A New Uncertainty Reduction Method for Fuel Fabrication Process and PWR cores with Erbia-Bearing Fuel

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1. Introduction

To achieve the super high burnup

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- **Increasing the 235U enrichment**

The 235U enrichment must be less than 5wt% in Japan

Erbia high burnup fuel

- 235U enrichment : >5wt%
- Erbia : Er_2O_3

Fuel Fabrication System, Reactor Core

- The erbia addition to the fuel with over 5wt% 235U enrichment retains the neutronics characteristics to those with under 5wt% fuel.

- We have no experience for the reactor cores and the fuel fabrication systems with full loaded erbia
- The use of erbia may produce additional uncertainty of the neutronics characteristics

The Uncertainty of neutronics chracteristics

- The cross section error
  - ➡ Evaluating with the **sensitivity coefficient** and the **cross section covariance matrix**

- The calculational method error
  - Monte-Carlo method
    - ➡ The standard deviation of the neutronics characteristics
  - Deterministic method
    - ➡
      - The self-shielding model
      - The energy condensation
      - The calculational mesh divisions
- The production error (manufacturing tolerance)

The Uncertainty evaluation

Calculation method error

$$R^c = R_t\left(1 + G_R \Delta\sigma + \Delta M + \Delta P\right)$$

Cross section error

Production error

$\Delta\sigma$: relative cross section error $\left(= \frac{d\sigma}{\sigma} \right)$

The sensitivity coefficient

$$G_R = \frac{dR}{R} \bigg/ \frac{d\sigma}{\sigma}$$

where R denotes the ρ , k_{eff} ...

The variance of R^c

$$Var\left(R^c\right) = G_R V_x G_R^t + Var\left(\Delta M\right) + Var\left(\Delta P\right)$$

Cross section covariance matrix

Improvement of prediction accuracy

1) Bias Factor Method

- $$\text{Bias Factor} = \frac{\text{Measured data of critical experiment}}{\text{Calculated data of critical experiment}}$$
- Bias factor is multiplied to calculated neutronics characteristics for a target system to improvement the accuracy
- Simple and easy to apply to design calculations

2) Cross Section Adjustment Method

- The method can utilize a wide range of neutronics characteristics
- Use of many experimental data
- Reliable cross section

2. Calculational Method

- **The effective neutron multiplication factor of the erbia bearing fuel system**

$$\frac{1}{k_{eff}} = \rho + \frac{1}{k_{eff,0}}$$

ρ : **the erbia worth**

$k_{eff,0}$: **the neutron multiplication factor of the system without the erbia**

- **The uncertainty of the k_{eff}**

$$\frac{1}{(k_{eff})^2} d\left(k_{eff}\right) = -\rho G_{\rho} \Delta\sigma + \frac{1}{k_{eff,0}} G_{k_{eff,0}} \Delta\sigma$$

The variance of the k_{eff}

The uncertainty of the erbia worth

$$Var(k_{eff}) = k_{eff}^{2}\left(G_1 V_{Er} G_1^{t} + G_2 V_{\sigma} G_2^{t}\right)$$

The uncertainty of the $k_{eff,0}$

→ Using the bias factor method or the generalized bias factor method

$$G_1 = G_{\rho}\,\rho \qquad G_2 = \frac{1}{k_{eff,0}} G_{k_{eff,0}}$$

V_{Er} : The cross section covariance matrix of Er

V_{σ} : The cross section covariance except for Er

Bias factor method

The $k_{eff,0}$ of the target core

$$\tilde{k}_{eff,0} = k_{eff,0} \times f$$

Bias factor

$$f = k^{e}_{eff,0,e} \big/ k^{c}_{eff,0,e}$$

The measured $k_{eff,0}$ for the mockup core

The calculated $k_{eff,0}$ for the mockup core

$$\tilde{k}_{eff,0} = k^{0}_{eff,0}\left(1 + G_{r,k_{eff,0}}\Delta\sigma\right) \times \frac{1+\Delta E_k}{1+G_{e,k_{eff,0}}\Delta\sigma}$$

The sensitivity coefficient of the system

The variance of the $\tilde{k}_{eff,0}$

$$Var\left(\tilde{k}_{eff,0}\right) = \Delta G_k V_\sigma \Delta G_k^{\,t} + Var\left(E_k\right)$$

$\Delta G_k = G_{k_{eff,0}} - G_{e,k_{eff,0}}$

$G_{e,k_{eff,0}}$: the sensitivity coefficient of experimental mockup core

$Var\left(E_k\right)$: the variance of the measured $k^{e}_{eff,0,e}$

Generalized bias factor method

- N experimental data

$$f_i = k^e_{eff,c,i} \Big/ k^e_{eff,e,i} \quad (i=1,2,3,\cdots,N)$$

- **Generalized bias factor** is determined by

$$\tilde{f} = \sum_{i=1}^{N} C_i f_i$$

- Weighting factor

$$\sum_{i=1}^{N} C_i = 1.0$$

- **The $k_{eff,0}$ of the target core by generalized bias factor method**

$$\tilde{k}_{eff,r} = k_{eff,r,0}\left(1+G_k\Delta\sigma\right)\times\sum_{i=1}^{N} C_i \frac{1+\Delta E_i}{1+G_{k,i}\Delta\sigma}$$

- **The variance of the $\tilde{k}_{eff,0}$**

$$Var(\tilde{k}^{c}_{eff,r}) = \left(G_{k,r} - \sum_{i=1}^{N} C_i G_{k,i}\right) V_x \left(G_{k,r} - \sum_{i=1}^{N} C_i G_{k,i}\right)^t + \sum_{i=1}^{N}\sum_{j=1}^{N} C_i C_j Var(\Delta E_i \cdot \Delta E_j)$$

- **The uncertainty of the erbia worth is reduced by using the cross section adjustment method**

→ **Utilizing the erbia sample worths measured by critical experiment**

The adjusted cross section

$$\sigma = \sigma_0 \left\{ 1 + V_{Er} G_{e,\rho}{}^t \left[G_{e,\rho} V_{Er} G_{e,\rho}{}^t + Var\left(E_\rho\right) \right]^{-1} \frac{\left[\rho_e^e - \rho_e^c(\sigma_0)\right]}{\rho_e^c(\sigma_0)} \right\}$$

σ_0 : **the cross section before adjustment**

$Var(E_\rho)$: **the variance of the measured erbia sample worth**

$G_{e,\rho}$: **the sensitivity coefficient of the erbia sample worth**

ρ_e^e : **the measured erbia sample worth**

$\rho_e^c(\sigma_0)$: **the erbia sample worth calculated by**

The covariance of the adjusted cross section

$$V_{Er}' = V_{Er} - V_{Er} G_{e,\rho}{}^t \left[G_{e,\rho} V_{Er} G_{e,\rho}{}^t + Var\left(E_\rho\right) \right]^{-1} G_{e,\rho} V_{Er}$$

- **The variance of $\rho_r^c(\sigma)$**

$$Var\left[\rho_r^c(\sigma)\right] = G_{r,\rho} V_{Er}' G_{r,\rho}{}^t$$

- **The variance of the effective neutron multiplication factor in the target system**

$$Var\left(\tilde{k}_{eff}\right) = k_{eff}{}^2 \left\{ G_{r,\rho} V_{Er}' G_{r,\rho}{}^t \rho^2 + \frac{1}{k_{eff,0}{}^2} \left[\Delta G_k V_\sigma \Delta G_k{}^t + Var\left(E_k\right) \right] \right\}$$

- **The uncertainty reduction (UR) is defined by**

$$UR = 1 - \frac{Var\left(\tilde{k}_{eff}\right)}{Var\left(k_{eff}\right)}$$

When UR becomes 1.0, the uncertainties are zero

3. Calculational Model

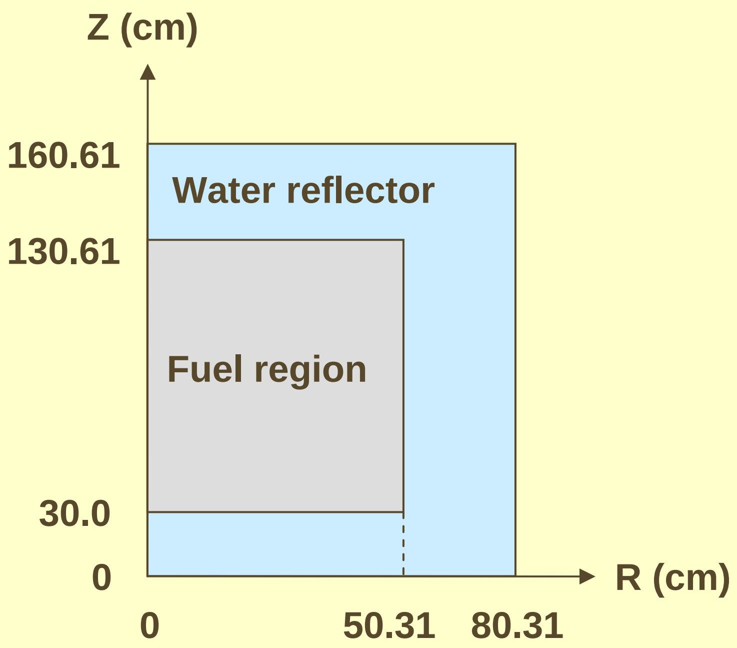


Fig. Calculational geometry of a blending machine (target system)

- **^{235}U enrichment : 5.4wt%**
- **The enrichment of Er relative to U : 0.3wt%**
- **Temperature : 300K**
- **H/U= 0 , 1**

- **Cross section set : JENDL-3.3**
- **Sensitivity coefficient : SAGEP**

Blending machine

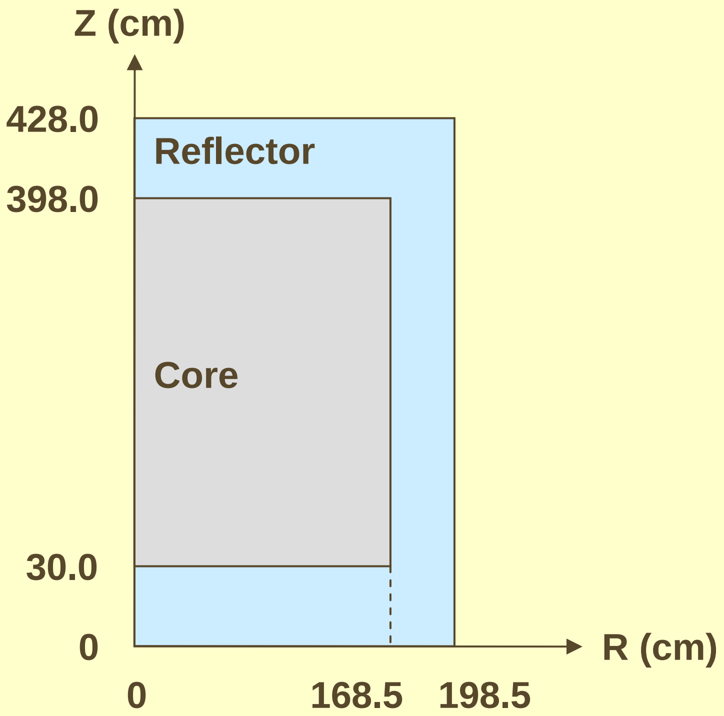


Fig. Calculational geometry of a PWR core (target system)

- ^{235}U enrichment : **5.4wt%**
- The enrichment of Er relative to U : **0.3wt%**
- Temperature : **300K**

- Cross section set : JENDL-3.3
- Sensitivity coefficient : SAGEP

PWR core

Fig. Erbia plate

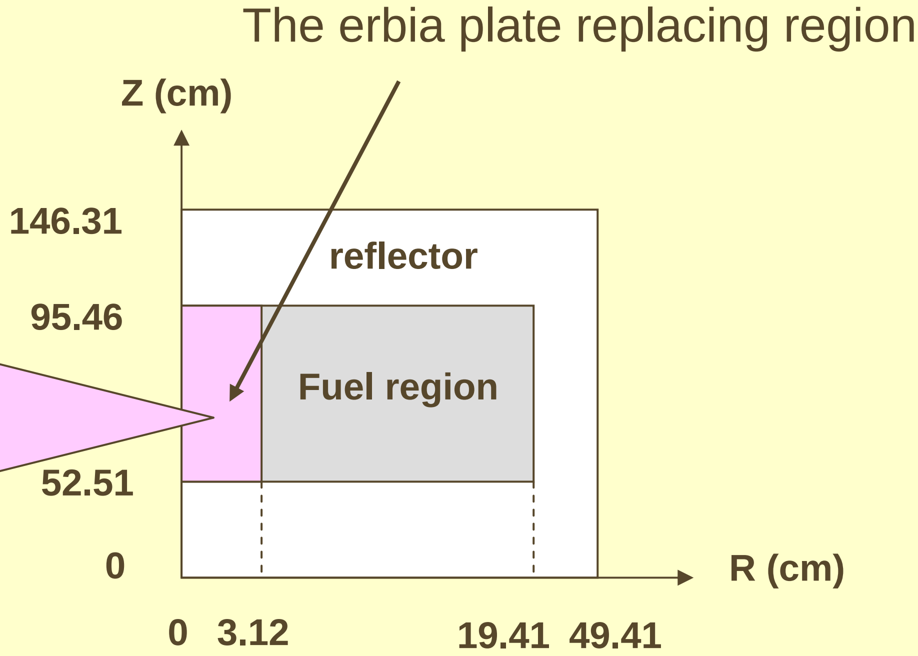


Fig. Calculational geometry of KUCA core

Table Calculated erbia sample worth and C/E value of KUCA experiment

Number of erbia plates	Calculated sample Worth (Δk/k)	C/E
3	$4.11X10^{-4}$	1.09
5	$7.07X10^{-4}$	1.13
9	$1.27X10^{-3}$	1.11
13	$1.44X10^{-3}$	1.12
17	$1.67X10^{-3}$	1.10

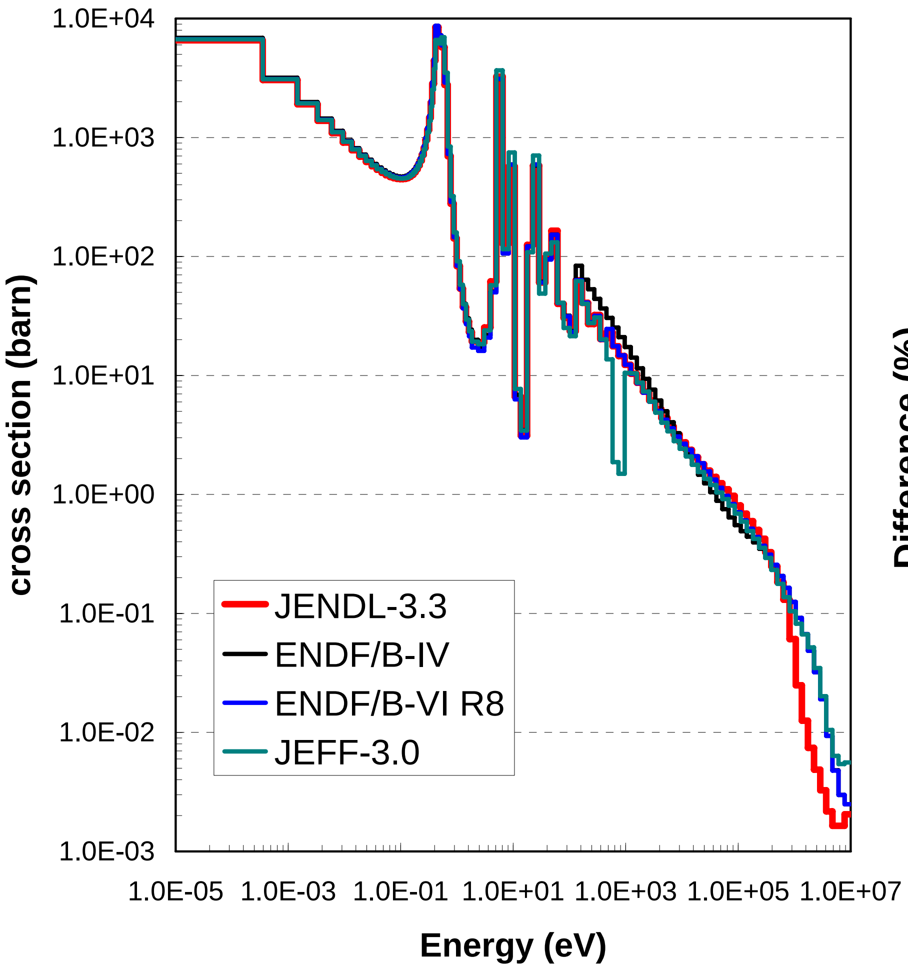


Fig. Comparison of ^{167}Er capture cross section in each nuclear data library

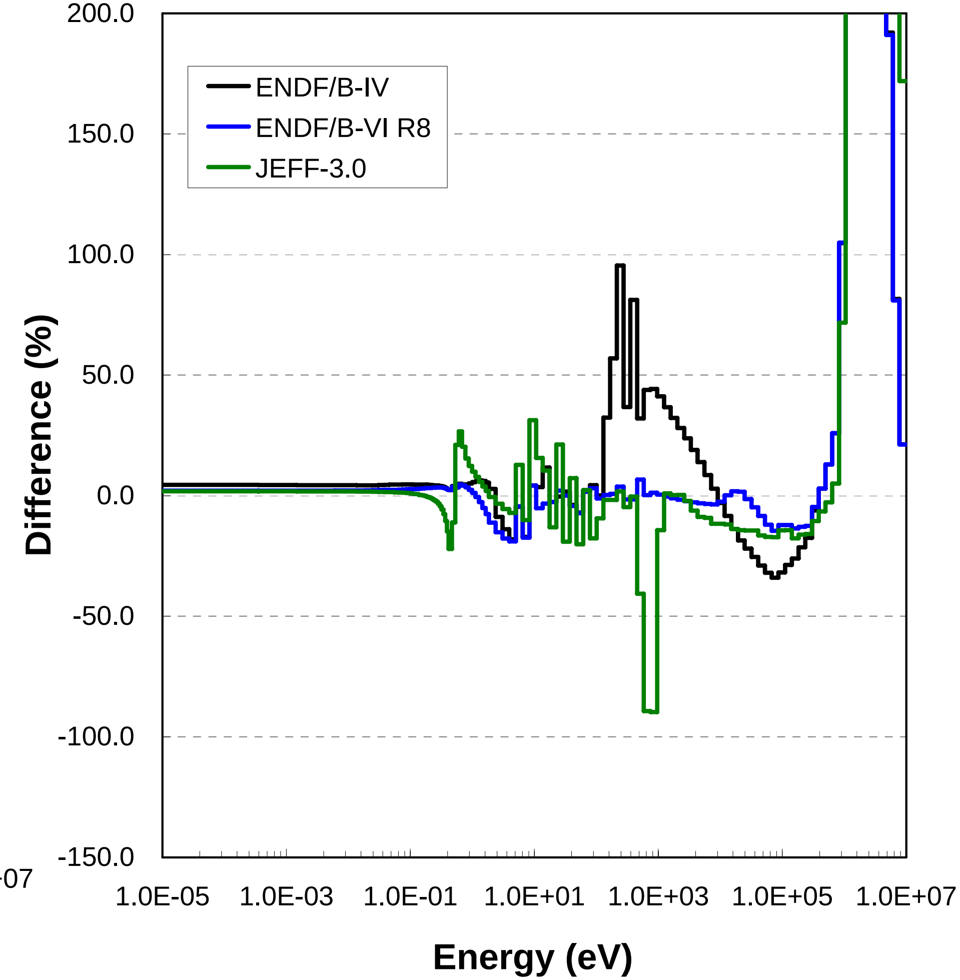


Fig. Difference of ^{167}Er capture cross section from JENDL-3.3

Er cross section errors ⇒ 10%

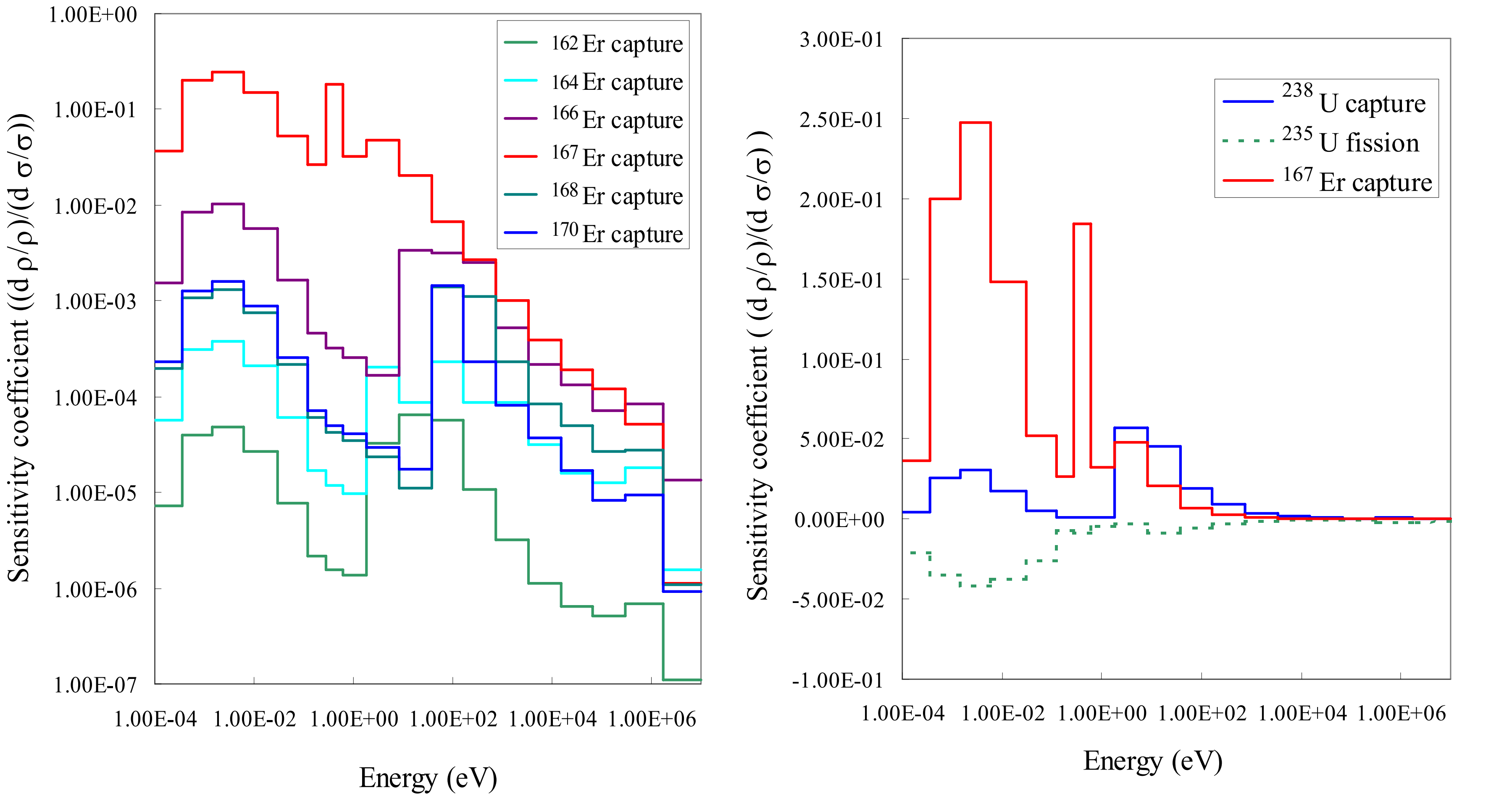


Fig. Comparison of sensitivity coefficients for erbia reactivity worth in KUCA

⇨ **167Er capture cross section is adjusted**

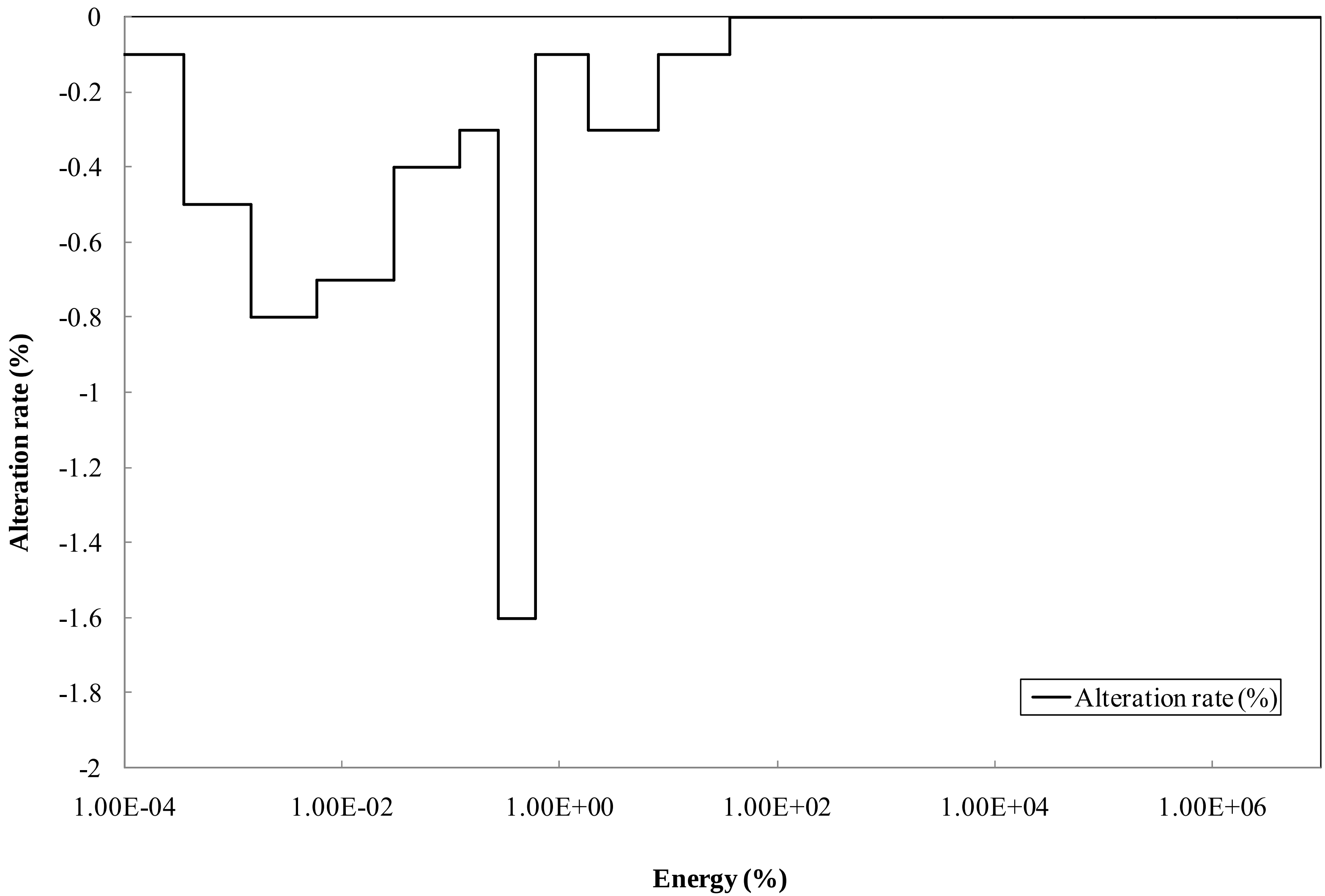


Fig. Alteration rate of 167Er capture cross section due to adjustment

4. Calculation Results

- **Evaluation of uncertainty reductions**

  Blending machine **in fuel fabrication process**

  PWR core

- **Method of uncertainty reduction**

Present method

Erbia worth : cross section adjustment method

keff : Generalized bias factor method

➡ **Two critical experiment data without erbia**

Bias factor method

➡ **critical experiment data with erbia is directly applied**

Without Experiment Data

Case of a blending machine

Table Comparison of prediction uncertainties and uncertainty reduction (UR) for k_{eff} in a blending machine (H/U=0)

	Uncertainty (%)	UR
Present method	0.495	0.604
Bias factor method	0.525	0.555
Case without experimental data	0.787	0.000

k_{eff} = 0.6210

$k_{eff,0}$ = 0.6361

ρ = $3.823\text{X}10^{-2}$ (Δk/k)

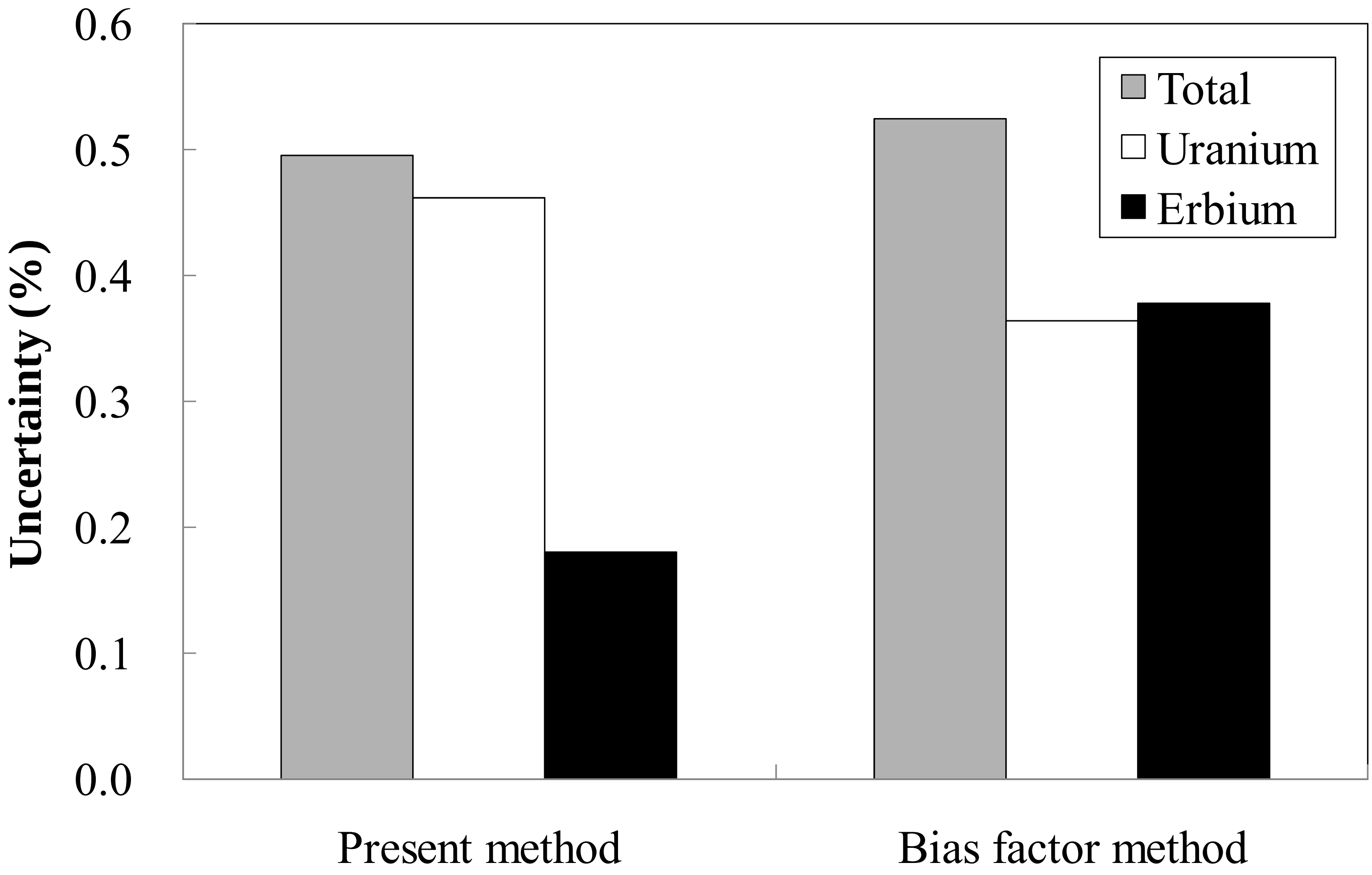


Fig. Components of prediction uncertainty of keff in a blending machine (H/U=0.0)

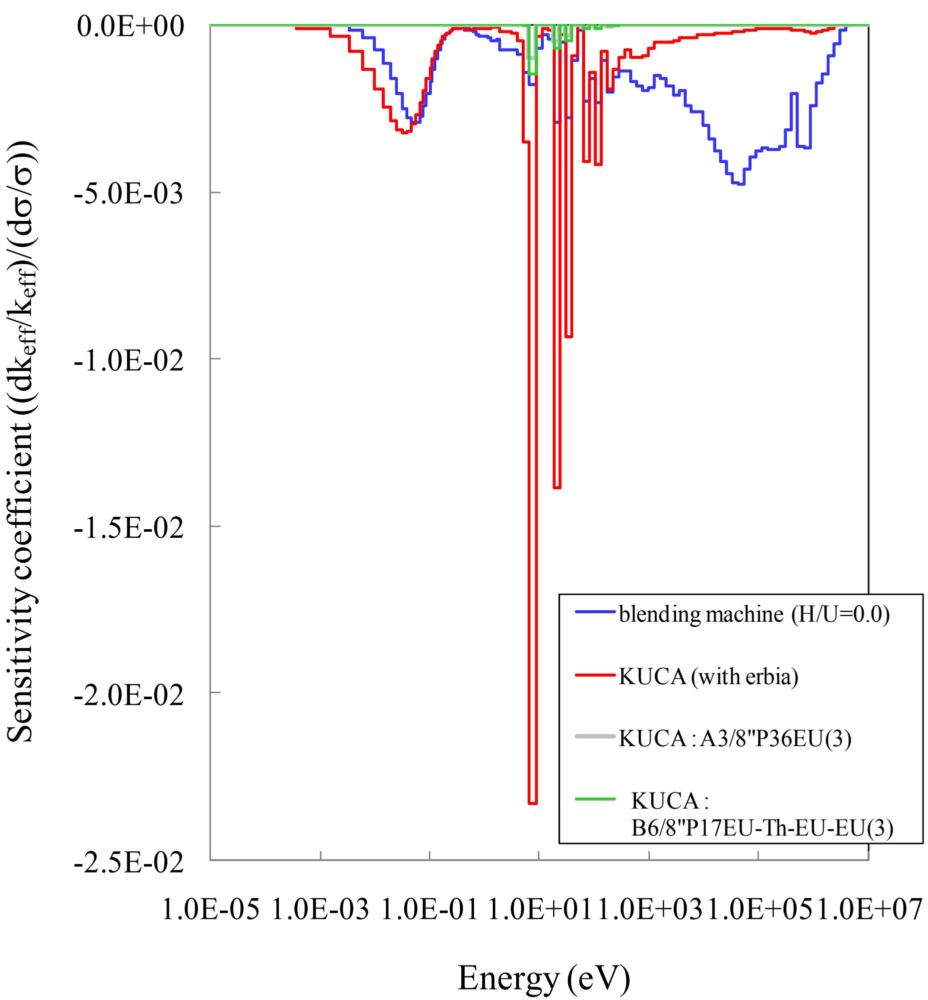


Fig. Sensitivity coefficients of keff with respect to 238U capture cross section

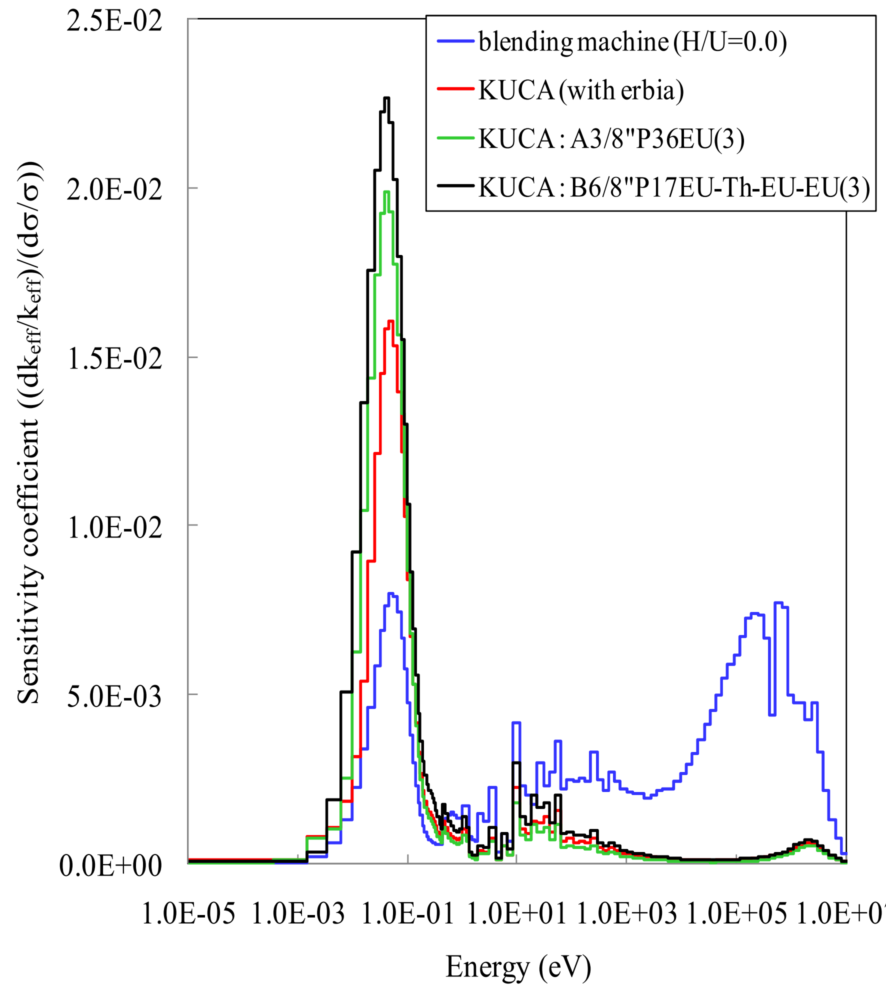


Fig. Sensitivity coefficients of keff with respect to 235U fission cross section

Table Comparison of prediction uncertainties and uncertainty reduction (UR) for k_{eff} in a blending machine (H/U=1)

	Uncertainty (%)	UR
Present method	0.414	0.760
Bias factor method	0.539	0.593
Case without experimental data	0.845	0.000

k_{eff} = 0.8026

$k_{eff,0}$ = 0.8287

ρ = 3.924×10^{-2} ($\Delta k/k$)

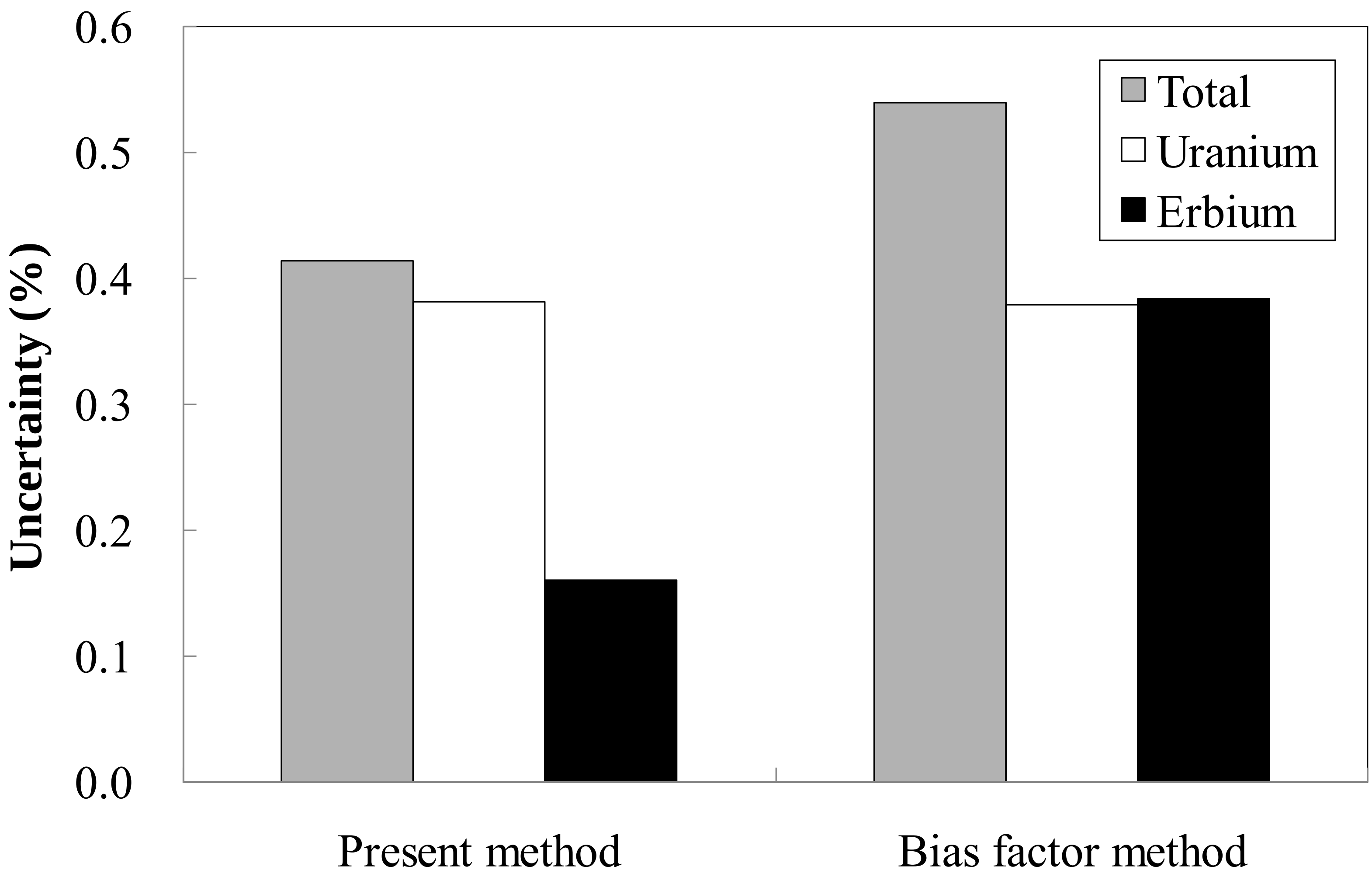


Fig. Components of prediction uncertainty of keff in a blending machine (H/U=1)

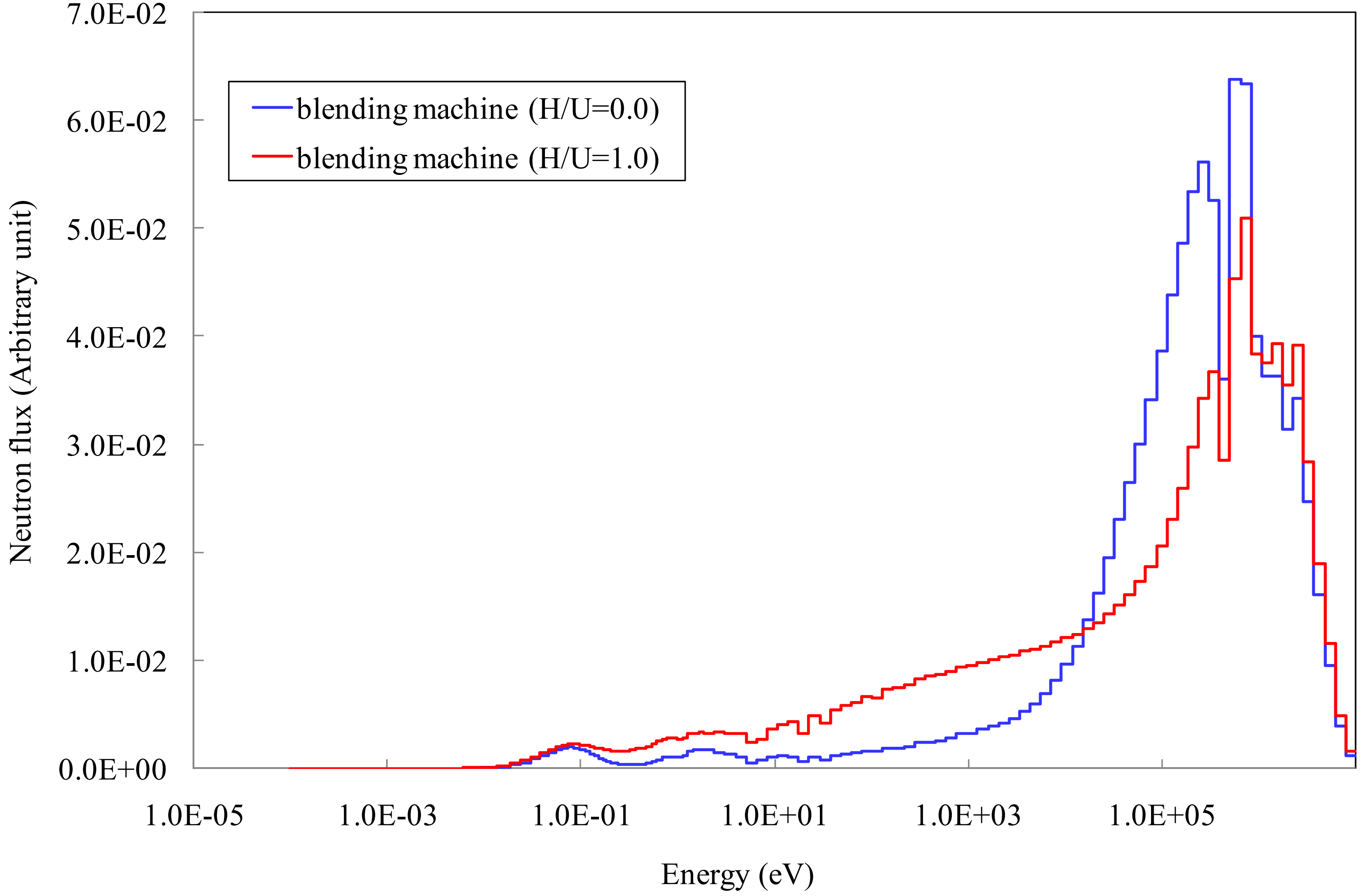


Fig. Comparison of neutron spectrum
between the case of H/U=0.0 and H/U=1.0

Case of a PWR core

Table Comparison of prediction uncertainties and uncertainty reduction (UR) for k_{eff} in a PWR core

	Uncertainty (%)	UR
Present method	0.322	0.865
Bias factor method	0.392	0.801
Case without experimental data	0.879	0.000

k_{eff} = 1.0072

$k_{eff,0}$ = 1.0479

ρ = 3.856×10^{-2} ($\Delta k/k$)

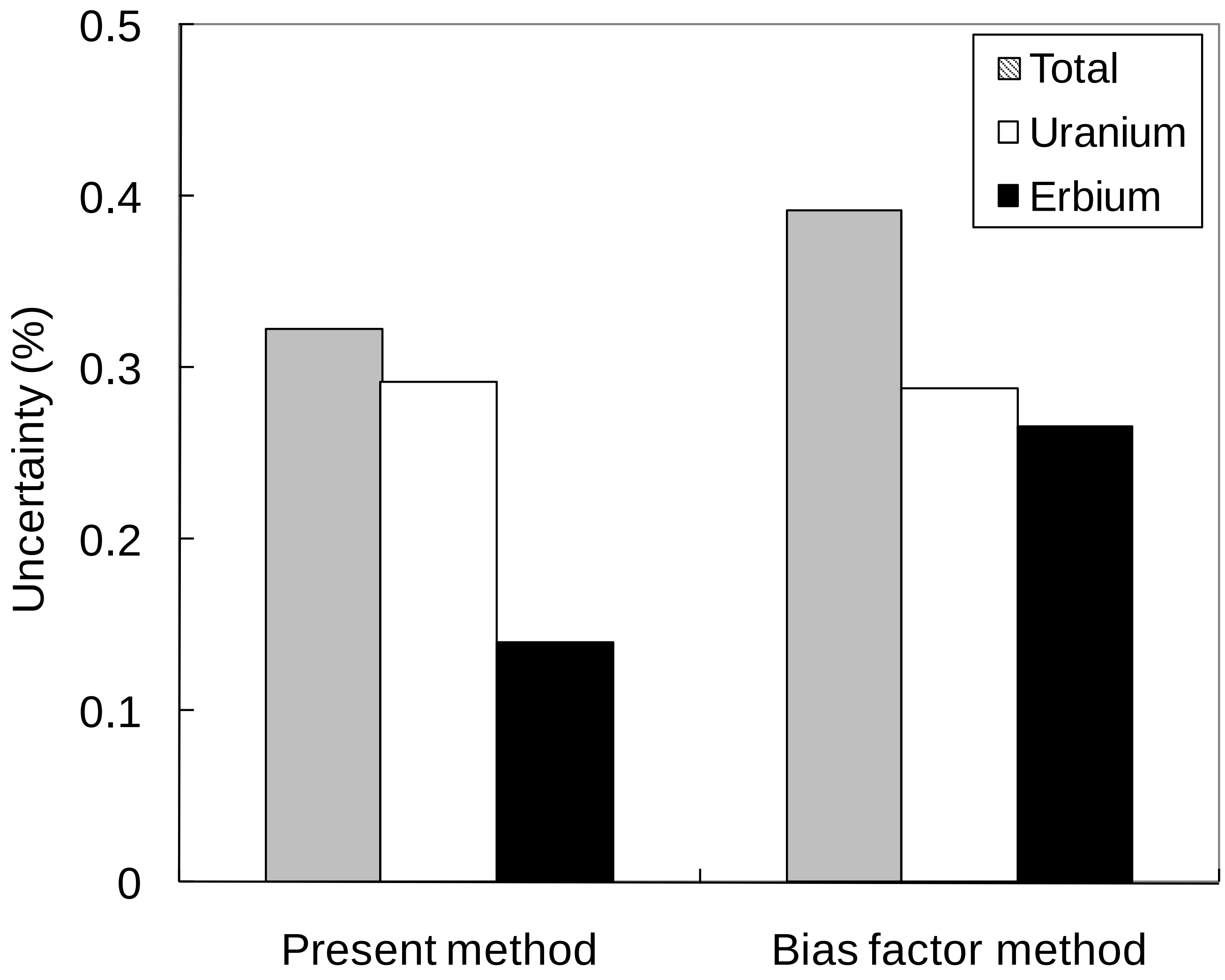


Fig. Components of prediction uncertainty of keff in a PWR core

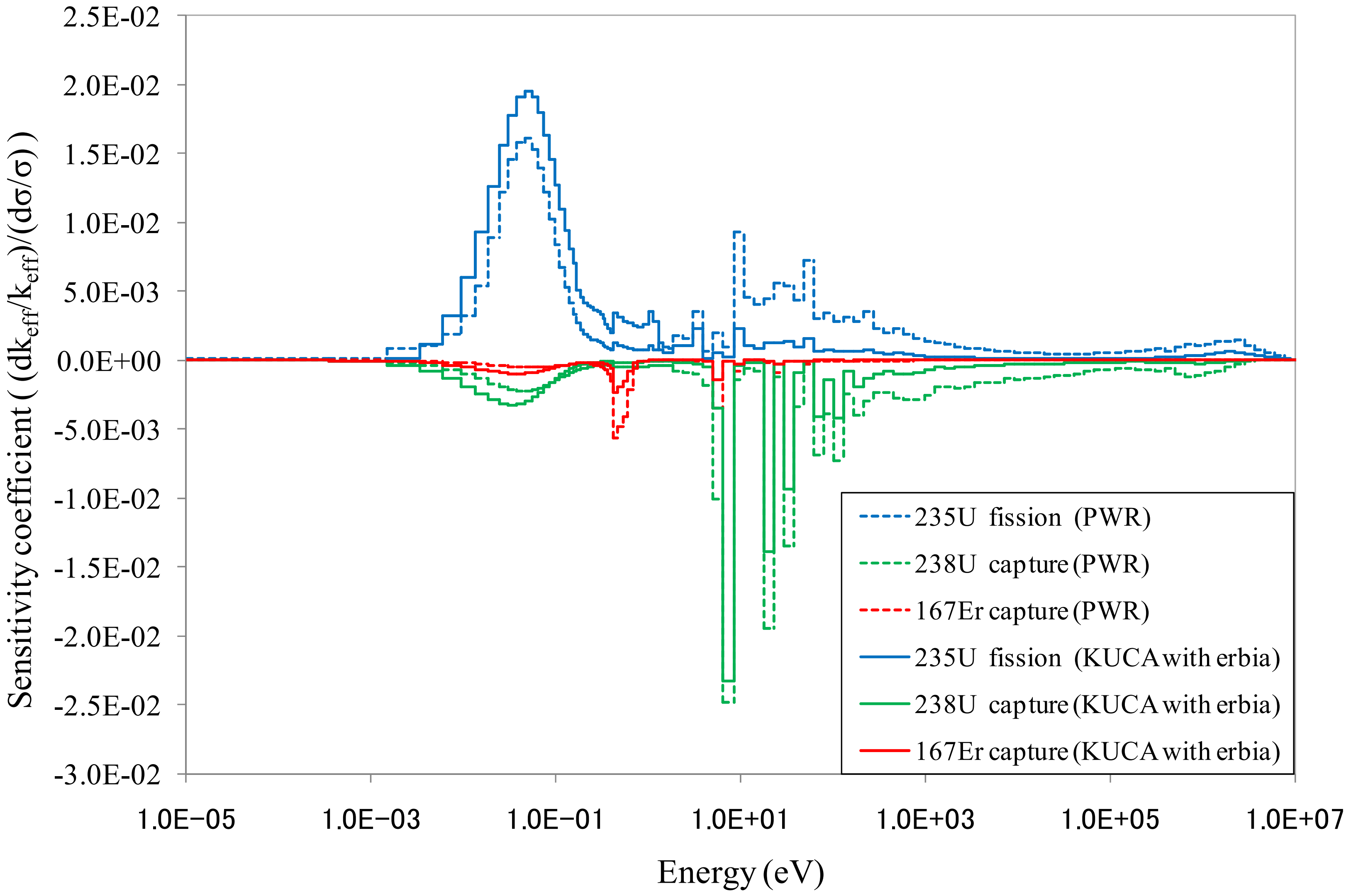


Fig. Sensitivity coefficients of keff for a PWR core and KUCA

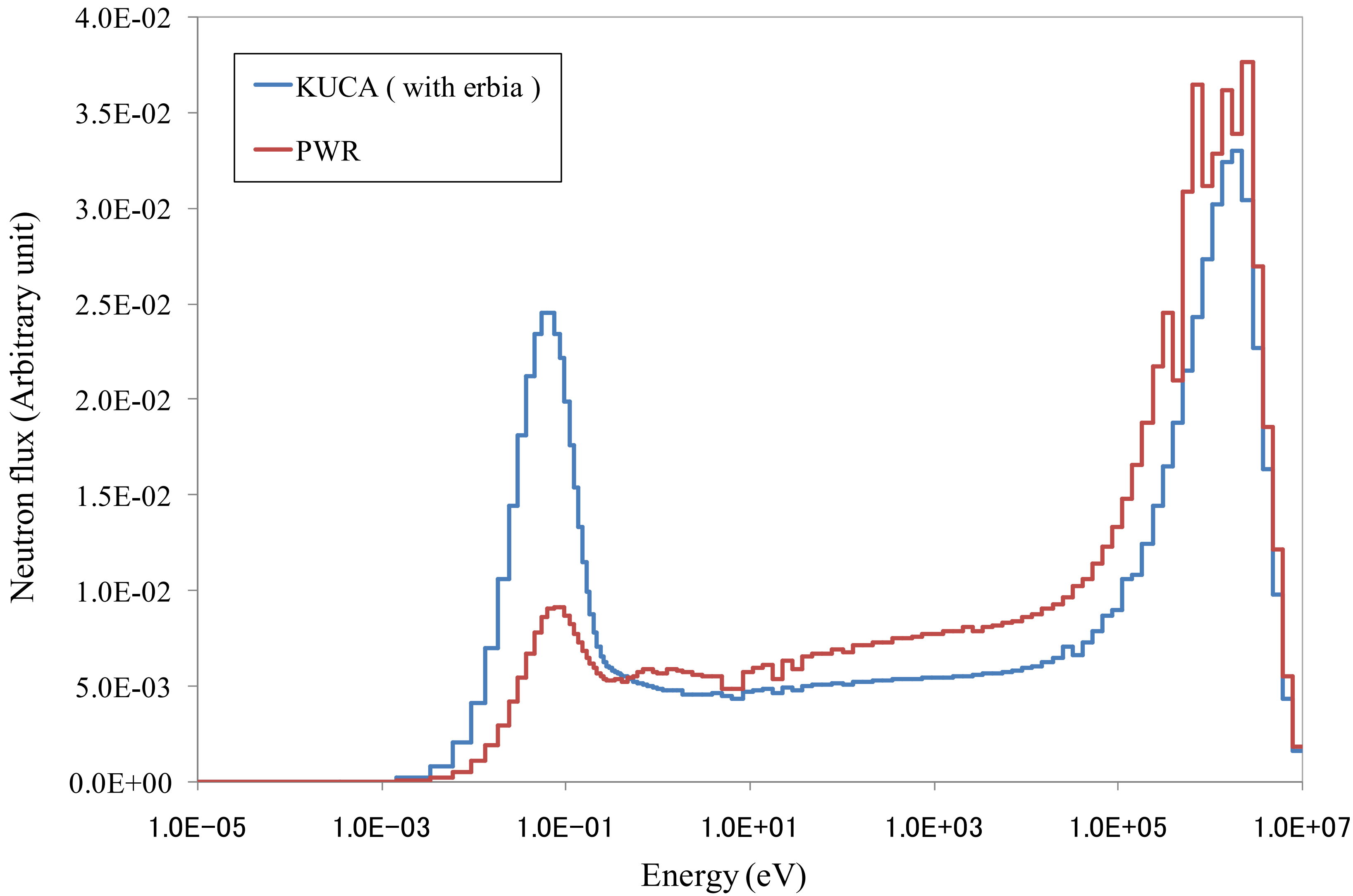


Fig. Comparison of neutron spectrum

5. Conclusion

- A new method was proposed.

  → Combining the bias factor method and the cross section adjustment method

- The present method was applied to evaluate the prediction uncertainty of neutronics characteristics of the blending machine and the PWR core loading the erbia bearing fuel.

- The uncertainty of erbia worth was reduced through the cross section adjustment method using the erbia sample worth measured in KUCA.

Blending machine

- **H/U=0**

  UR = 0.604 by the present method

  UR = 0.555 by the bias factor method

- **H/U=1**

  UR = 0.760 by the present method

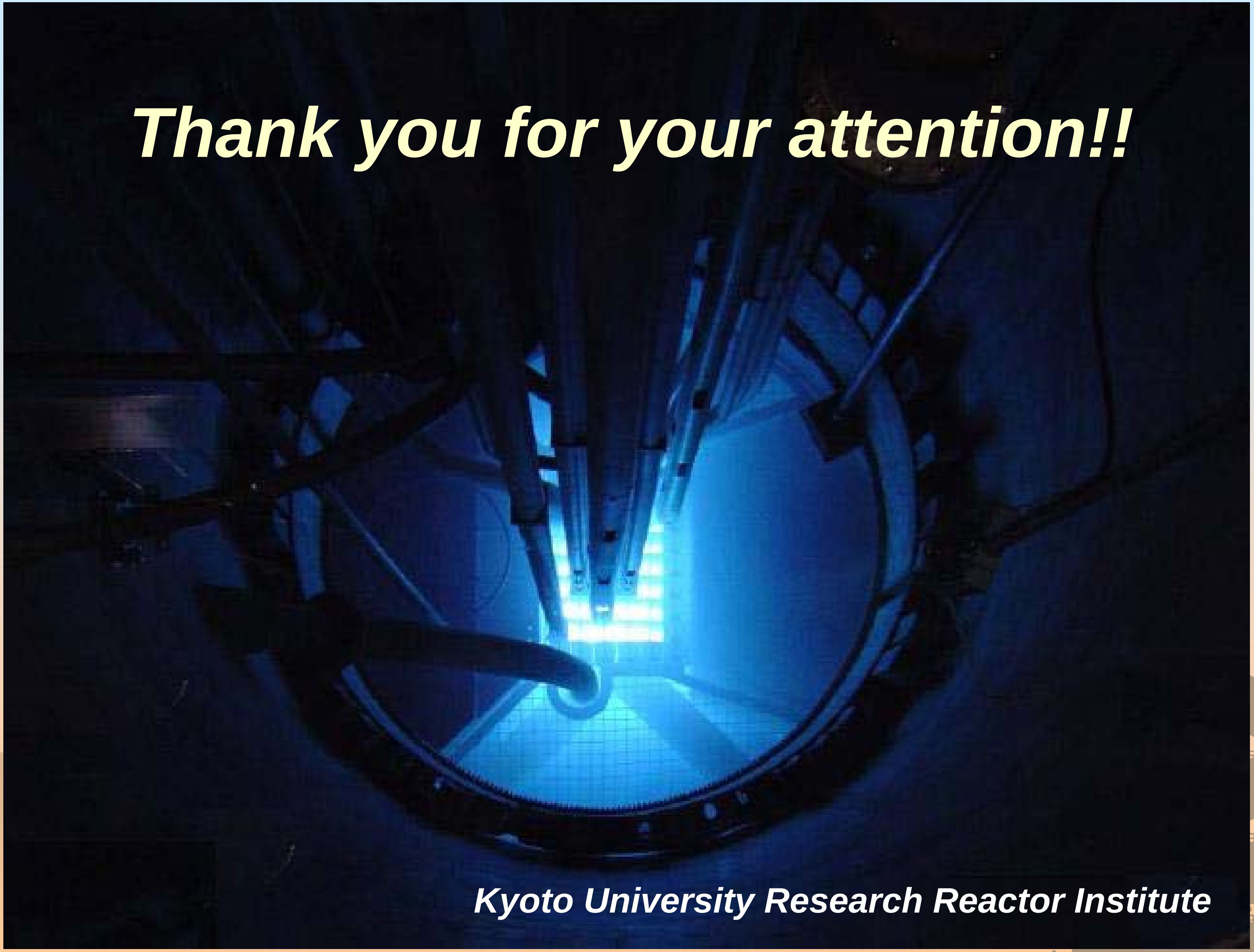
  UR = 0.593 by the bias factor method

PWR core

UR = 0.865 by the present method

UR = 0.801 by the bias factor method

The uncertainties were reduced by the present method


Thank you for your attention!!
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