

Aspects of Excited Baryon Phenomenology in the $1/N_c$ expansion of QCD

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PLAN

- Introduction & motivation
- Baryons in the $1/N_c$ expansion of QCD
- Strong decays of excited baryons
- Photoproduction amplitudes of excited baryons
- Simultaneous analysis of masses and strong decays in $[1^-, 70]$ -plet
- Summary & conclusions

INTRODUCTION & MOTIVATION

Most of our understanding of the excited baryon sector is largely based on data analyzed by models, most prominently the constituent quark model, which relation with QCD is, at least, unclear.

Although there has been some progress in the study of the excited baryon using lattice QCD simulations this remains to be a very hard problem .

In this situation it is important to have a model independent approach to the physics of excited baryons.

One possible systematic approach of this type is the $1/N_c$ expansion of QCD.

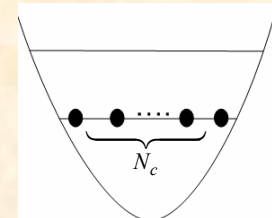
| | SU(6) irrep | SU(3) _f irrep | J ^P | S = 0 | S = - 1 | | S = - 2 | S = - 3 | |
|-----------------------|-----------------------------------|-----------------------------------|------------------------|-----------------------------------|--------------------|-----------|---------|---------|------------------|
| | | | | | I = 0 | I = 1 | | | |
| GS Baryons | 56 ⁺ (I=0) | ² 8 ⁴ 10 | 1/2 ⁺ | N(939) | Λ(1116) | Σ(1193) | Ξ(1318) | Ω(1672) | |
| | | | 3/2 ⁺ | Δ(1232) | | Σ(1385) | Ξ(1533) | | |
| Excited baryons | 70 ⁻ (I=1) | ² 8 | 3/2 ⁻ | N(1520) | Λ(1690) | Σ(1580)** | Ξ(1820) | | |
| | | | 1/2 ⁻ | N(1535) | Λ(1670) | Σ(1620)** | | | |
| | | | ⁴ 8 | 1/2 ⁻ | N(1650) | Λ(1800) | | | Σ(1750) |
| | | | | 5/2 ⁻ | N(1675) | Λ(1830) | | | Σ(1775) |
| | | | | 3/2 ⁻ | N(1700) | ? | | | Σ(1670) |
| | | ² 10 | 1/2 ⁻ | Δ(1620) | Λ(1405) Λ(1520) | | | | |
| | | | 3/2 ⁻ | Δ(1700) | | | | | |
| | | | ² 1 | 1/2 ⁻ | | | | | |
| | | | | 3/2 ⁻ | | | | | |
| | | | 56 ⁺ (I=0') | ² 8 ⁴ 10 | | | | | 1/2 ⁺ |
| 3/2 ⁺ | Δ(1600) | | | | Σ(1690)** | | | | |
| 56 ⁺ (I=2) | ² 8 ⁴ 10 | 3/2 ⁺ | N(1720) | Λ(1890) | ? | | | | |
| | | 5/2 ⁺ | N(1680) | Λ(1820) | Σ(1915) | | | | |
| | | 1/2 ⁺ | Δ(1910) | | Σ(2080)** | | | | |
| | | 3/2 ⁺ | Δ(1920) | | | | | | |
| | | 5/2 ⁺ | Δ(1905) | | | | | | |
| | | 7/2 ⁺ | Δ(1950) | | | | Σ(2030) | | |

BARYONS IN THE $1/N_c$ EXPANSION OF QCD

QCD has no obvious expansion parameter. However, t'Hooft ('74) realized that if one extends the QCD color group from $SU(3)$ to $SU(N_c)$, where N_c is an arbitrary (odd) large number, then $1/N_c$ may be treated as the relevant expansion parameter of QCD. To have consistent theory, QCD coupling constant $g^2 \sim 1/N_c$

For large N_c there are infinite mesons states, which are narrow and weakly coupled between each other. (t'Hooft '74)

Witten ('79) observed that baryons are formed by N_c "valence" quarks with $M_B \sim O(N_c^1)$ and $r_B \sim O(N_c^0)$. *In the Large N_c limit a Hartree picture of baryons emerges: each quark moves in a self-consistent effective potential generated by the rest of the (N_c-1) quarks.*



GS
Baryon

Moreover, in order to preserve unitarity for Large N_c , a dynamical spin-flavor arises in the baryon sector [[Gervais-Sakita '84](#); [Dashen-Jenkins-Manohar '93](#)]

$$S^i, T^a, X^{ia} = \frac{G_{ia}}{N_c} \quad \text{form contracted } \text{SU}(2 N_f) \text{ algebra}$$

Here, G_{ia} spin-flavor operator. E.g. in the quark representation $G_{ia} = \sum_j q_j^\dagger \sigma_i \tau_a q_j$

To derive a $1/N_c$ expansion of baryonic observables one can make use of the contracted algebra (“consistency relation method”).

Alternatively, one can use the usual $\text{SU}(2 N_f)$ algebra for large N_c . In this scheme (so-called “operator method”) GS baryons are taken to fill $\text{SU}(2 N_f)$ completely symmetric irrep.

Quark operator method: Any color singlet QCD operator can be represented at the level of effective theory by a series of composite operators ordered in powers of $1/N_c$

$$O_{eff} = \sum_{n,i} c_i^{(n)} \Phi_i^{(n)}$$

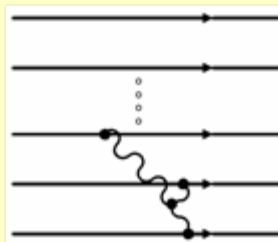
n -body operator obtained as the product of n generators of $SU(2 N_f)$, i.e. S^i, T^a, G^{ia}

Unknown coefficients to be fitted

Rules for N_c counting

- n -body operators need at least n quarks exchanging $(n-1)$ gluons according to usual rules it carries a suppression factor $(1/N_c)^{n-1}$

e.g. 3-body operator



$$g^4 \sim N_c^{-2}$$

- Some operators may act as coherent \rightarrow matrix elements of order N_c .

e.g. $G^{ia} \sim N_c$

As example we construct the operator for the GS baryon masses. Up to $1/N_c$ contributions, we get

$$M = c_1 N_c + c_{2,1} \frac{S^2}{N_c} + c_{2,2} \frac{T^2}{N_c} + c_{2,3} \frac{G^2}{N_c} + c_{3,1} \frac{S_i \{T_a, G_{ia}\}}{N_c^2}$$

It looks like there are many contributions up to this order. However, since for GS baryons operators are acting on fully symmetric irrep of SU(6), several reduction formulae appear (Dashen, Jenkins, Manohar, PRD49(94)4713; D 51(95), 3697).

$$2S^2 + 3T^2 + 12G^2 = \frac{5}{2} N_c (N_c + 6)$$

$$\{T_a, G_{ia}\} = \frac{2}{3} (N_c + 3) S_i$$

$$32S^2 - 27T^2 + 12G^2 = 0$$

Using these relations M simplifies to

$$M = c_1 N_c + c_2 \frac{S^2}{N_c}$$

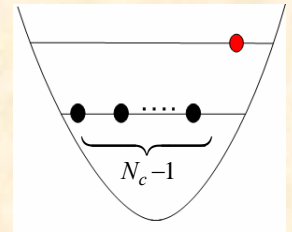
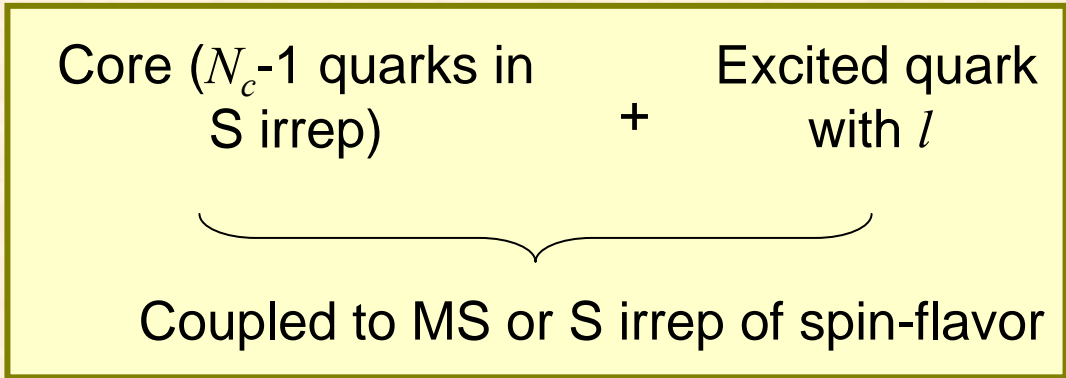
SU(3) flavor breaking can be also included in the analysis. One gets parameter free testable relations. Fitting empirical values one obtains predictions of c 's coefficients.

This type of analysis has been applied to study axial couplings, magnetic moments, etc. (Dai, Dashen, Jenkins, Manohar, PRD53(96)273, Carone, Georgi, Osofsky, PLB322(94)227, Luty, March-Russell, NPB426(94)71, etc).

Carone et al, PRD50(94)5793, Goity, PLB414(97)140; Pirjol and Yan, PRD57(98)1449; 5434 proposed to extend these ideas to analyze low lying excited baryons properties.

Take as convenient basis of states: multiplets of $O(3) \times SU(2N_f)$ (approximation since they might contain several irreps of $SU(2N_f)_c$ Schat, Pirjol, PRD67(03)096009, Cohen, Lebed, PLB619(05)115)

Excited baryon composed by



Low lying Excited Baryon

For lowest states relevant multiplets are $[1^-, 70]$, $[0^+, 56']$, $[2^+, 56]$...

In the operator analysis, effective n -body operators are now

$O^{(n)} = R \otimes \Phi^{(n)}$

where R is an $O(3)$ operator and $\Phi^{(n)}$ an $SU(2N_f)$ tensor

$$\Phi^{(n)} = \frac{1}{N_c^{n-1}} \lambda \otimes \underbrace{\Lambda_c \otimes \dots \otimes \Lambda_c}_{n-1} \quad \text{where} \quad \begin{cases} \lambda = t^a, s^i, g^{ia} \\ \Lambda_c = T_c^a, S_c^i, G_c^{ia} \end{cases}$$

This approach has been applied to analyze the excited baryon masses
Carlson, Carone, Goity and Lebed, PLB438 (98) 327; PRD59 (99) 114008.
Schat, Goity and NNS, PRL88(02)102002, PRD66 (02) 114014, PLB564 (03) 83
Matagne, Stancu, PRD71(05)014010

Remarks on [1,70]-plet masses

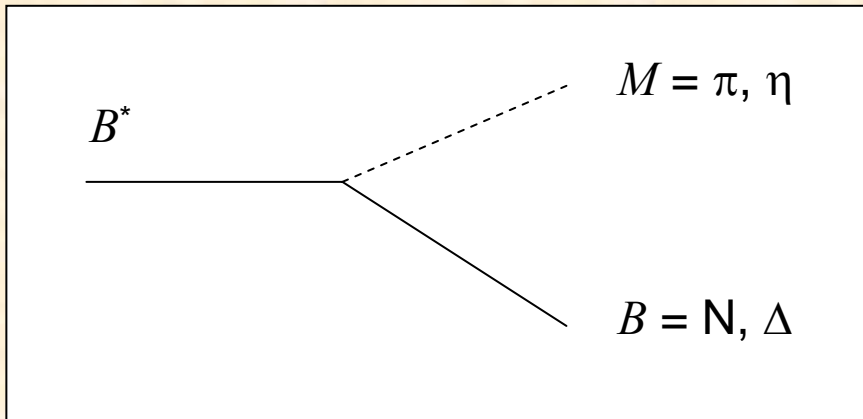
- Although $O(3) \times SU(6)$ symmetry is broken at $O(N_c^0)$, the corresponding operators (e.g. spin-orbit) have small coefficients.
- Hyperfine operator $S_c S_c / N_c$ of $O(N_c^{-1})$ give chief spin-flavor breaking.
- $\Lambda(1520) - \Lambda(1405)$ determined only by spin-orbit operator. For the rest of spin-orbit partners other operators appear. Important is $l^i t^a G_c^{ia} / N_c$

Remarks on [2⁺,56]-plet masses

- $O(3) \times SU(6)$ symmetry only broken at $O(1/N_c)$. Hyperfine operator $S_c S_c / N_c$ gives the most important contribution.
- Testable model independent relation well satisfied.

STRONG DECAYS

We consider decays of the type



where B^* are non-strange members of the $[1^-, 70]$, $[0^+, 56']$ and $[2^+, 56]$ multiplets.

For $[1^-, 70]$ partial waves are $l_M = S, D$

For $[0^+, 56]$ partial waves is $l_M = P$

For $[2^+, 56]$ partial waves are $l_M = P, F$

The decay widths are given by

$$\Gamma^{[l_M, i_M]} = \frac{k_M^{2l_M+1}}{8\pi^2 \Lambda^{2l_M}} \frac{M_B}{M_{B^*}} \left| \frac{\sum_n C_n^{[l_M, i_M]} \langle J, I \| (B^{[l_M, i_M]})_n \| J^*, I^*, S^* \rangle}{(2I^* + 1)(2J^* + 1)} \right|^2$$

where

$$B^{[l_M, i_M]} = \left(\xi^{(l)} \Phi^{[l, i_M]} \right)^{[l_M, i_M]}$$

acts orbital wf of excited quark

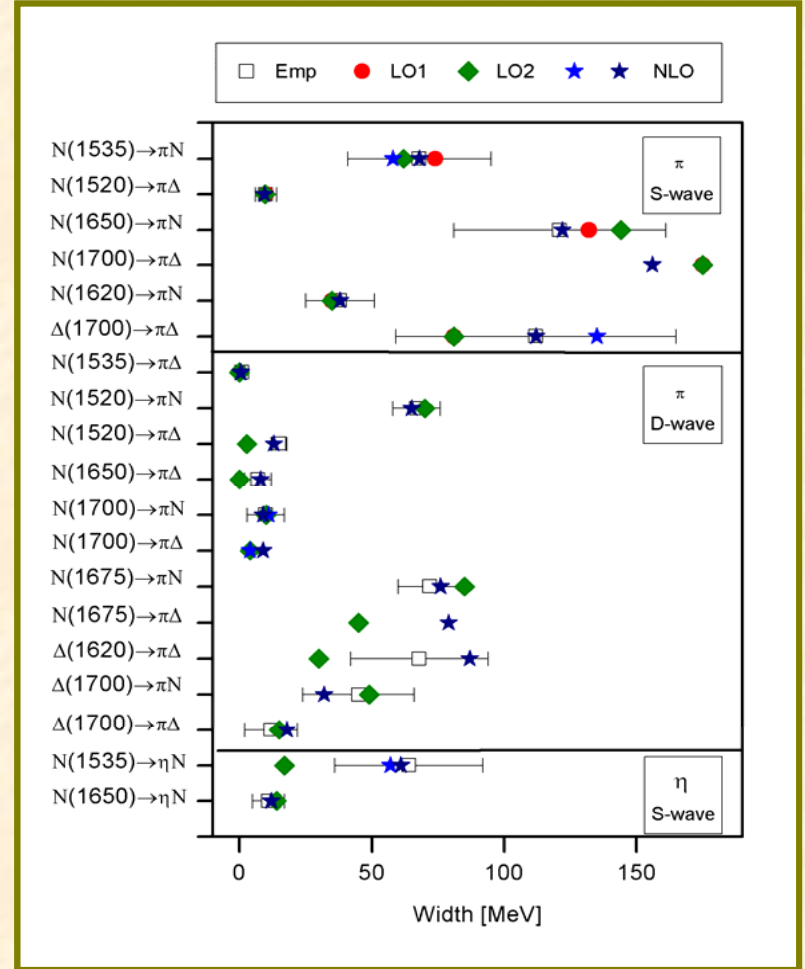
acts of spin-flavor wf of the excited quark – core system

Analysis of strong decays of non-strange ($1^-, 70$) resonances

Goity, Schat, NNS, PRD71(05)034016

Basis operators and fit parameters for the decay of non-strange baryons of the ($1^-, 70$)-plet. Errors in parenthesis. Square brackets imply that two solutions with very similar χ^2 exist.

| Operator | | #1 LO | #2 LO | #3 NLO |
|-----------------------|--|--------------------------|--------------------------|----------------------|
| $(\xi g)^{[0,1]}$ | | 31(3) | 31(3) | 23(3) |
| Pion | $\frac{1}{N_c} \left(\xi (s T_c)^{[1,1]} \right)^{[0,1]}$ | - | - | [7, 32]([30, 40]) |
| S wave | $\frac{1}{N_c} \left(\xi (t S_c)^{[1,1]} \right)^{[0,1]}$ | - | - | [21, 27](15) |
| | $\frac{1}{N_c} \left(\xi (g S_c)^{[1,1]} \right)^{[0,1]}$ | - | - | [-26, -67]([40, 65]) |
| $(\xi g)^{[2,1]}$ | | - | 4.6(0.5) | 3.4(0.3) |
| Pion | $\frac{1}{N_c} \left(\xi (s T_c)^{[1,1]} \right)^{[2,1]}$ | - | - | -4.5(2.4) |
| | $\frac{1}{N_c} \left(\xi (t S_c)^{[1,1]} \right)^{[2,1]}$ | - | - | [-0.01, 0.08](2) |
| | $\frac{1}{N_c} \left(\xi (g S_c)^{[1,1]} \right)^{[2,1]}$ | - | - | 5.7(4) |
| | $\frac{1}{N_c} \left(\xi (g S_c)^{[2,1]} \right)^{[2,1]}$ | - | - | 3.0(2.2) |
| Eta | $\frac{1}{N_c} \left(\xi (s G_c)^{[2,1]} \right)^{[2,1]}$ | - | [-1.86, -2.25](0.4) | -1.73(0.26) |
| | $(\xi s)^{[0,0]}$ | - | 11(4) | 17(4) |
| S wave | $\frac{1}{N_c} \left(\xi (s S_c)^{[1,0]} \right)^{[0,0]}$ | - | - | - |
| θ_1 | | 1.62(0.12) 0.29(0.11) | 1.56(0.15) 0.35(0.14) | 0.39(0.11) |
| θ_3 | | 3.01(0.07) 2.44(0.06) | [3.00, 2.44](0.07) | [2.82, 2.38](0.11) |
| χ^2_{dof} | | 0.25 2 | 1.5 10 | 0.9 3 |



- Clear dominance of 1B LO operator g^{ia} in π -decays as in χ QM

- NLO coefficients poorly determined due to large data error bars. More precise inputs needed to determine significance of NLO corrections

- $N^*(1535)$ ratio of decays to $N\eta/N\pi$ well reproduced.

Strong decays of positive parity resonances can be analyzed in a similar fashion
Goity, NNS, PRD71(05)034016, Goity, Jayalath, NNS, PRD80(09)074027

Remarks on strong decays of $[0^+, 56']$ states

- LO analysis rather poor.
- NLO essential for improving the ratio of the two $N(1440)$ decays. Also to get the right ordering of $\Delta(1660)$ decays.

Remarks on strong decays of $[2^+, 56]$ states

P wave decays

- LO analysis already gives excellent fit.
- In the NLO fit coefficients of 2B operators are compatible with zero.

F wave decays

- LO give a reasonable fit provided empirical errors taken at 30%.
- If in NLO fit errors taken as in PDG fit is quite poor. Main problem seems to be $\pi\Delta$ decay of $N(1680)$ for which only upper bound is given in PDG. If removed from the fit χ^2/dof quite small

PHOTOPRODUCTION AMPLITUDES

The helicity amplitudes of interest are defined as

$$A_\lambda = -\sqrt{\frac{2\pi\alpha}{\omega_\gamma}} \eta(B^*) \langle B^*, \lambda | \hat{e}_{+1} \cdot \vec{J}(\omega_\gamma \hat{z}) | N, \lambda - 1 \rangle$$

- $\lambda=1/2, 3/2$ is helicity along γ -momentum (z-axis)
- \hat{e}_{+1} is γ -polarization vector

$\eta(B^*)$ sign factor which depends on sign of strong amplitude $\pi N \rightarrow B^*$. When B^* can decay through 2 partial waves (e.g. S or D) \rightarrow undetermined sign ($\xi=S/D=\pm 1$)

$\vec{J}(\omega_\gamma \hat{z})$ can be represented in terms of effective multipole baryonic operators with isospin $I=0,1$. Then, the electric and magnetic components of the helicity amplitudes can be expressed as

$$A_\lambda^{ML} = \eta(B^*) \sqrt{\frac{3\alpha N_c}{4\omega_\gamma}} \left(\frac{\omega_\gamma}{m_\rho}\right)^L \sum_{n,I} g_n^{[L,I]} \langle B^*; [\lambda, I_3] | (B_n)_{[10]}^{[LI]} | N; [\lambda - 1, I_3] \rangle$$

$$A_\lambda^{EL} = \eta(B^*) \sqrt{\frac{3(L+1)\alpha N_c}{4(2L+1)\omega_\gamma}} \left(\frac{\omega_\gamma}{m_\rho}\right)^{L-1} \sum_{n,I} g_n^{[L,I]} \langle B^*; [\lambda, I_3] | (B_n)_{[10]}^{[LI]} | N; [\lambda - 1, I_3] \rangle$$

where

$$B^{[L,I]} = \left(\xi^{(I)} \Phi^{[I,I]} \right)^{[L,I]}$$

Acts orbital wf of excited q

Acts of spin-flavor wf of the excited quark – core system

Helicity amplitudes for $[1^-, 70]$ -plet non-strange resonances

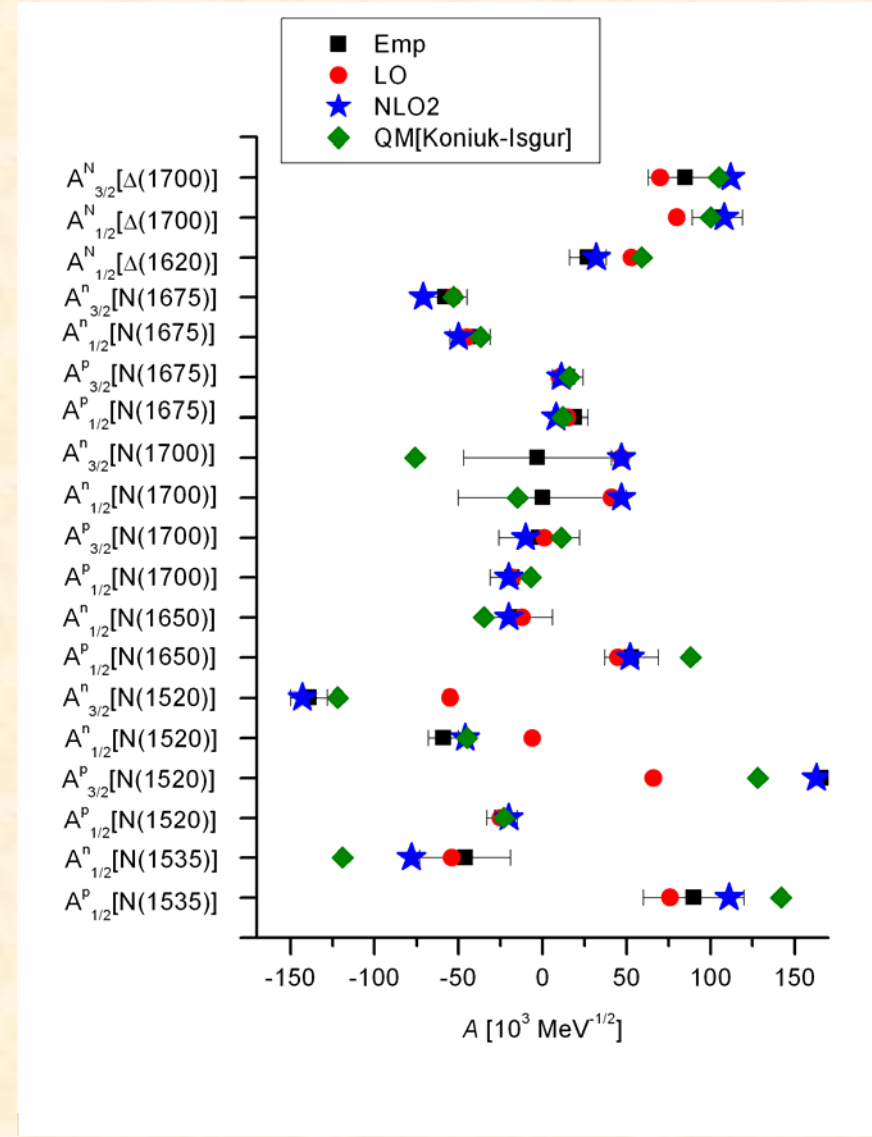
Goity, Matagne, NNS, Phys. Lett. B663 (08) 222

$\xi = -1$ and $\theta_3=2.82$ clearly favored by fits. Only this case is shown.

Basis operators and fit parameters of non-strange $[1^-, 70]$ baryons.
Errors are indicated in parenthesis.

| Operator | LO | NLO1 | NLO2 |
|--|-----------|-----------|-----------|
| $E1_1^S = (\xi^{[1,0]} s)^{[1,0]}$ | -0.4(0.2) | -0.3(0.2) | -0.3(0.2) |
| $E1_2^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[0,0]})^{[1,0]}$ | | 0.5(0.6) | |
| $E1_3^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[1,0]})^{[1,0]}$ | | 1.0(0.9) | |
| $E1_4^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[2,0]})^{[1,0]}$ | | 0.5(0.6) | |
| $E1_1^V = (\xi^{[1,0]} t)^{[1,1]}$ | 2.3(0.3) | 3.0(0.2) | 3.5(0.1) |
| $E1_2^V = (\xi^{[1,0]} g)^{[1,1]}$ | -0.7(0.4) | 0.4(0.3) | |
| $E1_3^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[2,1]})^{[1,1]}$ | 0.4(0.5) | -0.2(0.4) | |
| $E1_4^V = \frac{1}{N_c} (\xi^{[1,0]} (s T_c)^{[1,1]})^{[1,1]}$ | | -1.9(1.4) | |
| $E1_5^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[0,1]})^{[1,1]}$ $+ \frac{1}{4\sqrt{3}} E1_1^V$ | | -0.2(0.9) | |
| $E1_6^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[1,1]})^{[1,1]}$ $+ \frac{1}{2\sqrt{2}} E1_2^V$ | | 4.2(0.9) | 3.9(0.8) |
| $M2_1^S = (\xi^{[1,0]} s)^{[2,0]}$ | 0.8(0.2) | 1.5(0.3) | 1.3(0.2) |
| $M2_2^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[1,0]})^{[2,0]}$ | | -1.2(1.3) | |
| $M2_3^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[2,0]})^{[2,0]}$ | | -1.2(1.7) | |
| $M2_1^V = (\xi^{[1,0]} g)^{[2,1]}$ | 3.0(0.6) | 3.8(0.6) | 3.9(0.4) |
| $M2_2^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[2,1]})^{[2,1]}$ | -3.1(1.0) | -2.3(1.1) | -2.7(0.6) |
| $M2_3^V = \frac{1}{N_c} (\xi^{[1,0]} (s T_c)^{[1,1]})^{[2,1]}$ | | -0.1(1.1) | |
| $M2_4^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[1,1]})^{[2,1]}$ $+ \frac{1}{2\sqrt{2}} M2_1^V$ | | -1.5(2.4) | |
| $E3_1^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[2,0]})^{[3,0]}$ | | 0.3(0.8) | |
| $E3_1^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[2,1]})^{[3,1]}$ | 0.7(0.9) | 0.3(0.5) | |
| χ_{dof}^2 | 2.42 | - | 0.94 |
| dof | 11 | 0 | 13 |

2B



Helicity amplitudes of positive parity resonances can be analyzed in a similar way
Goity, NNS, PRL99(07)062002

Remarks on helicity amplitudes of $[0^+, 56']$ states

- To LO only M1 operator $G^{[1,1]}$ contributes leading to $\chi^2/\text{dof} \sim 2$.
- Inclusion of NLO 1B isosinglet M1 operator $S^{[1,0]}$ improves N(1440) amplitudes but $A^{3/2}_N[\Delta(1660)]$ still too large.
- 2B isovector M2 operator $[S, G]^{[2,1]}$ (usually not included in QM) required to get good overall agreement

Remarks on helicity amplitudes of $[2^+, 56]$ states

- To LO only 1B operators contribute: 1 isosinglet E2 and 3 isovector M1, E2, M3. The M3 operator $(\xi^{(2)} G)^{[3,1]}$ dominates. Fit not so good. $\chi^2/\text{dof} \sim 2.1$. Main reason is that large amplitude $A^{3/2}_N[\Delta(1680)]$ is badly underestimated.
- NLO fit favors $\xi' = \text{sign}(P/F) = -1$.
- A minimum number of 6 operators, 3 of which are LO (i.e. 1B isovector M1 is irrelevant), is needed to get $\chi^2/\text{dof} \sim 1$. One of the required NLO is 2B \rightarrow not included in QM.

SIMULTANEOUS ANALYSIS OF MASSES AND STRONG DECAYS OF $[1^-, 70]$ -PLET BARYONS

E. de Urreta and NNS, in preparation

Let us remind that in $[1^-, 70]$ -plet one can define the mixing angles (θ_1, θ_3) such that

$$\begin{pmatrix} N_J \\ N'_J \end{pmatrix} = \begin{pmatrix} \cos \theta_{2J} & \sin \theta_{2J} \\ -\sin \theta_{2J} & \cos \theta_{2J} \end{pmatrix} \begin{pmatrix} {}^2N_J \\ {}^4N_J \end{pmatrix} \quad \text{with } J = 1/2, 3/2$$

In the previously mentioned mass analysis mixing angles taken to be fixed at “standard values” $(\theta_1, \theta_3) = (0.61, 3.04)$

Alternatively, one make a fit in which they are taken as free parameters. This have been done in for $N_f = 2$ (Carlson et al, PRD59 (99) 114008) obtaining
 $\theta_1 = 0.55 \pm 0.37$; $\theta_3 = 3.00 \pm 0.21$

This has to be compared with e.g. the result obtained in the strong decay analysis
 $\theta_1 = 0.39 \pm 0.11$; $\theta_3 = (2.38, 2.82) \pm 0.11$.

It is thus important to see whether a simultaneous fit of both observables is possible.

We find that indeed a good fit is possible ($\chi^2/dof = 0.89$) leading to

$$\theta_1 = 0.47 \pm 0.06 ; \theta_3 = 2.74 \pm 0.07$$

- “Large” θ_3 is favored
- Smaller errors

SUMMARY & CONCLUSIONS

- The operator method for carrying the $1/N_c$ expansion has been shown to work for GS baryons and it seems to also work for excited baryons. The analyses of masses show small order N_c^0 breaking of spin-flavor symmetry. This is dominated by the subleading hyperfine interaction.
- For strong decays, in general, dominance of 1B LO operators. In some cases, as e.g. D wave decays of negative parity excited baryons, $1/N_c$ corrections not well established due to uncertainties in empirical partial widths. Roper baryons seem to require 2B operators.
- In the case of photoproduction amplitudes only a reduced number of the operators in the NLO basis turns out to be relevant. Some of these operators can be identified with those used in QM calculations. However, there are also 2B operators (not included in QM calculations) which are needed for an accurate description of the empirical helicity amplitudes.
- A simultaneous analysis of masses and strong decays of $[1^-, 70]$ –plet baryons is possible, leading to smaller uncertainties in mixing angles and removing existing ambiguities in analysis of strong decays alone.