COMPUTATIONAL EXPERIMENTS ON SIMULATION OF ONE-DIMENSIONAL OIL-DISPLACEMENT PROCESS

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Аннотация. В статье представлены результаты вычислительных экспериментов по моделированию одномерного процесса вытеснения нефти и прогнозированию значений коэффициента текущей нефтеотдачи в рамках математических моделей Баклея-Леверетта и Лиса-Рапопорта.

Ключевые слова: нефтевытеснение, двухфазная фильтрация, нагнетательная скважина, эксплуатационная скважина, математическая модель, функция насыщенности, дифференциальное уравнение, граничное условие, конечно-разностный метод, алгоритм, вычислительные эксперименты, распределение функции насыщенности, коэффициент текущей нефтеотдачи.

1. Statement of the problem. Various mathematical models of two-phase filtration in a porous medium have been developed, which are used for a theoretical study of the oil displacement process. They differ among themselves as completeness of the account of physical factors of the process, and the form of their account. For example, the Buckley-Leverett model does not take into account the difference between phase pressures, and the Leas-Rapoport model takes into account the action of capillary forces [1-2]. In the first model, the distribution of the saturation function of the displacing phase in the one-dimensional two-phase filtration problem without allowance for gravitational forces for a given volumetric flow rate of a two-phase liquid is described by a nonlinear equation

$$m \cdot \partial S / \partial t + W \cdot \partial \varphi(S) / \partial x = 0, \qquad (1)$$

where $S = S(x, t)$ is the required function, m - the porosity of the oil-bearing medium, W - the volume flow of the two-phase liquid, $\varphi(S)$ is a known function expressed in functions of the relative permeability of the displaced and displacing phases $f_1(S)$ and $f_2(S)$, the ratio of the dynamic viscosities of the phases $\mu_0 = \mu_1 / \mu_2$: $\varphi(S) = \mu_0 \cdot f_2(S) / [f_1(S) + \mu_0 \cdot f_2(S)]$, x and t, respectively, the spatial and time variables. In the second model, a parabolic equation

$$m \cdot \partial S / \partial t + W \cdot \partial \varphi(S) / \partial x = \partial [a(S) \cdot \partial S / \partial x] / \partial x \qquad (2)$$

is obtained with respect to the function $S(x, t)$, where $a(S) = -k_1 \cdot \varphi(S) \cdot P_k'(S) \geq 0$, $k_1 = k / \mu_1 \cdot f_1(S)$, k is the absolute permeability of the medium, $P_k(S)$ - is the capillary pressure function [1-3]. The functions $f_1(S)$ and $f_2(S)$ are determined from natural experiments on displacement and have the properties

$$f_1(\underline{S}) = 1,\ f_1(\overline{S}) = 0,\ f_2(\overline{S}) = 1,\ f_2(\underline{S}) = 0, \quad (3)$$

where $\underline{S}$ and $\overline{S}$ - respectively, the lower and upper limit value of the function $S(x, t)$.

In the Buckley-Leverett model, the injection condition of the displacing phase in the injection well Γ_H generates a boundary condition

$$S = \overline{S}\,[1\text{-}3]. \quad (4)$$

In the Leas-Rapoport model this condition has the form

$$W_2\,|_{\Gamma_H} = \left[\varphi(S)\cdot W - a(S)\cdot \partial S/\partial x\right]|_{\Gamma_H} = W = const \quad (5)$$

and generates a boundary condition

$$(\partial S/\partial x)|_{\Gamma_H} = -\left[1-\varphi(S)\right]\cdot W/a(S)\,|_{\Gamma_H}. \quad (6)$$

The selection condition on the production well of both phases in proportion to their phase permeability $k_i = k/\mu_i \cdot f_i(S),\ i = 1,2$ generates the boundary condition

$$(\partial S/\partial x)|_{\Gamma_Э} = 0\ [1]. \quad (7)$$

It is also assumed that a certain initial distribution of the saturation function of the displacing phase is given:

$$S(x, 0) = S_0(x). \quad (8)$$

2. Numerical method for solving of problems. Problems (1), (4), (8) and (2), (6), (7), (8) are solved numerically by a finite-difference method in the domain $D = \{0 \le x \le L;\ 0 \le t \le T\}$ which is covered by a uniform grid $x_i = i\cdot h, t_j = j\cdot\tau\ i = 0, 1, ..., N,\ j = 0, 1, ..., M$, h and τ are, respectively, the step of the difference grid in space and time. For the solving of problem (1), (4), (8) an explicit finite-difference scheme "central difference" $m\cdot(S_i^{j+1} - S_i^j)/\tau + W\cdot(\varphi_{i+1/2}^j - \varphi_{i-1/2}^j)/h = 0$ is used, where $\varphi_{i+1/2}^j = \varphi((S_{i+1}^j + S_i^j)/2)$, and for solving the problem (2), (6), (7), (8) is used a finite-difference scheme

$$m\cdot(S_i^{j+1} - S_i^j)/\tau + W\cdot(\varphi_{i+1/2}^j - \varphi_{i-1/2}^j)/h = \left[a_{i+1/2}^j\cdot(S_{i+1}^j - S_i^j) - a_{i-1/2}^j\cdot(S_i^j - S_{i-1}^j)\right]/h^2, \quad (9)$$

where $a_{i+1/2}^j = a((S_{i+1}^j + S_i^j)/2)$.

3. Algorithm for numerical realization of boundary conditions. The injection well is located at a point $x = 0$, and the operating well is at a point $x = L$. By virtue of the properties (3) of the functions of the relative phase permeability, it follows that in the Leas-Rapoport mode

$$\varphi(\underline{S}) = 0,\ \varphi(\overline{S}) = 1,\ a(\overline{S}) = a(\underline{S}) = 0 \quad (10)$$

Consequently, in the boundary condition (6) $\lim\limits_{S\to\underline{S}}(\partial S/\partial x)|_{\Gamma_H} = -\infty$. For values of a function $S = S(x, t)$ close to $\underline{S}$, the numerical realization of the boundary condition (6) causes certain difficulties, such as shredding the difference grid h in the vicinity of the injection well [1-4]. In the numerical solving of problem (2), (6), (7), (8) it is possible to realize the boundary condition not in the form (6), but in the form (5), with the help of the constructed difference and initial differential equations. A detailed exposition of such an algorithm for numerical realization of the boundary condition is given in [1-3].The difference equation (9) at the node $(i = 0, j + 1)$ takes the form

$$S_0^{j+1} = S_0^j + 2\cdot\tau/(m\cdot h)\cdot\left[(1-\varphi_{1/2})\cdot W + a_{1/2}^j\cdot(S_1^j - S_0^j)/h\right], \quad (11)$$

which allows to calculate the value of the saturation function on the injection well at each subsequent time step $(j+1)$ of the difference grid. If the initial saturation of the oil-bearing stratum by the displacing phase is equal to the residual $\underline{S}$, then the ratio (11) in the first step with respect to time takes the form $S_0^1 = S_0^0 + 2\cdot\tau/(m\cdot h)\cdot W$, whence it is seen that the saturation of the displacing phase on the injection well begins to increase with some finite rate in proportion to the injected volume of the displacing phase W and the time step of the difference grid τ, inversely proportional to the step of the difference grid along the space h and the coefficient of porosity m. Further, as the value of S_0^{j+1} is growing values of $\left(1 - \varphi_{1/2}^j\right)$ and $a_{1/2}^j$ decrease, and, according to (11), it is should be expected a slowdown in the growth of saturation of the displacing phase in the injection well. When the upper limiting value of the saturation of the displacing phase $\bar{S}$ is reached at the injection well and in its vicinity, according to the properties (10) of the functions $\varphi(S)$ and $a(S)$, the relation (11) takes the form $S_0^{j+1} = S_0^j$, i.e. the realization of the boundary condition (5) in the form (11) reflects the fact that the further saturation of the displacing phase on the injection well is stopped.

The boundary condition on the producing well is realized similarly to the boundary condition on the injection well and it is obtained the relation $S_N^{j+1} = S_N^j - 2\cdot\tau/(m\cdot h)\cdot\left[\left(\varphi_N^j - \varphi_{N-1/2}^j\right)\cdot W + a_{N-1/2}^j\cdot\left(S_N^j - S_{N-1}^j\right)/h\right]$ [1-3], from which it can be seen that the saturation of the displacing phase on the production well begins to increase only after the arrival of the displacement front, in this case $\left(S_N^j - S_{N-1}^j\right) < 0$, $\left(\varphi_N^j - \varphi_{N-1}^j\right) < 0$ and $S_N^{j+1} > S_N^j$.

4. Nature of the oil displacement process. Computational experiments on the numerical solving of a one-dimensional two-phase filtration problem with respect to the oil-displacement process are carried out. Functions of relative phase permeability of the form $f_1(S) = ((0{,}8 - S)/0{,}6)^3$, $f_2(S) = ((S - 0{,}2)/0{,}8)^3$, for which $\underline{S} = 0{,}2$ and $\bar{S} = 1$, as well as the function of capillary pressure $P_k(S) = \sigma\cdot\cos\theta\cdot\sqrt{m/k}\cdot(0{,}0072/S - S/2 + 0.391)$ are used [1-2]. The physical parameters and parameters of the computational algorithm have the following values: $L = 40$ sm, $m = 0.2$, $k = 0.0000000302$ md, $\theta =$ 0.5, $\mu_1 = 0.010$ poise, $\mu_2 = 0.001$ poise, $W = 0.012$sm^3/sec, $\sigma = 75$ dynes/sm, $N = 20$, $h = 2$ sм, $\tau =$ 0.02 sec, $\mu_0 = \mu_1/\mu_2 = 10$. Distributions of the displacing phase saturation obtained in the framework of the considered mathematical models of two-phase filtration at time t=180 seconds at $S(x, 0) = 0.2$ are shown in Fig.1. From the graph of the saturation function (upper line), it is seen that in the Buckley-Leverett model a relatively "piston" displacement character with a displacement front altitude of 0.4662 is observed. In calculations based on the Leas-Rapoport model (bottom line), it can be seen that in this model a "piston" displacement character with a displacement front height of 0.3326 is observed also, but less pronounced than in the Buckley-Leverett model. As the value of the saturation function approaches the residual value $\underline{S}$=0.2 (initial distribution), the value of the diffusion coefficient decreases and tends to 0 because of the properties of the function $a(S)$. Convective transfer under the action of the pressure gradient prevails over diffusion redistribution of phases under the action of capillary forces and the process of displacement becomes "piston". On the injection well, the value of the saturation function of the displacing phase increases with time at a certain finite rate, which agrees with the results of natural experiments [5-6]. At the production well, the saturation of the displacing phase begins to increase with the arrival of the displacement front.

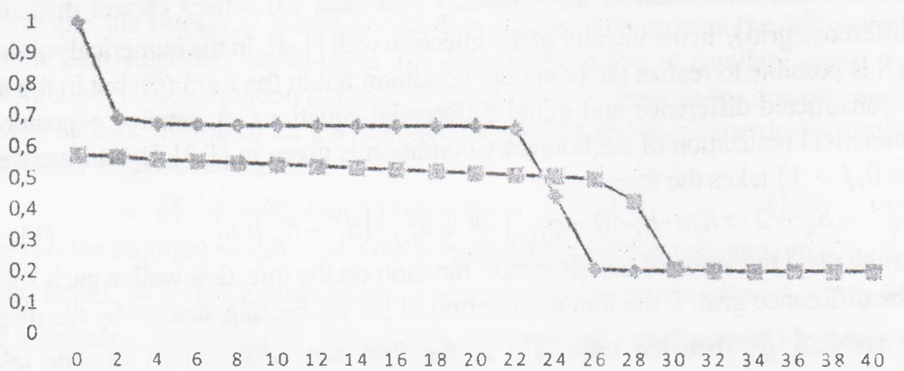

Fig.1. Distribution of the saturation function of the displacing phase

5. Dynamics of the current oil recovery factor. Figure 2 shows the dependence of the current oil recovery factor on time. The oil recovery factor $\kappa_0(t)$ at the current time t is the ratio of the currently extracted oil to its initial stock. It can be seen from the graphs that in the calculations for the Buckley-Leverett model (the upper line) until about time t = 300 seconds, the current oil recovery coefficient grows in direct proportion to time, and then a decrease in growth occurs. This time corresponds to the arrival time of the displacement front to the production well and the start of the joint selection of both phases, and displaced and displacing, the share of the oil phase in the bleed stream decreases, which is the reason for the decrease in the growth of the current oil recovery factor. In the framework of the Leas-Rapoport model (the bottom line), the linear growth of the current oil recovery coefficient is violated at the time t = 240 sec, earlier than in the Buckley-Leverett model. This reflects the fact that the Leas-Rapoport model takes into account the influence of capillary forces. Under the action of these forces, the process of displacement has a less "piston" character, the displacement front comes to the production well earlier than the Buckley-Leverett model, and earlier the joint selection of both phases begins.

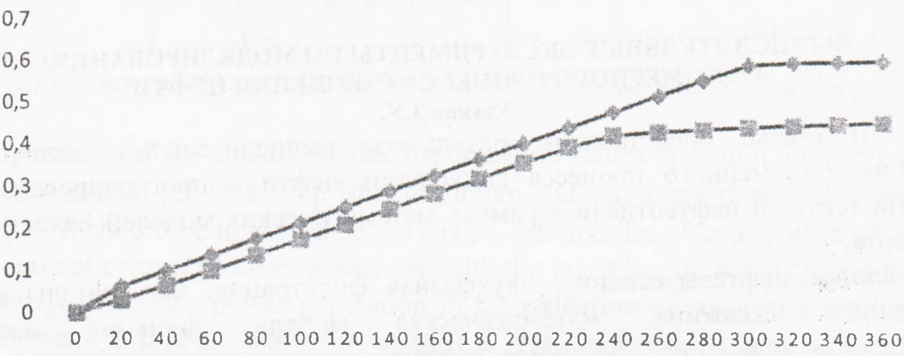

Fig. 2. Graph of the dependence of the current oil recovery factor on time.

7. Conclusion. The results of computational experiments on the computer show that with the functional dependencies, the values of the physical parameters and the parameters of the computational algorithm used in this paper, the Leas-Rapoport model, which takes into account the effect of capillary forces, gives results that are close to the results of natural experiments. The Buckley-Leverett model gives higher values of the current oil recovery factor. It is more applicable to modeling such processes of oil displacement, in which predominates convective transfer under the effect of pressure drop.

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Annotation. The paper presents the results of computational experiments on modeling a one-dimensional oil displacement process and forecasting the values of the current oil recovery coefficient within the framework of the mathematical models of Buckley-Leverett and Leas-Rapoport.

Keywords: oil displacement, two-phase filtration, injection well, production well, mathematical model, saturation function, differential equation, boundary condition, finite-difference method, algorithm, computational experiments, distribution of the saturation function, current oil recovery coefficient

ВЫЧИСЛИТЕЛЬНЫЕ ЭКСПЕРИМЕНТЫ ПО МОДЕЛИРОВАНИЮ ОДНОМЕРНОГО ПРОЦЕССА СМЕЩЕНИЯ НЕФТИ

Узаков З.У.

Аннотация. В статье представлены результаты вычислительных экспериментов по моделированию одномерного процесса вытеснения нефти и прогнозированию значений коэффициента текущей нефтеотдачи в рамках математических моделей Баклея-Леверетта и Лиса-Рапопорта.

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