$$\check{r}_{mc}=\begin{cases} \frac{3\tilde{\omega}}{(\Delta-2\tilde{\omega}^2)^{1/3}}+\frac{(\Delta-2\tilde{\omega}^2)^{1/3}}{\tilde{\omega}}, & \tilde{\omega}<\frac{2\sqrt{3}}{9} \\ 2\sqrt{3}\cos[\frac{1}{3}arc\cos(-\frac{2\sqrt{3}}{9}\frac{1}{\omega})], \tilde{\omega}\geq\frac{2\sqrt{3}}{9} & \end{cases} \qquad (16)$$

where $\Delta= (4\tilde{\omega}^4 - 27\tilde{\omega}^6)^{1/2}$. The obtained equation (16) is the original one.

As it was expected in the limiting case when $\tilde{\omega}$>> 1, i.e. when metric (1) coincides with the well known Schwarzschild metric, one can easily obtain the known result for minimal radius of circular orbits around the Schwarzschild black hole as $\check{r}_{mc} = 3$.

The minimum radius for a stable circular orbit will occur at point of inflexion of the function f(r), or in other words where the supplement conditions f(r) = f'(r) = 0

with the relation f''(r) ≥ 0 are satisfied. The numerical results for the values of ISCO radii for the different values of the parameter $\tilde{\omega}$in the case of $\tilde{\omega}$≥ 0.5 are given in the table I (the second line). From the results one can easily get in the limit of Schwarzschild spacetime $\omega r^2 \to \infty$ the standard value for ISCO radius as r_{ISCO}= 6M.

Table 1

The innermost stable circular orbits around black hole and the critic values of the momentum of the particles fallingdown to the central black hole in Hoˇrava-Lifshitz gravity.

$\tilde{\omega}$	0.5	1	2	4	6	8	10	12
r_{ISCO}	5.23655	5.66395	5.84024	5.92193	5.94834	5.9614	5.96918	5.97436
$\mathcal{L}^2_{cr}$	14.77	15.454	15.7395	15.8725	15.9156	15.9369	15.9496	15.9581

Constraints for the KS parameter from the Solar system tests were found as (5.660 $\pm$3.1) x10^{-26} cm^{-2}.The similar constraints for the parameter$\omega \simeq 1.25 \cdot 10^{-25}$ cm^{-2} have been found from the quantum interference effects . One can easily compare the obtained numerical results with observational data for ISCO radius for some candidates of rotating black holes. One can obtain the lower value for the parameter as $\omega \simeq 3.6 \cdot 10^{-24}$ cm^{-2}.

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IMAGE OF ROTATING KERR-LIKE BLACK HOLE

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Being apparently the simplest geometric theory of gravitation, in Einsteins general relativity, a number of fundamental problems regarding the nature of singularities, dark energy and dark matter, and quantization of gravitational interactions is appearing. To solve the mentioned problems, theorists are forced to use alternative theories of gravity. Recently a number of works has been devoted to the study of potentially observable processes around various non-Kerr black holes [1]. Therefore, a unified description of analogs of the Schwarzschild and Kerr solutions in various theories of gravity would be most useful for testing the alternatives to general relativity. Such a model was suggested by Johannsen and Psaltis [2], who considered deviations from the Schwarzschild and Kerr solutions and found a regular spacetime outside the event horizon, which reduces to the Kerr one, when the deformation parameters vanish. The Johannsen and Psaltis metric is not a vacuum solution of the Einstein equations but is obtained in a kind of perturbative way in order to include various possible deviations from the Kerr solution in alternative theories of gravity.

Here we will study massless particles motion in the vicinity of a black hole of mass *M* in the presence of deformation parameter described by the space-time metric ([2]):

$$ds^2 = \quad (1 \quad \frac{2Mr}{\ ^2})(1+h)dt^2 + \frac{\ ^2(1+h)}{\ + a^2 h \sin^2}dr^2 + \ ^2 d \ ^2$$
$$\frac{4aMr\sin^2}{\ ^2}(1+h)d \ \ dt + \sin^2 \ [\ ^2 + \frac{a^2(\ ^2+2Mr)\sin^2}{\ ^2}(1+h)]d \ ^2 \quad (1)$$

$$\Delta = r^2 + a^2 - 2Mr \ , \quad \Sigma^2 = r^2 + a^2\cos^2\theta, \quad h(r,\theta) = \varepsilon\frac{M^3 r}{\Sigma^4} \ . \quad (2)$$

Here a is angular momentum per total mass of black hole M and is the ε deformation parameter. Adopting the celestial coordinates is very convenient to describe the image(shadow) [3]:

$$= \lim_{r_0 \to \square}(\ r_0^2 \sin^2 \ _0 \frac{d}{dr}) \quad (3)$$

$$\beta = \lim_{r_0 \to \infty} r_0^2 \frac{d\theta}{dr} \quad (4)$$

since here an observer far away from the black hole is considered $r_0 \to \infty$, θ_0 is the angular coordinate of the observer, i.e. the inclination angle between the rotation axis of the black hole and the line of sight of the observer.

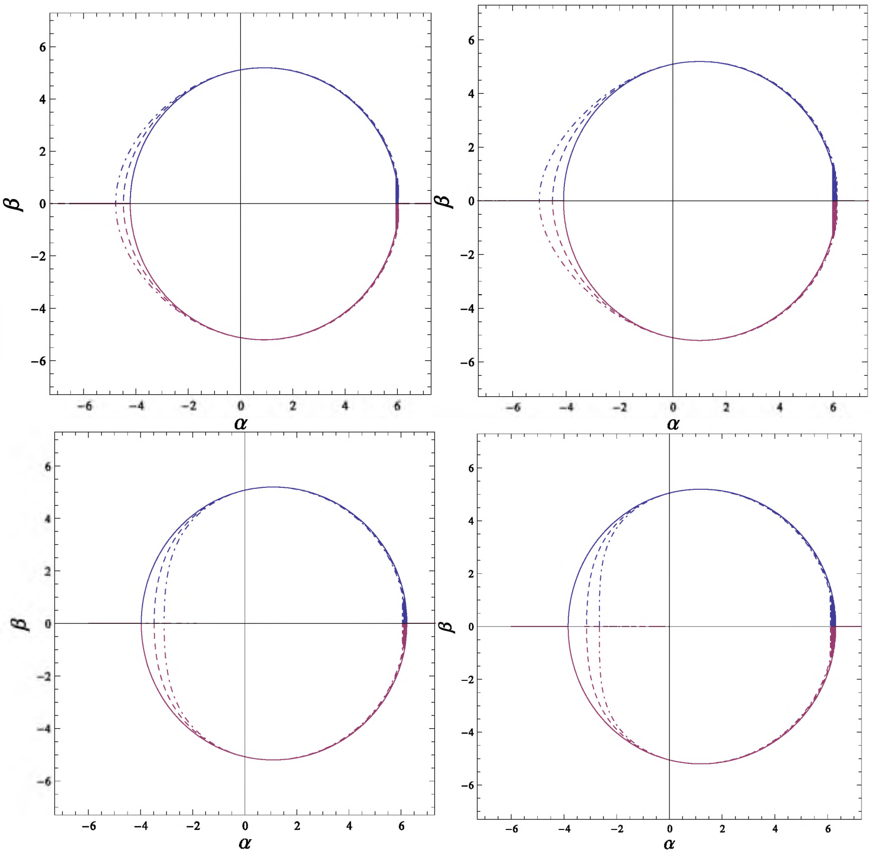

Fig. 1. Silhouette of the image(shadow) cast by a non-Kerr black hole situated at the origin of coordinates with inclination angle θ =π/2, having a rotation parameter a and a deformation parameter ε.

Upper row, left: *a/M* =0.45 , *ε*= 0 (solid line), *ε=-5* (dashed line), and *ε=-10* (dashed-dotted line). Upper row, right: *a/M* =0.5 , *ε=0* (solid line), *ε=-5* (dashed line), and *ε=-10* (dashed-dotted line). Lower row, left: *a/M* =0.55 , *ε=0* (solid line), *ε=5* (dashed line), and *ε=10* (dashed-dotted line). Lower row, right: *a/M* =0.6 , *ε=0* (solid line), *ε=5* (dashed line), and *ε=10* (dashed-dotted line). The shadow corresponds to each curve and the region inside it.

Fig.1 Silhouette of the image(shadow) cast by black hole situated at the origin of coordinates with inclination angle θ_0 =*π/2* having a black hole angular momentum a=0.5 and for the different value of deformation parameter *ε*. We have analyzed how the shadow of the black hole is distorted by the presence of the deformation parameter *ε*. We have shown that with increasing deformation parameter *ε*, the radius of the image (shadow) of the black hole decreases[4].

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SPIN DOWN OF ROTATING COMPACT MAGNETIZED STARS

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Neutron stars provide a natural laboratory to study extremely dense matter. In the interiors of such stars, the density can reach up to several times the nuclear saturation density $n_0 \approx 0.16 fm^{-3}$. At such high densities quarks could be squeezed out of nucleons to form quark matter. It has been suggested that strange quark matter that consists of comparable numbers of u, d, and s quarks may be the stable ground state of normal quark matter.

Here we will be concerned with the possibility to distinguish neutron star from the strange star from the spin down of pulsar.

Assume that the oblique rotating magnetized star is observed as radio pulsar through magnetic dipole radiation. Then the luminosity of the relativistic star in the case of a purely dipolar radiation, and the power radiated in the form of dipolar electromagnetic radiation, is given by [1]

$$L_{em} = \frac{{}^{4}_{R} R^6 \tilde{B}_0^2}{6c^3} \sin^2 \qquad (1)$$

where tilde denotes the general relativistic value of the corresponding quantity, subscript R denotes the value of the corresponding quantity at $r = R$ and χ is the inclination angle between magnetic and rotational axes. In this report we will use the spacetime of slowly rotating relativistic star which in a coordinate system (t, r, θ, φ) has the following form:

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2 - 2\omega(r) r^2 \sin^2\theta\, dt d\varphi \qquad (2)$$

where metric functions $\Phi(r)$ and $\Lambda(r)$ are completely known for outside of the star and given as:

$$e^{2\Phi(r)} = (1 - \frac{2M}{r}) = e^{-2\Lambda(r)} \qquad (3)$$

$\omega = 2J / r^3$, $J = I(M,R)\Omega$ is the total angular momentum of the star with total mass M and moment of inertia $I(M,R)\Omega$ is the angular velocity of the star.

The equivalent Newtonian expression for the rate of electromagnetic energy loss through dipolar radiation [2]

$$(L_{em})_{Newt} = \frac{\Omega_R^4 R^6 B_0^2}{6c^3} \sin^2\chi \qquad (4)$$

it is easy to realize that the general relativistic corrections emerging in expression (1) are due partly to the magnetic field amplification at the stellar surface

$$\frac{\tilde{B}_0}{B_0} = \frac{\tilde{B}_0 R^3}{2\mu} = f_R \quad , \qquad f_R = -\frac{3R^3}{8M^3}[\ln N_R^2 + \frac{2M}{R}(1 + \frac{M}{r})] \qquad (5)$$

and partly to the increase in the effective rotational angular velocity produced by the gravitational redshift

$$\Omega(r) = \Omega_R \frac{N_R}{R} = \Omega \sqrt{(\frac{R-2M}{r-2M})\frac{r}{R}} \qquad (6)$$

Expression (1) could be used to investigate the rotational evolution of magnetized neutron stars with predominant dipolar magnetic field anchored in the crust which converts its rotational energy into electromagnetic radiation.

Following the simple arguments proposed more than forty years ago [3], it is possible to relate the electromagnetic energy loss L_{em} directly to the loss of rotational kinetic energy E_{rot} defined as