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## **Introduction of Bootstrap Current Reduction in the Stellarator Optimization Using the Algorithm DAB**

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7 pp. 14 ref. 3 fig.

### **Abstract:**

This work is devoted to introduce new optimization criterion in the DAB (Distributed Asynchronous Bees) code. With this new criterion, we have now in DAB the equilibrium and Mercier stability criteria, the minimization of  $Bx_{grad}(B)$  criterion, which ensures the reduction of neoclassical transport and the improvement of the confinement of fast particles, and the reduction of bootstrap current. We have started from a neoclassically optimised configuration of the helias type and imposed the reduction of bootstrap current. The obtained configuration only presents a modest reduction of total bootstrap current, but the local current density is reduced along the minor radii. Further investigations are developed to understand the reason of this modest improvement.

## **Introducción de la Reducción de la Corriente de “Bootstrap” en la Optimización de Stellarators Usando el Algoritmo DAB**

Castejón, F.; Gómez-Iglesias, A.; Velasco, J. L.  
7 pp. 14 ref. 3 fig.

### **Resumen:**

Este trabajo consiste en la introducción de un nuevo criterio de optimización en el código DAB (Distributed Asynchronous Bees, es decir, Abejas Distribuidas Asíncronas). Con este nuevo criterio, el código DAB está ahora equipado con una serie de criterios de optimización a saber: los criterios de equilibrio y estabilidad Mercier, la minimización de la deriva proporcional a  $Bx_{grad}(B)$ , lo que asegura la reducción del transporte neoclásico y la mejora del confinamiento de partículas energéticas, y la minimización de la corriente de bootstrap.

Se ha obtenido una configuración magnética optimizada a partir de una configuración reactor tipo helias. Sin embargo, la configuración así obtenida presenta solo una modesta reducción de la corriente total de bootstrap, aunque la densidad de corriente local se reduce a lo largo de todo el radio menor. Se precisa seguir investigando para entender las razones de esta modesta mejora.





## 1 Introduction

The search for an optimized configuration that fulfils multiple optimization criteria is a key topic in the path to the stellarator reactor (see, e.g. <sup>1</sup>). The investigation of the relevance of the optimization criteria is a fundamental step to propose such a stellarator-based configuration for a reactor, so it makes sense to check the relevance of the different mechanisms in the working stellarators. There are different optimized stellarator configurations following several criteria such as diminishing the neoclassical transport or improving the stability. The Wendelstain 7-X <sup>2, 3</sup> is a helias type stellarator, which will start its operation in 2015 at IPP-Greifswald, Germany. LHD has been optimized by reducing the neoclassical transport by shifting the magnetic axis <sup>4</sup>, and QPS <sup>5</sup> and NCSX <sup>6</sup> designed in USA are examples of optimized magnetic configurations.

The reduction in neoclassical transport, the confinement of fast particles, based on the omnigenity property, the Mercier stability criterion and the ballooning stability are the main topics that are considered in the optimizations. Once the optimized configuration is found, the necessary coils to create the magnetic field are evaluated in order to make an engineering assessment of their complexity.

On top of those characteristics, the operation of the reactor using an island-based divertor, implies that the value of the rotational transform in the edge is unchanged during the operation. This property must be guaranteed by the reduction of toroidal current sources. The main one is the bootstrap current, which depends both on the plasma profiles and on the magnetic configuration.

The objective of this work is to introduce a new criterion on the stellarator optimization, which is a proxy for the reduction of bootstrap current and to use it in the optimization of a helias configuration. The algorithm DAB <sup>7</sup> will be used for this purpose.

This paper is organized as follows: the DAB algorithm is described in section 2, including the description of the searching strategy, the proxy that will be used is described in section and the optimization results are shown in section 3. The discussion and conclusions are drawn in section 4.

## 2. The DAB algorithm

The DAB (Distributed Asynchronous Bees) algorithm is a general purpose optimization algorithm based on metaheuristics. Metaheuristics is a combinatorial optimization process that tries to maximize or minimize an objective function defined by the user. DAB search is based on the swarm strategy <sup>8</sup> exploring the huge space of all the possible stellarators, starting from a defined configuration. DAB has been previously used to optimise a compact heliac stellarator <sup>9</sup> using three optimization criteria: the minimization of the neoclassical transport, the Mercier and the ballooning stability criteria. The first one implies the minimization of the average  $Bx_{grad}(B)$ , i. e., minimising the following objective function:

$$F_{\text{objective function}} = \sum_{i=1}^N \langle |\vec{B} \times \nabla B / B^3| \rangle_i. \quad (1)$$

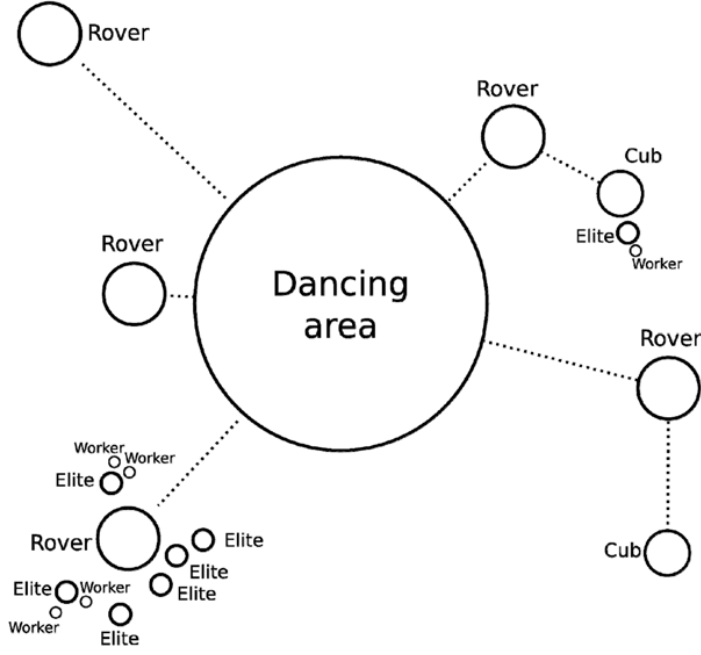
In this equation,  $i$  is the magnetic surface label and  $B$  is the magnetic field, tangent to the magnetic surface. A given configuration is an individual described by the Fourier components of the equilibrium, calculated using the VMEC code<sup>10</sup>. The last version of VMEC also includes the calculation of the Mercier stability criterion, which allows us to introduce this criterion in our optimization process. Thus, with the use of VMEC we already have two target functions. Moreover, the ballooning stability code COBRA<sup>11</sup> is introduced to include this criterion in the optimization process.

In order to perform a search of the huge space of all the possible stellarators, our algorithm can perform an exploration of the solution space or can use predefined candidate solutions. DAB is inspired by the behaviour of bees that are foraging and looking for flowers. In nature, bees look for new sources of food by exploring the space around the hive. As soon as a new food source is found, the bee communicates this information by the so-called waggle dance. In our case, a flower is a stellarator configuration satisfying the requirements being considered. The searching bees are asynchronous, since one does not need to wait for the others to finish their evaluation of the stellarator configuration, and distributed, since they can run on different processors without communication among them. These two characteristics make DAB a powerful and efficient tool both in parallel and distributed computing infrastructures.

DAB is based on two types of processes: (a) exploration and (b) exploitation, which are shown schematically in Figure 1.

(a) Exploration: this process explores the phase space, which has to be done in such a way that all the main areas of the solution space should be well-balanced-explored, with high enough dispersion. In our case, this balance is ensured by creating a set of new individuals, obtaining their distances to the previously selected individuals and choosing the configuration with the largest distance.

(b) Exploitation: it consists of introducing convergence in the algorithm when good configurations are found. In this case we divide this process into two more: (I) Mutation-based exploitation: following the behaviour of evolution in nature, by mutation or crossover of chromosomes (each of the Fourier modes). Mutation uses the previously good candidate solutions found. The number of chromosomes to be modified depends on a configurable mutation rate, which in fact limits the phase space to explore. (II) Local search exploitation: the best configuration found so far receives more computational resources and performs local searches using small modifications over just a few chromosomes. To prevent the best configuration from always getting more computational resources, a probability value is assigned to each candidate solution previously found.



**Figure 1.** Schematic view of exploration and exploitation .

## Bootstrap current in the optimization

The final objective of this work consisted of introducing the reduction of bootstrap current among the optimization criteria. As it has been stated above, the DAB algorithm takes into account the following optimization criteria: 1) MHD equilibrium and Mercier stability, which ensures to get robust and stable configurations; 2) reduction of  $B \times \text{grad}(B)$  drift, which implies the minimization of neoclassical transport and the improvement of fast particle confinement, since the transport of those particles is given mainly by such drift. Next step has been the inclusion of the minimization of the bootstrap current.

As it has been commented above, this last criterion is a key point for the helias configurations, since the divertor is based on the island concept, i.e., on keeping frozen the island structure during the operation. The main ingredients that affect island structure are the plasma pressure, which modifies the general magnetic structure of the configuration, and the toroidal currents. Hence, one important point for keeping constant the island structure is to minimise bootstrap current to levels that cause negligible changes in the edge rotational transform value or that can be compensated by a current drive method.

The bootstrap current is given by:

$$\frac{\langle \vec{j}_b \cdot \vec{B} \rangle}{Z_b e n B_0} = -L_{31}^b \left( \frac{1}{n} \frac{dn}{dr} - Z_b e \frac{E_r}{T_b} - \frac{3}{2} \frac{1}{T_b} \frac{dT_b}{dr} \right) - L_{32}^b \frac{1}{T_b} \frac{dT_b}{dr} \quad (2)$$

Where  $Z_b e$  is the charge of the species, -1 for the electrons,  $n$  is the plasma density,  $T_b$  is the temperature,  $E_r$  is the radial electric field, and the transport coefficients  $L_{ij}$  are given by:

$$L_{ij}^b(r, n, T_i, T_e, E_r) = \frac{2}{\sqrt{\pi}} \int_0^\infty dx^2 e^{-x^2} x^{1+2(\delta_{i,2}+\delta_{j,2})} D_{ij}(r, \nu^*, \Omega), \quad (3)$$

Here,  $x$  is the particle velocity normalised to the thermal velocity,  $\delta_{ij}$  is the Kronecker delta,  $\nu^*$  is the effective collision frequency,  $\Omega=Er/vB_0$  and  $D_{ij}$  are the neoclassical monoenergetic coefficients, calculated using the DKES code<sup>12</sup>. The bootstrap current only depends on the coefficient  $D_{13}=D_{23}$ .

According to Eq. (2), the bootstrap current depends on the actual profiles of plasma pressure and electrostatic potential. The possibility of tailoring those profiles is a way to reduce the current, but it is not valid for any plasma regime. On the opposite, we should try to look for a general procedure of minimising the current, independently of the plasma operation.

In order to reduce the current, we have introduced in DAB the minimization of the following objective function, as a new criterion:

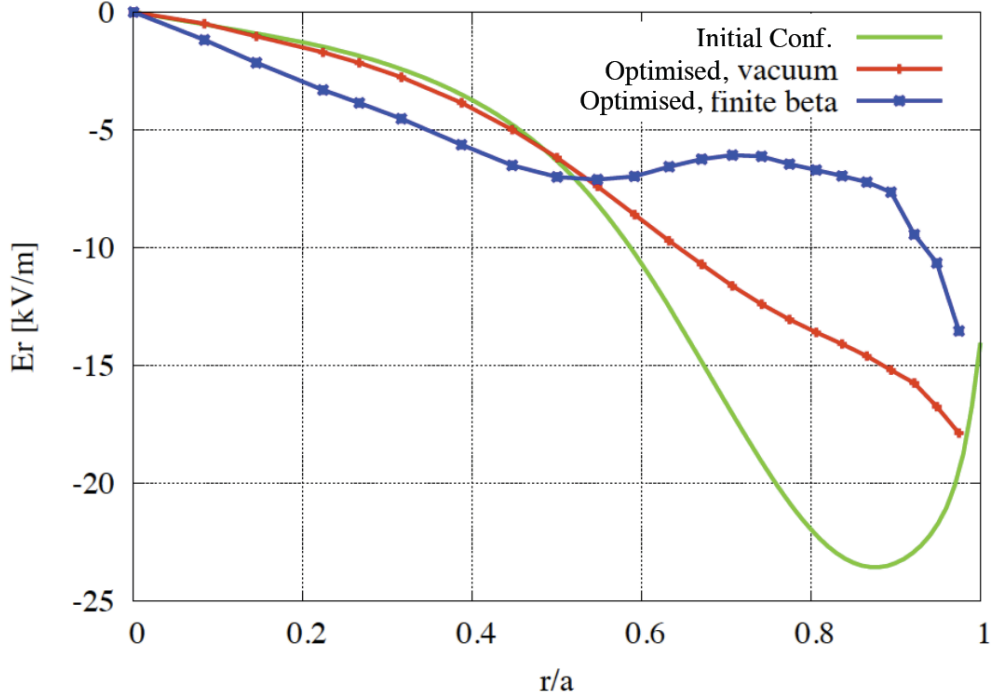
$$F_{jboot} = \sum_{i=1}^N \rho_i D_{13}(\rho_i) \quad (4)$$

Being  $D_{31}(\rho_i)$  the bootstrap coefficient of the neoclassical transport matrix on the magnetic surface of normalised radius  $\rho_i$ . The summation is taken for a large number of magnetic surfaces (100 typically). This coefficient is estimated using DKES code for the typical profiles expected in that fusion device.

With this new criterion, we have built a workflow among the following codes: DAB as the driver of the workflow, the code to estimate the Bxgrad(B) average, VMEC to estimate the equilibrium and the Mercier stability criterion, COBRA to estimate the ballooning instability growth rate, and the DKES code.

We start here from configuration Hydra-21, a reactor configuration based on Helias concept. The expected plasma characteristics are estimated using NTSS code<sup>13</sup>, so the device is already optimised to present a reduced neoclassical transport as well as bootstrap current.

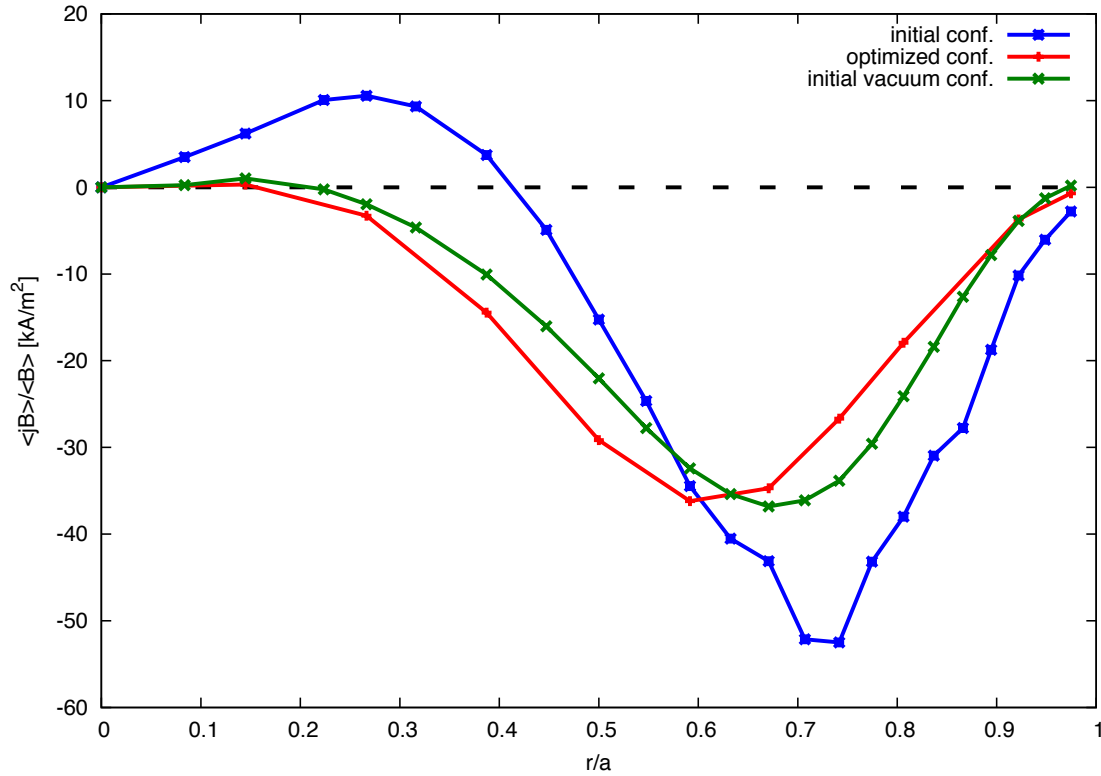
The plasma profiles are obtained running the predictive transport code NTSS, assuming Neoclassical transport plus a fraction of anomalous transport in the plasma edge. The bootstrap current for these plasmas is  $I_{boot}=-250$  kA.



**Figure 2.** Profiles of radial electric field estimated from the ambipolar condition: initial configuration (green), optimised configuration (red), optimised configuration for finite pressure and including the new current profile.

The code DAB is run on the EULER cluster at CIEMAT and we obtain the new optimised configuration with reduced bootstrap current. Then, the plasma profiles estimated with NTSS, are used for the calculation of the new bootstrap current profile. First of all, the electric field is calculated imposing the ambipolar condition, as in Ref. <sup>14</sup>, and it is shown in figure 2. The electric field for the initial configuration with given plasma profiles is seen to be negative along the minor radius, with a minimum at  $\rho \approx 0.9$ , so the plasma is in the ion root (in green in the picture). The new electric field for the DAB-optimised configuration is estimated imposing again the ambipolar condition and represented in Figure 2 (red curve). Using the estimated radial electric field, the bootstrap current is estimated for the new configuration and plotted in Figure 3. In this picture the original bootstrap current density is plotted in blue. The resulting bootstrap current is plotted in red and gives a value of  $I_{boot} = -197$  kA.

We calculate again the equilibrium with the new current profile, and re-calculate the equilibrium using the VMEC code. The DKES code is run again on the new equilibrium to find the new bootstrap current density profile, which happens to be very similar to the former one, and is plotted in Figure 3 in green. The integrated value of the current is  $I_{boot} = -190$  kA.



**Figure 3.** Profiles of bootstrap current density for: initial configuration (green), optimised configuration (red), optimised configuration for finite pressure and including the new current profile.

The integrated bootstrap current shows a modest decreasing in comparison with the starting configuration, but the current density values are substantially smaller in the optimised configurations.

There can be several causes for such a limited reduction of the bootstrap current, the most probable one is that the starting configuration is quite optimised itself. But further research continues to evaluate other possibilities like widening the exploration space or improving the bootstrap current proxy.

## Conclusion

A new optimization criterion has been added to the code DAB, in order that the minimization of bootstrap current can be considered for the optimization of stellarators. The monoenergetic bootstrap coefficients is calculated for a large number of magnetic surfaces (typically 100) using the DKES code. A target function is defined as a proxy for the bootstrap current: the addition of all the monoenergetic bootstrap coefficients for all the magnetic surfaces.

We started from the configuration Hydra-21, a healias based reactor. Running DAB code, a new configuration has been obtained. Considering all the target functions included in the optimization, the new configuration is Mercier-stable at least up to  $b=5\%$ , presents a reduced neoclassical transport, a good confinement of fast particles and a reduced bootstrap current.

The final value of the absolute value of the bootstrap current is, nevertheless, only 60 kA smaller than the one of the original configuration. The reason for this modest optimization is under investigation, but it could be due to the fact that the original one is already optimised for having a small value of the bootstrap current.

The current profile of the optimised configuration is smaller than the original one.

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