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The Residual Zonal dynamics in a Toroidally Rotating Tokamak

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Abstract

Zonal flows, initially driven by ion-temperature-gradient turbulence, may evolve due to the neoclassic polarization in a collisionless tokamak plasma. In this presentation, the form of the residual zonal flow is presented for tokamak plasmas rotating toroidally at arbitrary velocity. The gyro-kinetic equation is analytically solved to give the expression of residual zonal flows with arbitrary rotating velocity. The zonal flow level decreases as the rotating velocity increases. The numerical evaluation is in good agreement with the previous simulation result for high aspect ratio tokamaks.

I. INTRODUCTION

Zonal flows (ZFs) in magnetic confinement devices are symmetric low frequency electrostatic perturbations with long radial wave length. They are on the one hand excited by the nonlinear interaction of drift wave turbulence, and on the other hand regulate the turbulence level through the radially sheared plasma flow and meanwhile as a turbulence energy sink[1,2]. A recent numerical fluid simulation confirmed that ZFs play a dominant role in the saturation of turbulence in the core of tokamak plasmas where the safety factor is low3. Recent gyro-kinetic simulations of ion temperature gradient (ITG) modes indicated that the plasma heat flux grows to a high level without saturation when the plasma pressure ratio exceeds a threshold. The mechanism behind this phenomenon is the decrease of zonal flow level caused by the magnetic flutter in plasmas with high thermal/magnetic pressure ratio[4,5]. The reduction of turbulence transport levels was also observed in recent stellarator experiments as ZFs presented[6]. Because of the significant role played by ZFs in plasma turbulence evolution, the investigation of their driving, damping and time evolution is thus of critical importance in determination of turbulence levels for a tokamak discharge. Hence, in the past two decades, ZFs remain to be one of active research areas in fusion plasmas.

The long time (much longer than the ion transit/bounce time) evolution of the flow is determined by the neoclassic polarization, and is expressed as

$$\phi(t) = \left(1 + 1.6q^2 / \sqrt{\varepsilon}\right)^{-1} \phi(0) \quad (1)$$

in a large aspect ratio circular cross section approximation, where q is the safety factor and ε is the inverse aspect ratio. This result is called the residual zonal flow in later publications[8-10]. Rosenbluth and Hinton further suggested that this result be adopted in gyro-fluid codes to improve turbulence simulations.

Recent experiments have revealed that plasma rotation, or in other words the plasma mean mass flow, seems ubiquitous in tokamak plasmas, even without momentum injection[11,12]. Hence, it is necessary to extend the study for static plasmas to that for plasmas with rotations. The eigen-value problem of geodesic acoustic modes in such toroidally rotating plasmas was investigated by some authors using fluid models. The mean flow effect on zonal flow generation in slab geometry was considered by Lashkin[15]. Recent kinetic numerical simulations for rotating plasmas indicated that the residual zonal flow level decreases with increasing rotation[16].

In this work, we present a general expression for residual ZFs in a collisionless tokamak plasma rotating toroidally at arbitrary velocity. The gyro-kinetic equation for the rotating plasma is firstly solved as an initial value problem for high and low Mach numbers. Then a general form of residual ZFs in the intermediate velocity region is introduced through interpolation. The result in the present work is in good agreement with the kinetic numerical simulation for high aspect ratio tokamaks.

II. STARTING EQUATIONS

The linearized gyro-kinetic equation developed by Artun and Tang[18] for toroidally rotating plasmas is solved in response to an initial axisymmetric source potential driven by ITG turbulence. The velocity of equilibrium toroidal flow is $\mathbf{V} = \omega_R(\psi) R^2 \nabla\zeta$, with ζ the toroidal angle variable, R the major radius, ψ the label of magnetic surfaces and $\omega_R(\psi)$ the angular velocity. We introduce a new velocity variable $\mathbf{c} = \mathbf{v} - \mathbf{V}$ and define the guiding center position $\mathbf{X} = \mathbf{x} - \boldsymbol{\rho}$, where $\mathbf{x}$ is the particle position, $\boldsymbol{\rho} = \mathbf{b} \times \mathbf{c} / \Omega$ is the gyro-radius with $\mathbf{b}$ the unit vector along equilibrium magnetic field and $\Omega = eB/m$ is the gyro-frequency. The total particle distribution function is $f = F_0 + \delta f$, with F_0 the equilibrium distribution. The perturbation distribution δf is the summation of two parts, $\delta f = (e\phi)\partial F_0 / \partial E + \delta h$, where ϕ is the perturbation of electrostatic potential and E is the energy variable to be defined in Eq. (4). The equation for the non-adiabatic part δh, in the electrostatic case, is[17]

$$\left[\frac{\partial}{\partial t} + \left(c_{\|}\mathbf{b} + \mathbf{V} + \mathbf{C_D}\right)\cdot\frac{\partial}{\partial \mathbf{X}}\right]\delta h = -e\frac{\partial F_0}{\partial E}\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\frac{\partial}{\partial \mathbf{X}}\right)[J_0\phi(\mathbf{X})] - J_0\frac{e}{\Omega}\mathbf{b}\times\frac{\partial \phi}{\partial \mathbf{X}}\cdot\frac{\partial F_0}{\partial \mathbf{X}}$$
$$+ J_0\frac{e}{\Omega}\mathbf{b}\times\frac{\partial \phi}{\partial \mathbf{X}}\cdot\left[\left(c_{\|}\mathbf{b} + \mathbf{V}\right)\cdot\nabla\mathbf{V} + \nabla\mathbf{V}\cdot\left(c_{\|}\mathbf{b} + \mathbf{V}\right)\right]\frac{\partial F_0}{\partial E} + S_{nl} \qquad (2)$$

with the drift velocity and energy variable

$$\mathbf{C_D} = \frac{\mathbf{b}}{\Omega}\times\left[\frac{e}{m}\nabla\Phi_0 + \frac{c_\perp^2}{2}\nabla\ln B + \left(c_{\|}\mathbf{b} + \mathbf{V}\right)\cdot\left(\nabla\mathbf{V} + c_{\|}\nabla\mathbf{b}\right)\right] \qquad (3)$$

$$E = \frac{m}{2} c_{\parallel}{}^2 + \frac{m}{2} c_{\perp}{}^2 - \frac{m}{2} \mathbf{V}^2 + e\Phi_0 \qquad (4)$$

where $J_0 = J_0(k_\perp \rho)$ is the usual Bessel function, S_{nl} is the initial source contributed from the nonlinear interaction of ITG turbulence, Φ_0 is the equilibrium electrostatic potential. We have written potential perturbation in an eikonal form, *i. e.* $\phi(\mathbf{x}) = \phi_k \exp[iS(\mathbf{x}_\perp)]$, and defined $\mathbf{k}_\perp = \nabla S$. The equilibrium distribution is a function of two adiabatic invariables, the energy variable given by Eq. (4) and the magnetic moment variable defined as $\mu = (1/2B) m c_\perp{}^2$. The usual form of the equilibrium distribution is

$$F_0 = \frac{N_0(\psi)}{\mathbf{v}_T{}^2 \pi^{3/2}} \exp(-E/T) \qquad (5)$$

where $T = T(\psi)$ is temperature and $\mathrm{v}_T = (2T/m)^{1/2}$.

Thus far, particle species are not distinguished. For a simple plasma consisting of electrons and one species of ions with unit charge, the equilibrium quasi-neutrality condition requires the equilibrium potential to take the form[18-20]

$$\Phi_0 = \frac{m_i V^2}{2e(1+\tau^{-1})} \qquad (6)$$

where $\tau = T_e / T_i$. For plasmas with multiple ions the potential can only be obtained by a numerical procedure. In the present work, for simplicity, we adopt the form of Eq. (6) and the energy variable is thus written as

$$E = \frac{m}{2} c_{\parallel}{}^2 + \frac{m}{2} c_{\perp}{}^2 - \frac{m}{2(1+\tau)} \mathbf{V}^2 \qquad (7)$$

The perturbation distribution is also written in an eikonal form, *i. e.* $\delta h = \delta h_k \exp[iS(\mathbf{X}_\perp)]$. The perturbative potential is axisymmetric with toroidal mode number $n = 0$ and dominant poloidal mode number $m = 0$, so that the eikonal is only the function of ψ. It is obvious that terms involving $\mathbf{V} \cdot \frac{\partial}{\partial \mathbf{X}}$ and the second term on the right hand side of Eq. (2) are both equal to 0. Making use of the explicit form $\nabla\mathbf{V} = \omega_R R(\nabla R\nabla\zeta - \nabla\zeta\nabla R) + R^2\nabla\omega_R\nabla\zeta$, it is ready to obtain

$$\mathbf{C}_D \cdot \nabla\psi = \frac{mI}{e} c_\parallel \mathbf{b}\cdot\nabla\left(\frac{c_\parallel}{B}\right) + \frac{m}{e} c_\parallel \mathbf{b}\cdot\nabla\left(\omega_R{}^2\right) \qquad (8)$$

where we have taken the equilibrium magnetic field to be $\mathbf{B} = \nabla\zeta\times\nabla\psi + I\nabla\zeta$.

Eq. (2) is then reduced to

$$\left\{\frac{\partial}{\partial t} + \left[c_\parallel \mathbf{b}\cdot\nabla + c_\parallel \mathbf{b}\cdot\nabla(iQ)\right]\right\}\delta h_k = J_0 e\frac{F_0}{T}\frac{\partial\phi_k}{\partial t} + iJ_0\frac{mF_0}{T}S'\omega_R\phi_k c_\parallel \mathbf{b}\cdot\nabla R^2 + S_k F_0 \qquad (9)$$

where

$$Q = \frac{mS'}{e}\left(I\frac{c_\parallel}{B} + \omega_R R^2\right) \qquad (10)$$

Noticing that $S' = k_\perp / |\nabla\psi| = k_\perp / RB_p$, we get $Q \sim k_\perp\delta_b$, where δ_b is the ion orbit width. Q is used as a small parameter for the series expansions in the following section.

The potential is derived using the quasi-neutrality condition

$$-\frac{e}{T_i}n_0\phi_k + \int d^3\mathbf{v}J_0\delta h_{ik} = \frac{e}{T_e}n_0\phi_k + \int d^3\mathbf{v}\,\delta h_{ek} \qquad (11)$$

where the nonperturbative particle density is no longer a surface function, instead it is

$$n_0 = \int d^3\mathbf{v}F_0 = N_0(\psi)e^{\frac{mV^2}{2(T_i+T_e)}} \qquad (12)$$

Eqs. (9) and (11) form a closure to obtain the potential evolution. We have solved the long time evolution of potential for sonic flow cases in Ref. 17, and the form of residual ZFs is exactly the same as the R-H form to the order of $O(Q^2)$.

III. RESIDUAL ZONAL FLOWS AT LOW ROTATION VELOCITY

In this section, we consider a plasma rotating at very low speed, *i. e.* $\omega_R \sim \partial/\partial t \sim 0$ and the evolution time is much longer than the ion bounce/transit time. The leading order equation for ions, from Eq. (9), is then

$$\left[c_{\|}\mathbf{b}\cdot\nabla + c_{\|}\mathbf{b}\cdot\nabla(iQ)\right]\delta h_{k0} = 0 \qquad (13)$$

We have made an expansion of the distribution by the ordering of ω/ω_b. Writing the solution in the form $\delta h_{k0} = g\exp(-iQ)$, one obtains

$$\mathbf{b}\cdot\nabla g = 0 \qquad (14)$$

The next ordering components of Eq. (9) for ions yield

$$\frac{\partial}{\partial t}\left(ge^{-iQ}\right) + \left[c_{\|}\mathbf{b}\cdot\nabla + c_{\|}\mathbf{b}\cdot\nabla(iQ)\right]\delta h_{k1} = J_0 e\frac{F_{i0}}{T_i}\frac{\partial\phi_k}{\partial t} + iJ_0\frac{m_i F_{i0}}{T_i}S'\omega_R\phi_k c_{\|}\mathbf{b}\cdot\nabla R^2 + S_{ik}F_{i0} \quad (15)$$

Multiplying by e^{iQ} on both sides and taking the orbit average, one reaches

$$\frac{\partial}{\partial t}g = \overline{J_0 e^{iQ}}\frac{eF_{i0}}{T_i}\frac{\partial\phi_k}{\partial t} + i\frac{m_i F_{i0}}{T_i}S'\omega_R\phi_k\overline{J_0 e^{iQ}c_{\|}\mathbf{b}\cdot\nabla R^2} + \overline{e^{iQ}S_{ik}}F_{i0} \qquad (16)$$

where the orbit average is defined as

$$\overline{A} = \frac{\oint A\,dl/|c_{\|}|}{\oint dl/|c_{\|}|} \qquad (17)$$

the integration is carried out along the trajectory of particles while its poloidal projection forms a closed line.

The electron distribution is readily obtained from Eq. (9) to be

$$\delta h_{ek} = -(eF_{e0}/T_e)\phi_k \qquad (18)$$

For ions, we need to specify the source term. For ITG turbulence, we take $S_{ek} = 0$ and $S_{ik} = [\delta n(0)/n_0]\delta(t)$. The potential perturbation is accompanied by an initial density perturbation in a few gyro-periods due to the classical polarization and quasi-neutrality condition. Thus, we have[7] $\delta n(0) = (k_\perp\rho_i)^2(e/T_i)\phi_k(0)$, where $\rho_i = (T_i/m_i)^{-1/2}/\Omega_i$

Taking the Laplace transformation of Eq. (16), noting the time scale separation between the particle transition/bounce and the potential evolution, one obtains

$$G(p)=\frac{eF_{i0}}{T_i}J_0e^{iQ}\Phi_k(p)+\frac{1}{p}\left[\frac{eF_{i0}}{T_i}(k_\perp\rho_i)^2\overline{e^{iQ}}\phi_k(0)+i\frac{m_iF_{i0}}{T_i}S'\omega_R\overline{J_0e^{iQ}c_\parallel\mathbf{b}\cdot\nabla R^2}\Phi_k(p)\right]$$

(19)

where $G(p)=\int_0^{+\infty}g(t)e^{-pt}dt$ and the same operation to get $\Phi_k(p)$.

Making Laplace transformation and the surface average on Eq. (11), inserting Eqs. (18) and (19), one finally obtains

$$\Phi_k(p)=\frac{\Phi_k(0)(k_\perp\rho_i)^2\oint\frac{dl}{B}n_0}{p\left[\oint\frac{dl}{B}n_0-\oint\frac{dl}{B}\int d^3\mathbf{v}J_0e^{-iQ}\overline{J_0e^{iQ}}F_{i0}\right]-i\oint\frac{dl}{B}\int d^3\mathbf{v}\frac{m_iF_{i0}}{e}S'\omega_R\overline{J_0e^{iQ}c_\parallel\mathbf{b}\cdot\nabla R^2}}\qquad(20)$$

It is obvious that the second term in the denominator is zero since the term under integration is an odd function of velocity. The denominator can be further simplified. Using $d^3\mathbf{v}=d^3\mathbf{c}=\sum_{\sigma_v=\pm1}\frac{2\pi B}{m^2\,|c_\parallel|}d\mu dE$, we notice that

$$\oint\frac{dl}{B}\int d^3\mathbf{v}\overline{A}=\oint\frac{dl}{B}\int d^3\mathbf{v}A\quad(21)$$

Making the power expansion

$$J_0e^{iQ}\cong1-iQ-Q^2/2-(k_\perp\rho)^2/4\quad(22)$$

and keeping up to $O(Q^2)$ in the denominator of (20), one reaches

$$\Phi_k(p)=\frac{(k_\perp\rho_i)^2\langle n_0\rangle\phi_k(0)}{p\left[(k_\perp\rho_i)^2\langle n_0\rangle+\oint\frac{dl}{B}\int d^3\mathrm{v}\left(\overline{Q^2}-\overline{Q}^2\right)F_{i0}\right]}\qquad(23)$$

where $\langle n_0\rangle=\oint\frac{n_0dl}{B}$.

Eq. (23) has exactly the same form as that of R-H except for the definitions of Q and the orbit average.

We consider a large aspect ratio tokamak with circular cross section and set $I=B_0R_0$ in Eq. (10). Inserting (10) in (23), using (21) and making an inverse Laplace transformation, one can write the residual ZFs in the form of Eq. (1)

$$\phi(t)=\left(1+q^2F(M)/\sqrt{\varepsilon}\right)^{-1}\phi(0) \quad (24)$$

with $q=rB_0/R_0B_p$ the usual safety factor, $\varepsilon=r/R_0$ the inverse aspect ratio and

$$F(M)=F_1(M)+F_2(M) \quad (25a)$$

$$F_1(M)=\frac{1}{\langle n_0\rangle \mathrm{v}_t^{\,2}\varepsilon^{3/2}}\oint\frac{dl}{B}\int d^3\mathrm{v}\left[\left(c_{\parallel}B_0/B\right)^2-\overline{\left(c_{\parallel}B_0/B\right)}^2\right]F_{i0} \quad (25b)$$

$$F_2(M)=\frac{2(1+\tau)M^2}{\langle n_0\rangle\varepsilon^{3/2}}\oint\frac{dl}{B}\int d^3\mathrm{v}\left[(R/R_0)^4-\overline{(R/R_0)^2}^2\right]F_{i0} \quad (25c)$$

where we define the Mach number $M=\omega_R R_0/[2(T_e+T_i)/m_i]^{1/2}$ and $\mathrm{v}_t=(T_i/m_i)^{1/2}$.

Eqs. (24) (25) are derived under the assumption of slow rotation, *i. e.* $M<<1$. Obviously, the R-H result is recovered for $M=0$.

In the previous work, we have derived residual ZFs for $M\sim1$ in the form of Eq. (24) with $F_2(M)=0$. It is difficult to solve the problem for arbitrary flow velocity since the ordering expansion of Eq. (9) is not applicable except for $M\sim0$ and $M\sim1$. We can obtain the form of residual ZFs for arbitrary Mach numbers through an interpolation. One such a generalization is

$$F_2(M)=\frac{2(1+\tau)M^2}{\langle n_0\rangle\varepsilon^{3/2}}e^{-\xi M^2}\oint\frac{dl}{B}\int d^3\mathrm{v}\left[(R/R_0)^4-\overline{(R/R_0)^2}^2\right]F_{i0} \quad (26)$$

with ξ a large number to ensure that $F_2(M)\sim0$ for $M\sim1$. In the next section, we show that $F_2(M)$ is negligible in the whole range of *M.*

IV NUMERICAL RESULTS OF RESIDUAL ZFS AND COMPARISON WITH SIMULATION

In this section, we give some numerical results of residual ZFs for a large aspect ratio circular tokamak. The major radius $R=R_0+r\cos\theta$, the equilibrium magnetic field

$B = B_0/(1+\varepsilon\cos\theta)$. Then in the power expansion of ε we have

$$\langle n_0\rangle = 2\pi q R_0 N_0 e^{M^2}[1+(1.5+M^2)M^2\varepsilon^2+\cdots]/B_0 \qquad (27)$$

for convenience we introduce $\sigma = 1+(1.5+M^2)M^2\varepsilon^2+\cdots$.

It is straightforward to carry out the integration for the first parts in the square brackets of (25b) and (26) for any Mach number. Also in the power series of ε, one obtains

$$F_1(M) = \frac{1}{\sigma\varepsilon^{3/2}}\left\{\left[1+\left(1.5+3.5M^2+M^4\right)\varepsilon^2+\cdots\right]-\frac{B_0}{2\pi q R_0 N_0 e^{M^2}}\oint\frac{dl}{B}\int d^3\mathrm{v}\left(c_\parallel B_0/B\right)^2 F_{i0}\right\} \qquad (28)$$

and

$$F_2(M) = \frac{2(1+\tau)M^2}{\sigma\varepsilon^{3/2}}e^{-\varepsilon M^2}\left\{\left[1+\left(5+5.5M^2+M^4\right)\varepsilon^2+\cdots\right]-\frac{B_0}{2\pi q R_0}\oint\frac{dl}{B}\int d^3\mathrm{v}\left(R/R_0\right)^2 F_{i0}\right\} \qquad (29)$$

To perform integrations in (28) (29), we introduce two variables: $\hat{E} = E + A_M$ and $\lambda = \frac{\mu B_0}{\hat{E}}$, where $A_{M/m} = \frac{m\omega_R^{\,2} R_{M/m}^{\,2}}{2(1+\tau)}$ and $R_{M/m} = R_0(1\pm\varepsilon)$ is the maximum/minimum values of R. Then, from Eq. (7), we obtain the parallel velocity

$$|c_\parallel| = \left(\frac{2\hat{E}}{m}\right)^{1/2}\left[1-\frac{m\omega_R^{\,2}}{2\hat{E}(1+\tau)}\left(R_M^{\,2}-R^2\right)-\frac{\lambda B}{B_0}\right]^{1/2} \qquad (30)$$

The regions of passing and trapped particles can be figured out in $(\mu,\hat{E})$ as shown in Fig. 1 of Ref. 17. It is obvious that only passing particles contribute to the integral in (28) while all particles contribute in (29). The integration over passing particles is

$$\iint(\cdots)d\mu dE = \int_{\Delta A}^{+\infty}d\hat{E}\int_{\varepsilon+(1-\varepsilon)\Delta A/\hat{E}}^{1}(\cdots)(\hat{E}/B_0)d\eta \qquad (31)$$

where $\eta = 1-\lambda$ and $\Delta A = A_M - A_m$.

For small ε, we can use the expansion technique to carry out the integration in (28), the process is tedious but straightforward[8,10]. The final result involves the incomplete Gamma function $\int_{4M^2}^{+\infty}x^{3/2-k}e^{-x}dx$. Further analytical expression is impossible for arbitrary M values. For $M\sim 1$, we have made an adiabatic expansion of the incomplete Gamma function and given an approximate expression[17]

$$F_1(M) \cong 4.4M^3 + 0.32M + 0.66M^{-1} + 0.015M^{-3} + (0.5 + 2.5M^2 - 2M^4)\varepsilon^{1/2} + \cdots \quad (32)$$

For $M \sim 0$, we can make a Taylor series expansion in evaluating the incomplete Gamma function and get an approximate expression

$$F_1(M) \cong 1.6 + 3.34M^2 - 1.6M^4 + (0.5 - 2M^4)\varepsilon^{1/2} + (0.375 - 2.4M^2 + 3.9M^4)\varepsilon + \cdots \quad (33)$$

It is difficult to find such approximate expressions for $F_2(M)$ since it is much involved to have an integration over trapped particles in (29). We have also calculated $F_1(M)$ and $F_2(M)$ by numerically carrying out the integration in (28) and (29).

Plotted in Fig. 1 are the curves of $F_1(M)$ and $F_2(M)$, with parameters $q = 2.5$, $\varepsilon = 0.3$ and $\xi = 10$. The horizontal axis is labeled by M^2 instead of M since it is M^2 that appears in most formulas. Numerical and approximate expansion results of $F_1(M)$ are in good agreement. It is obvious that $F_2(M)$ is negligible since it accounts for less than 5% of $F(M)$ for the whole range of M. In Fig. 2 we plot the level of residual ZFs changing with the square Mach number, as well as the R-H result given by $F(M) \cong 1.6 + 0.5\varepsilon^{1/2} + 0.375\varepsilon + \cdots$. The level of ZFs decreases with increasing Mach number which is in qualitative agreement with a recent numerical gyro-kinetic simulation[16]. The physical explanation is that in a rotating plasma the fraction of trapped particles increases because of the inertial forces and as a result the neoclassic polarization effect increases to lower the residual ZFs. To have a quantitative comparison with the numerical simulation, we calculate residual ZFs using the same parameters as those in Ref. 16. Results are shown in Fig. 3 with the horizontal axis labeled by M. Making a careful comparison between Fig. 3 and the Fig. 12 of Ref. 16, we notice that good agreement is achieved for low ε values while higher discrepancies appear for higher ε values. It is understandable because we have made Taylor series expansions in ε to derive (28) and (29), higher ε may bring about larger errors in expansions.

V. SUMMARY AND DISCUSSION

In this work a general form of residual ZFs in a tokamak plasma toroidally rotating at arbitrary velocity is presented. We first solve the gyro-kinetic equation by ordering expansion for low rotation velocity and then apply the quasi-neutrality condition to derive the long time potential evolution. The form of residual ZFs at low Mach numbers is given by Eq. (24) together with Eqs. (25a-c). The generalized form is given by Eq. (24) together with Eqs. (25a,b) and (26). Numerical evaluation indicates that in Eq. (25a) F_2 is much lower than F_1 for the whole range of Mach numbers. If the F_2 term is neglected, the form of residual ZFs given by Eq. (24) is exactly the same as the R-H form except for the definition of orbit average. This is not a mere coincidency. The neoclassic polarization arises from the difference of guiding center orbits between ions and electrons. If we transform to the rotating framework, process of the neoclassic polarization would be the same except for that the drift of single particles is changed by inertial forces, which is embodied in the definition of orbit average. Although expressions are the same, the numerical result is different due to the different definition of orbit average, the different fraction of trapped particles and the non-uniformity of density on magnetic surfaces.

Numerical evaluation indicates that the level of residual ZFs decreases with increasing Mach numbers which is qualitative agreement with previous gyro-kinetic simulations. Quantitative agreement is satisfactory in high aspect ratio limit. The discrepancy in lower aspect ratio is due to the power series expansion in inverse aspect ratio in our calculation. So the general expression given by Eq. (24) is valid for arbitrary rotation velocity and can be adopted in plasma simulations.

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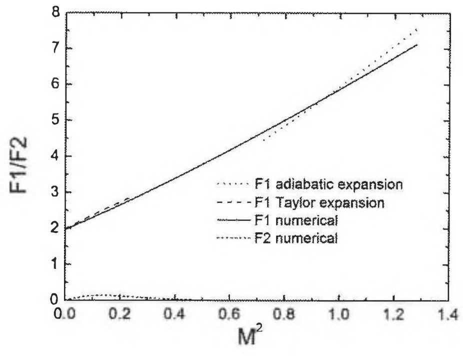

Fig. 1 Values of F_1 and F_2, with parameters $q = 2.5$, $\varepsilon = 0.3$ and $\xi = 10$.

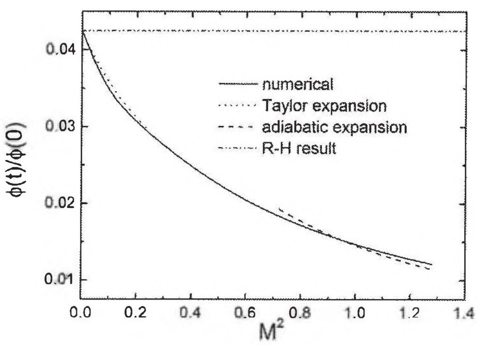

Fig. 2 The residual zonal flow level changes with squared Mach numbers with parameters $q = 2.5$, $\varepsilon = 0.3$ and $\xi = 10$. Solid line is the result by numerically carrying out the integration in (25b) and (26). Dot line is the result given by (24) and (33) with F_2 neglected. Dash line is given by (24) and (32).

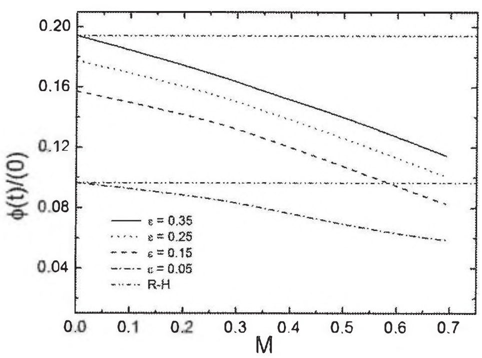

Fig. 3 The residual zonal flow level changes with Mach numbers with parameters $q = 1.1$ and $\xi = 10$ for different ε values.