

3-loop heavy flavor Wilson coefficients in deep-inelastic scattering[☆]

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Abstract

We present our most recent results on the calculation of the heavy flavor contributions to deep-inelastic scattering at 3-loop order in the large Q^2 limit, where the heavy flavor Wilson coefficients are known to factorize into light flavor Wilson coefficients and massive operator matrix elements. We describe the different techniques employed for the calculation and show the results in the case of the heavy flavor non-singlet and pure singlet contributions to the structure function $F_2(x, Q^2)$.

Keywords: Deep-inelastic scattering, heavy flavor

1. Introduction

The deep-inelastic precision data from HERA allow determinations of α_s at the 1% level [1], and precise measurements of the mass of the charm quark [2] and parton distribution functions. This applies to an even further extent to the proposed deep-inelastic scattering (DIS) experiments to be carried out in the future at new facilities, such as the EIC [3] and the LHeC [4], reaching much higher luminosities or energies than those available at HERA [5]. The corresponding analyses demand the knowledge of the anomalous dimensions and Wilson coefficients at 3-loop order for the structure function $F_2(x, Q^2)$, including the heavy flavor corrections. For $Q^2 \gg m^2$, where m is the mass of the heavy quark, the massive Wilson coefficients are known to factorize into the massless ones and massive operator matrix elements, allowing the computation of $F_2(x, Q^2)$ to the 1%

level [6]. The 3-loop light flavor Wilson coefficients are known [7]. Thus, it remains to calculate all the contributing massive OMEs.

In these proceedings we discuss the progress we have made in recent times in the calculation of these quantities [8–13], with special emphasis on the non-singlet [12] and pure singlet contributions [13]. By now, six out of eight operator matrix elements have been computed by us, namely, $A_{qq,Q}^{(3),PS}$, $A_{qg,Q}^{(3)}$, $A_{gq}^{(3)}$, $A_{qq,Q}^{(3),NS}$, $A_{qq,Q}^{(3),NS,TR}$ and $A_{Qq}^{(3),PS}$.¹ Only the operator matrix elements $A_{gg,Q}^{(3)}$ and $A_{Qg}^{(3)}$ remain to be calculated, although in these cases, we also have partial results for some of the color factors [10]. In the region $Q^2 \gg m^2$ also all Wilson coefficients for the structure function $F_L(x, Q^2)$ were calculated [9, 15]. Using these operator matrix elements, we have computed the massive Wilson coefficients $L_{q,(2,L)}^{NS}$, $L_{q,(2,L)}^{PS}$, $L_{g,(2,L)}^S$ and $H_{q,(2,L)}^{PS}$ at NNLO. Once $A_{Qg}^{(3)}$ is available, we will be able to obtain also $H_{g,(2)}^S$.

With the calculation of these operator matrix elements, we also obtain as a by-product the terms proportional to T_F of the 3-loop anomalous dimensions. In the case of transversity, we performed the first calculation ab initio. Our results agree with those given in the literature.

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¹For the notation see Ref. [14].

The operator matrix elements also yield the heavy-to-light transition relations in the variable flavor number scheme to 3-loop order. These relations define the parton distributions for $(n_f + 1)$ flavors from those of n_f light flavors by the OMEs. In particular, we now have the relation corresponding to $f_k(n_f+1, \mu^2) + f_{\bar{k}}(n_f+1, \mu^2)$ in the flavor non-singlet case.

In the next Section, we describe the different methods we used for the calculation of the operator matrix elements and the required Feynman integrals. In Section 3, we discuss our results and show the non-singlet and pure singlet contributions to $F_2(x, Q^2)$. Finally, in Section 4 we give some conclusions.

2. Calculation of the operator matrix elements

The operator matrix elements are computed in terms of Feynman diagrams using the standard Feynman rules of QCD, together with those for local operator insertions [14]. We generated the diagrams using QGRAF [16], the output of which was processed using a FORM [17] program [14], which ultimately allowed us to express the diagrams as a linear combination of scalar integrals.

The number of scalar integrals required for the calculation of the OMEs is quite large. We used integration by parts identities in order to express all integrals in terms of a relatively small set of master integrals. For this purpose we used the C++ program Reduze2 [18], which implements Laporta's algorithm [19]. Since this algorithm requires the integrals to be identified by definite indices, we rewrite the operator insertions by introducing an auxiliary variable x , multiplying the integrals by x^N and summing in N . For example, a line insertion is rewritten as [20]

$$(\Delta \cdot k)^{N-1} \rightarrow \sum_{N=1}^{\infty} x^{N-1} (\Delta \cdot k)^{N-1} = \frac{1}{1 - x\Delta \cdot k}, \quad (1)$$

where k is the momentum going through the line, and Δ is a light-like vector. In this way, the operator insertion is turned into an artificial propagator, making the application of Laporta's algorithm possible. Similarly, the 3-point and 4-point vertex insertions can be re-expressed in terms of products of such artificial propagators.

The master integrals were calculated using a variety of methods, namely,

- Hypergeometric functions [21]
- Mellin-Barnes integral representations [22].
- Hyperlogarithms for convergent integrals, [11, 23]

- Differential equations [24].

The representations lead to multiple nested sums which are summed using algorithms based on difference fields [25], implemented in the packages Sigma [26], HarmonicSums [27, 28], EvaluateMultiSums and SumProduction [29]. The choice of method depends on the complexity of the integral under consideration. The simplest integrals are calculated using hypergeometric functions, while the most complicated integrals we have encountered so far have been computed in a systematic manner using the differential equation method. This method ultimately leads to difference equations that are then solved using the packages Sigma and OreSys [30].

3. Results

All OMEs calculated so far have been found to be expressible in terms of (generalized) nested harmonic sums. In particular, $A_{qq,Q}^{(3),PS}$, $A_{qs,Q}^{(3)}$, $A_{gq}^{(3)}$, $A_{qq,Q}^{(3),NS}$ and $A_{qq,Q}^{(3),NS,TR}$ can all be expressed solely in terms of standard harmonic sums defined by [31]

$$S_{b,\vec{a}}(N) = \sum_{k=1}^N \frac{(\text{sign}(b))^k}{k^{|\vec{b}|}} S_{\vec{a}}(k), \quad (2)$$

with $S_{\emptyset} = 1$, $b, a_i \in \mathbb{Z} \setminus \{0\}$. In the case of $A_{Qq}^{(3),PS}$, we find for the first time in DIS generalized harmonic sums in the final result. They are defined by [28, 32]

$$S_{b,\vec{a}}(c, \vec{d})(N) = \sum_{k=1}^N \frac{c^k}{k^b} S_{\vec{a}}(\vec{d})(k), \quad (3)$$

where $S_{\emptyset} = 1$, $b, a_i \in \mathbb{N} \setminus \{0\}$, $c, d_i \in \mathbb{Z} \setminus \{0\}$. We have also found the emergence of inverse binomial sums [11, 33], such as

$$\binom{2N}{N} \sum_{k=0}^N \frac{k^A 2^k}{\binom{2k}{k}} S_1(k), \quad (4)$$

or

$$\sum_{i=1}^N \binom{2i}{i} (-2)^i \sum_{j=1}^i \frac{1}{j \binom{2j}{j}} S_{1,2} \left(\frac{1}{2}, -1; j \right). \quad (5)$$

This type of sums appears, e.g., in the terms proportional to T_F^2 in $A_{gg}^{(3)}$, as shown in [10].

The Mellin inversion of these quantities leads to harmonic polylogarithms in the case of Eq. (2). In the case of Eq. (3), they also lead to generalized harmonic polylogarithms [28] that we were able to re-express in terms of standard harmonic polylogarithms in a new variable $y = 1 - 2x$. On the other hand, the Mellin inversion

of inverse binomial sums leads to iterated integrals over square root-valued alphabets.

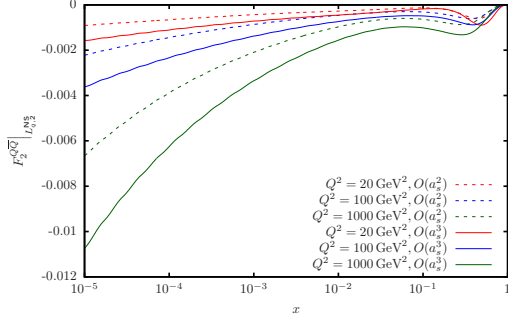


Figure 1: The flavor non-singlet contribution of the Wilson coefficient $L_{q,2}^{\text{NS}}$ to the structure function $F_2(x, Q^2)$ at 2- and 3-loop order using the ABM NNLO parton distribution functions (PDFs) [34] in the on-shell scheme for $m_c = 1.59$ GeV; from [12].

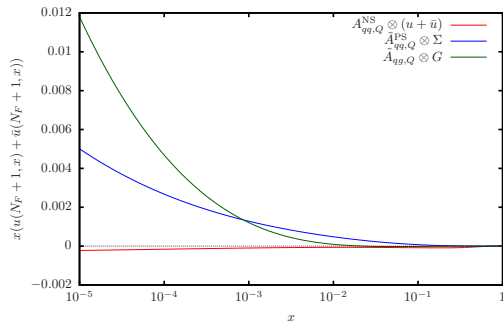


Figure 2: The contributions to the distribution $x(u + \bar{u})$ at 3-loop order for four flavors in the variable flavor number scheme matched at the scale $\mu^2 = 20 \text{ GeV}^2$ using the on-mass-shell definition of the charm quark mass $m_c = 1.59 \text{ GeV}$ and using the PDFs [34]. The contributions due to the non-singlet, singlet and gluon distributions are shown individually; from [12].

The heavy flavor contributions to the structure function $F_2(x, Q^2)$ are obtained by a Mellin convolution of the heavy flavor Wilson coefficients with the respective parton distributions. In Figure 1, we show the contribution up to three loops of the non-singlet heavy flavor Wilson coefficient $L_{q,2}^{\text{NS}}$ to $F_2(x, Q^2)$. We can see that they are smaller than 1% in the kinematic region of HERA. They will, however, become relevant at high luminosity machines such as the EIC, where all 3-loop Wilson coefficients will be of importance. As we mentioned in the introduction, the calculation of massive OMEs performed so far also allow us to obtain the transition relation for $n_f \rightarrow n_f + 1$ massless flavors of the flavor non-singlet distribution in the VNFS. In Figure 2, we show the different contributions to the 4-flavor distribution $x(u(x, \mu^2) + \bar{u}(x, \mu^2))$ as a function of x for

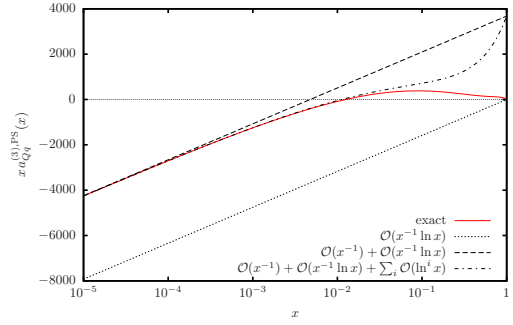


Figure 3: $x a_{Qq}^{(3),\text{PS}}(x)$ (solid red line) and leading terms approximating this quantity; dotted line: ‘leading’ small x approximation $O(\ln(x)/x)$, dashed line: adding the $O(1/x)$ -term, dash-dotted line: adding all other logarithmic contributions; from [13].

$\mu^2 = 20 \text{ GeV}^2$. We can see that the non-singlet contributions are very small, while the singlet and gluon contributions are much larger for small values of x . The distributions for down and strange quarks are therefore nearly the same.

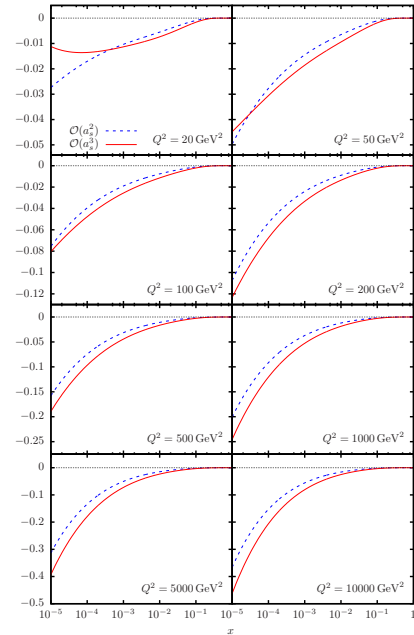


Figure 4: The charm contribution by the Wilson coefficient $H_{q,2}^{\text{PS}}$ to the structure function $F_2(x, Q^2)$ as a function of x and Q^2 choosing $Q^2 = \mu^2$, $m_c = 1.59 \text{ GeV}$ (on-shell scheme) using the PDFs [34]; from [13].

In Figure 3, we show the behavior of the constant part of the unrenormalized pure singlet OME, $a_{Qq}^{(3),\text{PS}}(x)$. The leading small x term $\propto \ln(x)/x$ does nowhere describe this quantity. In the small x region one has to add sev-

eral subleading terms to get a sufficient approximation. In Figure 4, the charm quark contribution to $F_2(x, Q^2)$ by the Wilson coefficient $H_{q,2}^{\text{PS}}$ is shown at $O(a_s^2)$ and up to $O(a_s^3)$ in dependence on x and several values of Q^2 , setting the factorization scale $\mu^2 = Q^2$. The $O(a_s^3)$ correction is quite significant in the region of low x , and at lower scales Q^2 the $O(a_s^2)$ corrections are larger than those at NNLO.

4. Conclusions

We have made considerable progress in recent years in the calculation of heavy flavor corrections to DIS at NNLO in the region of large virtualities. So far, we have calculated six out of eight massive operator matrix elements developing and applying a variety of mathematical and computer algebraic techniques. The results have been given in terms of (generalized) nested harmonic sums. As a by-product, we have calculated the terms proportional to T_F in the 3-loop anomalous dimensions confirming results in the literature. The corresponding massive Wilson coefficients have been calculated and their contributions to $F_2(x, Q^2)$ determined and analyzed.

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