

Simulating Beam Parameter using Malmberg–Penning Trap Device for Longitudinal Pulse Compression in Heavy Ion Inertial Fusion

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ABSTRACT

A simulating beam parameter is considered in a compact beam simulator constructed with a Malmberg–Penning trap device for heavy ion inertial fusion. The Malmberg–Penning trap device is expected as an experimental device to simulate longitudinal pulse compression in a driver system for heavy ion inertial fusion. Tune depression is estimated with Brillouin density limit, rigid–rotor Vlasov equilibrium, and a model considering with radial distribution of charge density. As a result, it is considered that these estimations are not suitable for the space charge strength of the condition in the device.

Keywords

Tune Depression, Malmberg–Penning Trap, Heavy Ion Inertial Fusion, Compact Simulator, Space–Charge–Dominated Beam, Beam Physics

1 Introduction

In inertial confinement fusion driven by intense heavy–ion–beam irradiation, so–called heavy ion inertial fusion (HIF), beam physics and dynamics should be cleared well for effective target pellet implosion [1]. Especially, the extreme longitudinal pulse compression scheme is required in the final stage of energy driver in HIF accelerator system [2–4].

Theoretically, it is predicted that the beam parameters passage through a risky regime to increase the emittance [5, 6]. For this reason, the emittance growth should be estimated to design the heavy ion accelerator for HIF system during the longitudinal pulse compression.

A Malmberg–Penning trap device [7, 8] is expected as an experimental device to simulate the longitudinal pulse compression for HIF driver system [9]. In the Malmberg–Penning trap device,

the magnetic flux density B_z applied in the axial z direction confines the charged particle in radial r direction, and the electrostatic potential applied at both the ends of axial direction reflects the charged particle. Electron devices scaled by the parameters of the heavy ion beam in HIF driver system were useful experimental device due to the compact size [10–12]. Not only the experimental work, but also the numerical approach was carried out [13], and the electron dynamics is simulated during the pulse compression manipulation [13].

In this study, the simulating beam parameter is estimated by using theoretical approach. One of most unique properties is "space–charge–dominated beam" for the HIF driver system. The simulating beam parameter in the Malmberg–Penning trap device as a compact simulator is indicated from the viewpoint of the space charge strength of the beam.

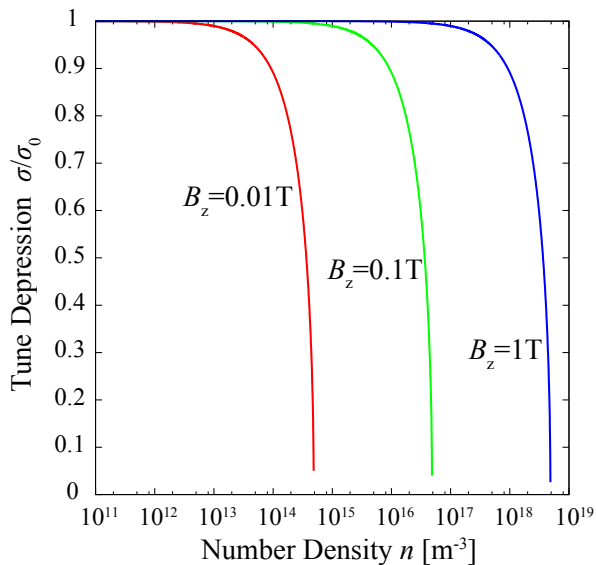


Figure 1: Tune depression estimated by Brillouin density limit.

2 Simulating Beam Parameter

To simulate the beam dynamics in HIF driver system, there are some indexes such as space charge wave velocity, generalized perveance, compression ratio, aspect ratio of bunch size, beam velocity divided by speed of light, and so on.

A tune depression is one of indexes to estimate the space-charge strength of the charged particle beams. The tune depression σ/σ_0 is explained by the ratio of the undepressed tune σ to the undepressed tune σ_0 [14–16]. For $\sigma/\sigma_0 \rightarrow 1$, the beam is in weak space-charge strength, so-called *emittance-dominated beam*. On the other hand, the beam is in strong space-charge strength, so-called *space-charge-dominated beam*, for $\sigma/\sigma_0 \rightarrow 0$.

3 Estimation of Tune depression with Brillouin density limit

Figure 1 shows the tune depression estimated by the Brillouin density limit [17]. The tune depression is estimated by [17]

$$\sigma/\sigma_0 = \sqrt{1 - \frac{\tilde{n}}{n_{\text{lim}}}} \quad (1)$$

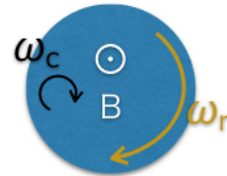


Figure 2: Rotation angular frequency ω_r in rigid-rotor Vlasov equilibrium and cyclotron frequency ω_c .

where \tilde{n} is the average density and n_{lim} is the density evaluated by the Brillouin density limit. In the case of electron, the limiting density is estimated as $n_{\text{lim}} = 4.86 \times 10^4 B_z^2 \text{ cm}^{-3}$, where B_z is in unit of Gauss.

As shown in Fig. 1, when the number density is assumed in $10^{11} \sim 10^{12} \text{ m}^{-3}$ with the solenoidal strength $B_z = 0.01 \sim 1 \text{ T}$ according to the experimental condition [9], and the tune depression $\sigma/\sigma_0 \sim 1$ is expected.

As mentioned in Ref. [17], this estimation is “based on the KV model and is not very accurate for low-density plasma.” For this reason, it is expected that the evaluation result in this assumption is not suitable for the parameters in this experimental setup.

4 Estimation of Tune depression with Rigid-Rotor Vlasov equilibrium

The tune depression is also described by using a dimensionless parameter Δ . The dimensionless parameter indicates the ratio of applied focusing strength to the space-charge defocusing strength [18]. In the case of Malmberg–Penning trap configuration, the dimensionless parameter Δ is written by [18]

$$\Delta = \frac{2(\omega_r \omega_c - \omega_r^2)}{\omega_p^2} - 1 \quad (2)$$

where ω_r is the rotation angular frequency in the assumption of rigid rotation, ω_c is the cyclotron frequency, and ω_p is the plasma frequency, respectively. Figure 2 shows the relation between the rotation angular frequency and the cyclotron frequency. For the assumption of the thermal equilibrium distribution, the dimensionless parameter

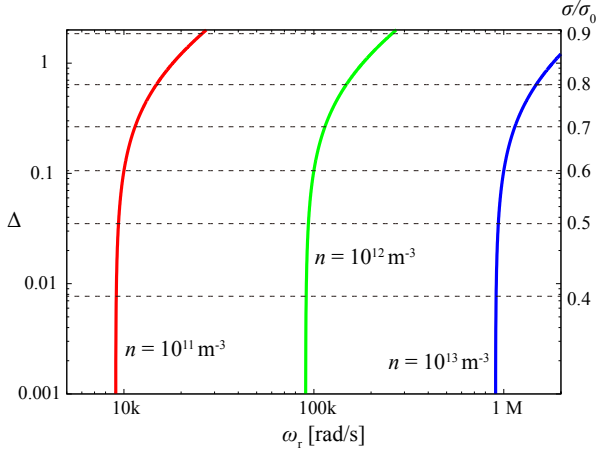


Figure 3: Tune depression σ/σ_0 and dimensionless parameter Δ as a function of rotation angular frequency ω_r for $T=1$ eV and $B_z=0.1$ T.

Δ is related by the tune depression as shown in Table V in Ref. [19], e.g., $\sigma/\sigma_0 = 0.9$ for $\Delta = 1.851$ and $\sigma/\sigma_0 = 0.1$ for $\Delta = 4.975 \times 10^{-12}$.

Figure 3 shows the tune depression estimated with the above dimensionless parameter. The temperature is assumed by 1 eV. As shown in Fig.3, the tune depression is controlled in the range of $\sigma/\sigma_0 = 1 \sim 0$ for number density $n = 10^{11} \sim 10^{12} \text{ m}^{-3}$ with the several-10 krad/s of the rotation angular frequency.

5 Estimation of Tune depression considering with radial distribution of charge density

The frequency of rigid rotation of the plasma column ω_r , the cyclotron frequency ω_c , (electron) plasma frequency ω_p are defined by

$$\omega_r = \frac{E_r}{rB_z} \quad (3)$$

$$\omega_c = \frac{eB_z}{m_e} \quad (4)$$

$$\omega_p^2 = \frac{n_0 e^2}{m_e \epsilon_0}. \quad (5)$$

Here B_z is the magnetic flux density in axial (z) direction, e is the elementary charge, m_e is the electron mass, n_0 is the number density at uniform inside the beam, ϵ_0 is the permittivity of free

space, and the electric field in radial (r) direction is

$$E_r = \frac{e}{\epsilon_0} \frac{1}{r} \int_0^r n(r) r dr = \frac{n_0 e}{\epsilon_0 r} \int_0^r \tilde{n}(r) r dr \quad (6)$$

where $n(r) = n_0 \tilde{n}(r)$.

Substituting Eqs.(3), (4), (5), and (6) into Eq.(2),

$$\Delta = 2 \left(1 - \frac{m_e n_0}{\epsilon_0 B_z^2} \frac{\tilde{E}_r}{r} \right) \frac{\tilde{E}_r}{r} - 1 \quad (7)$$

where

$$\frac{\tilde{E}_r}{r} = \frac{1}{r^2} \int_0^r \tilde{n}(r) r dr \quad (8)$$

depends on the density profile inside the beam. From Eq.(7), the dependence of Δ is written with

$$\Delta \propto 1 - \frac{n_0}{B_z^2}. \quad (9)$$

As a result, it is found that the condition for dense and weak strength of applied magnetic flux density creates the space-charge-dominated state.

If the distribution is flat top for strong space-charge-dominated state (tune depression $\sigma/\sigma_0 = 0$, temperature $T \rightarrow 0$), the number density of the beam is described with

$$\tilde{n}(r) = \begin{cases} 1 & (r < a_0) \\ 0 & (\text{otherwise}) \end{cases} \quad (10)$$

where a_0 is the zero-temperature beam radius [14]. Substituting Eq.(10) into Eq.(8), \tilde{E}_r/r inside the beam is given by

$$\frac{\tilde{E}_r}{r} = \frac{1}{2} = 0.5. \quad (11)$$

If the distribution becomes Gaussian for weak space-charge-dominated state (= emittance-dominated limit) ($\sigma/\sigma_0 = 1$, $T \rightarrow \infty$), the number density of the beam is assumed with

$$\tilde{n}(r) = \frac{n(0)}{n_0} \exp\left(-\frac{r^2}{\tilde{r}_0^2}\right) \quad (12)$$

where $\tilde{r}_0 = a_0/\sqrt{2}$. Substituting Eq.(12) into Eq.(8),

$$\frac{\tilde{E}_r}{r} = 1 - \frac{1}{e} \simeq 0.632... \quad (13)$$

for $r = \tilde{r}_0$ (beam edge radius of equivalent rms beam). Here $n(0)/n_0 = 2$ for $T \rightarrow \infty$ [14].

Figure 4 shows the Δ using Eq.(7) as a function of number density and magnetic flux density for

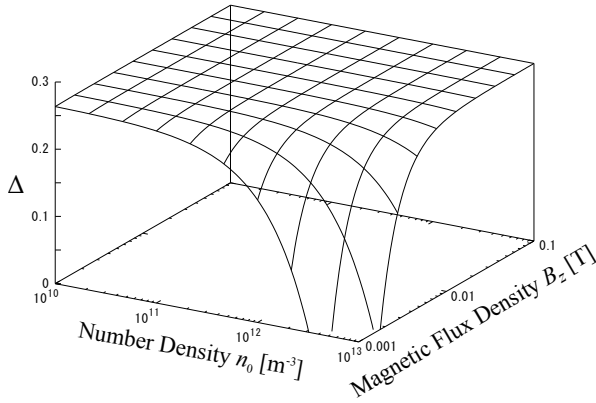


Figure 4: Dimensionless parameter Δ as a function of number density and magnetic flux density for $\tilde{E}_r/r = 0.632$.

$\tilde{E}_r/r = 0.632$ (i.e., at beam edge radius of equivalent rms beam for Gaussian distribution).

For this reason, the range of \tilde{E}_r/r is written by

$$\frac{1}{2} < \frac{\tilde{E}_r}{r} < 1 - \frac{1}{e} \quad (14)$$

or

$$0.5 < \frac{\tilde{E}_r}{r} < 0.632 \quad (15)$$

If in case of low density ($n_0 \rightarrow 0$) and/or extreme strong magnetic flux ($B_z \rightarrow \infty$) condition, the maximum value of Δ is given by

$$\Delta \simeq 2 \frac{\tilde{E}_r}{r} - 1 \simeq 0.264... \quad (16)$$

for $\tilde{E}_r/r = 0.632$. From Ref. [19], the dimensionless parameter $\Delta = 0.264$ corresponds to the tune depression $\sigma/\sigma_0 = 0.7$. Also, as shown in Fig.4, the tune depression is always below 0.7.

6 Conclusion

The simulating beam parameter was studied in the compact beam simulator constructed with the Malmberg–Penning trap device for HIF system. The tune depression was estimated with the Brillouin density limit, the rigid–rotor Vlasov equilibrium, and the model considering with the radial distribution of charge density.

The estimation of tune depression based on the Brillouin limit implied the quite weak in the space charge condition in the experimental device.

The estimation of tune depression based on the rigid–rotor Vlasov equilibrium indicated from

weak to strong space charge condition. However, the assumption of the rigid–rotor model is a little bit odd, because of the rotation is driven by $\mathbf{E} \times \mathbf{B}$ drift, and the electrical field \mathbf{E} depends on the charge density distribution in the radial direction. The rigid–rotor equilibrium fixes the linear electric field distribution.

The estimation of tune depression considering with the radial distribution of charge density was derived, and indicated that the tune depression becomes always below 0.7, theoretically.

As a result, it is considered that the above estimations are not suitable for the space charge strength of the condition in the device.

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