3-Loop Heavy Flavor Corrections to DIS with two Massive Fermion Lines

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Abstract. We report on recent results obtained for the massive operator matrix elements which contribute to the massive Wilson coefficients in deep-inelastic scattering for $Q^2 \gg m_i^2$ in case of sub-processes with two fermion lines and different mass assignment.

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INTRODUCTION

To determine the value of the strong coupling constant $\alpha_s(M_Z^2)$ from the world deep-inelastic scattering data [1] precisely, the structure function $F_2(x, Q^2)$ has to be described to $O(\alpha_s^3)$ accuracy at present. The corresponding next-to-next-to-leading order (NNLO) parton distributions are instrumental for accurate predictions of the weak boson and Higgs boson production cross sections at hadron colliders [2]. While the massless contributions are known to NNLO [3], in the massive case this has been achieved for a series of Mellin moments N only in the region $Q^2 \gtrsim 10 m_Q^2$ [4] [1], with m_Q the mass of the heavy quark. Based on the 2-loop massive operator matrix elements, which are known for general values of N [7] to the linear term in the dimensional parameter ε [8], and the massless 3-loop Wilson coefficients [9] one may compute all logarithmic contributions at 3–loop order [10]. In the kinematic region of HERA, however, these terms alone are not dominant over the yet unknown constant term for general values of N, as has been demonstrated for the moments calculated in [4]. The $O(\alpha_s^3)$ corrections to the structure function $F_L(x, Q^2)$ in the asymptotic region were calculated in [11, 4]. Heavy flavor corrections for charged current reactions were given in [12].

The heavy flavor contributions to the structure function $F_2(x, Q^2)$ at $O(\alpha_s^3)$ in case of one massive quark are described by the five massive Wilson coefficients in the asymptotic region $L_q^{\text{NS}}, L_q^{\text{PS}}, L_g^{\text{S}}, H_q^{\text{PS}}, H_g^{\text{S}}$ [4]. Two of these Wilson coefficients, L_q^{PS} and L_g^{S}, have been computed completely for general values of N in [13], cf. also [10]. In [13] the contributions to the color factors $O(T_F^2 N_f C_{A,F})$ of the Wilson coefficients $L_q^{\text{NS}}, H_q^{\text{PS}}, H_g^{\text{S}}$ were also calculated. The corresponding Feynman diagrams consist of graphs with two internal fermion lines, out of which one is massless and one massive. After applying algebraic relations [14] these contributions to the massive Wilson coefficients can be represented in terms of the known set of weight $\mathsf{w} = 4$ harmonic sums [15].

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[1] The $O(\alpha_s^2)$ corrections were given in [5]; see also the numerical implementation in Mellin space [6].

Related cases concern the $O(\alpha_s^3)$ contributions with two massive lines $\propto T_F^2 C_{F,A}$ with either equal or different masses. In this note we present first results for these contributions.

TWO MASSIVE QUARKS OF EQUAL MASS

We first consider the case of two quarks of equal mass in the $O(\alpha_s^3)$ operator matrix elements $\propto T_F^2$. In calculating the corresponding Feynman diagrams we apply the Mellin-Barnes representation [16] through which the Feynman parameter integrals can be expressed in terms of Meijer G-functions [17] in general. In these calculations, like those in [13], the use of modern summation algorithms as encoded in Sigma [18] are of essential importance. For the quarkonic flavor non-singlet case one obtains for the constant contribution to the operator matrix element :

$$
\hat{a}_{qq,Q}^{(3),NS}(N) = T_F^2 C_F \left\{ \frac{128}{27} S_4 - \frac{1024}{27} \zeta_3 S_1 + \frac{64}{9} \zeta_2 S_2 + \frac{256 \left(3N^2 + 3N + 2\right)}{27N(N+1)} \zeta_3 - \frac{320}{27} \zeta_2 S_1 - \frac{640}{81} S_3 \right.
$$
$$
\left. + \frac{8 \left(3N^4 + 6N^3 + 47N^2 + 20N - 12\right)}{27N^2(N+1)^2} \zeta_2 + \frac{1856}{81} S_2 - \frac{19424}{729} S_1 - \frac{4P_1(N)}{729N^4(N+1)^4} \right\}. \quad (1)
$$

The results in the flavor pure-singlet case are :

$$
\hat{a}_{Qq}^{(3),\text{PS}}(N) = \frac{T_F^2 C_F}{(N-1)N^2(N+1)^2(N+2)} \left\{ (N^2 + N + 2)^2 \left[\frac{32}{27} S_1^3 - \frac{512}{27} S_3 + \frac{128}{3} S_{2,1} - \frac{1024}{9} \zeta_3 \right. \right.
$$
$$
\left. - \frac{160}{9} S_2 S_1 + \frac{32}{3} \zeta_2 S_1 \right] - \frac{32 P_3(N) \zeta_2}{9N(N+2)} + \frac{32 P_4(N) S_2}{27N(N+2)(N+3)(N+4)(N+5)}
$$
$$
- \frac{32 P_5(N) S_1^2}{27N(N+1)(N+2)(N+3)(N+4)(N+5)} + \frac{64 P_6(N) S_1}{81N^2(N+1)^2(N+2)^2(N+3)(N+4)(N+5)}
$$
$$
\left. - \frac{64 P_7(N) S_1}{243N^3(N+1)^2(N+2)^3(N+3)(N+4)(N+5)} \right\}. \quad (2)
$$

Here $P_k(N)$ denote certain polynomials in N and $S_{\vec{a}} \equiv S_{\vec{a}}(N)$ are harmonic sums [19]. These relations generalize the moments being obtained previously in [4]. The contributions to the anomalous dimensions given in [3, 20] are confirmed.

TWO MASSIVE QUARKS OF UNEQUAL MASS

Beginning with $O(\alpha_s^3)$ graphs with internal massive fermion lines carrying unequal masses contribute. Since the mass ratio in case of the charm and bottom quarks is given by $x = m_c^2/m_b^2 \simeq 1/10$, one may expand the corresponding diagrams using this parameter. We first calculated the 2nd and 4th moment of the gluonic operator matrix element A_{Qg} extending the code qexp [21] to higher moments applying projectors

similar to those used in [4]. The unrenormalized operator matrix elements are given by :

$$\hat{A}_{Qg}^{(3)}(N=2) =$$

$$T_F^2 C_A \left\{ \frac{1}{\varepsilon^3} \frac{8960}{81} + \frac{1}{\varepsilon^2} \left[-\frac{1184}{243} + \frac{2240}{27} \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) \right] + \frac{1}{\varepsilon} \left[\frac{35800}{729} + \frac{1120}{27} \zeta_2 \right. \right.$$

$$+ \frac{1168}{27} \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) - \frac{296}{81} \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) + \frac{1024}{27} \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) \right]$$

$$+ \frac{156458}{2187} - \frac{1696}{81} \zeta_3 - \frac{148}{81} \zeta_2 + \frac{512608}{10125} x + \frac{3130072}{496125} x^2 + \frac{112173472}{843908625} x^3 + \frac{18710}{243} \ln\left(\frac{m_1^2}{\mu^2}\right)$$

$$- \frac{10}{3} \ln\left(\frac{m_2^2}{\mu^2}\right) + \frac{280}{9} \zeta_2 \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) + \left[\frac{14368}{675} x + \frac{12016}{4725} x^2 - \frac{328928}{2679075} x^3 \right] \ln(x)$$

$$+ \left[-\frac{70}{81} - \frac{16}{45} x - \frac{16}{45} x^2 - \frac{5104}{8505} x^3 \right] \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) + \left[-\frac{304}{81} + \frac{32}{45} x + \frac{32}{45} x^2 + \frac{10208}{8505} x^3 \right]$$

$$\times \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) + \frac{208}{27} \ln^2\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) + \frac{560}{27} \ln\left(\frac{m_1^2}{\mu^2}\right) \ln^2\left(\frac{m_2^2}{\mu^2}\right) + \frac{1544}{81} \ln^3\left(\frac{m_1^2}{\mu^2}\right)$$

$$\left. + \frac{1192}{81} \ln^3\left(\frac{m_2^2}{\mu^2}\right) \right\} + T_F^2 C_F \left\{ -\frac{1}{\varepsilon^3} \frac{4096}{81} + \frac{1}{\varepsilon^2} \left[\frac{5120}{81} - \frac{1024}{27} \left(\ln\left(\frac{m_2^2}{\mu^2}\right) + \ln\left(\frac{m_1^2}{\mu^2}\right) \right) \right] \right.$$

$$+ \frac{1}{\varepsilon} \left[\frac{28544}{729} - \frac{512}{27} \zeta_2 - \frac{256}{9} \left(\ln^2\left(\frac{m_2^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) + \frac{1280}{27} \left(\ln\left(\frac{m_2^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) \right] + \frac{128}{243}$$

$$+ \frac{3584}{81} \zeta_3 + \frac{640}{27} \zeta_2 - \frac{1517888}{30375} x + \frac{339785728}{10418625} x^2 + \frac{1653611968}{843908625} x^3 + \frac{256}{81} \ln\left(\frac{m_1^2}{\mu^2}\right) + \frac{13504}{243} \ln\left(\frac{m_2^2}{\mu^2}\right)$$

$$- \frac{128}{9} \zeta_2 \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) + \left[-\frac{45952}{2025} x - \frac{2056384}{99225} x^2 - \frac{11786368}{2679075} x^3 \right] \ln(x) + \left[\frac{1936}{81} \right.$$

$$+ \frac{896}{135} x + \frac{9536}{945} x^2 + \frac{47744}{8505} x^3 \right] \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) - \frac{256}{27} \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) \ln(x)$$

$$\left. + \left[\frac{1888}{81} - \frac{1792}{135} x - \frac{19072}{945} x^2 - \frac{95488}{8505} x^3 \right] \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) - \frac{1408}{81} \ln^3\left(\frac{m_1^2}{\mu^2}\right) - \frac{896}{81} \ln^3\left(\frac{m_2^2}{\mu^2}\right) \right\}$$

$$+ O(x^4 \ln(x)) , \tag{3}$$

where $m_1 < m_2, m_2^2 \ll \mu^2$. The 4th moment reads :

$$\hat{A}_{Qg}^{(3)}(N=4) =$$

$$T_F^2 C_A \left\{ \frac{1}{\varepsilon^3} \frac{287408}{2025} + \frac{1}{\varepsilon^2} \left[\frac{11614}{135} + \frac{71852}{675} \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_1^2}{\mu^2}\right) \right) \right] + \frac{1}{\varepsilon} \left[\frac{264315863}{1822500} + \frac{35926}{675} \zeta_2 \right. \right.$$

$$+ \frac{12287}{225} \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) + \frac{5807}{90} \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_1^2}{\mu^2}\right) \right) + \frac{3784}{75} \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) \right]$$

$$+ \frac{4887988511}{24300000} - \frac{47146}{2025} \zeta_3 + \frac{5807}{180} \zeta_2 + \frac{496855133}{7441875} x + \frac{2510388298}{468838125} x^2 + \frac{250077164867}{5616211899375} x^3$$

$$+ \frac{384762007}{2430000} \ln\left(\frac{m_1^2}{\mu^2}\right) + \frac{47956573}{810000} \ln\left(\frac{m_2^2}{\mu^2}\right) + \frac{17963}{450} \zeta_2 \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) + \left[\frac{1877399}{70875} x \right.$$

$$\left. + \frac{3269548}{1488375} x^2 - \frac{156082853}{1620840375} x^3 \right] \ln(x) + \left[\frac{532373}{16200} + \frac{707}{1350} x - \frac{284}{525} x^2 - \frac{744283}{935550} x^3 \right] \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) \right.$$

$$\left. + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) + \left[\frac{62893}{2025} - \frac{707}{675} x + \frac{568}{525} x^2 + \frac{744283}{467775} x^3 \right] \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) + \frac{7579}{675} \ln^2\left(\frac{m_1^2}{\mu^2}\right)$$

$$\times \ln\left(\frac{m_2^2}{\mu^2}\right) + \frac{17963}{675} \ln\left(\frac{m_1^2}{\mu^2}\right) \ln^2\left(\frac{m_2^2}{\mu^2}\right) + \frac{3817}{162} \ln^3\left(\frac{m_1^2}{\mu^2}\right) + \frac{74657}{4050} \ln^3\left(\frac{m_2^2}{\mu^2}\right) \Bigg\}$$

$$+ T_F^2 C_F \left\{ -\frac{1}{\varepsilon^3} \frac{297616}{10125} + \frac{1}{\varepsilon^2} \left[\frac{2947282}{151875} - \frac{74404}{3375} \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) \right] + \frac{1}{\varepsilon} \left[\frac{21662237}{9112500} \right. \right.$$

$$\left. - \frac{37202}{3375} \zeta_2 - \frac{18601}{1125} \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) + \frac{1473641}{101250} \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) \right]$$

$$- \frac{33406758667}{1093500000} + \frac{260414}{10125} \zeta_3 + \frac{1473641}{202500} \zeta_2 - \frac{119314474}{4134375} x + \frac{582667691}{37507050} x^2 + \frac{46049137562}{44929695195} x^3$$

$$- \frac{37307959}{2430000} \ln\left(\frac{m_1^2}{\mu^2}\right) + \frac{76621423}{4050000} \ln\left(\frac{m_2^2}{\mu^2}\right) - \frac{18601}{2250} \zeta_2 \left(\ln\left(\frac{m_1^2}{\mu^2}\right) + \ln\left(\frac{m_2^2}{\mu^2}\right) \right) + \left[\frac{368428}{39375} x \right.$$

$$\left. + \frac{2876423}{297675} x^2 + \frac{570093292}{324168075} x^3 \right] \ln(x) + \left[\frac{530371}{81000} + \frac{4456}{1125} x + \frac{27101}{4725} x^2 + \frac{1759616}{467775} x^3 \right]$$

$$\times \left(\ln^2\left(\frac{m_1^2}{\mu^2}\right) + \ln^2\left(\frac{m_2^2}{\mu^2}\right) \right) + \frac{18601}{3375} \ln\left(\frac{m_2^2}{\mu^2}\right) \ln\left(\frac{m_1^2}{\mu^2}\right) \ln(x) + \left[\frac{442267}{50625} - \frac{8912}{1125} x \right.$$

$$\left. - \frac{54202}{4725} x^2 - \frac{3519232}{467775} x^3 \right] \ln\left(\frac{m_1^2}{\mu^2}\right) \ln\left(\frac{m_2^2}{\mu^2}\right) - \frac{204611}{20250} \ln^3\left(\frac{m_1^2}{\mu^2}\right) - \frac{130207}{20250} \ln^3\left(\frac{m_2^2}{\mu^2}\right) \Bigg\}$$

$$+ O(x^4 \ln(x)) . \tag{4}$$

These contributions to the massive Wilson coefficients can be uniquely calculated in the fixed flavor number scheme with three massless quarks in the initial state. However, they cannot be attributed either to the charm or bottom distribution in a variable flavor scheme. [2] This shows one of the limitations of this intention despite the fact that operator matrix elements are process independent quantities. In (3, 4) power corrections contribute, however *not* of the kind $O(m_i^2/Q^2)$ but of $O(m_1^2/m_2^2)$. The computation of the corresponding contributions for general values of N is in progress.

We remark that the representation of the heavy flavor Wilson coefficients to $O(\alpha_s^3)$ are given in the fixed flavor number scheme. As has been shown in Refs. [23, 24] this choice is sufficient for the kinematic region at HERA.

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[2] As has been shown in [22] the application of a variable flavor scheme usually leaves no free choice of the matching scale μ^2. In particular it is often quite different from the respective heavy flavor mass m_Q^2.

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