \bar{t} \leftrightarrow \ell N_1)$ with the Higgs exchanged in the s -channel, and of the sum of the t and u -channel scatterings $\gamma_{\phi_{t+u}}^{\text{top}} \equiv \gamma(q_3 N_1 \leftrightarrow \ell t) + \gamma(\bar{t} N_1 \leftrightarrow \ell \bar{q}_3)$, are given in fig. 7.1(b) (adapted from [199]). In the case of $\gamma_{\phi_s}^{\text{top}}$, mild corrections are present only at high temperatures. However, in contrast to the previous cases, the most relevant corrections to $\gamma_{\phi_{t+u}}^{\text{top}}$ appear at low temperatures, reducing the scattering rates and suppressing the corresponding contributions to the $\Delta L = 1$ washout. This peculiar situation arises from the fact that in the zero temperature limit there is a large logarithmic enhancement $\sim \ln(M_{N_1}/m_\phi)$ from the quasi-massless Higgs exchanged in the t - and u -channels. This enhancement disappears when the Higgs thermal mass $m_\phi(T) \sim T \sim M_{N_1}$ is included.

(iv) $\Delta L = 1$ scatterings with the gauge bosons (see figs. 6.1(b) and 6.1(c)). The inclusion of thermal masses is required to avoid IR divergences that would arise when massless ℓ (and ϕ) states are exchanged in the t - and u -channels. A naive use of some cutoff for the phase space integrals to control the IR divergences can yield incorrect estimates of the gauge bosons scattering rates and would be particularly problematic at low temperatures, where gauge bosons scatterings dominate over top-quark scatterings.

7.3 CP asymmetries

As discussed in section 5.3, CP asymmetries arise from the interference of tree level and one-loop amplitudes, when the relevant couplings involved have complex phases, and the loop diagrams have an

absorptive part. This last condition is satisfied whenever the loop diagram can be cut in such a way that the particles in the cut lines can be produced on shell. In the N_1 decay asymmetry at zero temperature this is guaranteed by the fact that the decay products, the Higgs ϕ and the lepton doublet ℓ , coincide with the states circulating in the loops. However, at the high temperatures at which the N_1 's decay, the Higgs and the lepton doublets are in equilibrium with the hot plasma, and their interactions with the background particles modify the CP asymmetries and introduce a dependence on the temperature: $\epsilon \rightarrow \epsilon(T)$. Thermal corrections to CP asymmetries arise from various effects:

- i)* The possibility of absorption and re-emission of the loop particles by the medium requires the use of finite temperature propagators for computing the absorptive parts of the Feynman diagrams.
- ii)* The stimulation of decays into bosons and the blocking of decays into fermions due to the dense background requires a proper modification of the density distributions of the final states.
- iii)* Thermal motion of the decay particles with respect to the background breaks the Lorentz symmetry and affects the evaluation of the CP asymmetries.
- iv)* Thermal masses should be included in the finite temperature resummed propagators, and also modify the fermion and boson dispersion relations. Their inclusion yields the most significant modifications to the zero temperature results for the CP asymmetries.

The first three effects were investigated in [200]. A rather complete account of thermal corrections to the decay CP asymmetries, that includes also the effects of thermal masses, can be found in [60]. In principle, at finite temperature, there are additional effects related to new cuts that involve the heavy $N_{2,3}$ neutrino lines (see fig. 5.1). These new cuts appear because the heavy particles in the loops may absorb energy from the plasma and go on-shell. However, for hierarchical spectrum, $M_{2,3} \gg M_1$, the related effects are suppressed to a negligible level by a Boltzmann factor $\exp(-M_{2,3}/T)$ that, at the temperatures relevant for the N_1 decays, is tiny.

7.3.1 Propagators and statistical distributions

The real time formalism of thermal field theory [201,202] can be used to compute the particle propagators at finite temperature. In this formalism, ghost fields dual to each of the physical fields have to be introduced, and consequently the thermal propagators have 2×2 matrix structures. For the one-loop computations of the absorptive parts of the Feynman diagrams, the relevant propagator components are just those of the physical fields, that for fermions (ℓ) and bosons (ϕ) are:

$$S_\ell(p, m_\ell) = \left[\frac{i}{p^2 - m_\ell^2 + i0^+} - 2\pi n_\ell \delta(p^2 - m_\ell^2) \right] (\not{p} + m_\ell), \quad (7.1)$$

$$D_\phi(p, m_\phi) = \left[\frac{i}{p^2 - m_\phi^2 + i0^+} + 2\pi n_\phi \delta(p^2 - m_\phi^2) \right]. \quad (7.2)$$

The leading effects in *i)* are proportional to the factor $-n_\ell + n_\phi - 2n_\ell n_\phi$, where $n_{\ell,\phi} = [\exp(E_{\ell,\phi}/T \pm 1)]^{-1}$. This factor vanishes when the thermal masses of the leptons and of the Higgs are neglected, because the Bose-Einstein and Fermi-Dirac statistical distributions depend on the same argument, $E_\ell = E_\phi = M_1/2$, and consequently the thermal correction to the fermion propagator (n_ℓ), the thermal piece of the boson propagator (n_ϕ) and the product of the two thermal corrections ($n_\ell n_\phi$) cancel each other. This can be interpreted as a complete compensation between stimulated emission and Pauli blocking. As regards the effects in *ii)*, they lead to overall factors that cancel between numerator and denominator in the expression for the CP asymmetry ϵ .

In the supersymmetric case, the situation is more subtle. First, the singlet neutrino N_1 decays not only to the standard $\ell\phi$ final state, but also to their superpartners: $\tilde{\ell}\tilde{\phi}$. Given that both decay channels contribute to the imaginary part of both decay modes, the same cancellation as in the previous case occurs (when thermal masses are neglected). Second, a new source of lepton asymmetry comes from the

scalar neutrino \tilde{N}_1 that decays both into final fermions $\ell\tilde{\phi}$, and into final bosons $\tilde{\ell}\phi$. Thermal effects modify the asymmetry in each channel. One-loop diagrams with the $\tilde{\ell}\phi$ bosons contribute to the CP-asymmetry for decays into fermions, while one-loop diagrams with internal $\ell\tilde{\phi}$ fermions contribute to the CP-asymmetry for decays into bosons. As a result, the finite temperature propagator corrections to the partial asymmetries of the single decay channels do not vanish [200]. When the statistical functions that block and stimulate the final state emission (corrections of type *ii*) are taken into account, the branching fractions into fermion and boson final states differ. When the CP-asymmetries of the two channels are summed up, the effects of the two types of corrections *i*) and *ii*) compensate each other, and the zero temperature result is again reproduced [200].

7.3.2 Particle motion

We have seen that when a large hierarchy $M_{2,3} \gg M_1$ is assumed, the particle thermal masses are neglected, and the decaying particle is considered at rest in the thermal bath, there are no thermal corrections to the zero temperature results for the CP-asymmetries. However, since the decaying particle is moving with respect to the background with velocity $\vec{\beta}$, due to their statistics, the fermionic decay products are preferentially emitted in the direction anti-parallel to the plasma velocity (for which the thermal distribution is less occupied), while the bosonic ones are emitted preferentially in the forward direction (for which stimulated emission is more effective). This induces an angular dependence in the decay distribution at order $\mathcal{O}(\beta)$. In the total decay rate the $\mathcal{O}(\beta)$ anisotropy effects are integrated out, and only $\mathcal{O}(\beta^2)$ effects remain [200]. Therefore, while the inclusion of the effects of the thermal motion of the decaying particle do modify the zero temperature results, these corrections are numerically small [60, 200] and generally negligible.

7.3.3 Thermal masses

When the finite values of the light particle thermal masses are taken into account, the arguments of the Bose-Einstein and Fermi-Dirac statistical distributions are different. It is a good approximation [60] to use for the particle energies $E_{\ell,\phi} = M_1/2 \mp (m_\phi^2 - m_\ell^2)/2M_1$. Since now $E_\ell \neq E_\phi$, the prefactor $n_\ell - n_\phi + 2n_\ell n_\phi$ that multiplies the thermal corrections does not vanish anymore, and sizeable corrections become possible. The most relevant effect is that the CP-asymmetry vanishes when, as the temperature increases, the sum of the light particles thermal masses approaches M_1 [60]. This is not surprising, since the particles in the final state coincide with the particles in the loop, and therefore when the decay becomes kinematically forbidden, also the particles in the loop cannot go on the mass shell. The same happens in the supersymmetric case for the CP-asymmetry in N_1 decays. However, this is not the case for the decays of the scalar neutrino $\tilde{N}_1 \rightarrow \ell\tilde{\phi}$ and $\tilde{N}_1 \rightarrow \tilde{\ell}\phi$ for which the particles running in the loop are different from the particles in the final states. Since thermal masses are larger for the scalars than for the fermions, the \tilde{N}_1 CP-asymmetry vanishes when the decay $\tilde{N}_1 \rightarrow \ell\tilde{\phi}$ is still kinematically allowed. The relevant analytical expressions for this case and detailed numerical results can be found in [60].

When the temperature is large enough that the Higgs can decay (see section 7.2), there is a new source of lepton number asymmetry associated with the decay processes $\phi \rightarrow \ell N_1$. The CP-asymmetry in Higgs decays ϵ_ϕ can be up to one order of magnitude larger than the CP-asymmetry in N_1 decays [60]. This is mainly due to a kinematical suppression of the tree level decay rate appearing in the denominator of ϵ_ϕ that is roughly proportional to the thermal mass difference $m_\phi^2 - m_\ell^2$. While this represents a dramatic enhancement of the CP-asymmetry, ϵ_ϕ is non-vanishing only at temperatures $T \gtrsim T_\phi \sim 5M_1$, when the kinematical condition $m_\phi(T) > m_\ell(T) + M_1$ is satisfied. Therefore, in the strong washout regime, no trace of this effect survives. On the other hand, rather large λ couplings are required in order that Higgs decays can occur before the phase space closes: the decay rate can attain thermal equilibrium only when $\tilde{m} \gtrsim (T_\phi/M_1)^2 m_* \gg m_*$, and therefore, in the weak washout regime ($\tilde{m} \lesssim m_*$), these decays always remain strongly out of equilibrium. This means that only a small fraction of the Higgs particles have actually time to decay, and the lepton-asymmetry generated in this way is accordingly suppressed.

In summary, while the corrections to the CP-asymmetries can be significant at $T \gtrsim M_1$ (and quite large at $T \gg M_1$ for Higgs decays), in the low temperature regime, where the precise value of ϵ plays

a fundamental role in determining the final value of the baryon asymmetry, there are almost no effects, and the zero temperature results still give a reliable approximation.

Before concluding this section let us mention that effects similar to the ones described above can be expected also for the CP-asymmetries of scattering processes, like top-quark or gauge-boson scatterings. These asymmetries were considered in [61, 64, 85] in the approximation in which they are proportional to the CP-asymmetry in decays, and were further analyzed in [185] by going beyond this approximation, but still in the zero temperature limit. An important difference is that while $2 \leftrightarrow 2$ scatterings are always kinematically allowed, the absorptive part of the one-loop diagrams vanishes at the same thresholds when the decay CP-asymmetries vanish. This can have the peculiar effect of thermalizing the N_1 degree of freedom without producing an associated lepton asymmetry. To our knowledge, a study of the effects of thermal corrections to CP-asymmetries in scattering has not been carried out yet.

8 Spectator Processes

8.1 Introduction

During leptogenesis, various processes can modify the densities of particle species. Some of these, such as the heavy neutrino decays or various interactions that washout the lepton number, occur on a time scale comparable to the expansion rate of the Universe, and hence should be accounted for via appropriate Boltzmann equations. Other processes can be very fast (depending on the temperature considered) and their effect is to impose certain relations among the chemical potentials of various particle species. These processes include the gauge interactions, Yukawa interactions involving the heavier fermions, and the electroweak and QCD non-perturbative ‘sphaleron’ processes. They are called ‘spectator processes’ because they do not change lepton number directly. Instead, they affect it indirectly, by changing the densities of the lepton doublets and of the Higgs on which the rates of washout processes depend. The issue of spectator processes and their effects on leptogenesis was first raised in ref. [203]. A rather complete analysis of the numerical relevance of each spectator processes can be found in ref. [204]. The main results of this section can be read off from the last column in Table 1, where the effects of various spectator processes are quantified as a percentage variation of the final asymmetry resulting from leptogenesis.

We work in the scenario in which the singlet neutrino masses are hierarchical, $M_1 \ll M_{2,3}$, and the lepton asymmetry is generated mainly via the CP and lepton number violating decays of the lightest singlet neutrino N_1 . We restrict the discussion to the non-supersymmetric case, since no qualitative new features appear in the supersymmetric case.

Several spectator processes become relevant only in the temperature regime in which lepton flavour effects are also important. In particular, reactions mediated by the τ (μ) Yukawa couplings become faster than the expansion rate of the Universe below $T \sim 10^{12}$ GeV (10^9 GeV). Then it becomes important to know the flavour composition of the lepton asymmetry. For simplicity, in our discussion of spectator processes we assume that leptons and antileptons produced in the decays of N_1 are aligned with or orthogonal to some specific lepton and antilepton flavours ℓ_α and $\bar{\ell}_\alpha$ (with $\alpha = e, \mu, \tau$). That is, we assume that the only non-vanishing element in the matrix

$$P_{\alpha\beta} = \langle \ell_\alpha | \ell_{N_1} \rangle \langle \ell_\beta | \ell_{N_1} \rangle^* \quad (8.1)$$

that characterizes the flavour composition of the lepton ℓ_{N_1} into which N_1 decays, is $P_{\alpha\alpha} = 1$ while all the other diagonal and non-diagonal entries are 0. Accordingly, in the following we replace the notation ℓ_{N_1} with ℓ_α , that is appropriate to denote pure flavour states.

We divide the processes that generate and washout the $B/3 - L_\alpha$ asymmetry into three classes:

- i) N_1 decays and inverse decays, $N_1 \leftrightarrow \ell_\alpha \phi$, and on-shell N_1 -mediated $\Delta L = 2$ scatterings $\ell_\alpha \phi \leftrightarrow \bar{\ell}_\alpha \bar{\phi}$ (the on-shell part of the first diagram in fig. 6.1(d));
- ii) $\Delta L = 1$ Higgs-mediated scattering processes involving the top-Yukawa coupling, $\ell_\alpha N_1 \leftrightarrow q_3 \bar{t}$, $\ell_\alpha \bar{q}_3 \leftrightarrow N_1 \bar{t}$ and $\ell_\alpha t \leftrightarrow N_1 q_3$ (fig. 6.1(a)).
- iii) $\Delta L = 1$ scatterings with the gauge bosons, such a $\ell_\alpha N_1 \rightarrow A \bar{\phi}$, $\ell_\alpha A \rightarrow N_1 \bar{\phi}$ and $\ell_\alpha \phi \rightarrow A N_1$ with $A = W_i$ or B (figs. 6.1(b) and 6.1(c)).

Note that all the rates above depend (at tree level) on a single combination of neutrino Yukawa couplings that can be parameterized, as in eqn (4.7), by $\tilde{m}_{11} \equiv \tilde{m} = [\lambda \lambda^\dagger]_{11} v_u^2 / M_1$. Other (subleading) washout reactions couple to lepton states that are different from ℓ_{N_1} and $\bar{\ell}_{N_1}$. In particular, we refer here to the $\Delta L = 2$ scatterings which go through off-shell s -channel, $\ell \phi \leftrightarrow \bar{\ell} \bar{\phi}$, and t -channel, $\ell \ell \leftrightarrow \bar{\phi} \bar{\phi}$ (fig. 6.1(d)). In the temperature regime $T < M_1$, and including the contributions from $N_{2,3}$, the amplitude for these processes is proportional to the light neutrino mass matrix, $[m]_{\alpha\beta}$ of eqn (2.5). Consequently, the fastest rate couples to the lepton doublet containing the heaviest light neutrino state ν_3 . However, being of higher order in the λ couplings, these processes are generally negligible, with a possible exception in the high temperature regimes, $M_1 > 10^{13}$ GeV, where some Yukawa couplings are of order unity. Since

flavour effects, as well as the majority of the spectator processes, become relevant only at $T < 10^{13}$ GeV, in the following we consider ℓ_α and $\bar{\ell}_\alpha$ as the only relevant directions in flavour space.

By definition, washout processes are lepton number violating reactions that tend to destroy any excess of lepton or of antileptons. In general, they depend also on the abundances of other particle species.

The washout reactions that we consider are contained in the first three lines in eqn (6.56). Note that since we are assuming alignment conditions, $\ell_\alpha = \ell_\beta$, some of the terms vanish. Washout reactions that involve the gauge bosons (figs. 6.1(b), 6.1(c)) that appear in the last line in eqn (6.56) are equally important. However, since the chemical potential for gauge bosons vanishes, no further density asymmetries are associated with these reactions, and for simplicity we neglect them. As a result of our approximations and simplifications, washout rates are controlled by the following density asymmetries: Δy_{ℓ_α} for the lepton doublets, Δy_ϕ for the Higgs, Δy_{q_3} for the third generation quark doublet and Δy_t for the top-quark singlet. The latter two quantities always appear in the combination $\Delta y_t - \Delta y_{q_3}$. It is useful to recall that Δy_{ℓ_α} , Δy_ϕ and Δy_{q_3} represent the sum of the asymmetries in the two components of the relevant $SU(2)$ -doublet, and that (in the non-supersymmetric case) the densities of the lepton and baryon number charges are

$$Y_{\Delta L} = \sum_{\alpha=e,\mu,\tau} Y_{\Delta L_\alpha} = \sum_{\alpha} (\Delta y_{\ell_\alpha} + \Delta y_{e_\alpha}) Y^{\text{eq}}, \quad (8.2)$$

$$Y_{\Delta B} = \frac{1}{3} \sum_{i=1,2,3} (\Delta y_{q_i} + \Delta y_{u_i} + \Delta y_{d_i}) Y^{\text{eq}}, \quad (8.3)$$

where u_i, d_i denote the $SU(2)$ -singlet quarks of the i -th generation and e_α are the $SU(2)$ -singlet leptons. Similar relations hold also for the densities of the flavoured charges $Y_{\Delta_\alpha} \equiv Y_{\Delta B}/3 - Y_{\Delta L_\alpha}$, that are individually conserved by the electroweak sphalerons.

8.2 Detailed analysis

Relations among the asymmetry-densities Δy_{ℓ_α} , Δy_ϕ and $\Delta y_t - \Delta y_{q_3}$ are determined by the chemical equilibrium conditions enforced by the reactions that are faster than the expansion rate of the Universe. The dependence of the expansion rate on temperature is different from that of various particle interaction rates, and as the temperature drops down, more and more interactions become faster than the expansion rate and ‘enter into equilibrium’. Thus, the equilibrium conditions change with temperature.

Since leptogenesis takes place at temperatures $T \sim M_1$, the relevant constraints depend on the value of M_1 . As discussed in Appendix 14, the density of the charge $B/3 - L_\alpha$ (where in this section α is the flavour direction into which N_1 decays) is conserved by all standard model processes, but not by interactions involving N_1 . All the relevant asymmetry-densities can be expressed as linear functions of the charge asymmetry $Y_{\Delta_\alpha} \equiv \frac{1}{3}Y_{\Delta B} - Y_{\Delta L_\alpha}$, and the equilibrium conditions fix the coefficient of proportionality for each temperature range:

$$\Delta y_{\ell_\alpha} = -c_\ell \frac{Y_{\Delta_\alpha}}{Y^{\text{eq}}}, \quad \Delta y_\phi = -c_\phi \frac{Y_{\Delta_\alpha}}{Y^{\text{eq}}}. \quad (8.4)$$

The two coefficient c_ℓ and c_ϕ encompass all the effects of the relevant spectator processes (charged lepton and quark Yukawa interactions, and the electroweak and QCD sphalerons). When generalized to multiple generations, c_ℓ generalizes to a matrix (that is the inverse of the A -matrix [65] that is introduced at the end of section 9).

We distinguish between six relevant temperature ranges according to the set of interactions that are in equilibrium. For each such temperature regime we present the equilibrium conditions. We impose, when relevant, various conditions of flavour alignment and calculate the corresponding c_ℓ and c_ϕ defined in eqn (8.4). Note that c_ℓ and c_ϕ give a crude understanding of the impact of the respective asymmetries: c_ϕ/c_ℓ gives a rough estimate of the relative contribution of the Higgs to the washout, while $c_\ell + c_\phi$ gives a measure of the overall washout strength. The quantitative significance of the different spectator processes can be read off from the last column in Table 1 (adapted from ref. [204]) where for the different

temperatures, and assuming a strong washout regime with $\tilde{m} = 0.06$ eV, we give the percentage variations with respect to the case when all spectator processes are neglected ($c_\ell = 1$, $c_\phi = 0$).

1) *Only gauge and top-Yukawa interactions in equilibrium* ($T > 10^{13}$ GeV).

Since in this regime the electroweak sphalerons are out of equilibrium, no baryon asymmetry is generated during leptogenesis. Moreover, since the charged lepton Yukawa interactions are negligible, the lepton asymmetry is just in the left-handed degrees of freedom and confined in the ℓ_α doublet, yielding $Y_{\Delta L} = \Delta y_{\ell_\alpha} Y^{\text{eq}} = -Y_{\Delta\alpha}$. As concerns Δy_ϕ , although initially equal asymmetries are produced by the decay of the heavy neutrino in the lepton and in the Higgs doublets, the Higgs asymmetry is partially transferred into a chiral asymmetry for the top quarks ($\Delta y_t - \Delta y_{q_3} \neq 0$) implying $\Delta y_{\ell_\alpha} \neq \Delta y_\phi$. We see from the last column in Table 1 that the inclusion of the Higgs asymmetry yields a sizeable reduction in the surviving asymmetry.

2) *Strong sphalerons in equilibrium* ($T \sim 10^{13}$ GeV).

QCD sphalerons enter equilibrium at higher temperatures than the corresponding electroweak processes because of their larger rate ($\Gamma_{\text{QCD}} \sim 11(\alpha_s/\alpha_W)^5 \Gamma_{\text{EW}}$ [205]). These processes are likely to be in equilibrium already at temperatures $T_s \sim 10^{13}$ GeV [154, 205, 206]) and yield the constraint

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0. \quad (8.5)$$

Direct comparison with the previous case allows us to estimate the corresponding effects. The relation $Y_{\Delta L} = \Delta y_{\ell_\alpha} Y^{\text{eq}} = -Y_{\Delta\alpha}$, implying $c_\ell = 1$, holds also for this case. However, switching on the QCD sphalerons reduces the Higgs number asymmetry by a factor of 21/23. This effect yields a suppression of the washout that does not exceed the few percent level.

3) *Bottom- and tau-Yukawa interactions in equilibrium* (10^{12} GeV $\lesssim T \lesssim 10^{13}$ GeV).

The asymmetries in the $SU(2)$ -singlet b and e_τ degrees of freedom are populated. The corresponding chemical potentials obey the equilibrium constraints

$$\mu_b = \mu_{q_3} - \mu_\phi, \quad \mu_\tau = \mu_{\ell_\tau} - \mu_\phi. \quad (8.6)$$

Possibly, h_b and h_τ Yukawa interactions enter into equilibrium at a similar temperature as the electroweak sphalerons [154]. However, in order to quantify separately the impact of these two effects, we first consider the possibility of a regime with only gauge, QCD sphalerons and the Yukawa interactions of the whole third family in equilibrium. As concerns the flavour composition of the lepton asymmetry, we distinguish two alignment cases: first, when the lepton asymmetry is produced in a direction orthogonal to ℓ_τ ($P_{\tau\tau} = 0$) and second, when it is produced in the ℓ_τ channel ($P_{\tau\tau} = 1$). When $P_{\tau\tau} = 0$, the lepton asymmetry is produced in one of the two directions orthogonal to ℓ_τ and therefore it does not ‘leak’ into the $SU(2)$ singlet degrees of freedom, implying that $c_\ell = 1$ still holds. In the case where $P_{\tau\tau} = 1$, the washout effects are somewhat suppressed, since the lepton asymmetry is partially shared with e_τ that does not contribute directly to the washout processes. Our results for these two cases suggest that the effect on the final value of $Y_{\Delta\alpha}$ associated to the τ Yukawa interactions is of the order of 10%.

4) *Electroweak sphalerons in equilibrium* (10^{11} GeV $\lesssim T \lesssim 10^{12}$ GeV).

The electroweak sphaleron processes take place at a rate per unit volume $\Gamma_{EW}/V \propto T^4 \alpha_W^5 \log(1/\alpha_W)$ [148–150], and are expected to be in equilibrium from temperatures $\sim 10^{12}$ GeV, down to the electroweak scale or below [154]. Electroweak sphalerons equilibration implies

$$\sum_i (3\mu_{q_i} + \mu_{\ell_\alpha}) = 0. \quad (8.7)$$

As concerns lepton number, each electroweak sphaleron transition creates all the doublets of the three generations, implying that individual lepton flavour numbers are no longer conserved. As concerns

Equilibrium processes, constraints, coefficients and effects on Y_{Δ_α}

	T (GeV)	Equilibrium	Constraints	c_ℓ	c_ϕ	$\delta(Y_{\Delta_\alpha})$	
$B = 0$	–	no spectators	–	1	0	–	
	$\gg 10^{13}$	h_t , gauge	$B = \sum_i (2q_i + u_i + d_i) = 0$	1	$\frac{2}{3}$	–40%	
	$\sim 10^{13}$	+ QCD-Sph	$\sum_i (2q_i - u_i - d_i) = 0$	1	$\frac{14}{23}$	–37%	
	$10^{12 \div 13}$	+ h_b, h_τ	$b = q_3 - \phi, \begin{cases} P_{\tau\tau} = 0 \\ P_{\tau\tau} = 1 \end{cases}$ $\tau = \ell_\tau - \phi$	1 $\frac{3}{4}$	$\frac{3}{8}$ $\frac{1}{2}$	–27% –17%	
$B \neq 0$	$10^{11 \div 12}$	+ EW-Sph	$\sum_{i,\alpha} (3q_i + \ell_\alpha) = 0$	$\begin{cases} P_{\tau\tau} = 0 \\ P_{\tau\tau} = 1 \end{cases}$	$\frac{98}{115}$ $\frac{78}{115}$	$\frac{41}{115}$ $\frac{56}{115}$	–15% –10%
	$10^{8 \div 11}$	+ h_c, h_s, h_μ	$c = q_2 + \phi,$ $s = q_2 - \phi,$ $\mu = \ell_\mu - \phi$	$\begin{cases} P_{ee} = 1 \\ P_{\tau\tau} = 1 \end{cases}$	$\frac{151}{179}$ $\frac{344}{537}$	$\frac{37}{179}$ $\frac{52}{179}$	–3% +12%
	$\ll 10^8$	all Yukawas h_i, h_α	$P_{ee} = 1$	$\frac{442}{711}$	$\frac{16}{79}$	+27%	

Table 1: The relevant quantities in the various temperature regimes. Chemical potentials are labeled here with the same notation used for the fields: $\mu_{q_i} = q_i$, $\mu_{\ell_\alpha} = \ell_\alpha$ for the $SU(2)$ doublets, $\mu_{u_i} = u_i$, $\mu_{d_i} = d_i$, $\mu_{e_i} = e_i$ for the singlets and $\mu_\phi = \phi$ for the Higgs. The relevant reactions in equilibrium in each regime are given in the second column and the constraints imposed in the third. The conditions adopted for $P_{\alpha\alpha}$ are indicated (the appropriate constraints on the conserved quantities $\Delta_\beta = B/3 - L_\beta$ with $\beta = e, \mu, \tau$ should also be imposed). The values of the coefficients c_ℓ and c_ϕ are given in, respectively, the fourth and fifth column. For each regime (and assuming $\tilde{m} = 0.06$ eV in all the cases), the last column quantifies the percentage variation in the absolute value of Y_{Δ_α} with respect to the case when all spectator processes are neglected (first line in the table).

baryon number, electroweak sphalerons are the only source of B violation, implying that baryon number is equally distributed among the three families of quarks. In particular, for the third generation, $B_3 = B/3$ is distributed between the doublets q_3 and the singlets t and b . In Table 1 we give the coefficients c_ℓ and c_ϕ for the two aligned cases: (i) $P_{\tau\tau} = 0$ implying $Y_{\Delta_\tau} = 0$, and (ii) $P_{\tau\tau} = 1$ implying $Y_{\Delta_e} = Y_{\Delta_\mu} = 0$. We see that in this case the transfer of part of the lepton asymmetry to a single right-handed lepton (e_τ) can have a 5% enhancing effect on the final value of Y_{Δ_α} .

5) *Second generation Yukawa interactions in equilibrium* (10^8 GeV $\lesssim T \lesssim 10^{11}$ GeV).

In this regime, the h_c, h_s and h_μ interactions enter into equilibrium. We consider two cases of alignment: (i) $P_{ee} = 1$ implying $Y_{\Delta_\mu} = Y_{\Delta_\tau} = 0$, and (ii) $P_{ee} = 0$. To ensure a pure states regime we further assume complete alignment with one of the two flavours with Yukawa interactions in equilibrium, for definiteness $P_{\tau\tau} = 1$, and therefore $Y_{\Delta_e} = Y_{\Delta_\mu} = 0$. The difference in c_ℓ between the two aligned cases is larger than in the regimes 3 and 4, and accordingly the difference in the corresponding values of Y_{Δ_α} , of the order of 15%, is somewhat larger than in the cases in which just the third generation Yukawa couplings are in equilibrium.

6) *All Yukawa interactions in equilibrium* ($T \lesssim 10^8$ GeV).

In this regime, since all quark Yukawa interactions are in equilibrium (actually this only happens for

$T < 10^6$ GeV), the QCD sphaleron condition becomes redundant. Hence ignoring the constraint of eqn (8.5), as is usually done in the literature, becomes fully justified only within this regime. If, however, leptogenesis takes place at $T > 10^8$ GeV, as occurs for hierarchical singlet N 's, the constraint implied by the QCD sphalerons is non-trivial, even if the associated numerical effects are not large.

Due to the symmetric situation of having all Yukawa interactions in equilibrium we have just one possible flavour alignment (the other two possibilities being trivially equivalent). We take for definiteness $P_{ee} = 1$, implying $Y_{\Delta_\mu} = Y_{\Delta_\tau} = 0$. In this case c_ℓ is reduced by a factor of almost two with respect to the case in which the spectator processes are neglected ($c_\ell = 1$) and the final value of Y_{Δ_α} is correspondingly enhanced. The reason for the reduction in c_ℓ can be traced mainly to the fact that a sizable amount of B asymmetry is being built up at the expense of the L asymmetry, and also a large fraction of the asymmetry is being transferred to the right-handed degrees of freedom at the same time when inverse decays and washout processes are active, reducing the effective value of $\Delta y_{\ell e}$ that contributes to drive these processes.

To summarize, we considered the possible impact of the spectator processes. Conditions of flavour alignment/orthogonality were imposed, to ensure that these effects are cleanly disentangled from lepton flavour effects (these are numerically more significant, but of a different nature). A rough quantitative understanding of spectator processes can be obtained by relying on the fact that the surviving asymmetry is inversely proportional to the washout rate, as discussed in section 4. Hence, the final Y_{Δ_α} asymmetries obtained in the relevant temperature regimes will be inversely proportional to $c_\ell + c_\phi$. This suggests that numerical corrections related to the proper inclusion of spectator processes have at most $\mathcal{O}(1)$ effects [204]. Inspecting the table, we learn that when the electroweak sphalerons are not active and all Yukawa interactions (except those of the top-quark) are negligible, the Higgs contribution enhances the washout processes, leading to a smaller final Y_{Δ_α} asymmetry. As more and more spectator processes become fast (compared to the expansion rate of the Universe), the general trend is towards reducing the value of the washout coefficients and hence increasing the Y_{Δ_α} up to values that can be slightly larger than what is obtained when all spectator processes are neglected.

9 Flavour Effects

The aim of this section is to discuss what are flavour effects in leptogenesis, why they arise, and when they matter. It makes use of results from Sections 6, 8 and Appendices 13, 14 and 15, but should be independently readable. In Section 9.1, we introduce the puzzle of flavour in leptogenesis calculations, which is that the baryon asymmetry depends on the choice of lepton flavour basis. We also give heuristic rules for how to choose the correct basis. Sections 9.2 and 9.3 illustrate how flavour effects can modify the final baryon asymmetry. Spectator processes (including sphalerons) are neglected in Section 9.2, where we focus on how flavour dynamics can strongly enhance the asymmetries in lepton doublets $Y_{\Delta\ell_\alpha}$. The sum of the $Y_{\Delta\ell_\alpha}$ is usually of the order of the final baryon asymmetry; this is discussed in Section 9.3, where spectator effects are included, and the relation between the charge densities $Y_{\Delta\alpha}$ and the asymmetries for the various particle species is elucidated. Notice that it is important to distinguish the asymmetries in lepton doublets $Y_{\Delta\ell_\alpha}$ from the $B/3 - L_\alpha$ charge densities $Y_{\Delta\alpha}$, even though they are similar in our notation. In Section 9.4 we analyze the main phenomenological consequences of including the effects of the lepton flavours.

Historically, leptogenesis calculations were performed in the “single flavour approximation”, which consists of studying Boltzmann Equations for the $B - L$ asymmetry. Reference [65] considered the BE for the asymmetries in $B/3 - L_\alpha$, but did not emphasize that the results were significantly different from the single flavour approximation. In subsequent years, various authors [173, 207, 208] noticed that flavour effects could be used to enhance the baryon asymmetry in particular models. The flavoured BE are given in [66]. The importance of studying lepton asymmetries flavour by flavour, in order to obtain a “reliable” estimate of the baryon asymmetry, was recently highlighted in [61–64].

9.1 The flavour puzzle

In the introduction to Section 4, washout interactions were presented as a critical ingredient for thermal leptogenesis, and the importance of flavour effects was said to follow from the importance of washout: the washout interactions have lepton doublets in the initial state, so to compute the washout rates one needs to know which leptons are distinguishable. In Section 6 we became acquainted with the Boltzmann Equations, which describe the detailed evolution of asymmetry production and of washout processes. Upon solving the BE, one could expect to get a correct description of washout, and discover if flavour matters in leptogenesis. Unfortunately, this is not the case. The BE are not covariant in lepton flavour space, and solving them in different bases gives different answers.

A simple example can illustrate this problem. Suppose that N_1 decays to ℓ_μ and ℓ_τ with equal branching ratios, so that $\hat{\ell}_{N_1} = (\hat{\ell}_\mu + \hat{\ell}_\tau)/\sqrt{2}$ (see eqn (5.11)). Furthermore, assume that the asymmetries in both flavours are the same: $\epsilon_{\mu\mu} = \epsilon_{\tau\tau}$. Let us consider the simple BE eqn (6.30), and neglect the Higgs asymmetry Δy_ϕ and all spectator processes, that are irrelevant for the present discussion. In this approximation the total lepton asymmetry coincides with the asymmetry stored in the lepton doublets: $Y_{\Delta L} = Y^{\text{eq}} \sum_\alpha \Delta y_{\ell_\alpha} \equiv Y^{\text{eq}} \Delta y_\ell$. To write down the BE for the total lepton asymmetry we can proceed in two ways. We can chose to work in the basis of $\hat{\ell}_{N_1}$ and of two other flavours orthogonal to $\hat{\ell}_{N_1}$ that, by assumption, are not produced in N_1 decays. In this case the only CP asymmetry is $\epsilon_{\hat{\ell}_{N_1}} = \sum_\alpha \epsilon_{\alpha\alpha} \equiv \epsilon$, and furthermore $\tilde{m}_{\hat{\ell}_{N_1}} = \tilde{m}$. The BE for the total lepton asymmetry then is

$$\dot{Y}_{\Delta L} = \left[(y_{N_1} - 1)\epsilon - \frac{1}{2}\Delta y_\ell \right] \gamma_{N_1 \rightarrow 2}. \quad (9.1)$$

Alternatively, we can chose to work in the “flavour” (charged lepton mass eigenstate) basis, and in this case we shall write two BE for the μ and τ asymmetries. By summing them up we obtain

$$\begin{aligned} \dot{Y}_{\Delta L} = \dot{Y}_{\Delta L_\mu} + \dot{Y}_{\Delta L_\tau} &= \left[(y_{N_1} - 1)(\epsilon_{\mu\mu} + \epsilon_{\tau\tau}) - \frac{1}{2}\Delta y_{\ell_\mu} \frac{\tilde{m}_{\mu\mu}}{\tilde{m}} - \frac{1}{2}\Delta y_{\ell_\tau} \frac{\tilde{m}_{\tau\tau}}{\tilde{m}} \right] \gamma_{N_1 \rightarrow 2} \\ &= \left[(y_{N_1} - 1)\epsilon - \frac{1}{4}\Delta y_\ell \right] \gamma_{N_1 \rightarrow 2}, \end{aligned} \quad (9.2)$$

where we have used $\tilde{m}_{\mu\mu}/\tilde{m} = \tilde{m}_{\tau\tau}/\tilde{m} = 1/2$. Eqns (9.1) and (9.2) differ by a factor of 1/2 in the washout, and therefore they yield different baryon asymmetries. This simple example illustrates what was (cryptically) mentioned at the end of section 4: the rough estimates for the baryon asymmetry given in eqns (4.15) and (4.16) depended on the choice of the flavour basis.

To obtain a reliable result, we either need a formalism that is flavour covariant, or we need to guess which is the correct basis. Here we opt to guess, based on physical intuition. (A flavour covariant toy model, that motivates our guesses, is discussed in Appendix 15.)

Two basic points provide guidance in making the correct guess:

1. washout processes are critical in leptogenesis;
2. interactions whose timescale is very different from that of leptogenesis, drop out of the BE.

The second point is usual Effective Field Theory: interactions which are strong should be resummed, very weak interactions can be neglected. In particular, interactions that are much faster than the timescale of leptogenesis and of the Universe expansion rate, are “resummed” into thermal corrections (see Section 7) and into chemical equilibrium conditions (see Section 8). The second point relates to the first, because to perform a correct calculation of the washout rates, one should make the correct choice of basis for the lepton doublets which are in the initial state. The fast flavour-dependent interactions, mentioned in point 2. are precisely the ones that can resolve the flavour basis ambiguity.

The lepton doublets of different flavours are distinguished in the Lagrangian by their Yukawa couplings h_α . During leptogenesis, they will also be distinguishable, if the h_α mediated interactions are fast compared to those of leptogenesis and to the Universe expansion rate. Since the $h_{e,\mu,\tau}$ mediated interactions are of widely different strengths, they would, for instance, induce differences in the thermal masses of the different leptons. The answer to the “flavour puzzle” in the previous example is that when (some of) the charged lepton Yukawa interactions are “fast enough”, the “flavour” (charged lepton mass eigenstate) basis is the correct basis for the BE, and eqn. (9.2) should be used. In the opposite situation in which charged lepton Yukawa interactions are much slower than the Universe expansion rate, leptogenesis has no knowledge of lepton flavours, and the correct BE is eqn (9.1).

To be more quantitative, let us estimate the temperature below which lepton flavor effects cannot be neglected. The interaction rate for a charged lepton Yukawa coupling h_α [195, 209] can be estimated as

$$\Gamma_\alpha \simeq 5 \times 10^{-3} h_\alpha^2 T, \quad (9.3)$$

(for details see Appendix 14 around eqn (14.10)). The condition $\Gamma_{\tau(\mu)} \gtrsim H$ then implies that the rate for $h_{\tau(\mu)}$ mediated processes becomes faster than the expansion rate of the Universe below $T \sim 10^{12}$ GeV (10^9 GeV),²⁰ while for $T \gg 10^{12}$ GeV the charged lepton Yukawa couplings are irrelevant. Hence, above this temperature all flavour effects can be neglected, and the only “basis-choosing” interactions are those involving the neutrino Yukawa couplings λ .²¹

If leptogenesis occurs below $T \sim 10^{12}$ GeV, the h_τ -related reactions participate, along with the neutrino Yukawa, in determining a physical flavour basis (via, for instance, the “thermal mass matrix”). The conditions under which ℓ_τ is singled out as a distinct lepton are discussed in Section 15.4; see e.g. eqn (15.37). For instance, if at the time the lepton asymmetry starts to survive, the h_τ interactions are also faster than the N_1 inverse decays [180, 181], then the relevant basis states are ℓ_τ and the component in ℓ_{N_1} that is orthogonal to ℓ_τ , and the total lepton asymmetry is shared between these two flavours. This situation can have significant consequences. In particular, it can happen that the asymmetry in the τ flavour is larger in size than (and even opposite in sign to) the total lepton asymmetry generated in N_1 decays (see eqn (5.10)). If, in addition, the τ flavour is weakly washed-out, a sizeable fraction of its asymmetry can survive until the end of the leptogenesis era. An example of two cases in which flavour effects yield a large enhancement of the baryon asymmetry is given in figure 9.1.

²⁰In SUSY $h_\tau = m_\tau/(v_u \cos \beta)$, so the tau Yukawa is in equilibrium for $T < (1 + \tan^2 \beta) \times 10^{12}$ GeV.

²¹ $\Delta L = 2$ interactions mediated by N_i exchange could also become fast at large temperatures. Besides giving additional contributions to the washout, this also complicates the issue of the physical basis for the lepton doublets. See e.g. [65].

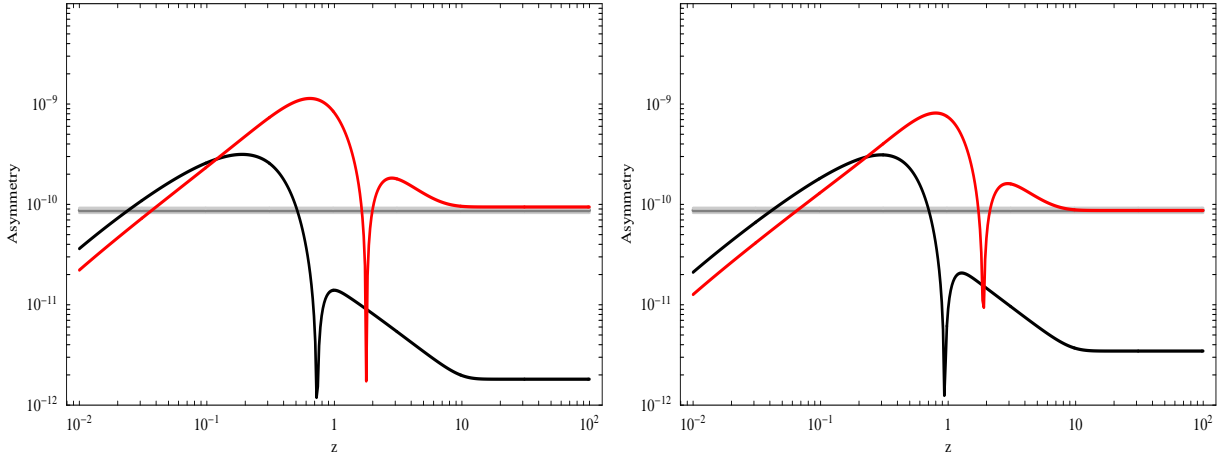


Figure 9.1: The total baryon asymmetry in the two flavour calculation (upper curves) and within the one-flavour approximation (lower curves) as a function of z , for two different sets of washout parameters. Left picture: $\tilde{m}_{\tau\tau}/m_* = 10$, $\tilde{m}_{\mu\mu}/m_* = 30$, $\tilde{m}_{ee}/m_* = 30$. Right picture: $\tilde{m}_{\tau\tau}/m_* = 10$, $\tilde{m}_{\mu\mu}/m_* = 30$, $\tilde{m}_{ee}/m_* = 10^{-2}$. In both cases $\epsilon_{\tau\tau} = 2.5 \times 10^{-6}$, $\epsilon_{\mu\mu} = -2 \times 10^{-6}$, $\epsilon_{ee} = 10^{-7}$ and $M_1 = 10^{10}$ GeV. These plots are from [64].

9.2 Enhancement of the $B - L$ asymmetry

To simplify the analysis, we focus in this Section on the asymmetries in lepton doublets, and neglect spectator and sphaleron processes. We make two assumptions to fix the flavour basis:

- (i) The rate of the h_τ -interactions Γ_τ is much larger than H and Γ_{ID} , the rates of, respectively, the expansion of the Universe, and the N_1 inverse decay;
- (ii) During the entire period of leptogenesis, the rate of the h_μ -interactions, Γ_μ , is either $\ll H, \Gamma_{ID}$, or $\gg H, \Gamma_{ID}$, so that the flavour basis does not change during leptogenesis.

When these two assumptions hold, it is a good approximation to work just with the projections of the ℓ_{N_1} densities onto (for $\Gamma_\mu \ll H, \Gamma_{ID}$) the two-flavour basis (ℓ_o, ℓ_τ) , where ℓ_o is the direction of the component in ℓ_{N_1} that is orthogonal to ℓ_τ , or (for $\Gamma_\mu \gg H, \Gamma_{ID}$) the three flavour basis $(\ell_e, \ell_\mu, \ell_\tau)$.

The flavour projectors correspond to the diagonal entries of the matrix P in eqn (8.1) (and \bar{P} for the CP conjugate states) [65], that is

$$\Gamma(N_1 \rightarrow \ell_\alpha \phi) = \Gamma(N_1 \rightarrow \ell_{N_1} \phi) P_{\alpha\alpha}. \quad (9.4)$$

The CP violating differences,

$$\Delta P_{\alpha\alpha} = P_{\alpha\alpha} - \bar{P}_{\alpha\alpha}, \quad (9.5)$$

are important quantities that account for the misalignment in flavour space of the states ℓ_{N_1} and $\bar{\ell}_{N_1}$. At tree level, $\Delta P_{\alpha\alpha} = 0$ and the flavoured washout parameters $\tilde{m}_{\alpha\alpha}$ can be simply written as

$$\tilde{m}_{\alpha\alpha} = \tilde{m} P_{\alpha\alpha}. \quad (9.6)$$

At the loop level, however, the $\Delta P_{\alpha\alpha}$ do not vanish in general. We use eqns (9.4,9.5,9.6) to rewrite the asymmetry in lepton flavour α , eqn.(4.2), as

$$\epsilon_{\alpha\alpha} = \epsilon \frac{\tilde{m}_{\alpha\alpha}}{\tilde{m}} + \frac{1}{2} \Delta P_{\alpha\alpha}. \quad (9.7)$$

The crucial observation is that, while washouts are dominated by the inverse-decay rates, which are $\mathcal{O}(\lambda^2)$ and do not involve the couplings to $N_{2,3}$, the CP-violating difference $\Gamma(N_1 \rightarrow \ell_\alpha \phi) - \Gamma(N_1 \rightarrow \bar{\ell}_\alpha \phi)$ is $\mathcal{O}(\lambda^4)$ and does involve also the couplings $\lambda_{\beta k}$ with $k = 2, 3$. Therefore, as shown by eqn (9.7), in general

$\epsilon_{\alpha\alpha}$ is not proportional to $m_{\alpha\alpha}$. This implies that, for fixed values of ϵ and \tilde{m} , one can have $\epsilon_{\alpha\alpha}$ large together with $\tilde{m}_{\alpha\alpha}$ small, and this can yield a strong enhancement of the asymmetry $Y_{\Delta\ell\alpha}$. Taking into account that $\sum_{\alpha} P_{\alpha\alpha} = \sum_{\alpha} \bar{P}_{\alpha\alpha} = 1$, we obtain the following ‘sum rules’:

$$\sum_{\alpha} \tilde{m}_{\alpha\alpha} = \tilde{m}, \quad \sum_{\alpha} \epsilon_{\alpha\alpha} = \epsilon. \quad (9.8)$$

The first term in eqn (9.7) corresponds simply to the projection of the total asymmetry ϵ onto the flavour α . The second contribution plays a more subtle role. It represents a type of CP-violation that is specific of the flavour CP-asymmetries and does not affect the value of ϵ .

The Boltzmann equation for Y_N is given in eqn (6.51):

$$\dot{Y}_N = -(y_{N_1} - 1)[\gamma_{N \rightarrow 2} + \gamma_{\text{top}}^{2 \leftrightarrow 2} + \gamma_A^{2 \leftrightarrow 2}]. \quad (9.9)$$

As concerns the Boltzmann Equations (6.57) for the $Y_{\Delta\alpha}$, we remarked at the end of Section 6 that, to write them in closed form, one has to include all the spectator processes. This induces a mixing between $Y_{\Delta\alpha}$ and $Y_{\Delta\beta}$ that complicates the expressions of the BE. For the sake of clarity, we provisionally neglect these effects (we discuss them in Section 9.3). This approximation allows us to consider, instead of coupled equations for the $Y_{\Delta\alpha}$ ’s, independent equations for each of the lepton doublet densities $Y_{\Delta\ell\alpha}$ ’s:

$$\dot{Y}_{\Delta\ell\alpha} \simeq -\dot{Y}_N \epsilon_{\alpha\alpha} + (\dot{Y}_{\Delta\ell\alpha})^w. \quad (9.10)$$

Note that since we neglect the redistribution of the asymmetry to singlet leptons, we took $(\dot{Y}_{\Delta\ell\alpha})^w = -(\dot{Y}_{\Delta\alpha})^w$ when using eqn (6.56). The solutions are of the form of eqn (4.15):

$$Y_{\Delta\ell} \equiv \sum_{\alpha} Y_{\Delta\ell\alpha} \sim 4 \times 10^{-3} \sum_{\alpha} \epsilon_{\alpha\alpha} \eta_{\alpha}(\tilde{m}, \tilde{m}_{\alpha\alpha}). \quad (9.11)$$

where the summation is over $\alpha = e, \mu, \tau$ for $T < 10^9$ GeV, and $\alpha = o, \tau$ for 10^9 GeV $< T < 10^{12}$ GeV. Approximate analytic solutions (reviewed in Appendix 16) to eqns (9.9) and (9.10) give

$$\eta_{\alpha} \simeq \begin{cases} \left[\left(\frac{m_*}{2\tilde{m}_{\alpha\alpha}} \right)^{-1.16} + \left(\frac{\tilde{m}_{\alpha\alpha}}{2.1m_*} \right)^{-1} \right]^{-1} & \tilde{m} > m_* \\ \frac{\tilde{m}}{2m_*} \frac{\tilde{m}_{\alpha\alpha}}{m_*} & \tilde{m} < m_*. \end{cases} \quad (9.12)$$

Eqns (9.11) and (9.12) allow us to estimate the lepton doublet asymmetry $Y_{\Delta\ell}$ in various washout regimes. In the case that all the flavours are in the strong or mildly strong washout regime, $\tilde{m}_{\alpha\alpha} \geq m_*$ for all α , the effects of the ΔP term in eqn (9.7) are most striking. The efficiency for $Y_{\Delta\ell\alpha}$ is approximately given by $\eta_{\alpha} \sim m_*/(2\tilde{m}_{\alpha\alpha})$. Inserting eqn (9.7) in eqn (9.11), we obtain

$$\frac{Y_{\Delta\ell}}{2 \times 10^{-3}} \sim \mathcal{N}_f \frac{m_*}{\tilde{m}} \epsilon + \sum_{\alpha} \frac{m_*}{\tilde{m}_{\alpha\alpha}} \frac{\Delta P_{\alpha\alpha}}{2}. \quad (9.13)$$

Here \mathcal{N}_f denotes the number of lepton flavours that are effectively resolved by the fast charged lepton Yukawa interactions. The first term represents an enhancement by a factor \mathcal{N}_f with respect to the doublet asymmetry that would be obtained by neglecting flavour effects. It is clearly independent of particular flavour structures and it would be the only effect in the special cases where $\epsilon_{\alpha\alpha} \propto \tilde{m}_{\alpha\alpha}$. Its origin is simple to understand. On one hand, the asymmetry in each flavour is reduced by a factor of the branching ratio, $\tilde{m}_{\alpha\alpha}/\tilde{m}$. On the other hand, the efficiency factor is increased by the same factor, because all the washout rates for $Y_{\Delta\ell\alpha}$ are reduced by it. The two factors cancel each other, and an asymmetry $Y_{\Delta\ell\alpha} \propto \epsilon m_*/\tilde{m}$ is generated in each flavour. The sum over flavours yields the \mathcal{N}_f enhancement.

In contrast, the second term depends crucially on the details of the flavour structure. Let us analyze its possible effects in the $\mathcal{N}_f = 2$ case, in which $\Delta P_{\tau\tau} = -\Delta P_{oo}$ (with $\Delta P_{oo} = \Delta P_{ee} + \Delta P_{\mu\mu}$). If we assume $\tilde{m} \gg m_*$, then the first term in the r.h.s of eqn (9.13) is strongly suppressed. However, flavour

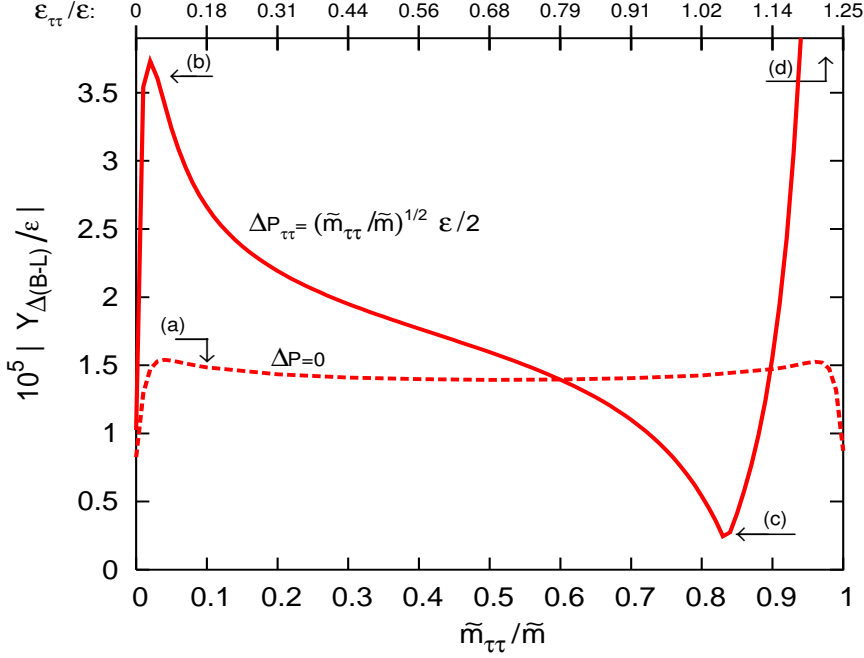


Figure 9.2: The absolute value of the final $|Y_{\Delta(B-L)}|$ (in units of $10^{-5}|\epsilon|$) as a function of $\tilde{m}_{\tau\tau}$. We use $M_1 = 10^{10}$ GeV and $\tilde{m} = 0.06$ eV. The dashed line corresponds to the case $\epsilon_{\alpha\alpha} \propto \tilde{m}_{\alpha\alpha}$ for which $\Delta P = 0$. The solid line gives an example of the results for $\Delta P \neq 0$. We take $\Delta P_{\tau\tau}/\epsilon \propto \sqrt{\tilde{m}_{\tau\tau}/\tilde{m}}$. The corresponding values of $\epsilon_{\tau\tau}/\epsilon$ are marked on the upper x -axis. The arrows with labels (a), (b), (c) and (d) correspond to the four situations discussed in the text. Note that around (c) $Y_{\Delta(B-L)}$ changes sign. (Figure adapted from ref. [63]).

configurations are possible for which $\tilde{m}_{\tau\tau} \sim m_*$ (or $\tilde{m}_{o\tau} \sim m_*$). Then, the production of an asymmetry $Y_{\Delta\ell_\tau}$ (or $Y_{\Delta\ell_o}$) driven by the second term occurs under ‘optimal’ conditions. This qualitative expectation is confirmed by numerical integration of the flavour dependent Boltzmann equations. Fig. 9.2 (adapted from ref. [63]) depicts a set of possible cases. The absolute value of the final $|Y_{\Delta(B-L)}|$ (in units of $10^{-5}|\epsilon|$) computed with $M_1 = 10^{10}$ GeV and $\tilde{m} = 0.06$ eV is shown as a function of $\tilde{m}_{\tau\tau}$. Two flavour-aligned cases at $\tilde{m}_{\tau\tau} = 0$ (*i.e.* no τ component in ℓ_{N_1}) and at $\tilde{m}_{\tau\tau} = \tilde{m}$ (*i.e.* $\ell_{N_1} \equiv \ell_\tau$) coincide with the results that would have been obtained by neglecting flavour. For $\tilde{m}_{\tau\tau}/\tilde{m} \neq 0, 1$ quite different results are obtained. Case (a) (dashed line) gives the results obtained by setting $\Delta P = 0$, that is by taking $\epsilon_{\alpha\alpha} \propto \tilde{m}_{\alpha\alpha}$. As predicted in our qualitative analysis, the final $B-L$ asymmetry is practically independent of the flavour structure, and gets enhanced by a factor $\sim \mathcal{N}_f = 2$ with respect to the flavour-aligned results.

For the more general cases where $\Delta P \neq 0$, the final doublet lepton asymmetry strongly depends on the particular flavour structure. To depict in a simple way the various possibilities, we adopt, as a convenient ansatz, the relation $\Delta P_{\tau\tau} \propto \sqrt{\tilde{m}_{\tau\tau}/\tilde{m}}$. This is based on the fact that, if we take the tree-level $N_1 \rightarrow \ell_\tau \phi$ decay amplitude to zero ($\lambda_{\tau 1} \rightarrow 0$) while keeping the total decay rate fixed [$(\lambda^\dagger \lambda)_{11} = \text{const.}$], $\tilde{m}_{\tau\tau}$ vanishes as the square of this amplitude ($\propto |\lambda_{\tau 1}|^2$) while $\Delta P_{\tau\tau}$ vanishes as the amplitude ($\propto \lambda_{\tau 1}$). We extrapolate this proportionality to finite values of $\tilde{m}_{\tau\tau}$ because this allows to represent the $\Delta P_{\tau\tau}$ effects in a simple two-dimensional plot of $Y_{\Delta(B-L)}$ versus $\tilde{m}_{\tau\tau}$.

On the upper x -axis of Fig. 9.2 we mark for reference the relative value of the τ CP-asymmetry $\epsilon_{\tau\tau}/\epsilon$ corresponding to the different values of $\tilde{m}_{\tau\tau}$. The most peculiar features are the two narrow regions marked with (b) and (d) where $|Y_{\Delta(B-L)}|$ is strongly enhanced. In (b) this happens around the ‘optimal’ value $\tilde{m}_{\tau\tau} \sim m_*$ that yields a strong enhancement, even though $\epsilon_{\tau\tau}/\epsilon$ is rather small. The region marked with (d) corresponds to $\tilde{m}_{o\tau} \sim m_*$ and to $\epsilon_{o\tau}$ of the opposite sign with respect to ϵ . Here the steep rise

of $|\Delta Y_{\Delta(B-L)}|$ can reach values up to one order of magnitude larger than the vertical scale of the figure. (Note, however, that the ansatz $\Delta P_{\tau\tau} \propto \sqrt{\tilde{m}_{\tau\tau}/\tilde{m}}$ does not yield the required behavior $\Delta P = 0$ at the boundary $\tilde{m}_{\tau\tau}/\tilde{m} = 1$ and therefore the continuous line in the plot should not be extrapolated too close to this limit.) It is worth noticing that the two peaks correspond to values of $Y_{\Delta(B-L)}/\epsilon$ of opposite sign. In fact, the $B - L$ asymmetry changes sign around point (c), and for $\tilde{m}_{\tau\tau}/\tilde{m} \gtrsim 0.85$ it is of the opposite sign with respect to what one would obtain neglecting flavour effects. Similar results are expected in the temperature regime below $T \sim 10^9$ GeV, when both the τ and the μ Yukawa reactions are in equilibrium. In this case even larger enhancements are possible because the dynamical production of the asymmetry of two flavours can occur in an ‘optimal’ regime.

In the weak washout regime for all the flavours, there can be no dynamical enhancement of the asymmetry and flavour effects are less important. The efficiencies for this regime are given in eqn (4.13) (or (9.12)). Using also eqn (9.7), we obtain

$$-\frac{Y_{\Delta\ell}}{2 \times 10^{-3}} \sim \frac{\tilde{m}^2}{m_*^2} \left[\epsilon \sum_{\alpha} \frac{\tilde{m}_{\alpha\alpha}^2}{\tilde{m}^2} + \frac{1}{2} \sum_{\alpha} \frac{m_{\alpha\alpha}}{\tilde{m}} \Delta P_{\alpha\alpha} \right]. \quad (9.14)$$

The first sum within square brackets is always ≤ 1 so this contribution by itself is suppressed with respect to the unflavoured case. The second sum is always less than or equal to the largest of the $\Delta P_{\alpha\alpha}$ ’s. Therefore, in this regime a necessary condition to have a large enhancement of the resulting doublet lepton asymmetry is that $\Delta P_{\alpha\alpha} \gg \epsilon$ for at least one lepton flavour.

Finally, in the case where one flavour β is weakly coupled to N_1 , $\tilde{m}_{\beta\beta}/m_* \ll 1$, but other flavours α are strongly coupled, $\tilde{m}_{\alpha\alpha}/m_* \sim \tilde{m}/m_* \gg 1$, the singlet neutrinos are brought into equilibrium by the reactions involving the strongly coupled flavours, so that $n_{N_1} \sim n_{\gamma}$. The efficiency factor η_{β} ‘loses’ the factor of \tilde{m}/m_* , which corresponds to a suppressed N_1 abundance [see eq. (4.13)]. Consequently, the asymmetry for the weakly coupled flavour β is given by

$$-\frac{Y_{\Delta\ell\beta}}{2 \times 10^{-3}} \sim \frac{\tilde{m}}{m_*} \left[\epsilon \frac{\tilde{m}_{\beta\beta}^2}{\tilde{m}^2} + \frac{m_{\beta\beta}}{\tilde{m}} \frac{\Delta P_{\beta\beta}}{2} \right]. \quad (9.15)$$

The asymmetry for the strongly coupled α flavours is again given by eqn (9.13) with $\mathcal{N}_f = 1$ or 2.

9.3 Mixing of lepton flavour dynamics

In Section 6 we derive BE for the charge densities $Y_{\Delta\alpha}$, without taking into account the SM spectator processes. For this reason, the dependence of the Δy_a ’s on the $Y_{\Delta\alpha}$ ’s is left implicit. In Section 8 we analyze the spectator processes, assuming that N_1 decays to pure (flavour) states, that is, neglecting flavour effects. In this section we combine these two issues, and we outline how to express the washout term in eqn (6.56) in terms of the $Y_{\Delta\alpha}$ (see Appendix 14 for a more complete treatment).

We assume that asymmetries can be generated in two flavour directions, which we take to be $\hat{\tau}$ and \hat{o} , where the latter is some linear combination of $\hat{\mu}$ and \hat{e} . Below $T \sim 10^{12}$ GeV, baryon and lepton numbers violating electroweak sphaleron processes, and h_{τ} -related processes are fast. In this situation, the charge densities that are (slowly) evolving because of the leptogenesis processes are

$$Y_{\Delta\alpha} \equiv \frac{1}{3} Y_{\Delta B} - Y_{\Delta L\alpha} = \frac{1}{3} \sum_{i=1,2,3} (\Delta y_{q_i} + \Delta y_{u_i} + \Delta y_{d_i}) Y^{\text{eq}} - (\Delta y_{\ell\alpha} + \Delta y_{e\alpha}) Y^{\text{eq}}. \quad (9.16)$$

Note that, in the temperature range of interest, all the asymmetry-densities in the right-hand-side of this equation are generally non-vanishing. In particular, N_1 decays generate a non-vanishing chemical potential for the Higgs particles which, in turn (see eqn (14.5)), induces a non-vanishing chemical potential to the top-quarks. The QCD sphaleron condition of eqn (8.5) further implies that this happens for all the lighter quarks. The only exceptions are $\Delta y_{e_{\mu}}$ above $T \sim 10^9$ GeV, and Δy_{e_e} that is not populated during standard leptogenesis.

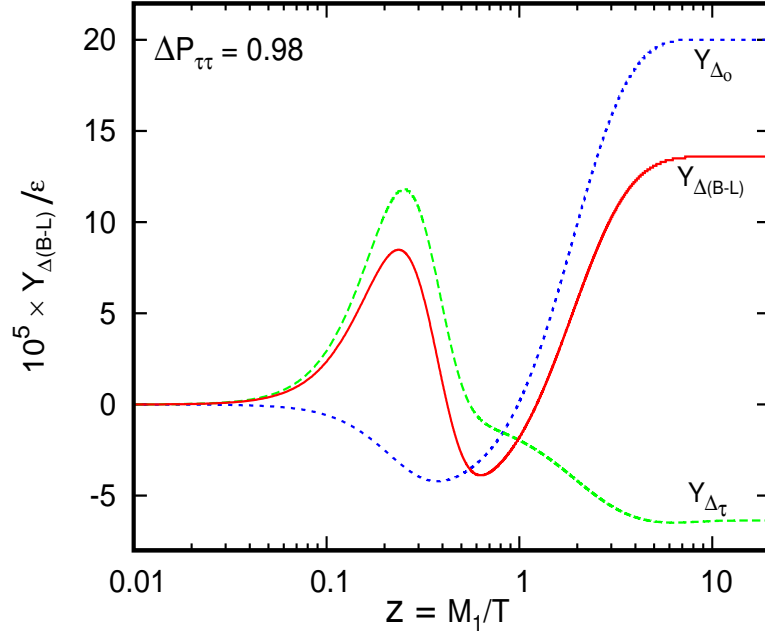


Figure 9.3: The evolution of the charges densities Y_{Δ_τ} (green dashed line), Y_{Δ_o} (blue dotted line) and $Y_{\Delta(B-L)}$ (red solid line) in units of $10^{-5} \epsilon$ as a function of $z = M_1/T$. We use $M_1 = 10^{10}$ GeV and $\tilde{m}_{l_1} = 0.06$ eV. The b - and τ -Yukawa reactions and the electroweak sphalerons are in equilibrium. The figure refers to the point labeled (d) in fig. 9.2 that corresponds to $\Delta P_{\tau\tau} = 0.98$ and $\epsilon_{\tau\tau} \approx 1.2 \epsilon$. (Figure adapted from ref. [63]).

The asymmetry densities for the quarks and for the $SU(2)$ -singlet leptons can always be re-expressed in terms of the asymmetry densities of the $SU(2)$ -doublet leptons. This procedure makes use of the condition of hypercharge neutrality, eqn (14.4), that involves all three generation fermions. Consequently, once the appropriate substitutions are implemented, one obtains that the densities of the Δ_α charges in eqn (9.16) correspond to linear combinations of the asymmetry densities of all the lepton doublets:

$$Y_{\Delta_\alpha} = -[A]_{\alpha\beta}^{-1} \Delta y_{\ell_\beta} Y^{\text{eq}}. \quad (9.17)$$

The matrix $[A]^{-1}$ can then be inverted to give Δy_{ℓ_α} in terms of the charge densities Y_{Δ_α} (see Appendix 14). The same is true also for the Higgs asymmetry. We can thus write:

$$\Delta y_{\ell_\alpha} = - \sum_{\beta} A_{\alpha\beta} \frac{Y_{\Delta_\beta}}{Y^{\text{eq}}}, \quad \Delta y_{\phi} = - \sum_{\beta} C_{\beta}^{\phi} \frac{Y_{\Delta_\beta}}{Y^{\text{eq}}}. \quad (9.18)$$

The matrix A (introduced in ref. [65]) and the vector C^{ϕ} (introduced in ref. [204]) generalize the coefficients c_{ℓ} and c_{ϕ} , defined in eqn (8.4) for the case of pure flavour states, to the general flavour case. Their numerical values are determined by the relevant set of chemical equilibrium conditions, and therefore depend on temperature.

As a consequence of eqns (9.18), the evolution of one density Y_{Δ_α} depends on the other charge densities, that is, the BE for the lepton flavours form a system of coupled equations. This phenomenon is sometimes referred to as ‘electroweak sphaleron flavour mixing’, and was studied in refs. [63, 210]. One example of the possible effects is depicted in fig. 9.3. The plot gives the evolution of Y_{Δ_τ} , Y_{Δ_o} and $Y_{\Delta(B-L)} = Y_{\Delta_\tau} + Y_{\Delta_o}$ for the flavour configuration corresponding to the point labeled with (d) in fig. 9.2. Y_{Δ_τ} is in the strong washout regime, $\tilde{m}_{\tau\tau} \approx 0.06$ eV, and the τ CP asymmetry is relatively

large, $\epsilon_{\tau\tau} \approx 1.2\epsilon$. For Y_{Δ_o} , the value $\tilde{m}_{oo} \approx 10^{-3}\text{eV}$ is close to optimal, and the asymmetry is smaller in size and opposite in sign to $\epsilon_{\tau\tau}$: $\epsilon_{oo} \approx -0.2\epsilon$. For the relevant temperature, $T \lesssim 10^{12}\text{GeV}$, electroweak sphaleron, h_b and h_τ reactions are in equilibrium, yielding the matrices of coefficients [in the $(Y_{\Delta_o}, Y_{\Delta_\tau})$ basis]:

$$A = \frac{1}{230} \begin{pmatrix} 196 & -24 \\ -9 & 156 \end{pmatrix}, \quad C^\phi = \frac{1}{115}(41, 56). \quad (9.19)$$

As apparent from fig. 9.3, Y_{Δ_o} gives the dominant contribution to the final $B - L$ asymmetry. A qualitative explanation goes as follows. For $z \gtrsim 0.1$, a large asymmetry builds up quickly in the τ flavour and contributes to the early washout of Y_{Δ_o} (the sign of Y_{Δ_τ} that is opposite to that of Y_{Δ_o} combines with the negative sign of $A_{o\tau} = -24/230$ yielding an increase of the washout rates). Thus, the early (negative) asymmetry in Y_{Δ_o} remains suppressed. It follows that the opposite sign o asymmetry generated from N_1 decays at $z \gtrsim 1$ is largely unbalanced. As concerns the τ asymmetry, typically of strong washout regimes, it ‘freezes-out’ at $z > 1$ with an absolute value that is sizably smaller than the values reached at earlier times.

Remarkably, as can be clearly seen in fig. 9.3, the $B - L$ asymmetry changes sign twice during its evolution. This peculiar behavior is qualitatively different from what would be obtained by neglecting the sphaleron induced flavour mixing, when there is a single change of sign. Other interesting washout effects induced by sphaleron mixing have been analyzed in ref. [211].

9.4 Phenomenological consequences

As discussed in Section 9.2, starting from a specific set of parameters within a given seesaw model, the inclusion of flavour effects in the calculation of the baryon asymmetry can change the result by factors of a few to orders of magnitude [63, 64]. In this section, we take a more ‘‘bottom-up’’ perspective of flavoured leptogenesis. As discussed below eqn (2.5), there are (currently) 14 unknown parameters in the three generation type I seesaw model, of which only a few are accessible to low energy experiments. It is interesting, from a phenomenological perspective, to constrain the unknown parameters by requiring that leptogenesis works successfully. We will find that the inclusion of flavour effects does not change the resulting constraints in a significant way.

9.4.1 The upper bound on the light neutrino mass scale

In the ‘‘single flavour’’ calculation, successful thermal leptogenesis requires a light ν mass scale $\lesssim 0.2\text{eV}$ [157, 212]. The argument leading to this bound does not apply in the flavoured calculation. Currently, there is no consensus in the literature on the value of the bound in the flavoured case. Analytic arguments (which we reproduce below) and the numerical analysis of Refs [62, 180, 210], suggest that flavoured leptogenesis can be tuned to work for m_{\min} up to $0.5 - 1\text{eV}$. This is of the order of current bounds from Large Scale Structure data and WMAP [119, 177–179]. The leptogenesis bound is relaxed because there is more CP violation available in flavoured leptogenesis. The upper limit of eqn (5.17) on the total CP asymmetry *decreases* like $\Delta m_{\text{atm}}^2/m_{\max}$, as the light neutrino mass scale m_{\max} increases. There is therefore an upper bound on m_{\max} ($\simeq m_{\min}$ for degenerate neutrinos). However, the limit (5.17) does not apply to the flavoured CP asymmetries, which can increase with the light neutrino mass scale, as shown in eqn (5.10).

The limit $m_\nu \lesssim 0.2\text{eV}$ [157], obtained in the single flavour approximation, can be understood as follows. For such a high mass scale, the light neutrinos are almost degenerate, with $m_1 \simeq m_2 \simeq m_3$. Due to the lower bound $\tilde{m}_1 \geq m_{\min}$, leptogenesis takes place in the strong washout regime. Taking the total CP asymmetry to be close to the upper bound of eqn (5.17), the final baryon asymmetry can be approximated as

$$Y_{\Delta B} \sim 10^{-3}\epsilon \frac{m_*}{\tilde{m}_1} \propto \frac{M_1 m_*}{v_u^2} \frac{\Delta m_{\text{atm}}^2}{m_{\min}^2}. \quad (9.20)$$

As the light neutrino mass scale is increased, M_1 and, correspondingly, the temperature of leptogenesis, must increase to compensate the $\Delta m_{\text{atm}}^2/m_{\min}^2$ suppression. The leptogenesis temperature is, however,

bounded from above, by the requirement of having lepton number violating processes out of equilibrium. For instance, the requirement that the $\Delta L = 2$ processes are out of equilibrium (we estimate $\Gamma \sim \gamma/n^{\text{eq}}$, with γ from eqn 13.26) gives

$$\frac{2m_{\min}^2 T^3}{\pi^2 v^4} \lesssim \frac{20T^2}{M_{\text{pl}}}, \quad (9.21)$$

which implies $M_1 \lesssim 10^{10}(\text{eV}/m_{\min})^2 \text{ GeV}$. There is therefore an upper bound on the baryon asymmetry which scales as $1/m_{\min}^4$. A large enough baryon asymmetry is obtained for $m_{\min} \lesssim 0.1 \text{ eV}$.

Consider now the flavoured case. We define $c^2 = P_{\tau\tau}$ and $s^2 = P_{oo}$ (with $s^2 + c^2 = 1$), so that $\Gamma_{\tau\tau} = c^2\Gamma_D$ and $\Gamma_{oo} = s^2\Gamma_D$. The individual flavour asymmetries, $\epsilon_{\alpha\alpha}$, satisfy the bound of eqn (5.10), while their sum satisfies the stricter bound of eqn (5.17). So, if we can arrange $-\epsilon_{oo} = \epsilon_{\tau\tau} \sim 3M_1 m_{\min}/(16\pi v^2)$, then for strong flavoured washout, and for sufficiently slow $\Delta L = 2$ processes, the final baryon asymmetry (from *e.g.* eqn (4.15)) is

$$Y_{\Delta B} \sim 10^{-3} \frac{3M_1 m_{\min} c^4 - s^4}{16\pi v_u^2 c^2 s^2} \sim 3 \times 10^{-11} \frac{c^4 - s^4}{c^2 s^2} \frac{M_1}{10^{10} \text{ GeV}}. \quad (9.22)$$

To show that the $m_{\min} \lesssim 0.2 \text{ eV}$ bound does not apply in the flavoured case, we must demonstrate that the three ingredients that entered the derivation of eqn (9.22) – (i) an appropriate set of flavoured CP asymmetries, (ii) negligible $\Delta L = 2$ processes, and (iii) strong flavoured washout – can be realized in a model.

(i) To obtain the desired decay rates and asymmetries we use the Casas-Ibarra [101] parametrization, in terms of m_i , M_j , U and the complex orthogonal matrix $R \equiv v D_m^{-1/2} U^T \lambda D_M^{-1/2}$ (see the discussion around eqn (2.12)). This ensures that we obtain the correct low-energy parameters. We take $o = \mu$ for simplicity, and write R as a rotation through the complex angle $\theta = \varphi + i\eta$. The flavour asymmetries, for maximal atmospheric mixing, are

$$\begin{aligned} \epsilon_{\mu\mu} &= \frac{3M_1 m_{\min}}{16\pi v^2} \frac{\text{Im}\{U_{\mu k}^* R_{k1} U_{\mu p} R_{p1}\}}{\sum_{\alpha} |\sum_j U_{\alpha j}^* R_{j1}|^2} = -\frac{3M_1 m_{\min}}{16\pi v^2} \frac{\sinh \eta \cosh \eta}{\sinh^2 \eta + \cosh^2 \eta} \cos 2\varphi, \\ \epsilon_{\tau\tau} &= \frac{3M_1 m_{\min}}{16\pi v^2} \frac{\text{Im}\{R_{k1} U_{\tau k}^* R_{p1} U_{\tau p}\}}{\sum_{\alpha} |\sum_j R_{j1} U_{\alpha j}^*|^2} = +\frac{3M_1 m_{\min}}{16\pi v^2} \frac{\sinh \eta \cosh \eta}{\sinh^2 \eta + \cosh^2 \eta} \cos 2\varphi \end{aligned} \quad (9.23)$$

The flavour dependent decay rates $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$ are proportional to

$$\begin{aligned} |\lambda_{\mu 1}|^2 &= \frac{M_1 m_{\min}}{2v^2} |R_{11} + R_{21}|^2 = \frac{M_1 m_{\min}}{2v^2} (\sinh^2 \eta + \cosh^2 \eta + \sin 2\varphi), \\ |\lambda_{\tau 1}|^2 &= \frac{M_1 m_{\min}}{2v^2} |R_{11} - R_{21}|^2 = \frac{M_1 m_{\min}}{2v^2} (\sinh^2 \eta + \cosh^2 \eta - \sin 2\varphi). \end{aligned} \quad (9.24)$$

We see that we can choose η and φ as required to obtain eqn (9.22) with $c^4 - s^4 \gtrsim s^2 c^2$.

(ii) Eqn (9.21) gives the condition for the $\Delta L = 2$ processes to be out of equilibrium: $M_1 \lesssim 10^{10}(\text{eV}/m_{\min})^2 \text{ GeV}$. Combined with eqn (9.22), this gives

$$Y_B \sim 3 \times 10^{-11} \frac{c^4 - s^4}{c^2 s^2} \left(\frac{\text{eV}}{m_{\min}} \right)^2. \quad (9.25)$$

(iii) In this strongly washed out area of parameter space, N_1 decays take place in the flavoured regime if $\Gamma_{\tau} \gg \Gamma_{ID}$ [180, 181] (see Appendix 15.4). Suppose $s^2 \ll c^2$. Then eqn (9.22) is a valid estimate, provided $\Gamma_{\tau} \gg \Gamma_{ID}$ when $s^2\Gamma_{ID} = H$:

$$\frac{\Gamma_{\tau}}{H} \simeq \frac{10^{12} \text{ GeV}}{T} \gg \frac{1}{s^2} = \frac{1}{B_{\phi\ell_o}^{N_1}} \quad (9.26)$$

We conclude that with careful tuning, thermal leptogenesis can work for $m_{\min} \lesssim \text{eV}$. This can be seen in Fig. 9.4. For the plot on the left, flavour effects are included, and leptogenesis can work for values $10^{-4} \text{ eV} \lesssim \tilde{m}_1 \lesssim 1 \text{ eV}$. At the upper bound we have $\tilde{m}_1 \simeq m_1$, so the cosmological bound is saturated.

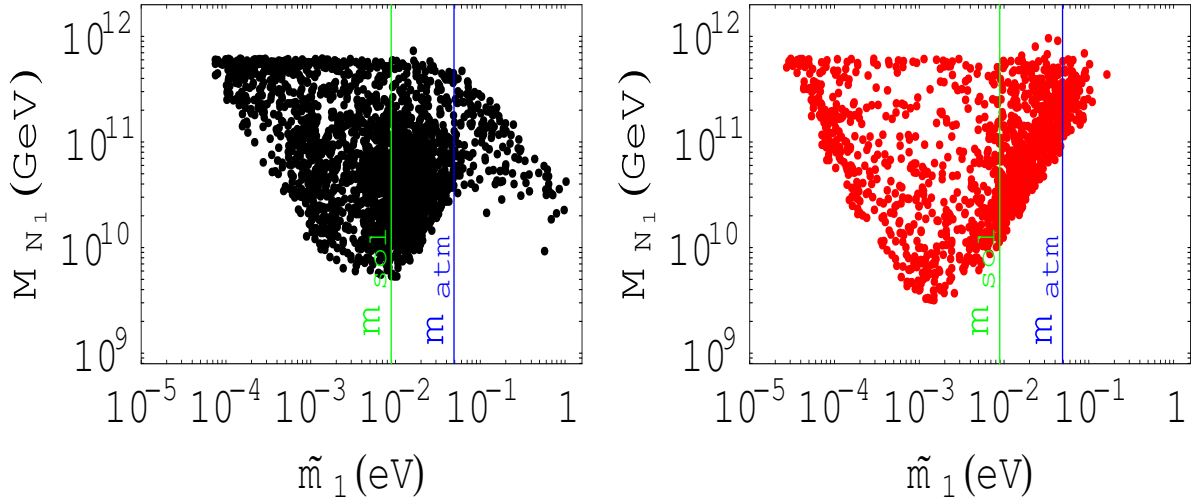


Figure 9.4: Points in the $M_1 - \tilde{m}_1$ plane where thermal leptogenesis generates a sufficient baryon asymmetry (updated from [210]). These plots clearly show the lower bound on M_1 arising when the N_i are hierarchical. The plot on the right corresponds to a single flavour calculation. The plot on the left includes flavour effects. The lower bound on M_1 is similar in both cases. Leptogenesis works for larger values of \tilde{m}_1 in the flavoured plot, which is obtained with two-flavour BE, and imposing the constraint (15.37).

9.4.2 Sensitivity of $Y_{\Delta B}$ to low energy phases

An important, but disappointing, feature of “single-flavour” leptogenesis is the lack of a model-independent relation between CP violation in the leptogenesis processes and the (observable) phases of the lepton mixing matrix U (see eqn (2.7)). With three singlet neutrinos N_i , thermal leptogenesis can work with no CP violation in U (see *e.g.* [213]) and, conversely, leptogenesis can fail in spite of non-vanishing phases in U [214]. In some specific models, however, it is possible to establish a relation [214, 215].

In unflavoured leptogenesis, it is simple to see that the baryon asymmetry is insensitive to phases of U [214]. Using the Casas-Ibarra parametrization of eqn (2.11) in eqn (5.13), ϵ can be written in a form where U does not appear:

$$\epsilon = -\frac{1}{16\pi v_u^2} \frac{\sum_{j,k,p} \text{Im}\{(R_{j1}^* m_j R_{jk})(R_{p1}^* m_p R_{pk})\}}{\sum_i m_i |R_{i1}|^2} M_k g\left(\frac{M_k^2}{M_1^2}\right). \quad (9.27)$$

Alternatively, in the top-down parametrization in terms of V_R , V_L and D_λ (see Section 2.1.1), ϵ depends on the phases in V_R , while the U phases *could* arise only from V_L .

The latter observation can be exploited to relate the U phases to flavoured leptogenesis in models where there is no CP violation in V_R ²² [63, 64, 217–220]. However, for flavoured leptogenesis with three N_i ’s, it is still true that the baryon asymmetry is not sensitive to phases of U [221], in the sense that the baryon asymmetry can be accounted for with the phases of U having any value. Conversely, if the phases of U are measured, the baryon asymmetry is still not constrained. This can be seen from the scatter plots of ref. [221], which show that, for any value of the U phases, the baryon asymmetry can be large enough.

In the two N_i model (see section 2.1.2), there are three phases. Relations between U phases and leptogenesis were obtained in unflavoured leptogenesis [106]. The relation between the flavoured CP asymmetries $\epsilon_{\alpha\alpha}$ and the phases of U , focusing on various texture models, was discussed in [222]. Possible

²²Such models may be difficult to construct [216].

relations between low energy observables and the baryon asymmetry, in the general case and for particular texture models, were discussed in [64].

In minimal left-right models with spontaneous CP violation, there is a single phase which controls all CP violation including that for leptogenesis [223, 224].

9.4.3 The lower bound on T_{reheat}

In the space of leptogenesis parameters, there is an envelope inside which leptogenesis *can* work. In the single-flavour calculation, the most important parameters are M_1 , Γ_D (or, equivalently \tilde{m}), ϵ and the light neutrino mass scale [212]. Including flavour gives the parameter space more dimensions ($M_1, \epsilon_{\alpha\alpha}, \Gamma_{\alpha\alpha}\dots$), but the envelope can still be projected onto the $M_1 - \tilde{m}$ plane by fixing ϵ to its maximum value of eqn (5.17), which is a function of M_1 .

For thermal leptogenesis to take place, an adequate number density of N_1 must be produced by thermal scattering in the plasma, suggesting $T_{\text{reheat}} \gtrsim M_1/5$ [59, 60]. There is a lower bound on M_1 , from requiring a large enough CP asymmetry. The corresponding bound on T_{reheat} depends on the fine details of reheating, the dynamics of which has been studied in refs. [59, 60]. To avoid these complications, the lower bound on M_1 is usually quoted.

For non-degenerate light neutrinos, the lowest allowed value of M_1 occurs at $\tilde{m} \sim m_*$, where analytic approximations are unreliable. It was shown in [129, 209, 210] that including flavour effects does not relax the bound with respect to the unflavoured case [60, 212], and we still have

$$M_1 > 2 \times 10^9 \text{ GeV}, \quad \text{non - degenerate } m_i. \quad (9.28)$$

This is shown in Fig. 9.4 [210].

10 Variations

The bulk of this review focuses on thermal leptogenesis, with hierarchical singlet neutrinos, and where the CP asymmetry is generated in the decays of the lightest singlet neutrino. In this chapter, we give brief overviews of various alternative scenarios of leptogenesis. More detailed presentations can be found in the original literature, to which we give appropriate references.

10.1 Supersymmetric thermal leptogenesis

In the framework of supersymmetric seesaw models, new leptogenesis mechanisms become possible, such as Affleck-Dine leptogenesis or soft leptogenesis. In this subsection, we focus on the standard thermal leptogenesis scenario with hierarchical singlet N 's, where the presence of supersymmetry (SUSY) makes only small qualitative and quantitative differences [54,225]. We have often referred to the supersymmetric modifications in previous chapters, so here we collect this information and summarize the differences that arise when calculating the baryon asymmetry in the supersymmetric type I seesaw.

Neglecting small SUSY-breaking terms, a singlet neutrino N_i and its superpartner, the singlet sneutrino \tilde{N}_i , have equal masses, equal decay rates $\Gamma_{N_i} = \Gamma_{\tilde{N}_i}$ and equal CP asymmetries $\epsilon_{N_i} = \epsilon_{\tilde{N}_i}$. In this approximation, estimating the $B - L$ asymmetry in the minimal supersymmetric standard model (MSSM) with respect to the SM case (at fixed values of the Yukawa couplings and M_{N_i} masses) amounts to a (careful) counting of a few numerical factors:

1. There are twice the number of states running in the loops, and thus the CP asymmetries ϵ_{N_i} and $\epsilon_{\tilde{N}_i}$ are roughly twice the SM value of ϵ_{N_i} (see the loop function $g(x_j)$ in eqn (5.15)).
2. Including the asymmetry generated in \tilde{N}_i decays gives another enhancement of a factor of two.
3. In the MSSM, the plasma is populated by twice as many particles as there are in the SM. More precisely, we have $g_*^{\text{MSSM}}/g_*^{\text{SM}} = 228.75/106.75 \approx 2.14$. Thus the lepton asymmetries to entropy ratios are reduced by 1/2.
4. Due to the additional $\tilde{\ell}\tilde{\phi}$ final states, the N_i decay rate is twice faster than in the SM. (For the scalar neutrinos there are also two channels $\tilde{N}_i \rightarrow \tilde{\ell}\tilde{\phi}, \ell\tilde{\phi}$.) In the strong washout regime, the associated inverse-decay reactions double the washout rates, reducing the asymmetry by a factor of two. In the weak washout regime, the production of N_i and \tilde{N}_i is more efficient, and this increases the asymmetry by the same factor.
5. The expansion rate of the Universe is proportional to $\sqrt{g_*}$ and is therefore roughly a factor of $\sqrt{2}$ faster in SUSY ($m_*^{\text{MSSM}} \approx \sqrt{2}m_*^{\text{SM}}$). This reduces the time during which the strong washout processes can erase the asymmetry, yielding a $\sqrt{2}$ enhancement. For weak washouts, it reduces the time for N_i and \tilde{N}_i production, yielding a $\sqrt{2}$ suppression of the final asymmetry.
6. Finally, in both the SM and the MSSM cases, the asymmetry in baryons $Y_{\Delta B}$ is of order 1/3 of the asymmetry in $B - L$. The exact relation differs only slightly: 28/79 for the SM *vs.* 8/23 for the MSSM (see eqns (14.17) and (14.18), respectively).

Putting all these factors together, we estimate:

$$\frac{(Y_{\Delta B})^{\text{MSSM}}}{(Y_{\Delta B})^{\text{SM}}}\Big|_{M_i, \lambda_\alpha, h_i} \approx \begin{cases} \sqrt{2} & \text{(strong washout),} \\ 2\sqrt{2} & \text{(weak washout).} \end{cases} \quad (10.1)$$

Thus, in the MSSM, in spite of the doubling of the particle spectrum and of the large number of new processes involving superpartners [186], one does not expect major numerical changes with respect to the non-supersymmetric case.

Supersymmetric thermal leptogenesis has a potential gravitino problem [76,77,79,80]. After inflation, the universe thermalizes to a reheat temperature T_{reheat} . Gravitinos are produced by thermal scattering

in the bath, and the rate is higher at higher temperatures. The gravitinos are long-lived; if there are lighter SUSY particles (the gravitino is not the LSP), the decay rate can be estimated as [79]

$$\Gamma \propto \frac{m_{\text{grav}}^3}{m_{\text{pl}}^2} \simeq \left(\frac{m_{\text{grav}}}{20T eV} \right)^3 \text{sec}^{-1}. \quad (10.2)$$

If too many gravitinos decay during or after Big Bang Nucleosynthesis (at $t \sim$ seconds), the resulting energetic showers in the thermal bath destroy the agreement between predicted and observed light element abundances [75, 78, 81]. There are several possible solutions to the gravitino problem:

1. The reheat temperature is low enough, $T_{\text{reheat}} \lesssim 10^6 - 10^{10}$ GeV [81, 226] that the gravitino density is small. The precise bound depends on the gravitino mass and SUSY spectrum.
2. The gravitinos decay before BBN; sufficiently heavy gravitinos can arise in anomaly-mediated scenarios [227].
3. Late time entropy production can dilute the gravitino abundance, but it also dilutes the $B - L$ asymmetry [228].
4. The gravitino is the LSP (and the dark matter), in which case the bound on T_{reheat} is less restrictive [229, 230]. The gravitino can be the LSP in gauge-mediated scenarios.

As discussed in Section 5.2, there is a lower bound on the reheat temperature $T_{\text{reheat}} \gtrsim 10^9$ GeV, to produce a big enough baryon asymmetry by thermal leptogenesis (in the three generation type I seesaw with hierarchical N_i masses). This is difficult to reconcile with the first solution described above [231], which is arguably the most plausible one in gravity mediated scenarios. A large enough baryon asymmetry can be produced at lower T_{reheat} , if $M_2 - M_1 \lesssim \Gamma_D$, the N_1 decay width.

10.2 Less hierarchical N 's

It is crucial for thermal leptogenesis that there are more than one singlet neutrino. In particular, the CP asymmetry that is produced in N_1 decays comes from interference between the tree diagram and the loop diagrams involving the heavier N_i as virtual particles in the loop. In the conventional picture, however, one assumes a strong hierarchy, $M_{i>1} \gg M_1$. Then, an upper bound on the CP asymmetry in N_1 decays applies (see eqn (5.10)), which provides much of the predictive power of the conventional leptogenesis scenario.

However, as mentioned in section 5.2, when the singlet neutrino masses are not very strongly hierarchical, a term that is higher order in $M_1/M_{i>1}$ can become important. Specifically, there is a contribution to the CP asymmetry that is proportional to $M_1^3/(M_2^2 M_3)$ and does not vanish when the light neutrino masses are degenerate [162, 163].

In addition to this effect of the heavier singlet neutrino masses on the CP asymmetry in the N_1 decays, there are also interesting effects related to the decays of the heavier singlet neutrinos $N_{2,3,\dots}$ themselves. These effects are discussed in the following subsection.

10.2.1 N_2 effects

When analyzing leptogenesis, the effects of N_2 and N_3 are often neglected. This is reasonable if $T_{\text{reheat}} < M_{2,3}$. However, if $T_{\text{reheat}} > M_{2,3}$, one should not assume that the L -violating interactions of N_1 would washout any lepton asymmetry generated at temperatures $T \gg M_1$ and, in particular, the asymmetry generated in the decays of $N_{2,3}$. Under various (rather generic) circumstances, the lepton asymmetry generated in $N_{2,3}$ decays survives the N_1 leptogenesis phase. Thus, it is quite possible that the lepton asymmetry relevant for baryogenesis originates mainly (or, at least, in a non-negligible part) from $N_{2,3}$ decays.

The possibility that N_2 leptogenesis can successfully explain the baryon asymmetry of the Universe has been shown in two limiting cases:

1. The “ N_1 -decoupling” scenario, in which the Yukawa couplings of N_1 are simply too weak to washout the N_2 -generated asymmetry [187, 208, 232].
2. The “strong N_1 -coupling” scenario, where N_1 -related decoherence effects project the lepton asymmetry from N_2 decays onto a flavour direction that is protected against N_1 washout [65, 71, 188, 207].

It is plausible that the role of N_2 leptogenesis cannot be ignored also in the intermediate range for N_1 couplings, but the analysis in this case is complicated and has not been carried out yet.

The N_1 -decoupling scenario is simple to understand. It applies when N_1 is weakly coupled to the lepton doublets (see eqn (4.7):

$$\tilde{m}_1 \ll m_*, \quad (10.3)$$

where $m_* = 1.66g_*^{1/2}8\pi v_u^2/m_{\text{pl}} \approx 10^{-3}$ eV and

$$\tilde{m}_i \equiv \frac{v_u^2(\lambda^\dagger \lambda)_{ii}}{M_i}. \quad (10.4)$$

In this case, the asymmetry generated in thermal N_1 leptogenesis is too small. Furthermore, the N_1 washout effects are negligible and, consequently, the asymmetry generated in N_2 decays survives.

We next discuss N_2 leptogenesis in the strong N_1 -coupling limit. We are interested in the case that a sizeable asymmetry is generated in N_2 decays, while N_1 leptogenesis is inefficient. We thus assume that the N_2 -related washout is not too strong, while the N_1 -related washout is so strong that it makes N_1 leptogenesis fail:

$$\tilde{m}_2 \not\gg m_*, \quad \tilde{m}_1 \gg m_*. \quad (10.5)$$

To further simplify the analysis, we impose two additional conditions [188]: thermal leptogenesis, and strong hierarchy, M_2/M_1 . These two conditions together guarantee that

$$n_{N_1}(T \sim M_2) \approx 0, \quad n_{N_2}(T \sim M_1) \approx 0. \quad (10.6)$$

Thus, the dynamics of N_2 and N_1 are decoupled: there are neither N_1 -related washout effects during N_2 leptogenesis, nor N_2 -related washout effects during N_1 leptogenesis.

The N_2 decays into a combination of lepton doublets that we denote by ℓ_2 :

$$|\ell_i\rangle = (\lambda^\dagger \lambda)_{ii}^{-1/2} \sum_\alpha \lambda_{\alpha i} |\ell_\alpha\rangle. \quad (10.7)$$

The second condition in (10.5) implies that already at $T \gtrsim M_1$ the N_1 -Yukawa interactions are sufficiently fast to quickly destroy the coherence of ℓ_2 . Then a statistical mixture of ℓ_1 and of the state orthogonal to ℓ_1 builds up, and it can be described by a suitable diagonal density matrix. On general grounds, one expects that decoherence effects proceed faster than washout. In the relevant range, $T \gtrsim M_1$, this is also ensured by the fact that the dominant $\mathcal{O}(\lambda^2)$ washout process (the inverse decay $\ell\phi \rightarrow N_1$) is blocked by thermal effects [60], and only scatterings with top-quarks and gauge bosons, that have additional suppression factors of h_t^2 and g^2 , contribute to the washout.

Let us consider the case where both N_2 and N_1 decay at $T \gtrsim 10^{12}$ GeV, so that flavour effects are irrelevant. We also neglect the effects of $\Delta L = 2$ interactions, which are generically in equilibrium at $T \gtrsim \text{few} \times 10^{12}$ GeV. (This scenario is interesting to consider, even though it may only occur in a narrow temperature range.) A convenient choice for an orthogonal basis for the lepton doublets is $(\ell_1, \ell_0, \ell'_0)$ where, without loss of generality, ℓ'_0 satisfies $\langle \ell'_0 | \ell_2 \rangle = 0$. Then the asymmetry ΔY_{ℓ_2} produced in N_2 decays decomposes into two components:

$$\Delta Y_{\ell_0} = c^2 \Delta Y_{\ell_2}, \quad \Delta Y_{\ell_1} = s^2 \Delta Y_{\ell_2}, \quad (10.8)$$

where $c^2 \equiv |\langle \ell_0 | \ell_2 \rangle|^2$ and $s^2 = 1 - c^2$. The crucial point here is that we expect, in general, $c^2 \neq 0$ and, since $\langle \ell_0 | \ell_1 \rangle = 0$, ΔY_{ℓ_0} is protected against N_1 washout. Consequently, a finite part of the asymmetry ΔY_{ℓ_2}

from N_2 decays survives through N_1 leptogenesis. A more detailed analysis [188] finds that ΔY_{ℓ_1} is not entirely washed out, and that the final lepton asymmetry is given by $Y_{\Delta L} = (3/2)\Delta Y_{\ell_0} = (3/2)c^2 \Delta Y_{\ell_2}$.

For $10^9 \text{ GeV} \lesssim M_1 \lesssim 10^{12} \text{ GeV}$, flavour issues modify the quantitative details, but the qualitative picture, and in particular the survival of a finite part of ΔY_{ℓ_2} , still hold. On the other hand, for $M_1 \lesssim 10^9 \text{ GeV}$, the full flavour basis $(\ell_e, \ell_\mu, \ell_\tau)$ is resolved and, in general, there are no directions in flavour space where an asymmetry is protected and Y_{ℓ_2} can be erased entirely.

The conclusion is that N_2 and N_3 leptogenesis cannot be ignored, unless at least one of the following conditions applies:

1. The asymmetries and/or the washout factors vanish, $\epsilon_{N_2}\eta_2 \approx 0$ and $\epsilon_{N_3}\eta_3 \approx 0$.
2. N_1 -related washout is still significant at $T \lesssim 10^9 \text{ GeV}$.
3. The reheat temperature is below M_2 .

In all other cases, the $N_{2,3}$ -related parameters play a role in determining the baryon asymmetry of the Universe. Consequently, relations between these parameters and neutrino masses are important. Such relations were obtained in Ref. [106,158], and they lead, for the case that the light neutrino masses have normal hierarchy, to the following consequences:

- In the framework with two singlet neutrinos, both N_1 and N_2 interactions are in the strong washout regime, with both $\tilde{m}_i \geq 0.009 \text{ eV}$ [106], and at least one $\geq 0.025 \text{ eV}$.
- In the framework with three singlet neutrinos, at least two N_i 's have interactions in the strong washout regime, with $\tilde{m}_i \geq 0.005 \text{ eV}$ and at least one $\geq 0.02 \text{ eV}$.

The lower bounds are stronger for inverted hierarchy, and even more so in the framework with three singlet neutrinos and quasi-degenerate light neutrinos.

10.2.2 Resonant leptogenesis

A resonant enhancement of the CP asymmetry in N_1 decay occurs when the mass difference between N_1 and N_2 is of the order of the decay widths. Such a scenario has been termed ‘resonant leptogenesis’, and has benefited from many studies in different formalisms²³ [61, 82, 85, 131, 132, 184, 189, 234–244]. As this review focuses on hierarchical N_i , we briefly list the idea and some references.

The resonant effect is related to the self energy contribution to the CP asymmetry. Consider, for simplicity, the case where only N_2 is quasi-degenerate with N_1 . Then, the self-energy contribution involving the intermediate N_2 , to the total CP asymmetry (we neglect important flavour effects [61]) is given by

$$\epsilon_{N_1}(\text{self - energy}) = -\frac{M_1}{M_2} \frac{\Gamma_2}{M_2} \frac{M_2^2 \Delta M_{21}^2}{(\Delta M_{21}^2)^2 + M_1^2 \Gamma_2^2} \frac{\text{Im}[(\lambda^\dagger \lambda)_{12}^2]}{(\lambda^\dagger \lambda)_{11}(\lambda^\dagger \lambda)_{22}}. \quad (10.9)$$

The resonance condition reads

$$M_2 - M_1 = \Gamma_2/2. \quad (10.10)$$

In this case

$$|\epsilon_{N_1}(\text{resonance})| \simeq \frac{1}{2} \frac{|\text{Im}[(\lambda^\dagger \lambda)_{12}^2]|}{(\lambda^\dagger \lambda)_{11}(\lambda^\dagger \lambda)_{22}}. \quad (10.11)$$

Thus, in the resonant case, the asymmetry is suppressed by neither the smallness of the light neutrino masses, nor the smallness of their mass splitting, nor small ratios between the singlet neutrino masses. Actually, the CP asymmetry could be of order one. (More accurately, $|\epsilon_{N_1}| \leq 1/2$.)

With resonant leptogenesis, the Boltzmann Equations are different. The densities of N_1 and N_2 are followed, since the both contribute to the asymmetry, and the relevant timescales are different. For instance, the typical time-scale to build up coherently the CP asymmetry is unusually long, of order

²³See [233] for a comparison of the different calculations.

$1/\Delta M$. In particular, it can be larger than the time-scale for the change of the abundance of the sterile neutrinos. This situation implies that quantum effects in the Boltzmann equations can be significant, in the weak or mild washout regime, for resonant leptogenesis [241, 242].

The fact that the asymmetry could be large, independently of the singlet neutrino masses, opens up the possibility of low scale resonant leptogenesis. Models along these lines have been constructed in Refs. [61, 236, 240, 245]. It is a theoretical challenge to construct models where a mass splitting as small as the decay width is naturally achieved. For attempts that utilize approximate flavour symmetries see, for example, [189, 237, 239, 243, 246], while studies of this issue in the framework of minimal flavour violation can be found in [131, 132, 244].

10.3 Soft leptogenesis

The modifications to standard leptogenesis due to supersymmetry have been discussed in section 10.1. The important parameters there are the Yukawa couplings and the singlet neutrino parameters, which are all superpotential terms (see eqn 2.21):

$$W^{(N)} = \lambda L H_u N^c + \frac{1}{2} M N^c N^c. \quad (10.12)$$

Supersymmetry must, however, be broken. In the framework of the supersymmetric standard model extended to include singlet neutrinos (SSM+N), there are, in addition to the soft supersymmetry breaking terms of the SSM, terms that involve the singlet sneutrinos \tilde{N} , in particular bilinear (B) and trilinear (A) scalar couplings. These terms provide additional sources of lepton number violation and of CP violation. Scenarios where these terms play a dominant role in leptogenesis have been termed ‘soft leptogenesis’ [86, 87, 247–258].

Soft leptogenesis would have taken place even with a single lepton generation. To simplify things we work, therefore, in the framework of a single generation SSM+N. The relevant soft supersymmetry terms in the Lagrangian are given in eqn (10.13):

$$\mathcal{L}_{\text{soft}}^{(N)} = - \left(A \lambda \tilde{L} \phi_u \tilde{N}^c + \frac{1}{2} B M \tilde{N}^c \tilde{N}^c + \text{h.c.} \right). \quad (10.13)$$

In addition, the electroweak gaugino masses play a role:

$$\mathcal{L}_{\text{soft}}^{(\lambda_2)} = - (m_2 \lambda_2^a \lambda_2^a + \text{h.c.}). \quad (10.14)$$

Here λ_2^a ($a = 1, 2, 3$) are the $SU(2)$ gauginos. The effects related to λ_1 , the $U(1)_Y$ gaugino, are similar to (and usually less important than) those of λ_2 , so we do not present them explicitly.

The Lagrangian derived from eqns (10.12), (10.13) and (10.14) has two independent physical CP violating phases:

$$\phi_N = \arg(AB^*), \quad (10.15)$$

$$\phi_W = \arg(m_2 B^*). \quad (10.16)$$

These phases give the CP violation that is necessary to dynamically generate a lepton asymmetry. If we set the lepton number of N^c and \tilde{N}^c to -1 , so that λ (and $A\lambda$) are lepton number conserving, then M (and BM) violate lepton number by two units. Thus processes that involve λ and M give the lepton number violation that is necessary for leptogenesis.

A crucial role in soft leptogenesis is played by the $\tilde{N} - \tilde{N}^\dagger$ mixing amplitude,

$$\langle \tilde{N} | \mathcal{H} | \tilde{N}^\dagger \rangle = M_{12} - \frac{i}{2} \Gamma_{12} \quad (10.17)$$

(M_{12} is the dispersive part of the mixing amplitude, while Γ_{12} is the absorptive part), which induces mass and width differences,

$$x \equiv \frac{\Delta M}{\Gamma} \equiv \frac{M_H - M_L}{\Gamma}, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma} \equiv \frac{\Gamma_H - \Gamma_L}{\Gamma}, \quad (10.18)$$

(Γ is the average width) between the two mass eigenstates, the heavy $|\tilde{N}_H\rangle$, and the light $|\tilde{N}_L\rangle$,

$$|\tilde{N}_{L,H}\rangle = p|\tilde{N}\rangle \pm q|\tilde{N}^\dagger\rangle. \quad (10.19)$$

The ratio q/p depends on the mixing amplitude ratio:

$$\left(\frac{q}{p}\right)^2 = \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}. \quad (10.20)$$

Also of importance are the decay amplitudes of \tilde{N} or \tilde{N}^\dagger into various final states X :

$$A_X = \langle X|\mathcal{H}|\tilde{N}\rangle, \quad \bar{A}_X = \langle X|\mathcal{H}|\tilde{N}^\dagger\rangle. \quad (10.21)$$

The decay width of the singlet sneutrino is given by (for $|M| \gg |A|$)

$$\Gamma = \frac{|M\lambda^2|}{4\pi}, \quad (10.22)$$

while the mixing parameters are given by

$$\begin{aligned} x &= \left| \frac{B}{M} \right| \frac{4\pi}{|\lambda|^2}, \\ y &= \left| \frac{A}{M} \right| \cos\phi_N - \frac{1}{2} \left| \frac{B}{M} \right|, \\ \left| \frac{q}{p} \right| &= \left(1 + \frac{2|A\lambda^2/(2\pi B)| \sin\phi_N}{1 - |A\lambda^2/(2\pi B)| \sin\phi_N + \frac{1}{4}|A\lambda^2/(2\pi B)|^2} \right)^{1/4}. \end{aligned} \quad (10.23)$$

The quantity of interest is the CP asymmetry in singlet sneutrino decay,

$$\epsilon = \frac{\Gamma(\tilde{L}) + \Gamma(L) - \Gamma(\tilde{L}^\dagger) - \Gamma(\bar{L})}{\Gamma(\tilde{L}) + \Gamma(L) + \Gamma(\tilde{L}^\dagger) + \Gamma(\bar{L})}, \quad (10.24)$$

where $\Gamma(X)$ is the time-integrated decay rate into a final state with leptonic content X . Here $L(\bar{L})$ is the (anti)lepton doublet and $\tilde{L}(\tilde{L}^\dagger)$ is the (anti)slepton doublet.

Qualitatively, the most special feature of soft leptogenesis is that it gets contributions that are related to CP violation in mixing. This is a phenomenon that is analogous to the CP violation in $K - \bar{K}$ mixing, where it leads to the $K_L \rightarrow \pi\pi$ decays, the process where CP violation was first discovered. A relative CP violating phase between M_{12} and Γ_{12} of eqn (10.17) gives [see eqn (10.19)] $|q/p| \neq 1$ which, in turn, leads to a situation where the mass eigenstates $\tilde{N}_{H,L}$ are not CP eigenstates. One such contribution is given by [86, 87]

$$\epsilon^{\text{mix}} = \frac{x^2}{4(1+x^2)} \left(\left| \frac{p}{q} \right|^2 - \left| \frac{q}{p} \right|^2 \right) \Delta_{sf} = \mathcal{O} \left(\frac{x}{1+x^2} \frac{m_{\text{susy}}}{M} \Delta_{sf} \right). \quad (10.25)$$

Here m_{susy} is the scale of the soft supersymmetry breaking terms ($m_2 \sim m_{\text{susy}}$ and $A, B \lesssim m_{\text{susy}}$), and

$$\Delta_{sf} \equiv \frac{\mathcal{N}_s(|A_{\tilde{L}}|^2 + |\bar{A}_{\tilde{L}^\dagger}|^2) - \mathcal{N}_f(|A_{\tilde{L}}|^2 + |\bar{A}_L|^2)}{\mathcal{N}_s(|A_{\tilde{L}}|^2 + |\bar{A}_{\tilde{L}^\dagger}|^2) + \mathcal{N}_f(|A_{\tilde{L}}|^2 + |\bar{A}_L|^2)}, \quad (10.26)$$

where \mathcal{N}_s (\mathcal{N}_f) are phase space factors for final states involving scalars (fermions). At zero temperature, $\Delta_{sf} = \mathcal{O}(m_{\text{susy}}^2/M^2)$, but for temperature at the time of decay that is comparable to the singlet neutrino mass we have $\Delta_{sf} \approx (\mathcal{N}_s - \mathcal{N}_f)/(\mathcal{N}_s + \mathcal{N}_f) = \mathcal{O}(1)$. The difference between \mathcal{N}_f and \mathcal{N}_s at finite temperature arises from the Pauli blocking of final state fermions and Bose-Einstein stimulation of decays into scalars.

The contribution (10.25) stands out among the soft leptogenesis contributions as the only one that is linear in the ratio m_{susy}/M . (All other contributions are quadratic in this ratio.) Thus, for $M \gg 10^2 m_{\text{susy}}$, this is the only contribution that is potentially significant. Indeed, it could account for the observed baryon asymmetry if the following conditions are all fulfilled:

1. The lightest singlet sneutrino is light enough, $M \lesssim 10^9$ GeV.
2. The Yukawa couplings are small enough, $\lambda \lesssim 10^{-4}$. The lighter is M , the smaller the Yukawa coupling must be.
3. The B parameter is well below its naive value, $B/m_{\text{susy}} \lesssim 10^{-3}$. The lighter is M , the more suppressed B must be.

For $M \lesssim 10^2 m_{\text{susy}}$, there are several other contributions that can be significant [86]. All of these contributions involve λ_2 and are therefore proportional to the weak coupling α_2 . In addition, as mentioned above, they are proportional to $(m_{\text{susy}}/M)^2$. In particular, there are contributions related to CP violation in decay (a phenomenon that is analogous to the one giving $\mathcal{R}e(\epsilon')$ in $K \rightarrow \pi\pi$ decays and to the vertex contribution in standard leptogenesis), and to CP violation in the interference of decays with and without mixing (analogous to $S_{\psi K}$ in B decays). These two contributions are suppressed by $\alpha_2(m_{\text{susy}}/M)^2$ but no other small factors. In particular, unlike the contribution ϵ^{mix} of eqn (10.25), these contributions do not vanish when $x \gg 1$ and therefore they allow $B \sim m_{\text{susy}}$.

Soft leptogenesis is an interesting scenario in the framework of the supersymmetric seesaw for several reasons. First, the relevant new sources of CP violation and lepton number violation appear generically in this framework. In this sense, soft leptogenesis is qualitatively unavoidable in the SSM+N framework, and the question of its relevance is a quantitative one. Second, if $M \lesssim 10^9$ GeV (in the supersymmetric framework, this range is preferred by the gravitino problem), then standard leptogenesis encounters problems, while soft leptogenesis can be significant. Third, if $M \lesssim 10^2 m_{\text{susy}}$, then it is almost unavoidable that soft leptogenesis plays an important role.

10.4 Dirac leptogenesis

The extension of the Standard Model with singlet neutrinos allows two different ways of giving the active neutrinos their very light masses. First, one can invoke the seesaw mechanism which gives Majorana masses to the light neutrinos. This extension has at least three attractive features:

- No extra symmetries (and, in particular, no global symmetries) have to be imposed.
- The extreme lightness of neutrino masses is linked to the existence of a high scale of new physics, which is well motivated for various other reasons (*e.g.* gauge unification).
- Lepton number is violated, which opens the way for leptogenesis.

The second way is to impose lepton number and give Dirac masses to the neutrinos. A-priori, one might think that all three attractive features of the seesaw mechanism are lost. Indeed, one must usually impose additional symmetries. But one can still construct natural models where the tiny Yukawa couplings that are necessary for small Dirac masses are related to a small breaking of a symmetry. What is perhaps most surprising is the fact that leptogenesis could proceed successfully even if the neutrinos are Dirac particles and lepton number is not broken [259, 260] (except, of course, by the sphaleron interactions). Such scenarios have been termed ‘Dirac leptogenesis’ [260–266]. Actually, the success of Dirac leptogenesis is closely related to the extreme smallness of the neutrino Yukawa couplings in this scenario.

An implementation of the idea is the following. A CP violating decay of a heavy particle can result in a non-zero lepton number for left-handed particles, and an equal and opposite non-zero lepton number for right-handed particles, so that the total lepton number is zero. For the charged fermions of the Standard Model, the Yukawa interactions are fast enough that they quickly equilibrate the left-handed and the right-handed particles, and the lepton number stored in each chirality goes to zero. This is not true, however, for Dirac neutrinos. The size of their Yukawa couplings is $\lambda \lesssim 10^{-11}$, which means that equilibrium between the lepton numbers stored in left-handed and right-handed neutrinos will not be reached until temperatures fall well below the electroweak breaking scale. To see this, note that the rate of the Yukawa interactions is given by $\Gamma_\lambda \sim \lambda^2 T$. It becomes significant when it equals the expansion rate of the Universe, $H \sim T^2/m_{\text{pl}}$. Thus, the temperature of equilibration between left-handed and

right-handed neutrinos can be estimated as $T \sim \lambda^2 m_{\text{pl}} \sim (\lambda/10^{-8}) T_{\text{EWPT}}$. By this time, the left-handed lepton number has been partially transformed into a net baryon number by the sphaleron interactions.

More specifically, consider a situation where the CP violating decays of some new, heavy particles have produced a negative lepton number in left-handed neutrinos, and a positive lepton number, of equal magnitude, in right-handed neutrinos. The sphalerons interact with the left-handed neutrinos, violating $B + L$ and conserving $B - L$. Consequently, part of the negative lepton number stored in the left-handed neutrinos is converted to a positive baryon number. At much lower temperatures, when sphaleron interactions (and B and L violation) are frozen, the remaining negative lepton number in left-handed neutrinos equilibrates with the positive lepton number stored in the right handed-neutrinos. Since, however, the negative lepton number in left-handed neutrinos has become smaller in magnitude than the positive lepton number in right-handed neutrinos, a net lepton number remains. The final situation is then a Universe with total positive baryon number, total positive lepton number, and $B - L = 0$.

A specific example of a natural, supersymmetric model with Dirac neutrinos is presented in Ref. [261]. The Majorana masses of the N -superfields are forbidden by $U(1)_L$ (lepton number symmetry). The neutrino Yukawa couplings are forbidden by a $U(1)_N$ symmetry where, among all the SSM+N fields, only the N superfields are charged. The symmetry is spontaneously broken by the vacuum expectation value of a scalar field χ that can naturally be at the weak scale, $\langle \chi \rangle \sim v_u$. This breaking is communicated to the SSM+N via extra, vector-like lepton doublet fields, $\phi + \bar{\phi}$, that have masses M_ϕ much larger than v_u . Consequently, the neutrino Yukawa couplings are suppressed by the small ratio $\langle \chi \rangle / M_\phi$. The CP violation arises in the decays of the vector-like leptons, whereby $\Gamma(\phi \rightarrow NH_u^c) \neq \Gamma(\bar{\phi} \rightarrow N^c H_u)$ and $\Gamma(\phi \rightarrow L\chi) \neq \Gamma(\bar{\phi} \rightarrow L^c \chi^c)$. The resulting asymmetries in N and in L are equal in magnitude and opposite in sign.

The main phenomenological implications of Dirac leptogenesis is the absence of any signal in neutrinoless double beta decays.

10.5 Triplet scalar leptogenesis

As explained in Subsection 2.2, one can generate see-saw masses for the light neutrinos by tree-level exchange of $SU(2)$ -triplet scalars. Since this mechanism necessarily involves lepton number violation, and allows for new CP violating phases, it is interesting to examine it as a possible source of leptogenesis [111, 249, 254, 267–281]. One obvious problem in this scenario is that, unlike singlet fermions, the triplet scalars have gauge interactions that keep them close to thermal equilibrium at temperatures $T \lesssim 10^{15}$ GeV. It turns out, however, that successful leptogenesis is possible even at a much lower temperature. This subsection is based in large part on Ref. [274], where further details and, in particular, an explicit presentation of the relevant Boltzmann equations can be found.

The CP asymmetry that is induced by the triplet scalar decays is defined as follows:

$$\epsilon_T \equiv 2 \frac{\Gamma(\bar{T} \rightarrow \ell\ell) - \Gamma(T \rightarrow \bar{\ell}\bar{\ell})}{\Gamma_T + \Gamma_{\bar{T}}}. \quad (10.27)$$

The overall factor of 2 comes because the triplet scalar decay produces two (anti)leptons.

To calculate ϵ_T , one should use the Lagrangian terms given in eqn (2.15). While a single triplet is enough to produce three light massive neutrinos, there is a problem in leptogenesis if indeed this is the only source of neutrinos masses: The asymmetry is generated only at higher loops and in unacceptably small.

It is still possible to produce the required lepton asymmetry from a single triplet scalar decays if there are additional sources for the neutrino masses, such as type I, type III, or type II contributions from additional triplet scalars. Define m_{II} (m_{I}) as the part of the light neutrino mass matrix that comes (does not come) from the contributions of the triplet scalar that gives ϵ_T :

$$m = m_{\text{II}} + m_{\text{I}}. \quad (10.28)$$

Then, assuming that the particles that are exchanged to produce m_I are all heavier than T , the CP asymmetry is given by

$$\epsilon_T = \frac{1}{4\pi} \frac{M_T}{v_u^2} \sqrt{B_L B_H} \frac{\text{Im}[\text{Tr}(m_{II}^\dagger m_I)]}{\text{Tr}(m_{II}^\dagger m_{II})}, \quad (10.29)$$

where B_L (B_H) is the tree-level branching ratio to leptons (Higgs doublets). If these are the only decay modes, *i.e.* $B_L + B_H = 1$, then $B_L/B_H = \text{Tr}(\lambda_L \lambda_L^\dagger)/(\lambda_H \lambda_H^\dagger)$. There is an upper bound on this asymmetry:

$$|\epsilon_T| \leq \frac{1}{4\pi} \frac{M_T}{v_u^2} \sqrt{B_L B_H \sum_i m_{\nu_i}^2}. \quad (10.30)$$

Note that, unlike the singlet fermion case, $|\epsilon_T|$ increases with larger m_{ν_i} .

As concerns the efficiency factor, it can be close to maximal, $\eta \sim 1$, in spite of the fact that the gauge interactions tend to maintain the triplet abundance very close to thermal equilibrium. There are two necessary conditions that have to be fulfilled by the two decay rates, $T \rightarrow \bar{\ell}\ell$ and $T \rightarrow \phi\phi$, in order that this will happen [274]:

1. One of the two decay rates is faster than the $T\bar{T}$ annihilation rate.
2. The other decay mode is slower than the expansion rate of the Universe.

The first condition guarantees that gauge scatterings are ineffective: the triplets decay before annihilating. The second condition guarantees that the fast decays do not lead to a strong washout of the lepton asymmetry: lepton number is violated only by the simultaneous presence of $T \rightarrow \bar{\ell}\ell$ and $T \rightarrow \phi\phi$.

Combining a calculation of η and the upper bound on the CP asymmetry (10.30), successful leptogenesis implies a lower bound on the triplet mass M_T varying between 10^9 GeV and 10^{12} GeV, depending on the relative weight of m_{II} and m_I in the light neutrino mass.

Interestingly, in the supersymmetric framework, “soft leptogenesis”, namely one that is driven by the soft supersymmetry breaking terms, can be successful even with the minimal set of extra fields – a single $T + \bar{T}$ – generating both neutrino masses and the lepton asymmetry [249, 254].

10.6 Triplet fermion leptogenesis

As explained in Subsection 2.3, one can generate see-saw masses for the light neutrinos by tree-level exchange of $SU(2)$ -triplet fermions. This mechanism necessarily involves lepton number violation, and allows for new CP violating phases so we should examine it as a possible source of leptogenesis [162, 270, 282]. This subsection is based in large part on Ref. [270], where further details and, in particular, an explicit presentation of the relevant Boltzmann equations can be found.

As concerns neutrino masses, the formalism and qualitative features are very similar to the singlet fermion case. As concerns leptogenesis, there are, however, qualitative and quantitative differences.

With regard to the CP asymmetry from the lightest triplet fermion decay, the relative sign between the vertex loop contribution and the self-energy loop contribution is opposite to that of the singlet fermion case. Consequently, in the limit of strong hierarchy in the heavy fermion masses, the asymmetry in triplet decay is 3 times smaller than in the singlet decay (see the discussion in Section 5.2 for the singlet case and for definitions). On the other hand, since the triplet has three components, the ratio between the final baryon asymmetry and $\epsilon\eta$ is 3 times bigger. The decay rate of the heavy fermion is the same in both cases. This, however, means that the thermally averaged decay rate is 3 times bigger for the triplet, as is the on-shell part of the $\Delta L = 2$ scattering rate.

A significant qualitative difference arises from the fact that the triplet has gauge interactions. The effect on the washout factor η is particularly significant for $\tilde{m} \ll 10^{-3}$ eV, the so-called “weak washout regime” (note that this name is inappropriate for triplet fermions). The gauge interactions still drive the triplet abundance close to thermal equilibrium. A relic fraction of the triplet fermions survives. The decays of these relic triplets produce a baryon asymmetry, with

$$\eta \approx M_1/10^{13} \text{ GeV} \quad (\text{for } \tilde{m} \ll 10^{-3} \text{ eV}). \quad (10.31)$$

The strong dependence on M_1 is a result of the fact that the expansion rate of the Universe is slower at lower temperatures. On the other hand, for $\tilde{m} \gg 10^{-3}$ eV, the Yukawa interactions are responsible for keeping the heavy fermion abundance close to thermal equilibrium, so the difference in η between the singlet and triplet case is only $\mathcal{O}(1)$.

Ignoring flavour effects, and assuming strong hierarchy between the heavy fermions, Ref. [270] obtained a lower bound on the mass of the lightest heavy triplet fermion:

$$M_1 \gtrsim 1.5 \times 10^{10} \text{ GeV} \quad (10.32)$$

When the triplet fermion scenario is incorporated in a supersymmetric framework, and the soft breaking terms do not play a significant role, the modifications to the above analysis is by factors of $\mathcal{O}(1)$.

11 Conclusions

During the last few decades, a large set of experiments involving solar, atmospheric, reactor and accelerator neutrinos have converged to establish that the standard model neutrinos are massive. The seesaw mechanism extends the standard model in a way that allows neutrino masses. It provides a nice explanation of the suppression of the neutrino masses with respect to the electroweak breaking scale, which is the mass scale of all standard model charged fermions. Furthermore, without any addition or modification, it can also account for the observed baryon asymmetry of the Universe. The possibility of giving an explanation of two apparently unrelated experimental facts – neutrino masses and the baryon asymmetry – within a single framework that is a natural extension of the standard model, together with the remarkable ‘coincidence’ that the same neutrino mass scale suggested by neutrino oscillation data is also optimal for leptogenesis, makes the idea that baryogenesis occurs through leptogenesis a very attractive one.

Leptogenesis can be quantitatively successful without any fine-tuning of the seesaw parameters. Yet, in the non-supersymmetric seesaw framework, a fine-tuning problem arises due to the large corrections to the mass-squared parameter of the Higgs potential that are proportional to the heavy Majorana neutrino masses (see Section 2.4). Supersymmetry can cure this problem, avoiding the necessity of fine tuning. However, the gravitino problem that arises in many supersymmetric models requires a low reheat temperature after inflation, in conflict with generic leptogenesis models (see Section 10.1). Thus, constructing a fully satisfactory theoretical framework that implements leptogenesis within the seesaw framework is not a straightforward task.

From the experimental side, the obvious question to ask is if it is possible to test whether the baryon asymmetry has been really produced through leptogenesis. Unfortunately it seems impossible that any direct test can be performed. To establish leptogenesis experimentally, we need to produce the heavy Majorana neutrinos and measure the CP asymmetry in their decays. However, in the most natural seesaw scenarios, these states are simply too heavy to be produced, while if they are light, then their Yukawa couplings must be very tiny, again preventing any chance of direct measurements.

The possibility of indirect tests from measurements of the asymmetries produced by leptogenesis is also ruled out. This is because there are too many high energy parameters that are relevant to leptogenesis and, even if we adopt the most optimistic point of view, there are at most four observables. These are the value of the baryon asymmetry that is known with a good accuracy (see Section 1.1), and the three cosmic neutrino flavour asymmetries that we do not know how to measure. A measurement of the total lepton asymmetry would be very valuable. If a value of the order of the baryon asymmetry were found, it would provide strong evidence that electroweak sphalerons have been at work in the early Universe.²⁴ Even if we have no reason to doubt that sphaleron processes occurred in the thermal bath with in-equilibrium rates, they are the fundamental ingredient of the whole leptogenesis idea, and thus the importance of an experimental test should not be underappreciated. Measurements of the single lepton flavour asymmetries would provide some information on leptogenesis parameters (on the flavour CP asymmetries if leptogenesis occurred in the unflavoured regime, or on a combination of the flavour CP asymmetries and of the flavour-dependent washouts otherwise). However, at present, we have not even observed the relic neutrino background, and the possibility of revealing $\mathcal{O}(10^{-10})$ asymmetries in this background seems completely hopeless.

Lacking the possibility of a direct proof, experiments can still provide circumstantial evidence in support of leptogenesis by establishing that (some of) the Sakharov conditions for leptogenesis (see Section 1.2) are realized in nature.

Planned neutrinoless double beta decay ($0\nu\beta\beta$) experiments (GERDA [285], MAJORANA [286], CUORE [287]) aim at a sensitivity to the effective $0\nu\beta\beta$ neutrino mass in the few $\times 10$ meV range. If they succeed in establishing the Majorana nature of the light neutrinos,²⁵ this will strengthen our confidence that the seesaw mechanism is at the origin of the neutrino masses and, most importantly, will

²⁴The opposite result would not disprove sphaleron physics: see [283, 284] for a model in which production of a large lepton asymmetry *after* sphalerons freeze-out can yield $Y_{\Delta L}$ orders of magnitude larger than $Y_{\Delta B}$.

²⁵This is likely to happen if neutrinos are quasi-degenerate or if the mass hierarchy is inverted

establish that the first Sakharov condition for the dynamical generation of a lepton asymmetry, that is that lepton number is violated in nature, is satisfied.

Proposed SuperBeam facilities [288, 289] and second generation off-axis SuperBeams experiments (T2HK [290], NO ν A [291]) can discover CP violation in the leptonic sector. These experiments can only probe the Dirac phase of the neutrino mixing matrix. They cannot probe the Majorana low energy or the high energy phases, but the important point is that they can establish that the second Sakharov condition for the dynamical generation of a lepton asymmetry is satisfied.²⁶

In contrast to the previous two conditions, verifying that the decays of the heavy neutrinos occurred out of thermal equilibrium (the third condition) remains out of experimental reach, since it would require measuring the heavy neutrino masses and the size of their couplings.

Given that we do not know how to prove that leptogenesis is the correct theory, we might ask if there is any chance to falsify it. Indeed, future neutrino experiments could weaken the case for leptogenesis, or even falsify it, in two main ways: Establishing that the seesaw mechanism is not responsible for the observed neutrino masses, or finding evidence that the washout is too strong.

By itself, failure in revealing signals of $0\nu\beta\beta$ decays will not disprove leptogenesis. Indeed, with normal neutrino mass hierarchy one expects that the rates of lepton number violating processes are below experimental sensitivity. However, if neutrinos masses are quasi-degenerate or inversely hierarchical, and future measurements of the oscillation parameters will not fluctuate too much away from the present best fit values, the most sensitive $0\nu\beta\beta$ decay experiments scheduled for the near future should be able to detect a signal [72].²⁷ If instead the limit on $|m_{\beta\beta}|$ is pushed below ~ 10 meV (a quite challenging task), this would suggest that either the mass hierarchy is normal, or neutrinos are not Majorana particles. The latter possibility would disprove the seesaw model and standard leptogenesis (see however section 10.4). Thus, determining the order of the neutrino mass spectrum is extremely important to shed light on the connection between $0\nu\beta\beta$ decay experiments and leptogenesis.

Long baseline neutrino experiments can achieve this result [293, 294] by exploiting the fact that oscillations in matter can determine the sign of the atmospheric mass difference Δm_{23}^2 . In fact, if the oscillations of atmospheric neutrinos involve the heaviest (lightest) state, corresponding to normal (inverted) hierarchy, then $\nu_\mu \leftrightarrow \nu_e$ oscillations are enhanced (suppressed) while $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$, oscillations are suppressed (enhanced).

Cosmology could also provide important information in establishing if the neutrino mass hierarchy is inverted. Cosmological observations are sensitive to the sum of the three neutrino masses $m_{\text{cosmo}} = m_1 + m_2 + m_3$. In the near future, improved CMB data and more precise large scale structure measurements can push the sensitivity on m_{cosmo} down to the 0.1 eV level. For an inverted hierarchy a signal should be detected around or above this value.

In summary, if it is established that the neutrino mass hierarchy is inverted and at the same time no signal of $0\nu\beta\beta$ decays is detected at a level $|m_{ee}| \lesssim 10$ meV, one could conclude that the seesaw is not at the origin of the neutrino masses, and that leptogenesis is not the correct explanation of the baryon asymmetry.

The neutrino mass scale has also more direct implications for leptogenesis. A quantitatively successful leptogenesis prefers a mass scale for the light neutrinos not larger than a few times the atmospheric neutrino mass scale (see Section 9.4.1). In particular, in the standard type I seesaw model (see section 2.1), if leptogenesis occurs in the unflavoured regime and the heavy Majorana neutrinos are sufficiently hierarchical, leptogenesis would fail to produce enough baryon asymmetry when the neutrino mass scale exceeds values as low as $\sim 0.1 - 0.2$ eV. Indeed, if past experiments had detected neutrino masses of order a few eV, leptogenesis, if not completely ruled out, would have certainly lost most of its theoretical appeal. In the future, a discovery of a neutrino mass at the β decay experiment KATRIN will also put

²⁶As discussed in [292], it is conceivable (but not natural) that the Dirac phase is solely responsible for the matter-antimatter asymmetry.

²⁷ $0\nu\beta\beta$ decay experiments are sensitive to the effective mass parameter $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$. In the inverted hierarchy scenario the $s_{13}^2 m_3$ contribution to $m_{\beta\beta}$ can be neglected, and approximating for simplicity $c_{13}^2 \simeq 1$ one gets $m_{\beta\beta} \approx m_1 e^{2i\beta} c_{12}^2 + m_2 e^{2i\alpha} s_{12}^2$. Since θ_{12} deviates from maximal mixing, we can then predict that $m_{\beta\beta}$ should be of the order of the atmospheric mass scale, independently of the values of the Majorana phases α and β .

the type I seesaw scenario for leptogenesis in serious doubt, since it will yield a mass value a bit too large (the claimed discovery potential is for an effective β -decay mass parameter $m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2$ of about 0.35 eV [295]). However, cosmological observations have already reached a sensitivity to m_{cosmo} somewhat below 1 eV [296–298], implying that it is unlikely that leptogenesis could be put to doubt by a direct detection of neutrino masses in laboratory experiments.

As concerns CP violation, a failure in detecting leptonic CP violation will not weaken the case for leptogenesis in a significant way. Instead, it would mean that the Dirac CP phase (or the θ_{13} mixing angle, or both) is small enough to render CP violating effects unobservable.

Finally, the CERN LHC has the capability of providing information that is relevant to leptogenesis. First, electroweak baryogenesis (see Section 1.2) can be tested at the LHC. It will become strongly disfavored (if not completely ruled out) if supersymmetry is not found, or if supersymmetry is discovered but the stop and/or the Higgs are too heavy. Eliminating various scenarios that are able to explain the baryon asymmetry will strengthen the case for the remaining viable possibilities, including leptogenesis. Conversely, if electroweak baryogenesis is established by the LHC (and EDM) experiments, the case for leptogenesis will become weaker.

Second, new physics discoveries at the LHC can play a fundamental role in establishing that the origin of the neutrino masses is not due to the seesaw mechanism, leaving no strong motivation for leptogenesis. This may happen in several different ways. For example (assuming that the related new physics is discovered), the LHC will be able to test if the detailed phenomenology of any of the following models is compatible with an explanation of the observed pattern of neutrino masses and mixing angles: supersymmetric R-parity violating couplings and/or L-violating bilinear terms [299,300]; leptoquarks [301, 302]; triplet Higgses [303,304]; new scalar particles of the type predicted in the Zee-Babu [305,306] types of models [307–309]. It is conceivable that such discoveries can eventually exclude the seesaw mechanism and rule out leptogenesis.

To conclude, the seesaw framework provides the most natural and straightforward explanation of the light neutrino masses and has, in principle, all the ingredients that are necessary for successful leptogenesis. This makes leptogenesis arguably the most attractive explanation for the observed baryon asymmetry. This scenario has limited predictive power for low energy observables, so it is unlikely to be directly tested. Yet, future experiments have the potential of strengthening, or weakening, or even falsifying the case for leptogenesis.

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12 Appendix: Notation

z	dimensionless time variable M_1/T	13.10
\dot{Y}	$(sH_1/z)(dY/dz)$	6.10
H, H_1	Hubble rate, Hubble rate at $T = M_1$	13.7, 13.11
s	entropy density	13.6
Y_X	ratio of X number density to entropy density	13.6
n_X^{eq}	number density for particle X in kinetic equilibrium	13.2, 13.5
Y_X^{eq}	Y_X in kinetic equilibrium	6.3, 13.6
Y_ℓ^{eq}	ratio of ν_α and e_α density to entropy density ($g_{\ell_\alpha} = 2$)	after 6.3
$Y_L^{\alpha\alpha}$	density of doublets and singlets of flavour α	6.7
y_X	Y_X/Y_X^{eq}	6.3
Δy_X	$y_X - y_{\bar{X}}$	6.3
Y_{Δ_α}	ratio of $B/3 - L_\alpha$ asymmetry to entropy density	6.9, 14.11
$Y_{\Delta B}$	ratio of baryon asymmetry to entropy density	1.2, 4.1, 16.10
g_*, g_{*S}	degrees of freedom in the thermal bath	13.8, 13.9
g_i	internal degrees of freedom of particle i	below 13.3
η_α	efficiency parameter for flavour α	4.1, 16.7, 16.8
$\epsilon_{\alpha\alpha}$	CP asymmetry in flavour α	5.1, 5.13
$[m]$	light neutrino mass matrix	2.2
$m_{\text{max}}, m_{\text{min}}$	largest, smallest light neutrino mass	5.10, 5.17
$\tilde{m}, \tilde{m}_{\alpha\alpha}, m_*$	(rescaled) N decay rates, Hubble expansion rate	4.7
A	matrix relating the $Y_{\Delta\ell_\alpha}$ to the Y_{Δ_α}	14.14 14.15
λ	neutrino Yukawa coupling	2.3
h_x	Yukawa coupling of particle x (not a neutrino)	2.3
v_u	Higgs vev ($v_u \simeq 174$ GeV)	2.3
Γ_D	total decay rate $\sum_\alpha \Gamma(N \rightarrow \phi\ell_\alpha, \bar{\phi}\bar{\ell}_\alpha)$	4.4, 13.20
Γ_{ID}	total inverse decay rate	4.10
$\Gamma_{\alpha\alpha}$	partial decay rate to flavour and antiflavour α : $\Gamma(N \rightarrow \phi\ell_\alpha, \bar{\phi}\bar{\ell}_\alpha)$	4.10
B_{xz}^X	branching ratio $\Gamma(X \rightarrow yz)/\Gamma_X^{\text{tot}}$	5.10
γ_B^A	rate density for $A \rightarrow B$	13.14
$\gamma_{N \rightarrow 2}$	rate density for two body decays of N_1	13.21
Γ_τ, γ_τ	rate, rate density of τ Yukawa interaction	14.10
$\Delta\gamma_B^A$	CP difference: $\gamma_B^A - \gamma_{\bar{B}}^{\bar{A}}$	6.2
$[A \leftrightarrow B]$	difference between process and its time reversed	6.4
$\gamma_B^{\prime A}$	γ_B^A with exchange of on-shell particle removed	6.5
$K_1(z), K_2(z)$	Bessel functions	13.19, 13.3
\mathcal{M}, \mathcal{A}	matrix element, amplitude	5.2
$\tilde{\delta}$	4-momentum conservation: $(2\pi)^4\delta^4(p_{a_1} + \dots - p_{b_1} - \dots)$	5.6
$d\Pi$	phase space $d^3p/(16E\pi^3)$	5.6

Table 2: Parameters and variables introduced in the text, and equations where they are defined. Simple BE for leptogenesis that neglect scattering processes are given in eqns (6.16). and (6.28). More complete BE that include top-quark and gauge bosons scatterings are given in eqns (6.51), (6.53), (6.54) and (6.56).

13 Appendix: Kinetic Equilibrium

The interested reader can find a more complete set of formulae for example in the Appendices of refs. [31, 310].

13.1 Number densities

Of the particles relevant to leptogenesis, all but the N_i 's carry charges under the standard model gauge group and are, therefore, subject to gauge interactions. We take the distributions $f_i(p)$ of the (non-singlet) particle species i to have their kinetic equilibrium form, because gauge interactions are fast in the relevant temperature range, $f_i(p) = (n_i/n_i^{\text{eq}})f_i^{\text{eq}}(p)$. (The N_i 's may also be produced in an equilibrium distribution because the initial state particles are in kinetic equilibrium, or kinetic equilibrium may be obtained due to fast Yukawa interactions, if λ is in the strong washout regime.) With some additional assumptions (see eqn 13.12), this allows to solve the BE for the total number density rather than mode by mode (as was done in [159]). The number densities of particles and anti-particles are considered separately. For some other applications, it may be the convention to count the particles and anti-particles together (for instance for g_* , g_{*S} , see eqns (13.8),(13.9)) so some care with factors of 2 is required in matching.

Using Maxwell-Boltzmann statistics for f_i ,

$$f_{i,\text{MB}}^{\text{eq}}(p) = e^{-(E_i - \mu_i)/T}, \quad (13.1)$$

the equilibrium number density of particles of species i is given by

$$n_{i,\text{MB}}^{\text{eq}} = \frac{g_i}{(2\pi)^3} \int d^3p f_{i,\text{MB}}^{\text{eq}}(p) = \begin{cases} \frac{g_i T^3}{2\pi^2} z_i^2 K_2(z_i) & z_i = \frac{m_i}{T} \neq 0, \mu = 0, \\ \frac{g_i T^3}{\pi^2} & m_i = 0. \end{cases} \quad (13.2)$$

Here $K_2(z)$ is a Bessel function (see ref. [311] eqns 8.432.3 and 8.472.2, and ref. [59] for useful analytic approximations):

$$z^2 K_2(z) = \int_z^\infty x e^{-x} \sqrt{x^2 - z^2} dx \rightarrow \begin{cases} 2 & z \ll 1 \\ \left(\frac{15}{8} + z\right) \sqrt{\frac{\pi z}{2}} e^{-z} & z \gg 1 \end{cases} \quad (13.3)$$

and g_i is the number of internal degrees of freedom of the *particle*. For example, $g_N = 2$ for the N_i 's, because they are Majorana fermions, $g_{e_R} = 1$ for the $SU(2)$ -singlet leptons, and $g_\ell = 2$ for the $SU(2)$ -doublet leptons and $g_\phi = 2$ for the Higgs. Antiparticle number densities have $g_{\bar{i}} = g_i$.

Using Fermi-Dirac (+) or Bose-Einstein (-) statistics,

$$f_{i,\pm}^{\text{eq}}(p) = \frac{1}{e^{(E_i - \mu_i)/T} \pm 1}, \quad (13.4)$$

gives the following equilibrium number densities:

$$n_{i,\pm}^{\text{eq}} = \frac{g_i}{(2\pi)^3} \int d^3p f_{i,\pm}^{\text{eq}}(p) \rightarrow \begin{cases} \frac{g_i T^3}{\pi^2} \times \begin{cases} \zeta(3) + \frac{\mu_i}{T} \zeta(2) + \dots & \text{(bosons)} \\ \frac{3}{4} \zeta(3) + \frac{\mu_i}{T} \frac{\zeta(2)}{2} + \dots & \text{(fermions)} \end{cases} & m_i \ll T, \\ n_{i,\text{MB}}^{\text{eq}} & \mu_i \ll T, m_i \gg T, \end{cases} \quad (13.5)$$

where $\zeta(2) = \pi^2/6$, $\zeta(3) = 1.202$, and $\zeta(4) = \pi^4/90$. Using Maxwell-Boltzmann instead of Fermi-Dirac statistics makes a difference in n^{eq} of order 10% at $T = m$.

13.2 Rates

Number densities decrease due to the expansion of the Universe. This can be factored out of the BE by taking Y_X to be the comoving number density:

$$Y_X = \frac{n_X}{s}, \quad s = \frac{g_{*S} 2\pi^2}{45} T^3. \quad (13.6)$$

The entropy density s is conserved per *comoving* volume:

$$\frac{ds}{dt} = -3Hs \quad .$$

The expansion rate of the Universe is given by

$$H = \sqrt{\frac{8\pi\rho}{3m_{\text{pl}}^2}} \simeq \frac{1.66\sqrt{g_*}T^2}{m_{\text{pl}}} \quad (13.7)$$

The density and the pressure of the gas of relativistic particles are given by

$$\rho = \frac{\pi^2}{30}g_*T^4, \quad P = \frac{\pi^2}{90}g_*T^4.$$

The (temperature-dependent) parameters $g_*(g_{*S})$ are the effective number of degrees of freedom contributing to the energy (entropy) density:

$$g_* = \frac{7}{8} \sum_{\text{fermions}} g_f \left(\frac{T_f}{T_\gamma}\right)^4 + \sum_{\text{bosons}} g_b \left(\frac{T_b}{T_\gamma}\right)^4 \quad (13.8)$$

$$g_{*S} = \frac{7}{8} \sum_{\text{fermions}} g_f \left(\frac{T_f}{T_\gamma}\right)^3 + \sum_{\text{bosons}} g_b \left(\frac{T_b}{T_\gamma}\right)^3 \quad (13.9)$$

where the particle and anti-particle should both appear in the sums over species. (Recall that we defined g_i as the internal degrees of freedom for the particle only.) The SM Higgs and anti-Higgs contribute $g_\phi + g_{\bar{\phi}} = 4$. Each flavour of neutrino and its anti-neutrino contribute $7/4$. At leptogenesis temperatures, $g_{*S} = g_*$. The contribution to the present entropy density s_0 comes from the photons at T_γ , and from the neutrinos and anti-neutrinos²⁸ at $T_\nu^3 \simeq (4/11)T_\gamma^3$. Thus $s_0 \propto g_{*S}T_\gamma^3$, with $g_{*S} = 2 + \frac{42}{8} \frac{4}{11} \simeq 3.9$.

A convenient time variable for leptogenesis in N_1 decay is

$$z \equiv M_1/T, \quad (13.10)$$

for which

$$\frac{dz}{dt} = -\frac{M_1}{T^2} \frac{dT}{dt} = zH(z) = \frac{H_1}{z}, \quad (13.11)$$

where $H_1 \equiv H(T = M_1)$ is a constant.

Suppose the number density of X -particles can be changed by various interactions. The time evolution of Y_X is

$$\begin{aligned} \frac{dY_X}{dt} = & \sum_{\text{int}} \left[- \int d\Pi_X f_X d\Pi_a f_a \dots |\mathcal{M}(X + a + \dots \rightarrow i + j + \dots)|^2 \tilde{\delta} d\Pi_i (1 \pm f_i^\pm) d\Pi_j (1 \pm f_j^\pm) \dots \right. \\ & \left. + \int d\Pi_i f_i^\pm d\Pi_j f_j^\pm \dots |\mathcal{M}(i + j + \dots \rightarrow X + a + \dots)|^2 \tilde{\delta} (1 \pm f_X) d\Pi_X (1 \pm f_a) d\Pi_a \dots \right], \quad (13.12) \end{aligned}$$

where $d\Pi_X$ and $\tilde{\delta}$ are defined in eqn (5.6), and $|\mathcal{M}|^2$ is summed over the internal degrees of freedom of the initial and final state. This equation simplifies if the particles are assumed to be in kinetic equilibrium, and if the final state Bose enhancement/Pauli blocking factors are ignored. Then one obtains:

$$\begin{aligned} \frac{dY_X}{dt} = & - \sum_{\text{int}} \left[\frac{Y_X Y_a \dots}{(sY_X^{\text{eq}})(sY_a^{\text{eq}}) \dots} \gamma(X + a + \dots \rightarrow i + j + \dots) \right. \\ & \left. - \frac{Y_i Y_j \dots}{(sY_i^{\text{eq}})(sY_j^{\text{eq}}) \dots} \gamma(i + j + \dots \rightarrow X + a + \dots) \right]. \quad (13.13) \end{aligned}$$

²⁸The momentum distribution of the neutrinos freezes out when they are relativistic, so even though their mass may be greater than their temperature, they contribute to s like a relativistic species.

Here γ is an *interaction density*:

$$\begin{aligned}\gamma_{ij\dots}^{Xa\dots} &\equiv \gamma(X + a + \dots \rightarrow i + j + \dots) \\ &= \int d\Pi_X f_X^{\text{eq}} d\Pi_a f_a^{\text{eq}} \dots |\mathcal{M}(X + a + \dots \rightarrow i + j + \dots)|^2 \tilde{\delta} d\Pi_i d\Pi_j, \quad (13.14)\end{aligned}$$

where we drop final state phase space factors. Recall that $|\mathcal{M}|^2$ is summed (not averaged) over the internal degrees of freedom of the initial states.

The rates in this Appendix are obtained in $T = 0$ field theory. Calculating rates at finite temperature [60] gives $O(1)$ effects on the number densities as a function of time, although it can change individual rates in a more significant way, as discussed in Section 7. It is useful, for the purpose of calculating decay and scattering rates, to have the expression for two body phase space:

$$\begin{aligned}\int \tilde{\delta} d\Pi_p d\Pi_q &= \int \frac{|\vec{p}_p|}{16\pi^2 \sqrt{s}} d\Omega_p \\ &= \frac{|\vec{p}_p - \vec{p}_q|}{8\pi \sqrt{s}} = \frac{\sqrt{(p_p \cdot p_q)^2 - m_p^2 m_q^2}}{4\pi s}, \quad (13.15)\end{aligned}$$

where the first line has the familiar form of the center of mass frame, while the second provides the Lorentz-invariant form.

Consider the two-body decay rate density,

$$\gamma_{\phi\ell\beta}^N + \gamma_{\bar{\phi}\bar{\ell}\beta}^N \equiv \gamma(N_1 \rightarrow \ell_\beta\phi, \bar{\ell}_\beta\bar{\phi}). \quad (13.16)$$

There are four possible final states, $\nu_\beta\phi^0, e_\beta\phi^+, \bar{\nu}_\beta\bar{\phi}^0, \bar{e}_\beta\bar{\phi}^-$. The four rates are all equal:

$$|\mathcal{M}(N_1 \rightarrow \nu_\beta\phi^0)|^2 = 2|\lambda_{\beta 1}|^2 p_N \cdot p_\ell = |\lambda_{\beta 1}|^2 (M_1^2 + m_\ell^2 - m_H^2).$$

We thus obtain for the decay rate density:

$$\begin{aligned}\gamma_{\phi\ell\beta}^N + \gamma_{\bar{\phi}\bar{\ell}\beta}^N &= \int \frac{d^3 p_N}{2E_N (2\pi)^3} e^{-E_N/T} 4|\lambda_{\beta 1}|^2 M_1^2 \int \tilde{\delta} d\Pi_\phi d\Pi_\ell \\ &= \frac{1}{2} \frac{T^2}{2\pi^2} \int_z^\infty e^{-x} \sqrt{x^2 - z^2} dx \frac{|\lambda_{\beta 1}|^2 M_1^2}{2\pi} \\ &= s Y^{\text{eq}} \frac{K_1(z)}{K_2(z)} \Gamma(N_1 \rightarrow \ell_\beta\phi, \bar{\ell}_\beta\bar{\phi}) \quad (13.17)\end{aligned}$$

$$= \frac{g_N T^3}{2\pi^2} z^2 K_1(z) \Gamma(N_1 \rightarrow \ell_\beta\phi, \bar{\ell}_\beta\bar{\phi}), \quad (13.18)$$

where K_1 is a Bessel function (see ref. [311], eqn 8.432.3):

$$zK_1(z) = \int_z^\infty e^{-x} \sqrt{x^2 - z^2} dx \rightarrow \begin{cases} 1 & z \ll 1 \\ \sqrt{\frac{\pi z}{2}} e^{-z} & z \gg 1 \end{cases} \quad (13.19)$$

and ($g_N = 2$)

$$\Gamma(N_1 \rightarrow \ell_\beta\phi, \bar{\ell}_\beta\bar{\phi}) = \frac{|\lambda_{\beta 1}|^2 M_1}{4\pi g_N}. \quad (13.20)$$

The inclusive two-body decay rate density appears frequently in our calculations:

$$\gamma_{N \rightarrow 2} = \sum_\beta (\gamma(N \rightarrow \phi\ell_\beta) + \gamma(N \rightarrow \bar{\phi}\bar{\ell}_\beta)) = \frac{g_N T^3}{2\pi^2} z^2 K_1(z) \Gamma_D. \quad (13.21)$$

In the literature it is frequently denoted by γ_D . It was evaluated with quantum statistics in [60], in which case it does not have a simple closed form, and differs by $\sim 10\%$.

Consider the scattering rate density,

$$\gamma_{mn}^{ij} \equiv \gamma(i + j \rightarrow m + n). \quad (13.22)$$

It is given by

$$\begin{aligned} \gamma_{mn}^{ij} &= \int d\Pi_i d\Pi_j f_i^{eq} f_j^{eq} \int |\mathcal{M}(i + j \rightarrow m + n)|^2 \tilde{\delta} d\Pi_m d\Pi_n \\ &= 4g_i g_j \int d\Pi_i d\Pi_j e^{-(E_i + E_j)/T} \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} \sigma((p_i + p_j)^2), \end{aligned} \quad (13.23)$$

where the $g_i g_j$ -factor appears because cross-sections are usually averaged over initial state internal degrees of freedom. The square root cancels the initial particle flux factor of the cross-section. It is convenient to separate the center of mass from initial state phase space integrals, by multiplying by $1 = \int d^4 Q \delta^4(Q - p_i - p_j)$. Using eqn (13.15) gives:

$$\begin{aligned} \gamma(i + j \rightarrow m + n) &= g_i g_j \int \frac{dQ_0 d^3 Q}{(2\pi)^4} \frac{e^{-Q_0/T}}{\pi s} [(p_i \cdot p_j)^2 - m_i^2 m_j^2] \sigma(Q^2) \\ &= \frac{g_i g_j}{32\pi^5} \int ds ds d\Omega \int_{\sqrt{s}} dQ_0 e^{-Q_0/T} \sqrt{Q_0^2 - s} \lambda\left(1, \frac{m_i^2}{s}, \frac{m_j^2}{s}\right) \sigma(s) \\ &= \frac{g_i g_j T}{32\pi^4} \int ds s^{3/2} K_1\left(\frac{\sqrt{s}}{T}\right) \lambda\left(1, \frac{m_i^2}{s}, \frac{m_j^2}{s}\right) \sigma(s), \end{aligned} \quad (13.24)$$

where $\lambda(a, b, c) = (a - b - c)^2 - 4bc$, $zK_1(z)$ is given in eqn (13.19), and we used $d^3 Q = \sqrt{Q_0^2 - s} ds d\Omega/2$ to obtain the second equality.

In the massless limit, $\lambda(1, x, y) \rightarrow 1$ and s can be integrated from 0 to ∞ , so the definite integral

$$\int_0^\infty x^n K_1(x) dx = 2^{n-1} \Gamma(1 + n/2) \Gamma(n/2) \quad (13.25)$$

is useful for obtaining γ . For interaction terms in the Lagrangian of dimension 4 and 5, one obtains

$$\gamma = \begin{cases} \frac{g_i g_j T^4}{8\pi^4} & \sigma = \frac{1}{s}, \\ \frac{g_i g_j T^6}{\pi^4 M^2} & \sigma = \frac{1}{M}. \end{cases} \quad (13.26)$$

14 Appendix: Chemical Equilibrium

In this section, the conditions of chemical equilibrium are used to derive the A -matrix and the factor of $12/37$ relating (within the Standard Model) ΔY_B to ΔY_{B-L} .

Various Standard Model interactions can change the number of particles of different species. For instance, the Lagrangian term $h_\alpha \bar{\ell}_\alpha \phi^c e_{R\alpha}$ changes a charged $SU(2)$ -singlet lepton into a Higgs and an $SU(2)$ -doublet lepton. If such interactions are fast, compared to the expansion rate H , they lead to an equilibrium state where the comoving number densities of the participating particles remain constant. This is described by conditions of chemical equilibrium (see chapter 10 of ref. [312]): The sum of the chemical potentials, over all particles entering the interaction, should be zero. For example, if the charged lepton Yukawa interaction is fast, we have

$$\mu_{e_\alpha} - \mu_{\ell_\alpha} + \mu_\phi = 0. \quad (14.1)$$

Thus, the set of reactions that are in chemical equilibrium enforce algebraic relations between various chemical potentials [41, 313].

The chemical potentials can be related to the asymmetries in the particle number densities, by expanding the distribution functions of eqn (13.5) for small μ/T :

$$n_f - n_{\bar{f}} = \frac{g_i T^3}{6} \mu \quad \text{fermions} \quad (14.2)$$

$$n_b - n_{\bar{b}} = \frac{g_i T^3}{3} \mu \quad \text{bosons} \quad (14.3)$$

where g_i is the number of degrees of freedom of the particle.

At the temperatures where the lepton asymmetry is generated, the interactions mediated by the top-quark Yukawa coupling h_t , and by the $SU(3) \times SU(2) \times U(1)$ gauge interactions, are always in equilibrium. This situation has the following consequences:

- All components of a gauge multiplet share the same chemical potential, and the chemical potential of gauge bosons is zero.
- Hypercharge neutrality implies

$$\sum_{i=1,2,3} (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i}) - \sum_{\alpha=e,\mu,\tau} (\mu_{\ell_\alpha} - \mu_{e_\alpha}) + 2\mu_\phi = 0. \quad (14.4)$$

- The equilibrium condition for the Yukawa interactions of the top-quark $\mu_t = \mu_{q_3} + \mu_\phi$ yields:

$$\Delta y_t - \frac{\Delta y_{q_3}}{2} = \frac{\Delta y_\phi}{4}, \quad (14.5)$$

where the relative factor $1/2$ between Δy_{q_3} and Δy_ϕ can be traced back to eqns (14.2)-(14.3): the relation of the number asymmetry to the chemical potential is different by a factor of 2 between bosons and fermions.

Using these relations, the asymmetries in B_i and L_α can also be expressed in terms of the chemical potentials, for example,

$$\begin{aligned} Y_{\Delta B_3} &= \frac{T^3}{6s} (2\mu_{q_3} + \mu_{t_R} + \mu_{b_R}) \quad , \\ Y_{\Delta L_\tau} &= \frac{T^3}{6s} (2\mu_{\ell_\tau} + \mu_{\tau_R}), \end{aligned} \quad (14.6)$$

and similarly for the lighter generations. We use the notation $Y_{\Delta X}$ for the *asymmetry* $Y_X - Y_{\bar{X}}$, because we think it is clearer, though not standard: for instance Y_B is usually the baryon asymmetry. However, this should not cause confusion.

The baryon flavour asymmetries can always be taken to be equal:

$$Y_{\Delta B_3} = Y_{\Delta B_2} = Y_{\Delta B_1}, \quad (14.7)$$

because they are conserved before flavour-changing quark interactions come into equilibrium, and we assume them to be equal as an initial condition. (Once flavour-changing quark interactions enter equilibrium, these asymmetries are driven to be equal.) The conservation rules (14.7) impose two conditions on the sixteen chemical potentials of the SM fields, and hypercharge neutrality, eqn (14.4), imposes a third.

We work in the approximation that each of the SM interactions is either negligible, in which case there are additional conserved quantum numbers, or in chemical equilibrium. Either case yields a restriction on the chemical potentials. The interaction rate densities for processes involving SM Yukawa couplings can be obtained from eqn (13.24), assuming the Yukawa coupling at one vertex, and a gauge or top Yukawa coupling at the other. Then the interaction rate for a Yukawa coupling that brings into equilibrium an $SU(2)$ -singlet r can be estimated as $\Gamma_r \simeq \sum_{i,j,n} \gamma(ij \rightarrow rn)/Y_j^{\text{eq}}$.

The rate of the charged lepton Yukawa interactions is important for leptogenesis. A careful study [195] of the relevant processes, which takes into account thermal masses, finds that the Higgs decay gives the fastest rate $\sim 10^{-2} h_\alpha^2 T$. Using the results obtained in [195, 314] for $\alpha = e$, we find that the h_τ, h_μ, h_e Yukawa interactions are in equilibrium for $T \lesssim 10^{12}, 3 \times 10^9, 10^5$ GeV, respectively. Here, as an example, we estimate the (subdominant) h_τ -related scattering rate at zero temperature. It can be calculated from interactions such as $q_3 + \ell_\tau \rightarrow \tau_R + t_R$ and $W + \ell_\tau \rightarrow \tau_R + H$. In the massless limit, the cross-sections for τ_R production satisfy

$$\sigma(\nu_\tau t_R^a \rightarrow \bar{\tau}_R b_L^a) = \sigma(\nu_\tau \bar{b}_L^a \rightarrow \bar{\tau}_R \bar{t}_R^a) = \sigma(b_L^a \bar{t}_R^a \rightarrow \bar{\nu}_\tau \tau_R) = \frac{h_t^2 h_\tau^2}{16\pi s}, \quad (14.8)$$

where a is a colour index, and

$$\sigma(\nu_\tau W^0 \rightarrow \tau_R \phi_d^+) = \sigma(\nu_\tau W^- \rightarrow \tau_R \phi_d^0) = \frac{g^2 h_\tau^2}{16\pi s}. \quad (14.9)$$

Including all these processes, summing over color and $SU(2)$ indices, and using eqn (13.26), we obtain

$$\Gamma_\tau = \frac{\gamma_\tau}{\frac{1}{2} n_\ell^{\text{eq}}} = \frac{(9h_t^2 + 2g^2)h_\tau^2 T^4 / 128\pi^4}{T^3 / \pi^2} \simeq 5 \times 10^{-3} h_\tau^2 T. \quad (14.10)$$

The temperature ranges in which the various SM interactions are faster than H are given in table 1.

Consider the temperature range 10^{12} GeV $> T > 10^9$ GeV, with SM particle content. The strong and electroweak sphalerons, and the top, bottom, tau and charm Yukawa interactions are taken to be in chemical equilibrium. This imposes six conditions, in addition to the three conditions of eqns (14.7) and (14.4). Moreover, the asymmetries in $\mu_R, e_R, u_R - d_R$, and $d_R - s_R$, are conserved and presumably zero. So the system of equations can be solved for the three asymmetries

$$Y_{\Delta\alpha} = \frac{1}{3} Y_{\Delta B} - Y_{\Delta L\alpha} \quad (14.11)$$

which are conserved in the SM, as a function of the μ_{ℓ_α} :

$$\begin{pmatrix} Y_{\Delta e} \\ Y_{\Delta\mu} \\ Y_{\Delta\tau} \end{pmatrix} = \frac{T^3}{6s} \begin{bmatrix} -22/9 & -4/9 & -4/9 \\ -4/9 & -22/9 & -4/9 \\ -2/9 & -2/9 & -139/45 \end{bmatrix} \begin{pmatrix} \mu_{\ell_e} \\ \mu_{\ell_\mu} \\ \mu_{\ell_\tau} \end{pmatrix} \quad (14.12)$$

However, in the range of temperatures that we consider, ℓ_μ and ℓ_e are indistinguishable, so their chemical potentials should be summed, $\mu_o = \mu_{\ell_e} + \mu_{\ell_\mu}$, giving

$$\begin{pmatrix} Y_{\Delta e} + Y_{\Delta\mu} \\ Y_{\Delta\tau} \end{pmatrix} = \frac{T^3}{6s} \begin{bmatrix} -26/9 & -8/9 \\ -2/9 & -139/45 \end{bmatrix} \begin{pmatrix} \mu_o \\ \mu_{\ell_\tau} \end{pmatrix} \quad (14.13)$$

The λ -Yukawa interactions can generate an asymmetry in the lepton doublets. However, SM interactions rapidly change the doublet densities, so it is better to write Boltzmann Equations for the asymmetries Y_{Δ_α} , which are conserved by all the SM interactions. The N interactions can add a lepton to the asymmetry, or remove a doublet lepton ℓ_α . Since the washout interactions can only destroy the part of the $B/3 - L_\alpha$ asymmetry stored in ℓ_α , we need to express the $Y_{\Delta_\alpha} \sim 2\mu_{\ell_\alpha} T^3/(6s)$ asymmetry as a function of the charges Y_{Δ_β} eqn (14.11). Inverting eqn (14.13) gives

$$\begin{pmatrix} Y_{\Delta_{\ell_e}} \\ Y_{\Delta_{\ell_\tau}} \end{pmatrix} = \begin{bmatrix} -417/589 & 120/589 \\ 30/589 & -390/589 \end{bmatrix} \begin{pmatrix} Y_{\Delta_e} + Y_{\Delta_\mu} \\ Y_{\Delta_\tau} \end{pmatrix} \quad (14.14)$$

and the matrix is called “the A -matrix” [65].

At temperatures below $\sim 10^9$ GeV, the strange and muon Yukawa interactions are in equilibrium, so the asymmetries in $d_R - s_R$, and in μ_R are no longer conserved. There are three distinguishable flavours, and the A -matrix is:

$$\begin{pmatrix} Y_{\Delta_{\ell_e}} \\ Y_{\Delta_{\ell_\mu}} \\ Y_{\Delta_{\ell_\tau}} \end{pmatrix} = \begin{bmatrix} -151/179 & 20/179 & 20/179 \\ 25/358 & -344/537 & 14/537 \\ 25/358 & 14/537 & -344/537 \end{bmatrix} \begin{pmatrix} Y_{\Delta_e} \\ Y_{\Delta_\mu} \\ Y_{\Delta_\tau} \end{pmatrix}. \quad (14.15)$$

These expressions for the A -matrix agree with refs. [63,64]. Ref. [63] defines Y_{Δ_ℓ} per gauge degree of freedom, that is 1/2 of what we use here. Therefore the A -matrices they quote differ by a factor of 2. Our expressions differ from ref. [62], where the u and d Yukawa interactions are taken in equilibrium, and from ref. [65], where the strong sphalerons are not included.

The coefficient relating Y_{Δ_B} to the Y_{Δ_α} should be calculated at the temperature when the sphalerons go out of equilibrium. The reason is that, after the period of leptogenesis, the Δ_α 's are conserved. Baryon number B is, however, not conserved in the presence of the EW sphalerons (see eqn 6.13), and the relation of Y_{Δ_B} to the Y_{Δ_α} depends on which other interactions are in equilibrium. Using the chemical equilibrium and charge conservation equations, one obtains the following relation, depending on whether the sphalerons go out of equilibrium above or below the electroweak phase transition [313]:

$$Y_{\Delta_B} = \sum_\alpha Y_{\Delta_\alpha} \times \begin{cases} \frac{24+4m}{66+13m} & T > T_{\text{EWPT}}, \\ \frac{32+4m}{98+13m} & T < T_{\text{EWPT}}, \end{cases} \quad (14.16)$$

where m is the number of Higgs doublets. For the Standard Model (where $m = 1$), one obtains [41,313]

$$Y_{\Delta_B}^{\text{SM}} = \sum_\alpha Y_{\Delta_\alpha} \times \begin{cases} \frac{28}{79} & T > T_{\text{EWPT}}, \\ \frac{12}{37} & T < T_{\text{EWPT}}. \end{cases} \quad (14.17)$$

In ref. [315], the relation is given for any value of the Higgs VEV, valid through the EWPT. In the minimal supersymmetric Standard Model (where $m = 2$), one obtains

$$Y_{\Delta_B}^{\text{MSSM}} = \sum_\alpha Y_{\Delta_\alpha} \times \begin{cases} \frac{8}{23} & T > T_{\text{EWPT}}, \\ \frac{10}{31} & T < T_{\text{EWPT}}. \end{cases} \quad (14.18)$$

15 Appendix: Evolution of Flavoured Number Operators

The aim of this appendix is twofold. First, to understand which flavour basis is appropriate for the Boltzmann Equations. Second, we investigate whether the physics that defines the preferred flavour basis introduces additional significant flavour-related effects on the baryon asymmetry.

To address the first question, one would like to have a formalism which is covariant under flavour transformations, where it is possible to “derive” the Boltzmann Equations. One possible approach is to study Schwinger-Dyson equations for the non-equilibrium Green’s functions of the particles involved in leptogenesis. The resulting Kadanoff-Baym [316] equations for N_1 two-point functions were first discussed in [317]. Since the N_1 population must be out of equilibrium for leptogenesis, the aim of this paper was to study quantum non-equilibrium effects. The Boltzmann Equations were found to be a good approximation for non-relativistic N_1 .²⁹ Non-equilibrium Schwinger-Dyson equations were also used in [241, 242] to show the importance of time-dependent CP violation in resonant leptogenesis. An elegant approach to flavour effects in leptogenesis would use the finite temperature Schwinger-Dyson equations for flavoured fermionic propagators, with a flavour off-diagonal $\epsilon_{\alpha\beta}$ as a source for flavour off-diagonal propagators. An alternative approach, which should in principle be equivalent, is to study the equations of motion, in field theory, for a flavoured number operator (following, for instance, chapter 17 of [319]. For a pedagogical introduction, see *e.g.* [320].) In this framework, however, neither the notation nor the formalism are transparent (similar to perturbative calculations without Feynman diagrams and rules). Here, we simplify this formalism and study a toy model of Simple Harmonic Oscillators (SHOs), then guess an extrapolation to field theory. We show that, in this toy model, fast interactions choose a basis where the equations of motion have the form of Boltzmann Equations. Thus, the flavour basis is determined by which interactions are fast at the time of leptogenesis.

The Boltzmann equations for number densities treat interactions as a series of quantum processes of a classical particle. They neglect quantum effects, such as oscillations. In field theory, effects of quantum mechanical evolution appear in the equations of motion for the (flavour-dependent) number operator

$$\hat{f}_{\Delta}^{\alpha\beta}(\vec{p}) = a_{+}^{\alpha\dagger}(\vec{p})a_{+}^{\beta}(\vec{p}) - a_{-}^{\beta\dagger}(\vec{p})a_{-}^{\alpha}(\vec{p}), \quad (15.1)$$

which counts the asymmetry in lepton doublets. (We denote the operator \hat{f} with a hat, to distinguish it from its expectation value.) The operator $a_{+}^{\alpha\dagger}$ ($a_{-}^{\alpha\dagger}$) creates lepton doublets (anti-lepton doublets) of flavour α . Notice the inverted flavour order between the particle and antiparticle number operators [321]. The operator is a matrix in flavour space, analogous to the density matrix of quantum mechanics, and is sometimes called a density matrix. The diagonal elements are the flavour asymmetries stored in the lepton doublets. The trace, which is flavour-basis-independent, is the total lepton asymmetry. The off-diagonals encode (quantum) correlations between the different flavour asymmetries.

Variants on such an operator have been studied in the context of neutrino oscillations in the early Universe [321–324], and, in particular, the generation of a lepton asymmetry by active-sterile oscillations [325–329]. The first discussion of such an operator in the context of thermal leptogenesis was made in ref. [65].

For simplicity, we replace $\hat{f}_{\Delta}^{\alpha\beta}(\vec{p})$ by the number operator of “flavoured” harmonic oscillators. Our aim is to separate the (possibly quantum) flavour effects from the (assumed classical) particle dynamics. That is, we extract the flavour structure from a toy model of harmonic oscillators, and input the particle dynamics and the Universe expansion, by analogy with the Boltzmann equations. The system contains harmonic oscillators of two “flavours”, coupled to match the interactions of the Lagrangian in eqn. (2.3). By taking expectation values of the number operator in a “thermal” state, this model can be reduced to a two-state quantum system. We then extrapolate the flavour structure of these equations of motion to obtain “flavoured” equations of motion for the lepton asymmetry number operator in the early Universe.

²⁹See however, [318] for numerical comparisons of the Kadanoff-Baym and Boltzmann Equations in a related model.

15.1 A toy model

Consider a system of two simple harmonic oscillators with “flavour” labels 3 and 2, which have different frequencies in the free Hamiltonian: $H_0 = (\omega_2 a_2^\dagger a_2 + \omega_3 a_3^\dagger a_3)I$. Usually, the number operator is introduced as $a_2^\dagger a_2 + a_3^\dagger a_3$. However, since we are interested in changing the basis of the SHOs, we introduce the number operator as a matrix \hat{f} :

$$\hat{f} = \begin{bmatrix} a_2^\dagger a_2 & a_2^\dagger a_3 \\ a_3^\dagger a_2 & a_3^\dagger a_3 \end{bmatrix}, \quad (15.2)$$

whose trace is the usual number operator. The commutation relations³⁰ are $[a_2, a_2^\dagger] = 1$, $[a_3, a_3^\dagger] = 1$. The number operator evolves according to the Hamiltonian equations of motion: $\frac{d\hat{f}}{dt} = +i[H, \hat{f}]$. Since our Hamiltonian conserves particle number, we can take expectation values – for instance, in a thermal bath – and reduce our toy model to a two-state system described by the amplitude of the particles to be 3’s, or 2’s. The density matrix of this two-state system, is

$$f \equiv \begin{bmatrix} \langle a_2^\dagger a_2 \rangle & \langle a_2^\dagger a_3 \rangle \\ \langle a_3^\dagger a_2 \rangle & \langle a_3^\dagger a_3 \rangle \end{bmatrix}$$

satisfying

$$\frac{df}{dt} = -i \begin{bmatrix} 0 & (\omega_2 - \omega_3)f^{23} \\ (\omega_3 - \omega_2)f^{32} & 0 \end{bmatrix}. \quad (15.3)$$

If at $t = 0$ the system is created in some state, then the probability to be found in this same state at a later time t is $\text{Tr}\{f(0)f(t)\}$, which can oscillate.

It is useful to make an analogy to the use of this formalism in the context of neutrino oscillations [330]. We take the expectation value of the neutrino number operator in a beam of neutrinos of momentum \vec{p} . This gives a density matrix f representing the flavour of the beam. For two neutrino species, this is a two-state quantum system. For an initial state $|\nu(t = 0)\rangle = |\nu_\mu\rangle = c_{23}|\nu_2\rangle + s_{23}|\nu_3\rangle$, (so that $\hat{f}(t = 0) = |\nu_\mu\rangle\langle\nu_\mu|$), the density matrix at $t = 0$ is given by

$$f(t = 0) = \begin{bmatrix} s_{23}^2 & c_{23}s_{23} \\ c_{23}s_{23} & c_{23}^2 \end{bmatrix}. \quad (15.4)$$

The solution to eqn. (15.3) with the initial condition (15.4) is

$$f(t) = \begin{bmatrix} s_{23}^2 & c_{23}s_{23}e^{-i\Delta t} \\ c_{23}s_{23}e^{+i\Delta t} & c_{23}^2 \end{bmatrix}, \quad \Delta = \frac{m_3^2 - m_2^2}{2E}. \quad (15.5)$$

We obtain the ν_μ survival probability:

$$P_{\mu \rightarrow \mu}(t) = \text{Tr}\{f(0)f(t)\} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta t \quad (15.6)$$

Returning to lepton flavour in the early Universe, consider doublets ℓ_α of momentum \vec{p} , in a temperature range where the rate associated to the τ Yukawa coupling is fast ($\Gamma_\tau \gtrsim H$), while the muon and electron Yukawa interactions can be neglected. The ℓ_α ’s have thermal masses [197, 331]:

$$m_\tau^2(T) \simeq \left(\frac{3g^2}{32} + \frac{g'^2}{32} + \frac{h_\tau^2}{16} \right) T^2, \quad m_{\beta \neq \tau}^2(T) \simeq \left(\frac{3g^2}{32} + \frac{g'^2}{32} \right) T^2. \quad (15.7)$$

Notice the difference with respect to neutrino oscillations in the lab: In the early Universe, the mass basis is the flavour basis. The contribution of the neutrino Yukawa coupling to the thermal mass has been neglected in eqn. (15.7), because we assume that $h_\tau \gg |\lambda_{\alpha 1}|$. For $h_\tau < |\lambda_{\alpha 1}|$, an additional term $\propto \lambda_{\alpha 1} \lambda_{\beta 1}^* f_{N_1}$ should be included. This case is discussed in section 15.3.

³⁰In studying flavour, we consider “bosonic” leptons. We anticipate no difficulties in generalizing to fermionic leptons.

The two harmonic oscillators in the early Universe can be labeled τ and o , where o is the projection onto e and μ of the direction into which N_1 decays: $\vec{\sigma} \propto \lambda_{e1}\hat{e} + \lambda_{\mu1}\hat{\mu}$. The eigenvalues of the free Hamiltonian are the particle energies $\omega(\vec{p})$, and the time evolution of the number operator is determined by the energy differences:

$$\omega_\tau^2 - \omega_o^2 = \frac{h_\tau^2 T^2}{16}. \quad (15.8)$$

By analogy with the neutrino oscillation example, we anticipate that “flavour oscillations” might affect the lepton asymmetry. Imagine the excitations of the oscillators to be the lepton *asymmetry*, carried by particles of energies $\omega_\tau \sim \omega_o \sim E$. Then $\omega_\tau - \omega_o \simeq h_\tau^2 T^2 / (32E)$. We treat the production and washout of the lepton asymmetry as, respectively, initial condition and subsequent measurement on the system. In other words, we consider the production of an asymmetry in some linear combination of o and τ , we allow it to evolve, and later turn on the inverse decays. Then the inverse decay rate can be suppressed or enhanced because the asymmetry changed flavour during time evolution (analogous to a survival probability in neutrino oscillations) [62]. Notice that the oscillation timescale is of order $(h_\tau^2 T)^{-1}$, which is parametrically the same as the timescale of the h_τ -mediated scattering rates that cause decoherence. This is different from, *e.g.* the MSW effect in matter, where the rate for decohering scattering can be neglected on the oscillation timescale.

The flavour-blind gauge contribution is neglected in eqn. (15.8). This is subtle, because the early Universe version of Δ (see eqn. (15.5)) depends on the energy of the lepton, and within the oscillation timescale, a lepton participates in many energy-changing gauge interactions. Ref. [62] used the thermally averaged energy $\langle E \rangle \simeq 3T$ to estimate Δ [325]. In path integral language, this means approximating the integral $(i \int E d\tau)$ along the path from one lepton-number violating interaction to the next, to be $i\langle E \rangle \int d\tau$. This is in agreement with the analytic and numerical analysis of [332], which indicates that fast gauge interactions do not affect the coherence of flavour oscillations: if the timescale for transitions between different energy levels is much shorter than the oscillation timescale, then a particle spends time at many different energies during an oscillation timescale. The probability to have an energy E is proportional to the distribution function f of eqn. (13.5). Therefore, on the oscillation timescale, the particles all have the thermal average energy, and oscillate coherently. In any case, this does not affect the principal claim of this appendix, that charged lepton Yukawa interactions choose the flavour basis for the Boltzmann Equations.

We now include interactions among the SHOs. This allows one to take into account the production and annihilation of particles, and can lead to decoherence. Recall that in ordinary neutrino oscillations, decoherence happens anyway after a few oscillation lengths, due to “spreading of the wave packet” (see *e.g.* [333]). Here we are looking for decohering interactions, which, in the quantum mechanical analogy, collapse the wavefunctions onto an eigenbasis. We will see that in this basis, the equations of motion look like Boltzmann Equations.

The decays and inverse decays due to the Yukawa interactions can be included by perturbing in an interaction Hamiltonian, H_I . We first consider the production and washout of leptons, due to λ , in a model of four harmonic oscillators: N , ϕ , and two leptons flavours $\{\ell_\alpha\}$. Although we use the index α , this discussion is covariant in flavour space, so any basis choice for doublet flavour space is valid. We are looking for behaviors analogous to the lepton number production and washout in the decay/inverse decay $N \leftrightarrow \phi \ell_\alpha$. We take ϕ and ℓ_α to be massless and N massive, and ignore the free Hamiltonian (considered previously) which does not change the particle numbers. The interaction Hamiltonian is

$$H_I = \lambda_{\alpha 1} a_N^\dagger a_\phi a_{\ell_\alpha} + \lambda_{\alpha 1}^* a_N a_\phi^\dagger a_{\ell_\alpha}^\dagger. \quad (15.9)$$

We perturbatively expand the Heisenberg equations of motion and obtain

$$\begin{aligned} \frac{\partial}{\partial t} f_\ell^{\alpha\beta} &= -[H_I, [H_I, f_\ell^{\alpha\beta}]] = -\lambda_{\rho 1}^* \lambda_{\alpha 1} [a_N a_\phi^\dagger a_{\ell_\rho}^\dagger, a_N^\dagger a_\phi a_{\ell_\beta}] + \lambda_{\rho 1} \lambda_{\beta 1}^* [a_N^\dagger a_\phi a_{\ell_\rho}, a_N a_\phi^\dagger a_{\ell_\alpha}^\dagger] \\ &= -\lambda_{\alpha 1} \lambda_{\rho 1}^* (f_\phi f_\ell^{\rho\beta} - f_N f_\phi \delta^{\rho\beta} - f_N f_\ell^{\beta\rho} - f_N \delta^{\beta\rho}) + (f_N \delta^{\alpha\rho} + f_N f_\phi \delta^{\alpha\rho} + f_N f_\ell^{\alpha\rho} - f_\phi f_\ell^{\alpha\rho}) \lambda_{\rho 1} \lambda_{\beta 1}^* \\ &= -\lambda_{\alpha 1} \lambda_{\rho 1}^* \left[f_\phi f_\ell^{\rho\beta} (1 + f_N) - f_N (1 + f_\phi) (\delta^{\rho\beta} + f_\ell^{\rho\beta}) \right] \end{aligned}$$

$$+ \left[f_N(1 + f_\phi)(\delta^{\alpha\rho} + f_\ell^{\alpha\rho}) - f_\phi f_\ell^{\alpha\rho}(1 + f_N) \right] \lambda_{\rho 1} \lambda_{\beta 1}^* . \quad (15.10)$$

This has the form of inverse decays and decays of N , with Bose enhancement factors (recall that the toy model leptons are bosons) for the final states. For simplicity, we now drop the Bose enhancement factors. A similar calculation can be performed for the tau Yukawa coupling \vec{h}_τ ($|\vec{h}_\tau| = h_\tau$). The interaction Hamiltonian is

$$H_{I,h} = h_\tau^\rho a_\phi^\dagger a_{\tau^c} a_{\ell_\rho} + h_\tau^{*\rho} a_\phi a_{\tau^c}^\dagger a_{\ell_\rho}^\dagger . \quad (15.11)$$

We assume that the processes $\phi \leftrightarrow \ell_\tau \tau^c$ are allowed, because the thermal mass of Higgs gets contributions from the top Yukawa interaction. (To avoid appealing to thermal masses, one could go to higher orders in H_I , and consider scattering such as $W\ell \rightarrow \tau^c \phi$.) We neglect Bose enhancement factors and take expectation values. We obtain, in the charged lepton mass eigenstate basis,

$$\begin{aligned} \frac{\partial}{\partial t} \begin{bmatrix} f_\ell^{oo} & f_\ell^{o\tau} \\ f_\ell^{\tau o} & f_\ell^{\tau\tau} \end{bmatrix} &= -f_{\tau^c} \begin{bmatrix} 0 & 0 \\ 0 & |h_\tau|^2 \end{bmatrix} \begin{bmatrix} f_\ell^{oo} & f_\ell^{o\tau} \\ f_\ell^{\tau o} & f_\ell^{\tau\tau} \end{bmatrix} - f_{\tau^c} \begin{bmatrix} f_\ell^{oo} & f_\ell^{o\tau} \\ f_\ell^{\tau o} & f_\ell^{\tau\tau} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & |h_\tau|^2 \end{bmatrix} \\ &+ 2f_\phi \begin{bmatrix} 0 & 0 \\ 0 & |h_\tau|^2 \end{bmatrix} . \end{aligned} \quad (15.12)$$

It is reassuring that the t -independent equilibrium solution of this equation imposes $f_{\tau^c} f_\ell^{\tau\tau} - f_\phi = 0$. Combined with eqn. (15.3), this gives damped oscillations for the off-diagonal elements:

$$\partial f_\ell^{o\tau} / \partial t = -i(\omega_o - \omega_\tau - i|h_\tau|^2 f_{\tau^c}) f_\ell^{o\tau} , \quad (15.13)$$

We conclude that exchanging doublet with singlet leptons destroys the coherence between different doublet flavours. In the ‘‘flavour’’ basis, the off-diagonal elements of the ℓ_α number operator vanish when the charged lepton Yukawa interactions are fast. The charged lepton Yukawa interactions are therefore the desired ‘‘basis-choosing’’ physics, that collapses the equations of motion of the density matrix to Boltzmann Equations for the number densities on the diagonal.

It is convenient to write these equations in the notation of projectors, introduced in eqn. (8.1). In an arbitrary basis, we define, at tree level, the projectors onto the decay direction of N_1 , and onto the τ flavour:

$$\overline{P}^{ab} = P^{ab} = \frac{\lambda_{a1} \lambda_{b1}^*}{\sum_d |\lambda_{d1}|^2}, \quad P_\tau^{ab} = \frac{h_\tau^a h_\tau^{*b}}{\sum_d |h_\tau^d|^2} . \quad (15.14)$$

When loop contributions are included in the decays of N_1 , a CP asymmetry can arise. It can be represented by a difference between P and \overline{P} , where \overline{P} is the projector onto the anti-lepton direction into which N_1 decays.

Equations for the anti-lepton density can be derived in a similar way. For simplicity, we take ϕ and N to be their own anti-particles, so there are now eight coupled oscillators: $N, \phi, \ell^\alpha, \ell^\alpha, \tau^c$ and $\bar{\tau}^c$. The expectation value of the difference between the two equations, again in the charged lepton mass eigenstate basis, is

$$\begin{aligned} \frac{\partial}{\partial t} f_\Delta^{\alpha\beta} &= -|\lambda_1|^2 P^{\alpha\rho} f_\phi f_\Delta^{\rho\beta} - |\lambda_1|^2 f_\phi f_\Delta^{\alpha\rho} P^{\rho\beta} + 2(f_N - f_N^{\text{eq}}) \epsilon^{\alpha\beta} \\ &- i(\omega_\alpha - \omega_\beta) f_\Delta^{\alpha\beta} - \frac{|h_\tau|^2}{2} (f_{\tau^c} + f_{\bar{\tau}^c}) (f_\Delta^{\alpha\tau} \delta_{\tau\beta} + \delta_{\tau\alpha} f_\Delta^{\tau\beta}) \\ &- \frac{|h_\tau|^2}{2} (f_{\tau^c} - f_{\bar{\tau}^c}) [(f_\ell + f_\ell^T)^{\alpha\tau} \delta_{\tau\beta} + \delta_{\tau\alpha} (f_\ell + f_\ell^T)^{\tau\beta}] , \end{aligned} \quad (15.15)$$

where $\rho, \alpha, \beta = o, \tau$. The purpose of this equation is to motivate the flavour index structure we use in the Boltzmann equations. We now briefly discuss the various terms.

- The first two terms describe washout by inverse decays. By construction, there is no washout by $\Delta L = 2$ scattering (we will introduce the resonant part by hand). Notice that the usual partial decay rates $\Gamma(N \rightarrow \phi \ell_\alpha)$ become the diagonal elements of a matrix. The total decay rate, which is flavour-basis invariant, is the trace of this matrix.

• The third term, which is $\propto \epsilon$, has been obtained artificially. It is well-known in field theory that no asymmetry is created at second order in λ (see eqn. 5.19). A CP asymmetry is proportional to the imaginary part of the loop amplitude times the imaginary part of the product of tree and loop coupling constants (see eqn. 5.5). We include the loop as an effective (non-unitary) interaction in the Hamiltonian: $(\delta\lambda)_{\alpha\Omega} a_N^\dagger a_\phi a_{\ell_\alpha} + (\delta\lambda)_{\alpha\Omega}^* a_N a_\phi^\dagger a_{\ell_\alpha}^\dagger$. Here Ω encodes the imaginary part of the loop amplitude; it is flavour independent, of little direct interest here, and can be obtained by matching to the calculation of $\epsilon_{\alpha\alpha}$ in section 5.1. Just as the decay rate Γ becomes a matrix in flavour space, the CP asymmetry ϵ also becomes a matrix in flavour space, proportional to $[(\delta\lambda)_{\alpha\Omega} \lambda_\beta^* - \lambda_\alpha (\delta\lambda)_{\beta\Omega}^*] \propto P^{\alpha\beta} - \overline{P}^{\alpha\beta}$. This gives

$$\epsilon_{\alpha\beta} \propto \text{Im}\{\lambda_\alpha (\delta\lambda)_{\beta\Omega}^* - (\delta\lambda)_{\alpha\Omega} \lambda_\beta^*\} = \frac{1}{v_u^2} \text{Im}\{\lambda_\alpha m_{\beta\rho}^* \lambda_\rho - m_{\alpha\rho} \lambda^{*\rho} \lambda_\beta^*\}. \quad (15.16)$$

Using the equilibrium condition,

$$2f_N^{\text{eq}}[\epsilon] = \frac{f_\phi}{2} ([\epsilon][f_\ell + f_\ell^T] + [f_\ell + f_\ell^T][\epsilon])$$

(where square brackets are matrices in flavour space), we obtain a production term $2(f_N + f_N^{\text{eq}})\epsilon^{\alpha\beta}$. This cannot be complete, because it gives an asymmetry in thermal equilibrium. The resonant part of $\Delta L = 2$ scattering should be subtracted [31], which ensures that $\text{Tr} f_\Delta$ vanishes in thermal equilibrium. Working in the charged lepton mass basis, it is straightforward to obtain eqn. (15.15) by following section 6.2, or ref. [31].

• The remaining terms of eqn. (15.15) describe the effects of the tau Yukawa coupling. The asymmetry oscillates in flavour space, as described at the beginning of the section, because the off-diagonal flavour matrix elements have time-dependent phases. We drop this oscillatory term from the equation of motion, when extrapolating to the early Universe. (See [62] for a discussion including these oscillations).

The tau Yukawa coupling also causes the off-diagonal elements to decay away, via the last two terms. This happens separately for the lepton and anti-lepton densities, as shown in eqn. (15.13). However, the equation is not so simple to solve for the asymmetry, because the last term depends also on the asymmetry in the $SU(2)$ -singlet τ^c s. The coupled equations for the singlet and doublet asymmetries should be solved. Instead, we attempt to include the singlet asymmetry via the A -matrix, as is done in the Boltzmann Equations, and argue in the following that the singlet asymmetry has little effect on the decay of the flavour-off-diagonal terms.

The last two terms of eqn. (15.15) drive the diagonal asymmetry $Y_{\Delta\ell_\tau}$ to the correct relation with $Y_{\Delta\tau}$, when the tau Yukawa coupling is in equilibrium. (In our toy model, where ϕ is a real scalar, this means $(f_{\tau^c} - f_{\bar{\tau}^c}) = f_\Delta^{\tau\tau}$. In the SM, there is a Higgs asymmetry [41, 65, 313].) This can be clarified by studying the evolution of

$$F^{\alpha\beta} = f_\Delta^{\alpha\beta} - P_\tau^{\alpha\beta} f_{\Delta\tau^c}, \quad (15.17)$$

where P_τ is the projector onto the τ direction in some arbitrary basis for lepton doublets, and $f_{\Delta\tau^c} = f_{\tau^c} - f_{\bar{\tau}^c}$. The trace of F is the total lepton number stored in o_L , τ_L and τ_R , so studying F in the toy model is analogous to replacing the asymmetry in lepton doublets with the asymmetry in $B/3 - L_\alpha$ [65] in usual leptogenesis calculations. For the remainder of this subsection, we focus on the decohering effects of the τ Yukawa. This means we neglect the oscillation term and the neutrino Yukawa λ . The equation for the doublet asymmetry is

$$\frac{\partial}{\partial t}[f_\Delta] = -|h_\tau|^2 \{ [P_\tau], [f_\Delta] \} \frac{(f_{\tau^c} + f_{\bar{\tau}^c})}{2} - |h_\tau|^2 \{ [P_\tau], [f_\ell + f_\ell^T] \} \frac{(f_{\tau^c} - f_{\bar{\tau}^c})}{2}, \quad (15.18)$$

where square brackets denote matrices in doublet space. The equation for the singlet asymmetry is

$$\frac{\partial}{\partial t} f_{\Delta\tau^c} = -|h_\tau|^2 (f_{\tau^c} + f_{\bar{\tau}^c}) f_\Delta^{\tau\tau} - |h_\tau|^2 f_{\Delta\tau^c} (f_\ell + f_\ell^T)^{\tau\tau}, \quad (15.19)$$

where $f_{\Delta}^{\tau\tau} = \hat{h}^\dagger \cdot [f_{\Delta}] \cdot \hat{h}$. The equation for the lepton asymmetry $F^{\alpha\beta}$ is

$$\begin{aligned} \frac{\partial}{\partial t}[F] &= -|h_{\tau}|^2 \left(\{[P_{\tau}], [f_{\Delta}]\} - 2[P_{\tau}]\text{Tr}[P_{\tau}f_{\Delta}] \right) \frac{(f_{\tau^c} + f_{\bar{\tau}^c})}{2} \\ &\quad - |h_{\tau}|^2 \left(\{[P_{\tau}], [f_{\ell} + f_{\bar{\ell}}^T]\} - 2[P_{\tau}]\text{Tr}[P_{\tau}(f_{\ell} + f_{\bar{\ell}}^T)] \right) \frac{(f_{\tau^c} - f_{\bar{\tau}^c})}{2}, \end{aligned} \quad (15.20)$$

where the second line is of the same order as the first (see eqn. 15.15). The trace of the right hand side is zero as expected, since $\text{Tr}(F)$ is conserved in the absence of λ . In the flavour basis, eqn. (15.20) can be rewritten as

$$\frac{\partial}{\partial t} \begin{bmatrix} f_{\Delta}^{oo} & f_{\Delta}^{o\tau} \\ f_{\Delta}^{\tau o} & f_{\Delta}^{\tau\tau} - f_{\Delta\tau^c} \end{bmatrix} = -\frac{|h_{\tau}|^2}{2} \begin{bmatrix} 0 & f_{\Delta}^{o\tau} \\ f_{\Delta}^{\tau o} & 0 \end{bmatrix} (f_{\tau^c} + f_{\bar{\tau}^c}) - \frac{|h_{\tau}|^2}{2} \begin{bmatrix} 0 & f_{\ell}^{o\tau} + f_{\bar{\ell}}^{\tau o} \\ f_{\ell}^{\tau o} + f_{\bar{\ell}}^{o\tau} & 0 \end{bmatrix} f_{\Delta\tau^c}. \quad (15.21)$$

The first term causes $f_{\Delta}^{o\tau}$ to decay. The second term is of the same order, and can be of either sign. To see that, nevertheless, $f_{\Delta}^{o\tau}$ and $f_{\Delta}^{\tau o}$ are driven to zero as h_{τ} comes into equilibrium, notice that eqn. (15.13) implies that $f_{\ell}^{\tau o} + f_{\bar{\ell}}^{o\tau}$ vanishes exponentially fast. So the problematic second term vanishes as h_{τ} comes into equilibrium, and the equilibrium (time-independent) solution is $f_{\Delta}^{\tau o} = 0$.

In summary, the Yukawa coupling h causes the flavour off-diagonal asymmetries to decay away, as it did for the off-diagonal number densities in eqn. (15.12). It has no effect on the asymmetry in either flavour. To obtain eqn. (15.22), we simply drop the second line (so we neglect the asymmetry in $SU(2)$ -singlets).

15.2 Extrapolating to the early Universe

A matrix equation describing the asymmetry ΔY_{ℓ} in the early Universe can be guessed by matching the flavour structure of eqn. (15.15) to the Boltzmann Equations:

$$\begin{aligned} \frac{d}{dz} \begin{bmatrix} Y_{\Delta}^{oo} & Y_{\Delta}^{o\tau} \\ Y_{\Delta}^{\tau o} & Y_{\Delta}^{\tau\tau} \end{bmatrix} &= \frac{z}{sH_1} \left(\gamma_{N \rightarrow 2} \left(\frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \begin{bmatrix} \epsilon^{oo} & \epsilon^{o\tau} \\ \epsilon^{\tau o} & \epsilon^{\tau\tau} \end{bmatrix} \right. \\ &\quad \left. - \frac{1}{2Y_{\ell}^{\text{eq}}} \left\{ \begin{bmatrix} \gamma_{N \rightarrow 2}^{oo} & \gamma_{N \rightarrow 2}^{o\tau} \\ \gamma_{N \rightarrow 2}^{\tau o} & \gamma_{N \rightarrow 2}^{\tau\tau} \end{bmatrix}, \begin{bmatrix} Y_{\Delta\ell}^{oo} & Y_{\Delta\ell}^{o\tau} \\ Y_{\Delta\ell}^{\tau o} & Y_{\Delta\ell}^{\tau\tau} \end{bmatrix} \right\} \right. \\ &\quad \left. - \frac{1}{2Y_{\ell}^{\text{eq}}} \left\{ [\gamma_{\tau}], \begin{bmatrix} Y_{\Delta}^{oo} & Y_{\Delta}^{o\tau} \\ Y_{\Delta}^{\tau o} & Y_{\Delta}^{\tau\tau} \end{bmatrix} \right\} + \frac{\text{Tr}[\gamma_{\tau} Y_{\Delta}]}{Y_{\ell}^{\text{eq}}} \delta_{\tau\tau} \right) \end{aligned} \quad (15.22)$$

where $\{\cdot, \cdot\}$ stands for anti-commutator. We now discuss each of the three terms on the right hand side of eqn. (15.22) in turn:

1. The first term describes the creation of the asymmetry. The CP asymmetry in the α -flavour is $\epsilon^{\alpha\alpha}$, and $\epsilon = \sum_{\alpha} \epsilon^{\alpha\alpha}$. Notice that $\epsilon^{\alpha\beta}$ is normalized by the total decay rate (so that $[\epsilon]$ transforms as a tensor under ℓ basis rotations). The matrix $[\epsilon]$ is now defined as

$$\begin{aligned} \epsilon_{\alpha\beta} &= \frac{1}{(16\pi)} \frac{1}{[\lambda\lambda^\dagger]_{11}} \sum_J \text{Im} \{ (\lambda_{1\alpha}) [\lambda\lambda^\dagger]_{1j} \lambda_{j\beta}^* - (\lambda_{1\beta}^*) [\lambda^* \lambda^T]_{1j} \lambda_{j\alpha} \} g \left(\frac{M_j^2}{M_1^2} \right) \\ &\simeq \frac{3}{(16\pi v^2)} \frac{M_1}{[\lambda\lambda^\dagger]_{11}} \text{Im} \{ \lambda_{\alpha} \lambda_{\sigma} [m_{\nu}^*]_{\sigma\beta} - \lambda_{\sigma}^* [m_{\nu}]_{\sigma\alpha} \lambda_{\beta}^* \} \quad (\text{for } M_1 \ll M_{2,3}), \end{aligned} \quad (15.23)$$

where $g(x)$ is the loop function of eqn. (5.14).

2. The second term is the washout of the asymmetry by decays and inverse decays. The matrix $[\gamma_{N \rightarrow 2}]$ is defined as

$$\gamma_{N \rightarrow 2}^{\alpha\beta} = \gamma_{N \rightarrow 2} P^{\alpha\beta} = \gamma_{N \rightarrow 2} \frac{\lambda_{1\alpha} \lambda_{1\beta}^*}{\sum_{\rho} |\lambda_{1\rho}|^2}, \quad (15.24)$$

where $\gamma_{N \rightarrow 2} = \sum_{\rho} \gamma_{N \rightarrow 2}^{\rho\rho}$ is the total (thermally averaged) decay rate. For $\alpha = \beta$, $\gamma_{N \rightarrow 2}^{\alpha\alpha}$ is the decay rate of N_1 to the α -flavour. This term is written as a function of the asymmetry in doublets, ΔY_{ℓ} , rather than the asymmetry in $B-L$: $Y_{\Delta} - \hat{h}\hat{h}^{\dagger}Y_{\Delta\tau^c}$. One way to proceed is to solve simultaneously this equation and the equation for $Y_{\Delta\tau^c}$. Alternatively, one can include the singlet asymmetry in an approximate way via an A -matrix, as one does in the Boltzmann equations. Since the h_{τ} -interaction is entering equilibrium, the A -matrix is time-dependent:

$$A^{\alpha\beta}(t) = A_{T>}^{\alpha\beta} e^{-\Gamma_{\tau} t} + A_{T<}^{\alpha\beta} (1 - e^{-\Gamma_{\tau} t}), \quad (15.25)$$

where $A_{T>}^{\alpha\beta}$ ($A_{T<}^{\alpha\beta}$) are the matrix elements before (after) h_{τ} comes into equilibrium, and $\Gamma_{\tau} = Y_{\ell}^{\text{eq}} \gamma(\ell \leftrightarrow \tau_R)$ is the interaction rate associated to the tau Yukawa coupling, see eqn. (14.10). We use this to obtain eqn. (15.29).

3. The last term describes the decay of the quantum correlations between o and τ , that happens as the h_{τ} interaction comes into equilibrium and makes the τ 's distinguishable. In the flavour basis, we have

$$[\gamma_{\tau}] = \begin{bmatrix} 0 & 0 \\ 0 & \gamma_{\tau} \end{bmatrix}, \quad (15.26)$$

where γ_{τ} is the interaction density for the τ Yukawa coupling (see discussion around eqn. 14.10):

$$\gamma_{\tau} \simeq 5 \times 10^{-3} h_{\tau}^2 T n_{\ell}^{\text{eq}}. \quad (15.27)$$

These interactions collapse the density matrix onto its diagonal elements in the flavour basis.

Following Stodolsky [322, 330], we write eqn. (15.22) in a more transparent way by expanding the various matrices on the basis provided by the σ -matrices, $\sigma^{\mu} = (I, \vec{\sigma})$. A hermitian 2×2 matrix Y can be written as

$$Y = Y^{\mu} \sigma_{\mu}, \quad Y^{\mu} = \frac{1}{2} \text{Tr}\{Y \sigma^{\mu}\}. \quad (15.28)$$

The asymmetry matrix in the flavour basis is then replaced by a four-vector with components $Y_{\Delta}^0 = (1/2)\text{Tr}[Y_{\Delta}]$, which represents half of the total lepton asymmetry, $Y_{\Delta}^z = (1/2)(Y_{\Delta}^{oo} - Y_{\Delta}^{\tau\tau})$, which is half of the difference of the asymmetries in the o and τ flavour densities, and $Y_{\Delta}^{x,y}$, which account for the quantum correlations between the lepton asymmetries.

In this notation, the Boltzmann equation (15.22) becomes a system of equations of the form

$$\begin{aligned} \frac{dY_{\Delta}^0}{dz} &= \frac{z}{sH_1} \left(\gamma_{N \rightarrow 2} \left(\frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \epsilon^0 - \frac{1}{Y_{\ell}^{\text{eq}}} \gamma_{N \rightarrow 2}^0 [AY_{\Delta}]^0 - \frac{1}{Y_{\ell}^{\text{eq}}} \vec{\gamma}_{N \rightarrow 2} \cdot [\vec{AY}_{\Delta}] \right), \\ \frac{d\vec{Y}_{\Delta}}{dz} &= \frac{z}{sH_1} \left(\gamma_{N \rightarrow 2} \left(\frac{Y_{N_1}}{Y_{N_1}^{\text{eq}}} - 1 \right) \vec{\epsilon} - \vec{\gamma}_D \frac{[AY_{\Delta}]^0}{Y_{\ell}^{\text{eq}}} - \frac{1}{Y_{\ell}^{\text{eq}}} \gamma_{N \rightarrow 2}^0 [\vec{AY}_{\Delta}] + \hat{\gamma}_{\tau} \times \vec{\gamma}_{\tau} \times \vec{Y}_{\Delta} \right). \end{aligned} \quad (15.29)$$

Had we kept the thermal masses in our equations, we would find that the components Y_{Δ}^x and Y_{Δ}^y precess around the z -direction with an angular velocity set by the thermal mass [62]. At the same time, such a precession is damped by the tau-Yukawa interactions at a rate $\sim \gamma_{\tau}$, as described by the term $\hat{\gamma}_{\tau} \times \vec{\gamma}_{\tau} \times \vec{Y}_{\Delta}$.

For $\gamma_{\tau} \gg \gamma_{N \rightarrow 2}$, the asymmetry \vec{Y}_{Δ} is projected onto $\vec{\gamma}_{\tau}$ before the washout interactions have time to act. So the last cross product term involving $\vec{\gamma}_{\tau}$ can be dropped from the equations, provided that $\vec{Y}_{\Delta} \parallel \vec{\gamma}_{\tau}$ is imposed. To summarize, we find that for $\gamma_{\tau} > \gamma_{N \rightarrow 2}$, the baryon asymmetry can be obtained by solving the Boltzmann Equations in the flavour basis.

15.3 Decoherence due to λ

In this section we consider the case where the N_1 decay rate is faster than, or of order of, the rate of τ Yukawa interactions [180, 181]. The evolution of the asymmetry density operator in this transition region was studied in [180], including the oscillations due to the thermal mass (15.7).

For $\gamma_{N \rightarrow 2} > \gamma_\tau$, the distinguished direction in the doublet-lepton flavour space is ℓ_{N_1} , the direction into which N_1 decays. Therefore the ‘‘single flavour’’ leptogenesis calculation applies, even if $\Gamma_\tau > H$. To see this effect analytically, we use the orthonormal basis $\{\hat{\ell}_{N_1}, \hat{\ell}_p\}$ of the plane spanned by ℓ_{N_1} and $\hat{\tau}$ (p stands for perpendicular: $\hat{\ell}_{N_1} \cdot \hat{\ell}_p = 0$). In this basis for the two-dimensional flavour space, the four-vector components are $\epsilon^0 = \epsilon^z = \epsilon/2$, $\gamma_{N \rightarrow 2}^0 = \gamma_{N \rightarrow 2}^z = \gamma_{N \rightarrow 2}/2$, and $\gamma_{N \rightarrow 2}^x = \gamma_{N \rightarrow 2}^y = 0$. The lepton asymmetry matrix $Y_{\Delta\ell}^{ij}$ can be written as the four-vector $Y_\ell^{\text{eq}}(\Delta y_\ell^0, \Delta y_\ell^x, \Delta y_\ell^y, \Delta y_\ell^z)$. The equations (15.29) become

$$\frac{2sH_1}{\tilde{z}} \frac{dY_{\Delta}^0}{d\tilde{z}} = \gamma_{N \rightarrow 2} (y_{N_1} - 1) \epsilon - \gamma_{N \rightarrow 2} (\Delta y_\ell^0 + \Delta y_\ell^z) \quad (15.30)$$

$$\frac{2sH_1}{\tilde{z}} \frac{dY_{\Delta}^z}{d\tilde{z}} = \gamma_{N \rightarrow 2} (y_{N_1} - 1) \epsilon - \gamma_{N \rightarrow 2} (\Delta y_\ell^0 + \Delta y_\ell^z) + 2(\hat{\gamma}_\tau \times \vec{\gamma}_\tau \times \Delta \vec{y}_\ell) \cdot \hat{z} \quad (15.31)$$

$$\frac{sH_1}{\tilde{z}} \frac{dY_{\Delta}^y}{d\tilde{z}} = \gamma_{N \rightarrow 2} (y_{N_1} - 1) \epsilon^y - \frac{\gamma_{N \rightarrow 2}}{2} \Delta y_\ell^y + (\hat{\gamma}_\tau \times \vec{\gamma}_\tau \times \Delta \vec{y}_\ell) \cdot \hat{y} \quad (15.32)$$

$$\frac{sH_1}{\tilde{z}} \frac{dY_{\Delta}^x}{d\tilde{z}} = \gamma_{N \rightarrow 2} (y_{N_1} - 1) \epsilon^x - \frac{\gamma_{N \rightarrow 2}}{2} \Delta y_\ell^x + (\hat{\gamma}_\tau \times \vec{\gamma}_\tau \times \Delta \vec{y}_\ell) \cdot \hat{x}. \quad (15.33)$$

where the time variable $z = M/T$ has exceptionally been represented as \tilde{z} , to avoid confusion with the four-vector index. In the above equations, the A -matrix has been neglected for two reasons. First, it includes the effects of other interactions, in the approximation that they are fast compared to N_1 decays, while we are interested in the case where the interactions rates of N_1 , τ_R and the sphalerons are of the same order. Second, it is simpler to consider the equations for the doublet asymmetry $Y_{\Delta\ell}$ rather than the $B/3 - L_\alpha$ asymmetries $Y_{\Delta\alpha}$. The A -matrix is included in [180].

The final baryon asymmetry is proportional to the total asymmetry in the doublets, which is the trace $Y_{\Delta\ell}^0 = Y_\ell^{\text{eq}} \Delta y_\ell^0$. The second washout term in eqn. (15.30) depletes the asymmetry less effectively for $\Delta y_\ell^0 + \Delta y_\ell^z \simeq 0$. When $\Delta y_\ell^0 + \Delta y_\ell^z < 0$, it contributes to generating the total asymmetry by reprocessing a flavour asymmetry. This change of sign must be accomplished by γ_τ , before the washout interactions have time to act.

Consider an asymmetry produced in N_1 decay. In a short timescale, before the ℓ have time to interact via λ or h_τ , $\Delta y_\ell^a \propto \epsilon^a$ (where $a = 0, x, y, z$). Flavour effects can modify the evolution of the asymmetry if Δy_ℓ^z is determined by the h_τ interactions (recall that $\hat{z} = \hat{\gamma}_{N \rightarrow 2}$):

$$(\hat{\gamma}_\tau \times \vec{\gamma}_\tau \times \vec{\epsilon}) \cdot \hat{\gamma}_{N \rightarrow 2} = (\hat{\gamma}_\tau \cdot \vec{\epsilon} \hat{\gamma}_\tau \cdot \hat{\gamma}_{N \rightarrow 2} - \epsilon^0) |\gamma_\tau| \gg \gamma_{N \rightarrow 2} \epsilon^0. \quad (15.34)$$

Notice that $|\vec{\epsilon}| \gg \epsilon^0$ is possible, corresponding to $\epsilon_{00} \simeq -\epsilon_{\tau\tau} \gg \epsilon$. Eqn. (15.34) is satisfied for $|\vec{\epsilon}| \gamma_\tau \gg \gamma_{N \rightarrow 2} \epsilon$, provided that certain misalignment conditions hold. For the purpose of rough estimates, this condition can be taken to be $\gamma_\tau \gg \gamma_{N \rightarrow 2}$, because $\vec{\epsilon}$ is washed out by $\gamma_{N \rightarrow 2}$, so the approximation that $\Delta y_\ell^j \propto \epsilon^j$ is only valid in a time-step shorter than the N_1 inverse decay time. The misalignment conditions are expected: if for instance N_1 decays only to τs ($\hat{\tau} \cdot \hat{\ell}_{N_1} = 1$), the single-flavour analysis is correct. The misalignment is probably more transparent in flavour space than in 4-vector space.

15.4 Summary

The charged lepton Yukawa couplings are relevant for leptogenesis, because they can determine the initial state of washout interactions, and washout is crucial for thermal leptogenesis.

In this appendix, we explicitly saw that the equations of motion of the ‘‘asymmetry number operator’’, which are covariant in flavour space, are projected onto the flavour basis by the interactions of the charged lepton Yukawa couplings. In this flavour basis, the equations for the diagonal elements of the asymmetry operator become the familiar Boltzmann Equations, in which the charged lepton Yukawa interactions do not appear. We expect this projection to occur if the charged lepton Yukawa interactions are fast compared to leptogenesis timescales. Indeed, the flavour off-diagonal elements are negligible in the equations when

$$\gamma_\tau \gg \gamma_{N \rightarrow 2} \quad . \quad (15.35)$$

Intuitively, this makes sense: if a lepton ℓ interacts many times via the h_τ Yukawa coupling before interacting with N_1 , it will participate in the washout process as a flavour eigenstate ($\hat{\ell}_\tau$ or $\hat{\ell}_o$).

The temperature ranges where flavour effects should be included can be estimated by comparing rates, rather than rate densities. The timescale for leptogenesis is H^{-1} , because the “non-equilibrium” is provided by the Universe expansion, so the Yukawa couplings can be neglected when they are out-of-equilibrium, that is, slower than H . Comparing H to the rates for h_τ - or h_μ -mediated interactions, such as $q_L \bar{t}_R \rightarrow \ell_\tau \bar{\tau}_R$, one finds

$$\Gamma_\alpha \simeq 10^{-2} h_\alpha^2 T > H \text{ for } T < \begin{cases} 10^{12} \text{ GeV} & \alpha = \tau \\ 10^9 \text{ GeV} & \alpha = \mu. \end{cases} \quad (15.36)$$

Below $T \sim 10^{12}$ GeV, h_τ is in equilibrium, and there can be two distinguishable flavours down to $T \sim 10^9$ GeV. Below $T \sim 10^9$ GeV, h_μ is also in equilibrium, and there can be three.

To impose the flavour basis in the washout processes, the charged lepton Yukawa interactions should also be faster than the inverse decays of N_1 . (Notice that the Boltzmann factor $e^{-M_1/T}$, heuristically included in Γ_{ID} , appears in the rate density $\gamma_{N \rightarrow 2}$ of eqn. (15.35).) We thus define a temperature T_{fl} , at which $\Gamma_\tau = \Gamma_{ID}$. Below T_{fl} , the lepton mass eigenstates are the flavour states. In flavoured leptogenesis, one can estimate a temperature T_B , after which the baryon asymmetry is generated. If $T_B < T_{\text{fl}}$, then it is consistent to calculate the baryon asymmetry with flavoured Boltzmann Equations.

With strong washout for all flavours, it is more convenient to keep track of the fraction of the N_1 population remaining, $\propto e^{-M_1/T}$. At T_{fl} , when the thermal mass eigenstates become the flavour states, this fraction is $\sim \Gamma_\tau/\Gamma_D$. The fraction remaining when a flavoured lepton asymmetry can survive is $\sim H/\Gamma_{\alpha\alpha}$, where $\Gamma_{\alpha\alpha} = \min\{\Gamma_{o\alpha}, \Gamma_{\tau\tau}\}$. (Γ_τ is the h_τ -mediated scattering rate, and $\Gamma_{\tau\tau} = \Gamma(N_1 \rightarrow \tau\phi)$.) So leptogenesis can be calculated using BE written the flavour basis, when

$$\frac{\Gamma_\tau}{\Gamma_D} \gg \frac{H}{\Gamma_{\alpha\alpha}}. \quad (15.37)$$

(Recall that we are discussing the case of $\Gamma_{\alpha\alpha} > H$.) For instance, for $M_1 \sim 10^{10}$ GeV, and $\tilde{m} \sim m_{\text{atm}}$, one finds that $\Gamma_\tau(M_1) \gtrsim \Gamma_D(M_1)$, so the baryon asymmetry is generated after T_{fl} (for strong washout in all flavours).

There are, however, regions of parameter space where $T_B > T_{\text{fl}}$ [181], such as the strong washout regime at large M_1 . In this case, the baryon asymmetry could be divided into three parts:

- i*) An unflavoured contribution from the decays taking place before $\Gamma_{ID} \sim \Gamma_\tau$. In strong washout and assuming $\Gamma_\tau > H$, these are negligible.
- ii*) A contribution from the transition region where $\Gamma_{ID} \sim \Gamma_\tau$, to be obtained by solving the equations of the covariant asymmetry density operator.
- iii*) A flavoured contribution from the decays taking place after $\Gamma_{ID} \sim \Gamma_\tau$.

In the limit where *ii*) is neglected, the final baryon asymmetry can be estimated analytically: It is the flavoured asymmetry that is produced in the decays of the N_1 's that remained at T_{fl} . See [180] for a detailed discussion based on solving the equations for the asymmetry density operator.

16 Appendix: Analytic Approximations to $Y_{\Delta B}$

In this Appendix, we reproduce analytic estimates of the baryon asymmetry, derived in ref. [64]. These are approximate solutions of the BE which take into account decays, inverse decays, and $\Delta L = 1$ scatterings with N_1 on an external leg. CP violation in all these processes is included. The off-diagonal elements of the A -matrix are neglected (they are included in Section 9.3), as is $\Delta L = 2$ scattering, so the equations for the $B/3 - L_\alpha$ asymmetries are decoupled. This simplifies the solutions. Ref. [64] estimates the overall uncertainty related to the various approximations to be of order 30%.

A detailed discussion of analytic approximations (as well as numerical evaluations) can be found in ref. [59], for single-flavour leptogenesis and without CP violation in $\Delta L = 1$ scatterings. We here modify the formulae of [59] to include flavour and CP violation in scattering, providing simple and useful approximations.

The Boltzmann equations can be written as follows:

$$\frac{d}{dz} Y_{N_1} = -S(z), \quad (16.1)$$

$$\frac{d}{dz} Y_{\Delta\alpha} = \epsilon_{\alpha\alpha} S(z) - W_{\alpha\alpha}(z) Y_{\Delta\alpha}, \quad (16.2)$$

where the source and washout functions can be read from eqns (6.54) and (6.56). Their functional form can be found in ref. [59], or, with finite temperature effects, in ref. [60].

As discussed earlier, the values of the light neutrino mass-squared differences suggest that $\tilde{m} > m_*$. In this strong washout regime, there is a useful approximation to $Y_{N_1}(z) - Y_{N_1}^{\text{eq}}(z)$ at large z (see chapter 6.4 of ref. [310]). Indeed, the asymmetry is produced when the inverse decays go out of equilibrium, which happens at $z \gg 1$. The relevant equation is then (6.16):

$$\frac{d}{dz} [Y_{N_1} - Y_{N_1}^{\text{eq}}] = -\frac{z}{sH_1} [Y_{N_1} - Y_{N_1}^{\text{eq}}] \frac{\gamma_{N \rightarrow 2}}{Y_{N_1}^{\text{eq}}} - \frac{d}{dz} Y_{N_1}^{\text{eq}}. \quad (16.3)$$

When the interaction rate $\gamma_{N \rightarrow 2}/(sY_{N_1}^{\text{eq}})$ is fast compared to H , the differential equation is “stiff” and can be solved approximately by setting the right-hand-side to zero, and taking $dY_{N_1}^{\text{eq}}/dz \simeq -Y_{N_1}^{\text{eq}}$:

$$[Y_{N_1} - Y_{N_1}^{\text{eq}}] \simeq \frac{sY_{N_1}^{\text{eq}} H_1}{\gamma_{N \rightarrow 2}} \frac{H_1}{z} Y_{N_1}^{\text{eq}} \simeq \frac{zK_2(z)H_1}{4g_*\Gamma_D}. \quad (16.4)$$

The last equality uses eqn (13.2) and, from eqn (13.17), $\gamma_{N \rightarrow 2}/(sY_{N_1}^{\text{eq}}) \simeq \Gamma_D$.

Using the trick of rewriting eqn (16.2) as $(fY)' = fS\epsilon_{\alpha\alpha}$, with $f' = W_{\alpha\alpha}f$, it has the formal solution

$$Y_{\Delta\alpha}(z) = \epsilon_{\alpha\alpha} \int_0^z dx S(x) e^{-\int_x^z dy W_{\alpha\alpha}(y)}. \quad (16.5)$$

We are interested in $Y_{\Delta\alpha}(z \rightarrow \infty)$. This can be approximated via the “steepest descent”, or “saddle-point” evaluation of an integral along the contour C [334]:

$$J(x) = \int_C e^{xF(t)} dt \simeq \sqrt{\frac{2\pi}{-xF''(t_0)}} e^{xF(t_0)}, \quad (16.6)$$

where t_0 is an extremum of the integrand. In the case of eqn (16.2), the extremum is at $W_{\alpha\alpha} = d/dz(\ln|S|)$. Using the saddle point approximation, and simple forms for the functions S and $W_{\alpha\alpha}$, ref. [64] obtained the following approximations for the efficiency parameter :

$$\eta_\alpha = \left[\left(\frac{|A_{\alpha\alpha}| \tilde{m}_{\alpha\alpha}}{2.1m_*} \right)^{-1} + \left(\frac{m_*}{2|A_{\alpha\alpha}| \tilde{m}_{\alpha\alpha}} \right)^{-1.16} \right]^{-1} \quad (\tilde{m} > m_*) \quad (16.7)$$

$$= \frac{|A_{\alpha\alpha}| \tilde{m}_{\alpha\alpha}}{2.5m_*} \frac{\tilde{m}}{m_*} \quad (\tilde{m} < m_*) \quad (16.8)$$

where m_* and $\tilde{m}_{\alpha\alpha}$ are defined in eqn (4.7), and the $A_{\alpha\alpha}$ are defined in eqn (9.18). The $B/3 - L_\alpha$ asymmetries are given by

$$Y_{\Delta\alpha} = 4 \times 10^{-3} \epsilon_{\alpha\alpha} \eta_\alpha \quad (16.9)$$

and finally the baryon asymmetry (in the SM) is

$$Y_{\Delta B} \simeq \frac{12}{37} \sum_{\alpha} Y_{\Delta\alpha}. \quad (16.10)$$

The temperature range where tau Yukawa interactions are faster than the expansion rate H is $T < (1 + \tan^2 \beta) \times 10^{12}$ GeV ($\tan \beta = 1$ in the SM). If the tau Yukawa interactions are also faster than Γ_{ID} when the asymmetries are generated (see Appendix 15.4), then there are two relevant CP asymmetries – $\epsilon_{oo} = \epsilon_{ee} + \epsilon_{\mu\mu}$ and $\epsilon_{\tau\tau}$ – and two relevant partial decay rates – $\tilde{m}_{oo} = \tilde{m}_{ee} + \tilde{m}_{\mu\mu}$ and $\tilde{m}_{\tau\tau}$. So the sum in eqn (16.10) is over $\alpha = o, \tau$ and one should use eqn (14.14) for the $A_{\alpha\alpha}$. For $T < (1 + \tan^2 \beta) \times 10^9$ GeV, the muon Yukawa interactions are also faster than H and there are three distinguishable flavours. The sum in eqn (16.10) is over $\alpha = e, \mu, \tau$, and one should use eqn (14.15) for the $A_{\alpha\alpha}$.

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