Design of geometric phase measurement on EAST Tokamak

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Abstract:

The optimum scheme for geometric phase measurement in plasma is proposed. The theoretical values of geometric phase for the probe beams of EAST Polarimeter/Interferometer (POINT) system are calculated by path integration in parameter space and the influences of some controllable parameters on geometric phase are evaluated. The feasibility and problems of distinguishing geometric effect in the POINT signal are assessed.

Keywords: Geometric phase measurement; Faraday rotation angle; Ray tracing equations;

1. Introduction:

Geometric phase is firstly discovered in quantum mechanics, afterwards proved to exist in classical mechanics. Lately J. Liu and H. Qin prove that the variation of the propagation direction of circularly polarized waves results in geometric phase, which also contributes to the Faraday rotation, in addition to the standard dynamical phase. This research can amend the diagnostic accuracy of POINT system in tokamak, the diagnostic principle of which is to obtain the current density profile by reversal of the Faraday rotation angle. The present data processing method for POINT system only takes the dynamical phase which depends on the plasma dispersion relation into account in the Faraday rotation angle while ignoring the geometric phase which depends on the wave propagation path. As a result the geometric effect is an error source in the tokamak current diagnosis and the influence would be higher in the future ITER and other fusion reactors with higher parameters. Verification and assessment of the geometric quotient in Faraday rotation experimentally is of physical significance and feasible in hardware with the POINT system.

Theories about geometric phase are abundant while relevant experiments are comparatively rare. One typical experiment designed for verifying the existence of geometric phase is performed by comparing the Faraday rotation angle of a linearly polarized laser before and after transmitting through helical fibers of different deformation modes, which inspires that the light propagation path should be explicit to calculate the theoretical value of geometric phase in plasma.

In this paper, the beam path in plasma is simulated by ray-tracing equations with the fourth-order Runge-Kutta method employed, afterwards the geometric phase is computed by integrating along wave vector in parameter space with Polynomial fitting.

Given that various parameters have different levels of impact on geometric effect and appropriate combination gives the optimum experimental conditions for geometric phase measurement in plasma, the simulation results can be divided into three parts: Firstly, the refraction and frequency of incident wave effect on geometric phase are demonstrated; Secondly, the density distribution and safety factor q effect on geometric phase are estimated; Thirdly, the toroidal installation error effect and the retro reflector effect on geometric phase are investigated. The feasibility of distinguishing geometric effect in the POINT signal is assessed, and the optimum scheme for geometric phase measurement in plasma is proposed.

2. Significance and hardware conditions for geometric phase measurement on EAST Tokamak

As an important diagnosis in tokamak, three waves polarimeter-interferometer (PI) technique utilizes two collinearly circular polarized lasers of counter-rotating to acquire current density profile by reversal of the Faraday rotation angle and one linearly polarized laser as reference light to obtain electron density profile by reversal of phase shift. Considering that the geometric phases for left and right circularly polarized lights in the same path are equal and opposite, the geometric amendment would exist in the Faraday rotation angle and counteract in the phase shift for the linearly polarized probe beam, which can be decomposed into left and right circularly polarized lights. Therefore in inhomogeneous plasma the rotation angle of polarization direction for linearly polarized probe beam contains two parts: the dynamic phase which depends on the plasma dispersion relation, and the geometric phase which depends on the wave propagation path. Considering that geometric effect is greater with higher plasma parameters, verification and assessment of the geometric quotient in Faraday rotation would be more and more necessary in the future discharge of fusion device.

The present far-infrared double-pass, radially-viewing, multichannel EAST POINT system gives reasonable current density profiles with Faraday rotation angle resolution of about 0.1 degree, which lays a solid foundation for the hardware design of geometric measurement.

3. Theoretical calculations for geometric phase in plasma

In collisionless cold plasma the dispersion relations for right and left circularly polarized waves paralleled to the magnetic field are:

$$N^2 = 1 - \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega(\omega \pm \omega_{c\alpha})} \quad \cdots\cdots \quad (1)$$

Where $\omega_{p\alpha}$ and $\omega_{c\alpha}$ are the plasma frequency and gyrofrequency for particles of species α, respectively. And the right and left here are defined with respect to the background field. When the characteristic variation length scale and variation time scale of the plasma are much longer than the wave-length and the period of incident wave, the Ray tracing equations can be used:

$$
\left.
\begin{aligned}
\dot{\vec{k}} &= \frac{\partial D}{\partial \vec{r}} \Big/ \frac{\partial D}{\partial \omega} = F_a \\
\dot{\vec{r}} &= -\frac{\partial D}{\partial \vec{k}} \Big/ \frac{\partial D}{\partial \omega} = F_b
\end{aligned}
\right\} \cdots\cdots \quad (2)
$$

Here $D = k^2 - \frac{\omega^2}{c^2} N^2$ characterizes the plasma properties. With equation (1) substituted, we can get:

$$
\left.
\begin{aligned}
\frac{\partial D}{\partial \vec{r}} &= \frac{1}{c^2} \sum_\alpha \left[\frac{\omega \omega_{p\alpha}^2}{(\omega \mp \omega_{c\alpha}) n_\alpha} \cdot \frac{\partial n_\alpha}{\partial \vec{r}} \pm \frac{\omega \omega_{p\alpha}^2 q_\alpha}{(\omega \mp \omega_{c\alpha})^2 m_\alpha} \cdot \frac{\partial B}{\partial \vec{r}} \right] \\
\frac{\partial D}{\partial \omega} &= \frac{1}{c^2} \left[-2\omega \mp \sum_\alpha \frac{\omega_{p\alpha}^2 \omega_{c\alpha}}{(\omega \mp \omega_{c\alpha})^2} \right] \\
\frac{\partial D}{\partial \vec{k}} &= 2\vec{k}
\end{aligned}
\right\} \cdots \quad (3)
$$

With cylindrical approximation, the magnetic field can be expressed as:

$$
\vec{B} \approx \frac{B_0 R_0}{R} \vec{e}_z + \frac{r B_t}{qR} \vec{e}_\varphi \cdots\cdots \quad (4)
$$

The density profile and safety factor profile are assumed as:

$$
q = c_1 [(x - R_0)^2 + y^2]^{N_q/2} + c_2 \cdots\cdots \quad (5)
$$

$$
n_\alpha = c_4 [(x - R_0)^2 + y^2]^{N_n/2} + c_3 \cdots\cdots \quad (6)
$$

With Fourth-order Runge-Kutta method employed, the ray tracing equations can be expressed as follows:

$$
\left.
\begin{aligned}
\vec{k}(n+1) &= \vec{k}(n) + \Delta t F_a[\vec{r}(n)] \\
\vec{r}(n+1) &= \vec{r}(n) + \frac{\Delta t}{6} \Big\{ F_{b1}[\vec{r}(n), \vec{k}(n)] + 2F_{b2}[\vec{r}(n), \vec{k}(n)] \\
&\quad + 2F_{b3}[\vec{r}(n), \vec{k}(n)] + F_{b4}[\vec{r}(n), \vec{k}(n)] \Big\} \\
F_{b1}[\vec{r}(n), \vec{k}(n)] &= F_b[\vec{r}(n), \vec{k}(n)] \\
F_{b2}[\vec{r}(n), \vec{k}(n)] &= F_b[\vec{r}(n) + \frac{\Delta t}{2} F_{b1}[\vec{r}(n), \vec{k}(n)], \vec{k}(n) + \frac{\Delta t}{2} F_a[\vec{r}(n)]] \\
F_{b3}[\vec{r}(n), \vec{k}(n)] &= F_b[\vec{r}(n) + \frac{\Delta t}{2} F_{b2}[\vec{r}(n), \vec{k}(n)], \vec{k}(n) + \frac{\Delta t}{2} F_a[\vec{r}(n)]] \\
F_{b4}[\vec{r}(n), \vec{k}(n)] &= F_b[\vec{r}(n) + \Delta t F_{b3}[\vec{r}(n), \vec{k}(n)], \vec{k}(n) + \Delta t F_a[\vec{r}(n)]]
\end{aligned}
\right\} (7)
$$

With equations (2)~(7), the trajectory of probe beam in plasma can be decided.

Considering the wave vector changing slowly because of adiabatic approximation, the expression of type I geometric phase can be calculated by polynomial fitting shown as equation (8).

$$\theta_{gI} = m\int_{incident\ point}^{emergent\ point} \frac{k_z(K_x dK_y - K_y dK_x)}{K(K_x^2 + K_y^2)} \ldots\ldots \quad (8)$$

4. Conclusions

In summary, the refraction effect in inhomogeneous plasma results in different levels of path deflection for left and right circularly polarized waves, and hence leads to observable geometric effects of different quotient in the Faraday rotation measurement. The incident frequency, incidence position, density profile, safety factor profile, toroidal and poloidal installation angles can influence the geometric effect in varying degrees. Controlling the incident frequency or density profile in the right position can increase the geometric phase to 0.01°, while the present Faraday rotation angle resolution is 0.1°, not big enough to distinguish the geometric effect. Increasing the toroidal installation angle can add the geometric angle up to 0.1°; nevertheless the cotton mutton effect would increase as well. Given these, the geometric phase is difficult to distinguish in the present POINT system. Next step three improvement approaches can be adopt to realize the geometric measurement experiment. Firstly, the system resolution can be improved to 0.05° by collinear adjustment; secondly, an optical path can be desired for Geometric measurement, for example, increasing the path length in plasma or increasing the density and density gradient; Thirdly, deliberately controlling the included angle between detected beam and toroidal magnetic field would work if the cotton mutton effect and geometric effect can be told apart. At the same time, the geometric phase generated by discrete optical systems can be calculated by the theory of M. Kitano, given that most mirrors can be considered as ideal conductor for far infrared light.

5. Reference

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