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RESEARCH AND DEVELOPMENT

THEORY OF NUCLEAR MAGNETIC MOMENTS

LT-35

BY

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CHALK RIVER, ONTARIO

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I Introduction

The purpose of these notes is to give an account of some attempts at interpreting the observed values of nuclear magnetic moments. There is no attempt at a complete summary of the field as that would take much more space than is used here. In many cases the arguments are only outlined and references* are given for those interested in further details.

A discussion of the theory of nuclear magnetic moments necessitates many excursions into the details of the nuclear models because the magnetic moments have a direct bearing on the validity of these models. However the main emphasis here is on those features which tend to explain the magnetic moments and other evidence is not discussed unless it has a direct bearing on the problem.

In the first part of the discussion the Shell Model of the nucleus is used, as this model seems to correlate a large body of data relating to the heavier nuclei. Included here are the modifications proposed to explain the fact that the experimental magnetic moments do not fit quantitatively with the exact predictions of the Shell Model. The next sections deal with some of the more drastic modifications introduced to explain the

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*References are quoted by the author's name and the last two digits of the year in which the paper appeared. The list of references is arranged in alphabetical order.

large nuclear quadrupole moments and the effect of these modifications on the magnetic moments. Finally we turn to more detailed investigations of the light nuclei, in particular the Conjugate nuclei.

II Definitions and Basic Formulae

(A) Magnetic Moment and g-Factor

The classical interaction energy between a current distribution $\bar{I}$ and a slowly varying (in space) electromagnetic field includes a term $\bar{m}\cdot\bar{H}$, where $\bar{H}$ is the magnetic field at the origin and $\bar{m}$ is the magnetic dipole moment density of the current (Rosenfeld 47 page 390);

$$\bar{m} = \frac{1}{2c}\left(\bar{r} \times \bar{I}\right)$$

Quantum mechanically $\bar{m}$ becomes the magnetic moment operator. Setting the z axis in the direction of $\bar{H}$, the magnetic moment of the system is defined as the expectation value of the z component of the magnetic moment operator in the state of maximum magnetic quantum number. It is always proportional to the total angular momentum I and the proportionality constant is called the g-factor;

$$\text{magnetic moment} = \mu\mu_o = g\mu_o I$$

where

$$\mu_o = 1 \text{ magneton} = \frac{e\hbar}{2Mc} \; .$$

The magnetic moment is fixed by giving the value of μ and specifying the type of magneton, which depends on the mass of some particle.

(B) Magnetic Moment of a Single Particle

For a particle with charge e and mass M the current is given by $\bar{I} = e\bar{v} = \frac{e}{M}\,\bar{p}$, where $\bar{p}$ is the momentum of the particle.

Then the orbital magnetic moment operator is given by the equation

$$\bar{m}^{\ell} = \frac{1}{2c}(\bar{r} \times \bar{I}) = \frac{e}{2Mc}(\bar{r} \times \bar{p}) = \frac{eh}{2Mc}\ \bar{\ell} = \mu_o \bar{\ell} \ ,$$

where $\bar{\ell}$ is the orbital angular momentum operator (divided by $\hbar$).

If in addition the particle has intrinsic spin $\bar{s}$ (with eigenvalue $s = \frac{1}{2}$) the Dirac theory gives an intrinsic magnetic moment operator (Dirac 47, page 265)

$$\bar{m}^{s} = \frac{eh}{Mc}\ \bar{s} = 2\mu_c \bar{s} \ (\bar{s} = \tfrac{1}{2}\ \bar{\sigma} \text{ of Dirac}).$$

When there is no orbital motion the magnetic moment consists only of the spin part and the total angular momentum is $\bar{J} = \bar{s}$ so that

$$\text{m.m.} = \langle 2\mu_o s_z \rangle = 2\mu_o s = \mu_o$$

and the Dirac spin g-factor is given by $g_s = 2$. The electron fits very well into this picture except for small radiative corrections (Schwinger 49, Luttinger 48). The neutron and proton, however, are found to have anomalous magnetic moments as given in table I (Klinkenberg 52, Mack 50).

The magnetic moment operator for a nucleon in an orbit is then given by the equation

$$\bar{m} = g_{\ell}\ \mu_o \bar{\ell} + g_s \mu_o \bar{s},$$

where $\bar{j} = \bar{\ell} + \bar{s}$ and $j = \ell \pm \frac{1}{2}$

We have introduced g_ℓ to take care of the fact that the neutron has zero charge and hence zero orbital moment ($g_\ell = 1$ for a proton, $g_\ell = 0$ for a neutron). The appropriate g_s value for the nucleon is given in table I. Using this operator:

$$\mu = \frac{m.m.}{\mu_o} = g_\ell \langle \ell_z \rangle + g_s \langle s_z \rangle$$

At this point we make use of a formula which is very useful in calculating magnetic moments and which will be used again. If $\bar{J} = \bar{J}_1 + \bar{J}_2$ and $\bar{J}$, $\bar{J}_1$ and $\bar{J}_2$ are angular momentum operators it can be shown (Condon & Shortley 35, page 64) that

$$\langle J_{1z} \rangle = \left\langle \frac{J^2 + J_1^{\,2} + J_2^{\,2}}{2J^2} J_z \right\rangle = \left[\frac{j(j+1) + j_1(j_1+1) - j_2(j_2+1)}{2j(j+1)} \right] J \ldots (\alpha)$$

if j, j_1, j_2, j_z are all good quantum numbers. Then the magnetic moment of a nucleon in an orbit is given in nuclear magnetons by

$$\mu = g_\ell \left[\frac{j(j+1) + \ell(\ell+1) - \frac{3}{4}}{2j(j+1)} \right] j + g_s \left[\frac{j(j+1) + \frac{3}{4} - \ell(\ell+1)}{2j(j+1)} \right] j \ldots (\beta)$$

where $j = \ell \pm \frac{1}{2}$.

The g-factor is obtained as $g_j = \frac{\mu}{j}$. From this point on all magnetic moment values will be quoted in nuclear magnetons. Table II gives the magnetic moments and g-factors for nucleons in various single particle states using both the measured values and the Dirac values for g_s.

III Magnetic Moments on the Individual Particle Model

(A) Even-Even Nuclei

Even-even nuclei are those with an even number of protons and an even number of neutrons. It is a well known fact (Mack 50) that these nuclei have zero spin and consequently zero magnetic moment. This lends weight to the assumption that in the ground state the spins of pairs of protons or neutrons are anti-parallel and that in some way the orbital contributions of these pairs to the total angular momentum tends to cancel. From the point of view of L-S coupling and the liquid drop model this is reasonable because the ground state of a spinning drop might be expected to have zero angular momentum. In the alpha-particle model the nucleon orbits are postulated as localized in space and a net angular momentum means that one of the alpha particles is contributing kinetic energy (Gamow & Critchfield 49) so that this would not be the ground state of the nucleus. Calculations based on the Hartree individual particle model (Feenberg & Phillips 37b) indicate that as a general rule the ground state has the lowest total orbital angular momentum. Thus we may qualitatively account for the lack of nuclear spin in even-even nuclei in all of the models. At worst we can take it as an assumption for the analysis which follows.

(B) Odd Nuclei - The Schmidt Lines

The basic assumption here is that there is no coupling between the odd nucleon and the core. Since the core is equivalent to an even-even nucleus it has spin zero and hence the odd nucleon is entirely responsible for the spin (I) and magnetic moment (μ) of the nucleus.

Thus: $$I = \ell \pm \tfrac{1}{2}$$

and by formula (β) for the magnetic moment of a spin-half particle in an orbit we have the two cases;

$$\mu = \mu_s + g_\ell (I - \tfrac{1}{2}) \quad , \; I = \ell + \tfrac{1}{2}$$

$$\mu = \left(\frac{I}{I+1}\right)\left[-\mu_s + g_\ell(I+\tfrac{3}{2})\right], \; I = \ell - \tfrac{1}{2}$$

where g_ℓ and μ_s are the appropriate values for a proton or a neutron (given in table I). Plotting μ vs. I for the odd proton and odd neutron cases separately we get two pairs of lines (labelled by S in figures I and II). These are called the Schmidt lines after the man who first derived them (Schmidt 37). Using the Dirac values for the spin g-factors we obtain another pair of lines which de Shalit (51) calls the Dirac lines. These lines fall between the Schmidt lines and are labelled by D in figures I and II. The points for plotting all of these lines are those tabulated in table II.

The experimental* values of μ for odd nuclei lie in between the Schmidt and Dirac lines with very few exceptions (Figure I), whereas on the strict single particle model using the free particle g_s values they would be expected to fall on the Schmidt lines. Two conclusions are usually drawn from this distribution:

(1) The total spin of these nuclei is indeed one-half, as assumed.

(2) A value of ℓ can be unambiguously assigned to the single particle state of the odd nucleon from the measured spin and magnetic moment of the nucleus because the magnetic moment is always in one or the other of the regions corresponding to $\ell = I \pm \frac{1}{2}$.

The shell model of the nucleus (Feenberg & Hammack 49; Mayer 49, 50; Nordheim 49) is usually constructed by the use of these j (=I) and ℓ values for the specification of the single particle states. This model correlates a wide body of nuclear data including the magic numbers, beta decay transitions and isomeric states, and this lends support to the above procedure. The fact that the μ do not fall on the Schmidt lines then remains to be explained.

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* All μ values used are taken from Klinkenberg's review article R.M.P. 24, 63, 1952.

(C) Odd-Odd Nuclei - j-j Coupling

The assumption here is that we have a spinless even-even core and two nucleons (a neutron and a proton) acting as individual particles. Using the shell model assignments for these particles we can calculate the magnetic moment from the measured spin by using j-j coupling (Feenberg 49, Klinkenberg 52).

$$I = \bar{j}_n + \bar{j}_p$$

$$\mu = \langle (g_p j_p + g_n j_n)_z \rangle$$

where g_n and g_p are taken from table II and depend on the states of the neutron and proton respectively.

$$\mu = \tfrac{1}{2}(g_n + g_p) \langle I_z \rangle + \tfrac{1}{2}(g_n - g_p) \langle (\bar{j}_n - \bar{j}_p)_z \rangle$$

Using formula (a) we obtain

$$\mu = \tfrac{1}{2}(g_n + g_p) I + \tfrac{1}{2}(g_n - g_p) \left[\frac{(j_n - j_p)(j_n + j_p + 1)}{I(I + 1)} \right]$$

In table III the calculated μ values are compared to the measured ones. Note the particular simple form of μ when $j_n = j_p$. From the table we see that in these cases the calculated and measured values agree much better than when $j_n \neq j_p$. In addition. the agreement is much better than in the case of the odd nuclei and the Schmidt lines. This is somewhat puzzling because the single particle states were obtained from these same Schmidt lines.

The puzzle can be resolved (Talmi 51) if the assumption is made that the deviations from the Schmidt lines are due to quenching of the spin g-factors, i.e. inside nuclear matter the spin g-factor of a nucleon is not the same as it is in free space but lies somewhere between the Dirac value and the measured value in free space. In the case discussed above, $\mu = \frac{1}{2}(g_n + g_p)$, and this is the same as the free particle value because the quenching is equal and opposite for neutrons and protons.

Mizushima and Umezawa (52) have extended the j-j coupling model by considering closed shell ± 3 nucleons, the closed shells being those given by the shell model (Klinkenberg 52). Assuming that isotopic spin is a good quantum number and using group theoretical methods they have obtained the values shown in table IV. For closed shell ± 1 nucleon they obtain the Schmidt values and in addition the success with odd-odd nuclei is preserved.

(D) Interaction* Moments and Quenching

One of the attempts at explaining the deviations from the Schmidt lines while retaining the single particle interpretation involves the possibility of interaction magnetic moments. It was

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* The term interaction moment has been adopted in preference to the usually used term, exchange moment, because it has more general validity (Austern & Sachs 51).

first pointed out by Siegert (37) that the existence of charge exchange forces between nucleons together with the differential law of conservation of charge requires that electric currents must flow in the space between interacting protons and neutrons. These exchange currents can interact with a magnetic field and this interaction implies a magnetic moment which is additional to the spin and orbital magnetic moments.

A model of this effect can be constructed on the basis of the meson theory of nuclear forces. If the nuclear force is indeed due to a charged meson field surrounding the nucleons, then it is the virtual exchange of these charged mesons between nucleons which constitutes the exchange current and is responsible for at least part of the nuclear force. These charged mesons can also be used to qualitatively explain the anomalous magnetic moments of the proton and neutron. The physical nucleon is assumed to consist of a Dirac particle surrounded by a cloud of charged mesons and it is this cloud which produces the anomalous part of the magnetic moments. However, detailed meson theories have not yet been able to explain quantitatively either the nuclear force or the anomalous magnetic moment.

It turns out (Osborne and Foldy 50) that the exchange force does not completely determine the exchange current. Only the longitudinal (irrotational, zero curl) part is uniquely determined while the transverse (solenoidal, zero divergence) part depends on more detailed assumptions concerning the theory used to calculate the exchange current. In particular it depends on which meson theory is used.

Miyazawa (51) has calculated the interaction moment contribution for heavy nuclei, approximating the core by a Fermi gas. The transverse part of the moment was calculated using symmetrical pseudoscalar meson theory and the longitudinal part, which depends directly on the exchange force, was calculated using the phenomenological potential of Christian and Hart (50). He was able to match the general trend of the deviations by a small readjustment of the constants in his expressions.

It is possible to proceed without making any reference to the exact nature of the meson theory by using the following device (Stern 49). In deriving the two body interaction from meson theory a function of the interparticle distance related directly to the more detailed properties of the theory is first derived and then various operations are applied to it to determine the spatial dependence of the central and tensor parts of the interaction potential and the spatial dependence of both the longitudinal and transverse parts of the interaction moment operator. Starting from a phenomenological interaction potential we can invert the operation and calculate the above function, i.e. we get in effect a phenomenological description which in some way includes the details of a meson theory. This function can then be used to determine the transverse part of the interaction moment. Preliminary calculations (Villars, unpublished) using the Fermi gas model for the core have given interaction moments which are too small by a factor of ten and specialization to a finite nucleus does not change this result.

Russek and Spruch (to be published in the physical review) have carried the phenomenological method a step farther. By invariance and symmetry considerations one can derive the most general interaction moment operators which can arise from a charge exchange potential (Osborne & Foldy 50). Under the restriction that these operators behave properly under time reversal (Kynch 51) and with the arbitrary requirement that they be derivable from a second order meson calculation, only three contributions remain. The two transverse contributions are determined only to within arbitrary functions of the interparticle distance. Using Gaussian wells with the usual nuclear range for all three space functions and shell model harmonic oscillator wave functions for the particles, Russek and Spruch were able to fit the general trend of the deviations by selecting proper strengths for the three contributions.

In addition they were able to qualitatively confirm other regularities among the magnetic moments. Their expressions for the interaction moments of odd proton nuclei are proportional to the neutron density so that the addition of two protons to an odd proton nucleus lowers the neutron density and hence gives a smaller interaction moment, provided the ℓ and j of the odd proton are not changed by the addition. A similar situation exists for the odd neutron nuclei. The experimental evidence shows that in most cases the addition of two neutrons to an odd neutron nucleus or two protons to an odd proton nucleus tends to push the magnetic moments towards the Schmidt lines (table V). The above mechanism also explains why odd neutron nuclei tend to deviate less from the Schmidt lines than do odd proton nuclei with the same ℓ and j and comparable A.

There is another way of looking at interaction moments which has an interesting extension. This is the quenching mechanism (Bloch 51, de Shalit 51) already discussed in connection with j-j coupling. The meson cloud around a nucleon is the cause of its anomalous magnetic moment and when other nucleons are present the clouds are modified so that the magnitude of the moment of a single nucleon is less than its value in free space. This would cause the magnetic moment to fall between the Schmidt and Dirac lines as is observed. It has been shown by Miyazawa (51) that the quenching effect is equivalent to the interaction moment for the usual type of meson theory. However additional quenching may be obtained using nonlinear meson theories (Schiff 51) and it is believed (Bloch 51) that this may be important.

V Modification of the Independent Particle Model

(A) Evidence

There are two reasons why it seems necessary to modify the strict independent particle model. These have to do with the seemingly random scatter of the experimental points between the Schmidt lines (interaction moments have only been able to give the general trend) and the large quadrupole moments (Q) of many nuclei. Most of the modifications so far have dealt with Q.

The quadrupole moments of heavy nuclei, especially those near Z = 73, cannot be accounted for by a single particle model. In some cases the measured values are too large by as much as a factor of thirty-five and even in nuclei with closed shell ± 1 nucleon Q is sometimes several times larger than the maximum value obtainable from a single particle in an orbit (Townes, Foley & Low 49). This suggests that as many as 30 protons must be responsible for Q, and points towards some sort of distortion of the core (Bohr 51a). In this event it is possible for the core to have angular momentum and so contribute to the magnetic moment.

(B) Liquid Drop Model

If we make the simple assumption that the nucleus acts as a liquid drop with a uniform charge density we immediately run into difficulty. By direct classical calculation the magnetic moment of a spinning drop is given as $\mu = \frac{Z}{A} I$ (Way 39) which is the right order of magnitude but is by no means as detailed as necessary.

In addition, since a spinning drop has the form of a pancake, it does not predict positive quadrupole moments which are observed in some cases.

It has been pointed out (Inglis 38) that if we assume that the orbital angular momentum is shared by many particles as in the liquid drop model, effective orbital g-factors of 1/8 for the neutron and 7/8 for the proton give lines on the Schmidt diagram which fit the general trend of the data, but there are no convincing arguments to justify this. Margenau and Wigner (40) adopted the pure liquid drop model value of Z/A for both neutron and proton orbital g-factors but this results in lines which do not come as close to the data as do the Schmidt lines.

(C) Spheroidal Model

It should not be necessary to abandon completely the individual particle model because of its many successes. We can retain the single particle interpretation provided the core is allowed to contribute some of the quadrupole moment. This can be achieved on the basis of the shell model by assuming that the average potential for the nucleons has a spheroidal shape (Rainwater 50). Then for odd nuclei the core will also have a spheroidal shape, contributing to* the quadrupole moment and also to the magnetic moment* by interaction with the odd particle.

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*The nucleus has no electric dipole moment or magnetic quadrupole moment, hence there is no confusion.

Preliminary calculations have been made by Foldy and Milford (50) assuming that the odd particle polarizes the core, giving rise to surface waves. This causes an exchange of angular momentum between the core and the particle and the results are an improvement on the Schmidt lines except for $I = \frac{1}{2}$ and $I > 5/2$. The exact significance of this calculation is not clear because of the approximations.

If we ignore the origin of the core deformations and just quantize the angular momentum in analogy with molecular structure (Bohr 51a) we obtain some interesting results. Assuming that the shape of the core changes slowly with respect to the single particle motion and that the nucleus does not rotate as a rigid body (to avoid low lying excited states) there are several possible kinds of coupling between the orbital and spin angular momenta of the single particle, the angular momentum of the core and the total angular momentum. Most of these give some improvement on the Schmidt lines and at the same time allow for a large quadrupole moment. In particular if the orbital angular momentum of the single particle is coupled to the angular momentum of the core to give a total orbital angular momentum $\bar{L}$, and this is then added to the spin $\bar{s}$ of the single particle, we arrive at something very close to the Schmidt values while retaining our explanation of the large quadrupole moments. The other coupling cases give lines which are qualitatively somewhat better than the Schmidt lines but of course do not account for the scatter of the points.

(D) Mixture of States $\ell = I \pm \frac{1}{2}$

We could assume that the reason the points lie between the Schmidt lines is that there is a mixture of the single particle states of opposite parity $\left[\ell = I \pm \frac{1}{2} \right]$. This would require ambiparity of the core and in addition does not agree with the shell model which demands a large energy difference between the two states. In spite of these arguments Schawlow and Townes (51) have considered this assumption for nuclei with $Z < 50$ where one might assume that coulomb effects are small so that neutron and proton wave functions are similar. Hence for the odd nuclei with either Z or N equal to some given odd number one might expect that the spin I would be the same and that the magnetic moments would lie at the same fractional distance between the appropriate Schmidt lines. This in fact does occur for the sets of nuclei with Z or N equal to 5, 13, 15, 17, 29, 37, 49, to within $\pm$ 0.1 n.m. and on this basis Schawlow and Townes predict some magnetic moments which have not yet been measured (table VI).

(E) Remarks

Several models have been able to qualitatively account for the general characteristics of the data on magnetic moments of heavy nuclei but none of them offer any explanation as to the cause of the scatter of the points. It seems that this depends on the detailed nature of the nuclear wave functions and hence on the exact form of the nuclear forces. The theory of exchange moments is the only one which can make use of these wave functions and so it seems this idea holds the most promise although we must certainly include something of the nature of a spheroidal core to give agreement with the large measured quadrupole moments. Probably a combination of all these possibilities with a better understanding of the theory behind the nuclear two body force will be necessary before exact quantitative agreement is achieved. In addition there is always the possibility that it may be necessary to bring in many body forces as exemplified by non-linear meson theories.

VII <u>Light Nuclei</u>

(A) Conjugate Nuclei

A pair of odd nuclei of the same Mass number (A) are said to be conjugate if the number of neutrons in one is equal to the number of protons in the other and vice versa. That is to say we could turn a nucleus into its conjugate by changing all neutrons to protons and all protons to neutrons. It is immediately seen that nuclei with Z=N=A/2 are <u>self conjugate</u>.

For conjugate nuclei it is possible to obtain a relation for the sum of the two magnetic moments if the usual light nucleus assumptions are made.

(1) The coulomb force is negligible compared to nuclear forces.

(2) The nuclear forces are charge independent.

Under these conditions the wave functions of two conjugate nuclei should be identical, which means their Spin ($\bar{I}$) should be the same. With this in mind we may write the magnetic moments in their most general form (Sachs 46).

$$\mu_1 = \left\langle \left(\sum_{\pi_1} \ell_{\pi_1} + g_{s\pi} \sum_{\pi_1} \bar{s}_{\pi_1} + g_{s\nu} \sum_{\nu_1} \bar{s}_{\nu_1} \right)_z \right\rangle$$

$$\mu_2 = \left\langle \left(\sum_{\pi_2} \ell_{\pi_2} + g_{s\pi} \sum_{\pi_2} \bar{s}_{\pi_2} + g_{s\nu} \sum_{\nu_2} \bar{s}_{\nu_2} \right)_z \right\rangle$$

where $g_{s\pi}$ and $g_{s\nu}$ are the spin g-factors of the proton and neutron respectively. Since the wave functions of 1 and 2 are the same

$$(\mu_1 + \mu_2) = \left\langle \left(\sum_{\pi} \ell_{\pi} + g_{s\pi} \sum_{\pi} \bar{s}_{\pi} + g_{s\nu} \sum_{\nu} \bar{s}_{\nu} \right)_z \right\rangle$$

$$= \left\langle (\bar{L} + (g_{s\nu} + g_{s\pi}) \bar{S})_z \right\rangle \qquad (1)$$

Now the nuclear spin is $\bar{I} = \bar{L} + \bar{S}$ and by Algebraic rearrangement

$$(\mu_1 + \mu_2) = \tfrac{1}{2}(1 + g_{s\nu} + g_{s\pi}) \left\langle (\bar{L} + \bar{S})_z \right\rangle + \tfrac{1}{2}(1 - g_{s\nu} - g_{s\pi}) \left\langle (\bar{L} - \bar{S})_z \right\rangle$$

$$(\mu_1 + \mu_2) = (\tfrac{1}{2} + \mu_d^{\circ})\, I + (\tfrac{1}{2} - \mu_d^{\circ}) \left\langle L_z - S_z \right\rangle, \quad \mu_d^{\circ} = \tfrac{1}{2}(g_{s\pi} + g_{s\nu})$$

and using the formula (α) separately on L_z and S_z

$$(\mu_1+\mu_2) = (\tfrac{1}{2}+\mu_d{}^{\circ})\ I + (\tfrac{1}{2}-\mu_d{}^{\circ}) \left\langle L^2 - S^2 \right\rangle \frac{I}{I(I+1)} \qquad (2)$$

Now we expand the common wave function of the nuclei in terms of eigenfunctions of L and S.

$$\Psi = \sum_{LS} C_{LS}\ \Psi_{LS}\ , \qquad \sum |C_{LS}|^2 = 1 \qquad (3)$$

where $C_{LS} = 0$ unless $I = |L-S|\ ,........,\ (L+S)$.

Then

$$\left\langle L^2 - S^2 \right\rangle = \sum_{LS} |C_{LS}|^2 \left[L(L+1) - S(S+1) \right]$$

which, together with (2) and (3) gives

$$\frac{4I(I+1)}{(1-2\mu_d{}^{\circ})} \left(\frac{\mu_1+\mu_2}{2I} - \mu_d{}^{\circ} \right) = \sum_{LS} |C_{LS}|^2 \left[I(I+1)+L(L+1)-S(S+1) \right] \qquad (4)$$

In this expression $\mu_d{}^{\circ} = \mu_{s\pi} + \mu_{s\nu}$ is a measured quantity as is the spin I. If we had outside information as to the proportions of the various LS states (the C_{LS}) we could then calculate the quantity $(\mu_1 + \mu_2)$. In practice it is more convenient to work in the reverse direction.

(B) The Deuteron H^2

This is the simplest example of a self conjugate nucleus. Using the measured values I = 1 and $\mu_d = 0.8573_6$ in equation (4) with $\mu_1 = \mu_2 = \mu_d$ we obtain the relation

$$0.235_9 = 6|c_{21}|^2 + 2|c_{11}|^2 + 4|c_{10}|^2 + 0|c_{01}|^2 \qquad (5)$$

Notice that this equation does not involve $|c_{01}|^2$, which can be obtained from $\sum |c_{LS}|^2 = 1$ once the others are known.

It is well known that the deuteron ground state is mostly $^3S_1$ and since parity is a good quantum number we must rule out the $^1P_1$ and $^3P_1$ states and set $|c_{11}|^2 = |c_{10}|^2 = 0$. Then relation (5) requires that

$$|c_{21}|^2 = .0393_1 = \text{fraction of } {}^3D_1 \text{ state} .$$

Note that if $\mu_d = \mu_d^\circ$, the sum of the neutron and proton magnetic moments, we would have obtained the result that there is no D state present. In fact the first prediction of this non-additivity was given by Rarita and Schwinger (41) in connection with the quadrupole moment of the deuteron which would be zero if there were no D state present. Fitting the measured quadrupole moment they required that the fraction of D state be 0.039 which agrees very well with the value obtained above. Later more extensive calculations have not changed this estimate.

However it is thought that this agreement may be largely fortuitous (Bloch, Nicodemus & Staub 48) because relativistic effects resulting from the motion of the nucleons could cause deviations from additivity in the same direction and of comparable magnitude as those introduced through the admixture of D state. The estimates vary widely (Margenau 40, Caldirola 46, Sachs 47b, Primakoff 47, Breit 47, Breit & Bloch 47) depending on the assumptions about the neutron proton force, and thus no definite conclusions can be drawn.

(C) Other Self Conjugate Nuclei

Other light nuclei which might be expected to obey the conditions necessary for application of formula (4) are Li^6, B^{10} and N^{14}

Nucleus	I	g	$\sum_{LS} \lvert C_{LS}\rvert^2 \left[I(I+1)+L(L+1)-S(S+1)\right]$	States (S = 0,1)
Li^6	1	.8219	.610	$^3S_1$ $^1P_1$ $^3P_1$ $^3D_1$
B^{10}	3	$.600_1$	17.676	$^3D_3$ $^1F_3$ $^3F_3$ $^3G_3$
N^{14}	1	$.4036_5$	5.016	$^3S_1$ $^1P_1$ $^3P_1$ $^3D_1$

We make the <u>assumption</u> that states of total spin higher than S = 1 are not present. This is equivalent to saying that at most two particles are responsible for the total spin while all the other spins cancel out. Then using (4) one can obtain upper limits for the percentages of the various states by setting the others temporarily equal to zero (Sachs 46).

This information might be useful in conjunction with other details of the nuclear wave functions.

(D) The Conjugate Pair $He^3 - H^3$

The pair $He^3 - H^3$ is the simplest example of conjugate nuclei. The measured quantities are $I = \frac{1}{2}$ and $\mu_1 + \mu_2 = .8512_3$ which, on using equation (4), gives

$$0.11_3 = 3\left|C_{2,\frac{3}{2}}\right|^2 - \left|C_{1,\frac{3}{2}}\right|^2 + 2\left|C_{1,\frac{1}{2}}\right|^2$$

If we assume that $|C_{0,\frac{1}{2}}|^2$ is the largest component and set

$$\left|C_{1,\frac{3}{2}}\right|^2 + \left|C_{1,\frac{1}{2}}\right|^2 = 0$$

we arrive at :-

$$\left|C_{2,\frac{3}{2}}\right|^2 = .037_6 = \text{fraction of } {}^4D_{\frac{1}{2}} \text{ state}$$

which agrees with the value estimated by Gerjuoy & Schwinger (42) by fitting the binding energy variationally.

Making symmetry assumptions about the triton (H^3) wave functions Sachs & Schwinger (46) derive :-

$$\mu(H^3) = \mu_p - \tfrac{3}{2}\left|C_{2,\frac{3}{2}}\right|^2 (\mu_n + 2\mu_p - \tfrac{1}{2}) = 2.714 \text{ n.m.}$$

$$\mu(He^3) = \mu_n - \tfrac{3}{2}\left|C_{2,\frac{3}{2}}\right|^2 (2\mu_n + \mu_p - 1) = -1.861 \text{ n.m.}$$

where we have used $\left|C_{2,\frac{3}{2}}\right|^2 = 0.37_6$ as estimated above.

As is expected these two values add up to the correct sum ($\mu(H^3) + \mu(He^3) = .853$) but their individual values are out by $\pm$ 0.265 n.m. The measured value of $\mu(H^3)$ is greater than μ_p and not less as is predicted here.

This discrepancy might be due to large admixtures of ^4P and ^2P states (Sachs 47a,c, 48a) but even in the most extreme case this cannot quite account for the observed moments (Anderson 48, Avery & Sachs 48).

We are then led to consider the interaction moments discussed in Section III (D). The theory predicts that these will be equal and opposite for conjugate nuclei which is necessary to fit the above discrepancy. Sachs (Sachs 48b) has calculated the longitudinal part of the interaction moment using a phenomenological exchange potential and his value of 0.04 n.m. is too small. This can be interpreted to mean that most of the interaction moment comes from the transverse part of the exchange current which depends on the details of the theory used to calculate the exchange interaction. In particular it depends on which meson theory one uses.

The transverse interaction moment has been calculated using the Moller-Rosenfeld mixture (Thellung & Villars 48) (Their value is too small by a factor of ten.) and agreement is not obtained. But using symmetrical pseudoscalar theory Villars (47) obtains the range of values 0.08 → 0.31 n.m, which is consistent with the "observed" interaction moment of 0.265 n.m. This has been interpreted as evidence in favor of the existence of interaction moments, but it is by no means conclusive because of the well known difficulties in meson theory and because of the possibility of relativistic effects.

TABLE I

NUCLEON MAGNETIC MOMENTS and g-FACTORS

	μ_s(measured)	g_s(measured)	g_s(Dirac)	g_ℓ
proton	2.7926	5.5852	2	1
neutron	-1.9129	-3.8258	0	0

TABLE II

SINGLE PARTICLE MAGNETIC MOMENTS and g-FACTORS

j	ℓ	State	μ_n	μ_p	g_n	g_p	μ_n^D	μ_p^D
$\frac{1}{2}$	0	$s_{\frac{1}{2}}$	-1.913	2.793	-3.826	5.585	0.0000	1.000
	1	$p_{\frac{1}{2}}$	0.638	-0.264	1.275	-0.528	0.0000	0.333
$\frac{3}{2}$	1	$p_{\frac{3}{2}}$	-1.913	3.793	-1.275	2.528	0.0000	2.000
	2	$d_{\frac{3}{2}}$	1.148	0.124	0.765	0.083	0.0000	1.200
$\frac{5}{2}$	2	$d_{\frac{5}{2}}$	-1.913	4.793	-0.765	1.917	0.000	3.000
	3	$f_{\frac{5}{2}}$	1.366	0.863	0.547	0.345	0.000	2.143
$\frac{7}{2}$	3	$f_{\frac{7}{2}}$	-1.913	5.793	-0.547	1.655	0.000	4.000
	4	$g_{\frac{7}{2}}$	1.489	1.717	0.425	0.491	0.000	3.111
$\frac{9}{2}$	4	$g_{\frac{9}{2}}$	-1.913	6.793	-0.425	1.510	0.000	5.000
	5	$h_{\frac{9}{2}}$	1.565	2.624	0.348	0.583	0.000	4.091
$\frac{11}{2}$	5	$h_{\frac{11}{2}}$	-1.913	7.793	-0.348	1.417	0.000	6.000
	6	$i_{\frac{11}{2}}$	1.619	3.560	0.294	0.647	0.000	5.077

TABLE III

MAGNETIC MOMENTS OF ODD-ODD NUCLEI BY j-j COUPLING

nucleus	I	neutron state	proton state	μ(calc.)	μ(observed)
Li^6	1	$p_{\frac{3}{2}}$	$p_{\frac{3}{2}}$	0.63	0.8221
B^{10}	3	$p_{\frac{3}{2}}$	$p_{\frac{3}{2}}$	1.88	1.801
N^{14}	1	$p_{\frac{1}{2}}$	$p_{\frac{1}{2}}$	0.37	0.403
Na^{22}	3	$d_{\frac{5}{2}}$	$d_{\frac{5}{2}}$	1.73	1.7458
K^{40}	4	$f_{\frac{7}{2}}$	$d_{\frac{3}{2}}$	-1.68	-1.29
Rb^{86}	2	$g_{\frac{9}{2}}$	$f_{\frac{5}{2}}$	-2.12	-1.68
Lu^{176}	9 10 11	$i_{\frac{13}{2}}$	$h_{\frac{11}{2}}$	3.94 4.60 5.25	4.2 .8

TABLE IV

MAGNETIC MOMENTS OF CLOSED SHELL ± 3 NUCLEI

nucleus	I	state	T	μ(calculated)	μ(observed)	μ(Schmidt)
Li^7	$\frac{3}{2}$	$(2p_{\frac{3}{2}})^3$	$\frac{1}{2}$	3.07	3.26	3.79
Be^9	$\frac{3}{2}$	$(2p_{\frac{3}{2}})^{-3}$	$\frac{1}{2}$	-1.14	-1.18	-1.91
Cl^{35}	$\frac{3}{2}$	$(3d_{\frac{3}{2}})^3$	$\frac{3}{2}$	0.30	0.82	0.12
Mg^{25}	$\frac{5}{2}$	$(3d_{\frac{5}{2}})^{-3}$	$\frac{1}{2}$	-1.06 2.76	-0.96	-1.91

TABLE V

VARIATIONS IN THE DEVIATIONS OF THE MAGNETIC MOMENTS FROM THE SCHMIDT LINES FOR NUCLEI OF THE SAME AND j DIFFERING BY TWO NUCLEONS OF THE ODD NUCLEON TYPE

odd Z		
Cl^{37} K^{39}	$d_{\frac{3}{2}}$	.29
Cs^{137} La^{139}	$g_{\frac{7}{2}}$	.06
odd N		
Mo^{95} Mo^{97}	$d_{\frac{5}{2}}$	.02
Cd^{111} Cd^{113}	$s_{\frac{1}{2}}$	.02
Sn^{115} Sn^{117}	$s_{\frac{1}{2}}$	.08
Sn^{117} Sn^{119}	$s_{\frac{1}{2}}$	.05
Te^{123} Te^{125}	$s_{\frac{1}{2}}$	.15
Ba^{135} Ba^{137}	$d_{\frac{3}{2}}$	.10
Nd^{143} Nd^{145}	$f_{\frac{7}{2}}$	-.38*
Sm^{147} Sm^{149}	$?_{\frac{5}{2}}$	-.05*

*The minus sign means that the addition of two neutrons has pushed the magnetic moment away from the Schmidt lines instead of towards them.

TABLE VI

MAGNETIC MOMENTS OF ODD NUCLEI BY INTERPOLATION BETWEEN THE SCHMIDT LINES

odd number	nucleus	I	(predicted)
11	$_{10}Ne^{21}$	$\frac{3}{2}$	-.60
19	$_{16}S^{35}$	$\frac{3}{2}$	-1.0
23	$_{20}Ca^{43}$	$\frac{7}{2}$	-1.38
25	$_{22}Ti^{47}$	$\frac{5}{2}$	-1.79
27	$_{22}Ti^{49}$	$\frac{7}{2}$	-0.96
31	$_{26}Fe^{57}$	$\frac{3}{2}$	-0.66
33	$_{28}Ni^{61}$	$\frac{3}{2}$	0.10
41	$_{32}Ge^{73}$	$\frac{9}{2}$	-1.39
51	$_{40}Zr^{91}$	$\frac{5}{2}$	-0.99

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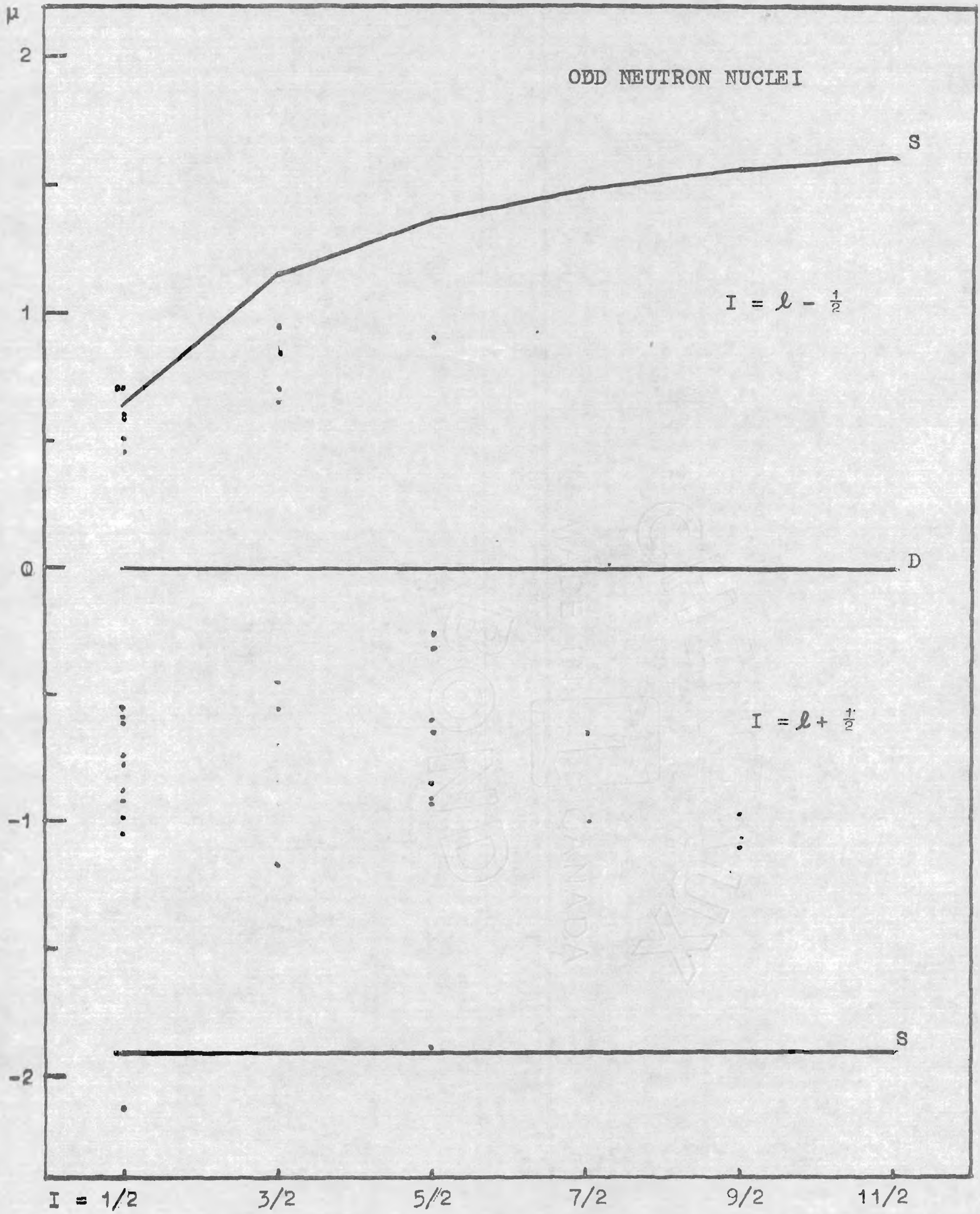

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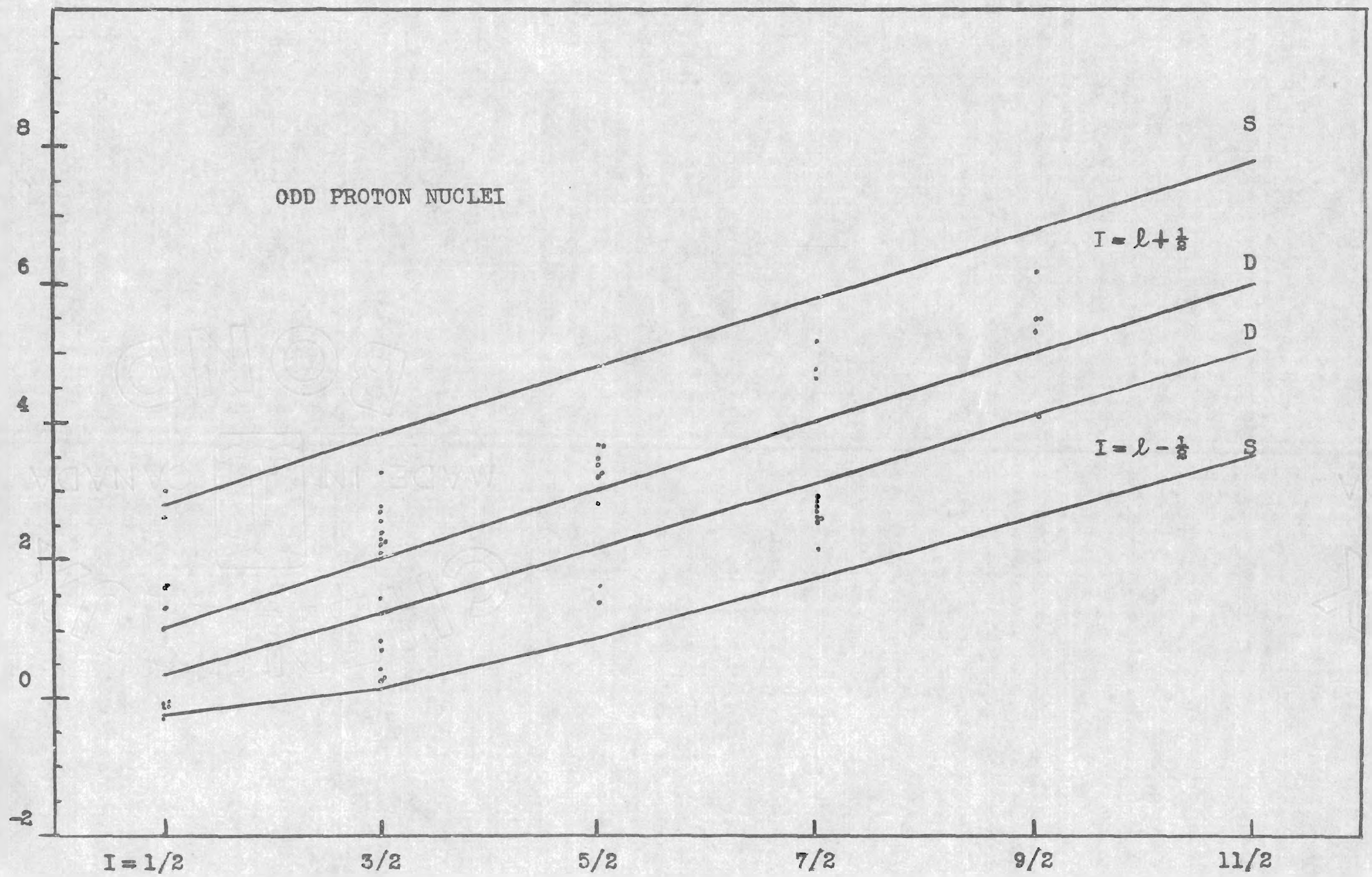
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