Simulation of Long-term Dynamic Behavior of Runaway Electrons

Yulei Wang, Jian Liu, Ruili Zhang, Yang He

Abstract

The secular dynamics of runaway electrons in Tokamak electromagnetic field is studied. The radiation effect is added into a relativistic volume-preserving algorithm to gain long-term stability of calcucation. The results shows that the method we used is able to reveal the behavior of a runaway electron in configuration space.

Runaway electrons play an important role in the safe operation of Tokamak. Through experimental observations, it has been proved that large mounts of runaway electrons are generated during fast plasma shut down[1] and the disruption[2] in large Tokamaks. Accelerated continuously by electic field[3], a runaway electron may carry considerably high energy up to tens of MeVs, which becomes a serious threat to the life of all the plasma-facing components(PFCs). Besides experimenters, extensive theorists have been devoted to the study of runaway electrons. Because the physical process of runaway electrons has multi-scale in both time and space, theorical and simulation studies are mostly based on guiding-center model. Through averaging out the fast gyromotion and reducing the timespace scale by two or three magnitude orders, gyrocenter approximation saves lots of computation resources and some macroscope dynamical behavior of runaway electrons can be effectively predicted analytically[4-6] and numerically[7, 8].

In our resent work, a probe particle model is established for simulating the secular dynamical behavior of runaway electrons in Tokamaks. Radiation effect is added into the relativistic motion equation while collision resistance is ignored because of the high speed of runaway electrons[3, 9]. In order to gain long-term stability, we have combined radiation terms with a relativisctic volume-preserving algorithm(VPA) [10]. VPAs are a series of advanced geometric algorithm that can preserve the phase-space volume of dynamical system no matter how many steps the calcucation needs. Although the combination of radiation and VPA weakens the geometric property of algorithm, the stability of secular computation remains. During our long-term simulation typically requiring tens of billion steps, some macroscope behaviors of a runaway electron, including circle orbit of passing particles and neo-classical drift of runaway orbit, match well with the results of guiding center theory[7, 11, 12].

The motion of a runaway electron is governed by the relativistic Lorenz force formula[13]. Within our consideration, the collision resistance is ignorable because the speed of simulated runaway electrons is considerably high[3, 9]. And all the energy dissipation comes from the radiaton emitted through the interaction between the electron and background field. In our model, a radiation drag force providing all the radiation energy loss is constructed, which leads to the motion equation with radiation,

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v}, \tag{1}$$

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}_R ,$$

$$\mathbf{p} = \gamma \mathrm{m}_0 \mathbf{v} ,$$

where **x, v** and **p** are, respectively, the positon, velocity and mechanical momentum of a runaway electron, q_e denotes the charge of an electron, m_0 is the rest mass of an electron, **E** and **B** are the electric and magnetic field, and the Lorentz factor γ is defined as

$$\gamma = \sqrt{1 + \frac{p^2}{\mathrm{m}_0^2 \mathrm{c}^2}} = \frac{1}{\sqrt{1 - (v/\mathrm{c})^2}} . \tag{2}$$

Finally, $\mathbf{F}_\mathrm{R}$ denotes the radiation drag force, namely,

$$\mathbf{F}_\mathrm{R} = -P_R \frac{\mathbf{v}}{v^2} . \tag{3}$$

The radiation power for relativistic charged particle, the Larmor formula[13], is

$$P_R = \frac{q_e^2}{6\pi\epsilon_0 \mathrm{c}} \gamma^6 \left[\left(\frac{\mathbf{a}}{\mathrm{c}}\right)^2 - \left(\frac{\mathbf{v}}{\mathrm{c}} \times \frac{\mathbf{a}}{\mathrm{c}}\right)^2 \right] , \tag{4}$$

where ϵ_0 is the permittivity in vacuum, c is the speed of light in vacuum, $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ denotes the acceleration of the particle. According to Eqs. (2)-(4), radiation drag force is a function of velocity and acceleration, namely, $\mathbf{F}_\mathrm{R} = \mathbf{F}_\mathrm{R}(\mathbf{v}, \mathbf{a})$.

Since the dynamics of a runaway electron has multi-scale in time and space, it is necessary to compute tens of billion steps for a complete simulation. In order to guarantee secular stability, we discretize the motion Eq. (1) based on a relativistic volume-preserving algorithm, namely,

$$\begin{aligned}
\mathbf{a}^k &= \frac{\mathbf{v}^k - \mathbf{v}^{k-1}}{\Delta t} , \\
\mathbf{F}_R^k &= \mathbf{F}_R^k(\mathbf{v}^k, \mathbf{a}^k) , \\
\mathbf{x}^{k+1/2} &= \mathbf{x}^k + \frac{\Delta t}{2} \frac{\mathbf{p}^k}{\sqrt{\mathrm{m}_0^2 + (p^k/\mathrm{c})^2}} , \\
\mathbf{p}^- &= \mathbf{p}^k + \frac{\Delta t}{2} \left(q_e \mathbf{E}^{k+1/2} + \mathbf{F}_R^k \right) , \\
\mathbf{p}^+ &= \mathbf{p}^- + q_e \Delta t \left(\frac{\mathbf{p}^- + \mathbf{p}^+}{2\sqrt{\mathrm{m}_0^2 + (p^-/\mathrm{c})^2}} \times \mathbf{B}^{k+1/2} \right) , \\
\mathbf{p}^{k+1} &= \mathbf{p}^+ + \frac{\Delta t}{2} \left(q_e \mathbf{E}^{k+1/2} + \mathbf{F}_R^k \right) , \\
\mathbf{x}^{k+1} &= \mathbf{x}^{k+1/2} + \frac{\Delta t}{2} \frac{\mathbf{p}^{k+1}}{\sqrt{\mathrm{m}_0^2 + (p^{k+1}/\mathrm{c})^2}} .
\end{aligned} \tag{5}$$

Without radiation terms, the discreted equation is exactly a time-symmetry volume-preserving algorithm(VPA) [10]. Known as excellent property of conservation, volume-preserving algorithms belong to a branch of advanced geometric algorithms that have been widely used in plasma physics[7, 8, 14-18]. Although the added radiation terms in Eq. (5) weakens the geometric property of VPA, the long-term stability of the algorithm remains, which can be prove by the simulation results.

The motion of a runaway electron is calculated in a background electromagnetic field in the form of

$$\mathbf{B} = \frac{B_0 R_0}{R}\mathbf{e}_\xi - \frac{B_0\sqrt{(R-R_0)^2+z^2}}{qR}\mathbf{e}_\theta,$$
$$\mathbf{E} = E_l\frac{R_0}{R}\mathbf{e}_\xi, \tag{6}$$

where $R = \sqrt{x^2+y^2}$, ξ and z denote, respectively, the radial distance, the azimuth and the height of cylindrical coordinate system, $\mathbf{e}_\xi$ and $\mathbf{e}_\theta$ are the toriodal and poloidal direction of the torus, and q is the safty factor. We choose the parameters based on EAST Tokamak[19]. The center magnetic field intensity is chosen to be $B_0 = 3\mathrm{T}$, while the center loop voltage is $E_l = 3\mathrm{V/m}$. The major radius is set as $R_0 = 1.7\mathrm{m}$. And the safty factor is $q = 2$. The motion of a typical runaway electron, with initial momentum $p_{\parallel 0} = 5\mathrm{m_0c}$, $p_{\perp 0} = 1\mathrm{m_0c}$ and initial position $r = 0.1\mathrm{m}$, $\theta = \xi = 0$, is computed. Snapshots of the runaway drifting orbit on the poloidal plane at $t = 0\sim0.045s$ are depicted in Fig. 1. The drift of circle orbit tells about the macroscope dynamics of a runaway electron. The drift velocity, about $2\mathrm{m/s}$, matches well with the result of guiding center theory[7] while the radiation effect is weak enough for the first $0.1\mathrm{s}$. The result serves as an evidence for the secular stability of the discrete format given by Eq. (5).

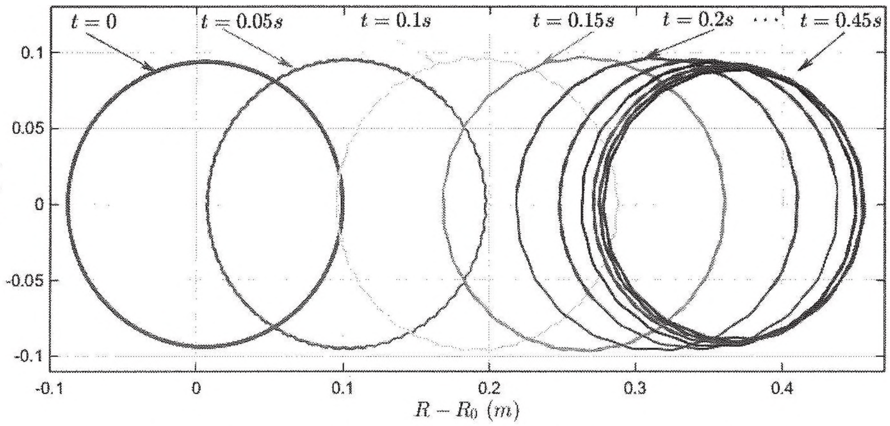

Fig. 1 Snapshots of the runaway drifting orbit projected on the poloidal plane from $t = 0$ to $t = 0.045\mathrm{s}$.

References

1. Yoshino, R., et al., *Fast plasma shutdown by killer pellet injection in JT-60U with reduced heat flux on the divertor plate and avoiding runaway electron generation.* Plasma phys. control. fusion, 1997. **39**(2): p. 313.
2. Jaspers, R., et al., *Disruption generated runaway electrons in TEXTOR and ITER.* Nuclear fusion, 1996. **36**(3): p. 367.
3. Dreicer, H., *Electron and ion runaway in a fully ionized gas. I.* Physical Review, 1959. **115**(2): p. 238.
4. Martın-Solıs, J., et al., *Momentum–space structure of relativistic runaway electrons.* Physics of Plasmas (1994-present), 1998. **5**(6): p. 2370-2377.
5. Martin-Solis, J., et al., *Energy limits on runaway electrons in tokamak plasmas.* Phys. Plasmas, 1999. **6**(1): p. 238-252.

6. Bakhtiari, M., G.J. Kramer, and D.G. Whyte, *Momentum-space study of the effect of bremsstrahlung radiation on the energy of runaway electrons in tokamaks.* Phys. Plasmas, 2005. **12**(10): p. 102503.
7. Guan, X., H. Qin, and N.J. Fisch, *Phase-space dynamics of runaway electrons in tokamaks.* Phys. Plasmas, 2010. **17**(9): p. 092502.
8. Liu, J., et al., *What is the fate of runaway positrons in tokamaks?* Phys. Plasmas, 2014. **21**(6): p. 064503.
9. Connor, J. and R. Hastie, *Relativistic limitations on runaway electrons.* Nuclear Fusion, 1975. **15**(3): p. 415.
10. Zhang, R., et al., *Volume-preserving algorithm for secular relativistic dynamics of charged particles.* Physics of Plasmas (1994-present), 2015. **22**(4): p. 044501.
11. Wesson, J., *Tokamaks*. Vol. 149. 2011: Oxford University Press.
12. Knoepfel, H. and D.A. Spong, *RUNAWAY ELECTRONS IN TOROIDAL DISCHARGES.* Nuclear Fusion, 1979. **19**(6): p. 785-829.
13. Jackson, J.D. and J.D. Jackson, *Classical electrodynamics*. Vol. 3. 1962: Wiley New York etc.
14. He, Y., et al., *Volume-preserving algorithms for charged particle dynamics.* Journal of Computational Physics, 2015. **281**: p. 135-147.
15. Qin, H. and X. Guan, *Variational symplectic integrator for long-time simulations of the guiding-center motion of charged particles in general magnetic fields.* Physical review letters, 2008. **100**(3): p. 035006.
16. Qin, H., et al., *Why is Boris algorithm so good?* Physics of Plasmas (1994-present), 2013. **20**(**8**): p. 084503.
17. Xiao, J., et al., *A variational multi-symplectic particle-in-cell algorithm with smoothing functions for the Vlasov-Maxwell system.* Physics of Plasmas (1994-present), 2013. **20**(10): p. 102517.
18. Zhang, R., et al., *Canonicalization and symplectic simulation of the gyrocenter dynamics in time-independent magnetic fields.* Physics of Plasmas (1994-present), 2014. **21**(3): p. 032504.
19. Wu, S., *An overview of the EAST project.* Fusion Engineering and Design, 2007. **82**(5): p. 463-471.