

Modeling the UO₂ ex-ADU pellet process

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Abstract

Studies on modeling uranium dioxide (UO₂) powder and pellet processes from ammonium diuranate (ADU)- derived uranium dioxide powder (UO₂ ex-ADU powder) were reported in the paper. A mathematical model describing the effect of the fabrication parameters on specific surface area (SSA) of UO₂ powders was built up. The Brandon model is used to describe the relationship between the essential fabrication parameters [reduction temperature (T_R), calcination temperature (T_C), calcination time (t_C) and reduction time (t_R)] and SSA of the obtained UO₂ powder product. Response surface methodology (RSM) based on face centered (CCF), one type of quadratic central composite design (CCD), was used to model the pellet process. The experimental studies on the UO₂ pellet process determined region of experimental planning as follows: conversion of ADU into UO₂ powder at various temperatures of 973 K, 1023 K and 1073 K and sintering of UO₂ pellets at temperatures of 1923 K, 1973 K and 2023 K for times of 4 h, 6 h and 8 h. On the base of the proposed model, the relationship between the technological parameters and density of the UO₂ pellet product was suggested to control the UO₂ ex-ADU pellet process as desired levels.

Key Words: UO₂ ex-ADU, UO₂ pellet process, modeling;

1. Introduction

In nuclear fuel technology for light water reactors (LWRs), uranium dioxide (UO₂) is the essential material for the fabrication of ceramic fuel that has been widely used in both pressurized water reactors (PWR) and boiling water reactors (BWR). Uranium in the form of UO₂ ceramic pellets has been used as fuel in more than three quarters of the total installed capacity of nuclear power plants [1-3].

The manufacture of the UO₂ nuclear fuel pellets includes the conversion of UF₆ into UO₂ powder and the fabrication of UO₂ pellets from such UO₂ powder [1-3]. In regard to the conversion of UF₆ into UO₂ powder, many wet and dry conversion methods have been developed. In a former wet conversion, UF₆ was hydrolyzed in water to form uranyl fluoride – fluoride acid (UO₂F₂-HF) solution. Subsequently, the solution was precipitated through either an ammonium di-uranate (ADU) route or an ammonium uranyl carbonate (AUC) route. These ADU and/or AUC powders are then calcinated and reduced into UO₂ powders.

The parameters of the UO₂ preparation strongly affect the final characteristics of UO₂ powder [4] and, therefore, have an effect on UO₂ pelletizing. Specific surface area (SSA) of the UO₂ powder is one of the most important characteristics affecting the activity and the correspondence of the powder during UO₂ ceramic pellet fabrication. The SSA is a function of grain size, aggregation and agglomeration, morphology and structure of the powder [5-6]. Therefore, SSA is considered as the most important feature to assess sinterability of the UO₂ powder. In an effort to control the SSA of UO₂ powder, we established a mathematical model to describe the relationship between its SSA and the process parameters for the calcination and reduction that were employed for UO₂ powder fabrication via ADU route.

An important prerequisite for stabilizing and controlling the UO₂ pellet process is to find quantitative relationships between product characteristics and process parameters. For UO₂ pellet process the density is one of the most important product characteristics [7-8]. There are many factors affecting directly and indirectly the final density of the pellets, including technological parameters, machine, operator empowerment, process review and etc. The most important factors affecting directly the UO₂ pellet process are technological parameters, including material parameters of calcination – reduction conversion of ADU into UO₂ ceramic powder (temperature and time for calcination and reduction) and process parameters of UO₂ pellet sintering (sintering temperature and time) [7-8]. In the study, a model for the UO₂ ex-ADU pellet process was established to assess the systematic relationship between the technological parameters and the density of UO₂ ex-ADU pellets that could apply to nuclear fuel fabrication and design. Three of the most important technological parameters including conversion temperature, sintering temperature, and sintering time were studied; and RSM based on CCF type of CCD improved by Box and Hunter was empirically used to study on and model the interactive effect of the technological parameters (independent variables) on the UO₂ pellet density (response variable). The model showed the contribution of individual parameter that controls the density of the UO₂ pellet products through those important parameters. So, the purpose of the present study is to assess the effects of the three technological parameters on the UO₂ ex-ADU pellet process, using RSM based on CCF type of CCD for designing the experiments to minimize the experimental runs, for developing the model to optimize the UO₂ ex-ADU pellet process conditions and for assessing the effect of the parameters on the pellet density to control the process.

2. Experiments

2.1. Experimental methods

The ADU powder was precipitated by the reaction of ammonium hydroxide with a synthetic solution containing UO₂F₂ and HF with U:F molar ratio of 1:6. The calcination of ADU into U₃O₈ and the reduction of U₃O₈ into UO₂ powder were carried out in an apparatus consisting of a rotary tube furnace 1300°C (Nabertherm, Germany) and hydrogen-nitrogen-steam supply system. The calcination was carried out over a range of time and temperatures in an atmosphere of nitrogen and steam (1:1 in molar ratio). After the calcination finished, the subsequent reduction was carried out in a reducing atmosphere of hydrogen and nitrogen gases (3:1 in molar ratio). The final product was UO₂ powder. The specific surface area (SSA) of the obtained UO₂ powder was measured by the Brunauer–Emmett–Teller (BET) method (Coulter SA 3100, USA).

Sintering was carried out with UO₂ pellets prepared from UO₂ powder samples at the various conversion temperatures. The UO₂ powder samples first were blended with 10 wt.% and 0.25 wt.% of U₃O₈ and porous former (ammonium oxalate), respectively; and then compacted green pellets in a die of 11.3 mm in diameter by using a hydraulic single acting press (Carver, USA) and pressing at 350 to 400 MPa, lubricating on die surface with a mixture of zinc stearate and acetone. Sintering was performed at temperature of 1923 K, 1973 K and 2023 K for time of 4h, 6 h and 8 h in a high temperature furnace 1800 °C (Nabertherm, Germany) with a molybdenum heating sheet. A flow of high-purity hydrogen gas was used for a reducing atmosphere in sintering.

Density, the most important characteristic of the sintered pellet, was determined by hydrostatic (or Archimed) method [4].

2.2. Modeling method

RSM based on CCF type of CCD was empirically used to model the the UO₂ pellet process. The total number of required experimental runs was: $(2^k + 2k + n_0) = 17$, where k is the number of factors (k =3), n₀ is the number of replications at the center points (n₀ = 3). The UO₂ pellet

density (Y, in 10^3 kg/m^3) was taken as the response variable and described in the form given in Eq. (1).

$$Y = b_0 + \sum_{i=1}^k b_i X_i + \sum_{i=1}^k b_{ii} X_i^2 + \sum_{i,j=1(i \neq j)}^k b_{ij} X_i X_j \quad (1)$$

The UO_2 pellet process were estimated through the regression analysis and response surface plots of the independent variables (X_i) and each dependent variable (Y).

3. Results and discussion

3.1. Modeling the UO_2 ex-ADU powder process

Multiple regression analysis for the establishment of Brandon equation

In order to master preparing the UO_2 powders whose properties are appropriate to the UO_2 ceramic pellet fabrication and on the basis of experimental data that describe the effects of process conditions on SSA of UO_2 powder, a statistical modeling method using Brandon multiple regression model is used. The form of Brandon mathematical equation is as follows:

$$y = a \cdot f_1(x_1) f_2(x_2) \dots f_j(x_j) \dots f_k \quad (2)$$

Where, y denotes the SSA of UO_2 powder, $f_j(x_j)$ are the functions presenting the effect of process parameter x_j on SSA (y), and a is a constant.

In Brandon equation, the series of functions $f_j(x_j)$ are presented in a descending order of the relevance of process factors.

In order to establish Brandon equation, an experimental data set $\{y; x_1, x_2, \dots, x_k\}$ is used for determining the regression function $y = f_1(x_1)$. From $f_1(x_1)$, a new data set is obtained by evaluating:

$$\hat{y}_1 = \frac{y}{f(x_1)} \quad (3)$$

As a result, \hat{y}_1 is independent on x_1 but is affected by x_2, x_3, \dots, x_k :

$$\hat{y}_1 = a \cdot f_1(x_1) \cdot f_2(x_2) \dots f_j(x_j) \dots f_k(x_k) \quad (4)$$

The others $f_j(x_j)$ are calculated in the same way with $f_1(x_1)$, we obtain:

$$\hat{y}_k = \frac{y_{k-1}}{f(x_k)} = \frac{y}{f_1(x_1) \cdot f_2(x_2) \dots f_k(x_k)} \quad (5)$$

Our experimental data indicated that four parameters (factors) affecting SSA of UO_2 powder are in a descending order as follows: reduction temperature T_R , calcination temperature T_C , calcination time t_C , and reduction time t_R . Thus, we established Brandon model by determining corresponding parameters in that order.

By using the method of least squares and Solver tool of Microsoft Excel, the function $f_1(T_R)$ is determined in the equation as follows:

$$f_1(T_R) = 5.2506 - 0.0023 \cdot T_R \quad (6)$$

\hat{y}_1 was calculated as follows:

$$\hat{y}_1 = \frac{y}{f_1(T_R)} = \frac{SSA(Ex.)}{f_1(T_R)} \quad (7)$$

With the same calculation, the other functions of T_C , t_C , and t_R were obtained as bellows:

$$f_2(T_C) = 3.1369 - 0.0031 \cdot T_C \quad (8)$$

$$f_3(t_C) = 0.8899 + 0.031 \cdot t_C \quad (9)$$

$$f_4(t_R) = 0.9324 - 0.0166 \cdot t_R \quad (10)$$

The corresponding independent functions \hat{y}_1 were:

$$\hat{y}_2 = \frac{\hat{y}_1}{f_2(T_C)} \quad (11)$$

$$\hat{y}_3 = \frac{\hat{y}_2}{f_3(t_C)} \quad (12)$$

$$\hat{y}_4 = \frac{\hat{y}_3}{f_4(t_R)} \quad (13)$$

All of these values are reported in **Table 1**.

The constant a in Brandon equation was calculated from average of y_4 to be 1.00006.

Thus, Brandon function describing the effect of the process parameters on the SSA of the UO_2 powder is in the form:

$$y(SSA) = a \cdot f_1(T_R) \cdot f_2(T_C) \cdot f_3(t_C) \cdot f_4(t_R) \quad (14)$$

$$y(SSA) = 1.00006 \cdot (5.2506 - 0.0023 \cdot T_R) \cdot (3.1369 - 0.0031 \cdot T_C) \cdot (0.8899 + 0.031 \cdot t_C) \cdot (0.9324 + 0.0166 \cdot t_R) \quad (15)$$

$SSA_{(Cal.)}$ values of the UO_2 powder are shown in **Table 1**.

Test Brandon mathematical model by Wilcoxon's rank sum test.

The Wilcoxon rank-sum test is a nonparametric alternative to the two-sample (for example A and B) test that we wish that the data of measurements in population A is the same as that in B. We have two groups:

Group $SSA_{(Ex.)}$: $X_1, X_2, X_3, \dots, X_{n1}$; distribution \dot{y}

Group $SSA_{(Cal.)}$: $Y_1, Y_2, Y_3, \dots, Y_{n2}$; distribution \hat{y}

Null Hypothesis: $SSA_{(Ex.)} = SSA_{(cal.)}$

Herein, $SSA_{(Ex.)}$ is experimentally obtained SSA. The two groups are combined into one group (for example W_T) W_T of $W_{(1)}, W_{(2)}, W_{(3)}, \dots, W_{(n1+n2)}$; order data in the combined group $W_{(1)} \leq W_{(2)} \leq \dots \leq W_{(n1+n2)}$; and then assign ranks (as in **Table 2**).

Thus, sum of ranks S of group \hat{y} is calculated as follows:

$$S=2+4+5+12+13+14+15+17+18+21+23+25+26+27=222$$

Table 2.

Order of all observations in the combined sample and assign ranks of the group W_T ($SSA_{(Cal.)}$ data are underlined)

| | | | | | | | | | | |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-------|-------|
| W_T | 2.868 | 2.899 | 2.917 | 2.994 | 3.182 | 3.34 | 3.424 | 3.478 | 3.514 | 3.538 |
| Rank | 1 | <u>2</u> | 3 | 4 | <u>5</u> | 6 | 7 | 8 | 9 | 10 |
| W_T | 3.549 | 3.552 | 3.613 | 3.613 | 3.624 | 3.626 | 3.674 | 3.735 | 4.07 | 4.199 |
| Rank | 11 | <u>12</u> | <u>13</u> | <u>14</u> | <u>15</u> | 16 | <u>17</u> | <u>18</u> | 19 | 20 |
| W_T | 4.205 | 4.333 | 4.338 | 4.43 | 4.471 | 4.604 | 4.771 | 5.921 | | |
| Rank | <u>21</u> | 22 | <u>23</u> | 24 | <u>25</u> | <u>26</u> | <u>27</u> | 28 | | |

Table 1.

Experimental and calculated data of function $f_1(T_R)$ and \hat{y}_1 ; $f_2(T_C)$ and \hat{y}_2 ; $f_3(t_C)$ and \hat{y}_3 ; $f_4(t_R)$ and \hat{y}_4 ; and $SSA_{(Cal.)}(\hat{y})$ used to establish Brandon mathematical model

| Sample | T_R (°C) | t_R (hr.) | T_C (°C) | t_C (hr.) | $SSA_{(Ex.)}(\hat{y})$ (m ² /g) | $f_1(T_R)$ | \hat{y}_1 | $f_2(T_C)$ | \hat{y}_2 | $f_3(t_C)$ | \hat{y}_3 | $f_4(t_R)$ | \hat{y}_4 | $SSA_{(Cal.)}(\hat{y})$ (m ² /g) |
|--------|---------------|----------------|---------------|----------------|---|------------|-------------|------------|-------------|------------|-------------|------------|-------------|--|
| M1 | 550 | 5 | 650 | 4 | 4.430 | 3.986 | 1.111501 | 1.122 | 0.990731 | 1.014 | 0.977149 | 1.015 | 0.962329 | 4.604 |
| M2 | 600 | 5 | 650 | 4 | 4.333 | 3.871 | 1.119465 | 1.122 | 0.997829 | 1.014 | 0.984150 | 1.015 | 0.969224 | 4.471 |
| M3 | 650 | 5 | 650 | 4 | 5.921 | 3.756 | 1.576579 | 1.122 | 1.405276 | 1.014 | 1.386010 | 1.015 | 1.364990 | 4.338 |
| M4 | 700 | 5 | 650 | 4 | 3.478 | 3.641 | 0.955337 | 1.122 | 0.851535 | 1.014 | 0.839861 | 1.015 | 0.827123 | 4.205 |
| M5 | 600 | 2 | 700 | 3 | 4.070 | 3.871 | 1.051517 | 0.967 | 1.087513 | 0.983 | 1.106433 | 0.966 | 1.145851 | 3.552 |
| M6 | 600 | 3 | 700 | 3 | 3.340 | 3.871 | 0.862915 | 0.967 | 0.892456 | 0.983 | 0.907982 | 0.982 | 0.924437 | 3.613 |
| M7 | 600 | 4 | 700 | 3 | 3.514 | 3.871 | 0.907870 | 0.967 | 0.938949 | 0.983 | 0.955284 | 0.999 | 0.956432 | 3.674 |
| M8 | 600 | 5 | 700 | 3 | 3.538 | 3.871 | 0.914070 | 0.967 | 0.945362 | 0.983 | 0.961809 | 1.015 | 0.947221 | 3.735 |
| M9 | 700 | 3 | 600 | 5 | 4.199 | 3.641 | 1.153381 | 1.277 | 0.903267 | 1.045 | 0.864453 | 0.982 | 0.880119 | 4.771 |
| M10 | 700 | 5 | 700 | 4 | 3.626 | 3.641 | 0.995990 | 0.967 | 1.030086 | 1.014 | 1.015964 | 1.015 | 1.000555 | 3.624 |
| M11 | 700 | 3 | 700 | 5 | 3.549 | 3.641 | 0.974839 | 0.967 | 1.008211 | 1.045 | 0.964888 | 0.982 | 0.982374 | 3.613 |
| M12 | 650 | 4 | 750 | 2 | 2.917 | 3.756 | 0.776707 | 0.812 | 0.956653 | 0.952 | 1.004993 | 0.999 | 1.006201 | 2.899 |
| M13 | 650 | 4 | 750 | 3 | 2.868 | 3.756 | 0.763660 | 0.812 | 0.940583 | 0.983 | 0.956947 | 0.999 | 0.958097 | 2.994 |
| M14 | 650 | 4 | 750 | 5 | 3.424 | 3.756 | 0.911705 | 0.812 | 1.122928 | 1.045 | 1.074675 | 0.999 | 1.075966 | 3.182 |

Mean rank (μ_T) of distribution \hat{y} is:

$$\mu_T = \frac{n_2(n_1 + n_2 + 1)}{2} = \frac{14(14 + 14 + 1)}{2} = 203$$

And the variance is:

$$\sigma_T^2 = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} = \frac{14 \cdot 14 (14 + 14 + 1)}{12} = 473.66$$

$$\sigma_T = \sqrt{\sigma_T^2} = \sqrt{473.66} = 21.76$$

95% reliability of μ_T is: $\mu_T \pm 1.96 \cdot \sigma_T$

$$\mu_T - 1.96 \cdot \sigma_T = 203 - 1.96 \cdot 21.76 = 160.35$$

$$\mu_T + 1.96 \cdot \sigma_T = 203 + 1.96 \cdot 21.76 = 245.65$$

The sum of ranks S of group \hat{y} is 222, in reliability range from 160.35 to 245.65, so two group $SSA(Ex.)$ and $SSA(Cal.)$ are asserted to be the same.

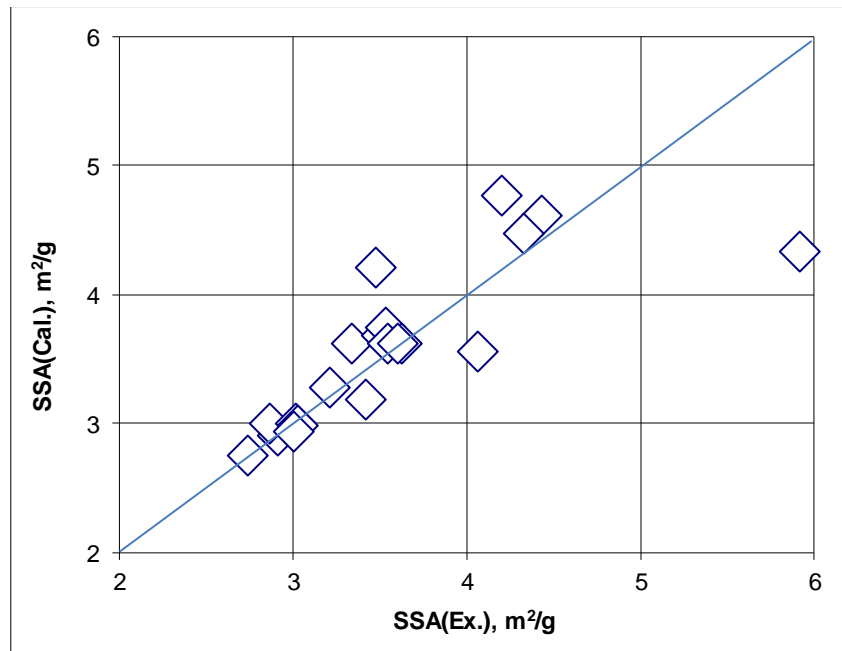


Fig. 1. The plot comparing $SSA(Ex.)$ with $SSA(Cal.)$ of the UO_2 powder.

Figure 2 is the plot comparing $SSA(Ex.)$ with $SSA(Cal.)$ of the UO_2 powder indicating the agreement of the proposed calculation with the experimental data. Thus, we suppose that the Brandon mathematical model is capable to describe the effect of the factors on the SSA of the UO_2 powder that was obtained from the calcination and reduction of ADU.

Table 3.

Characteristics of the UO_2 powder

| Inspection items | UO_2 ex-ADU | Methods |
|-----------------------------------|-------------------------------|------------------|
| SSA | 2.5 – 6.0 m ² /g | BET |
| Bulk density (g/cm ³) | 1.42 ± 0.11 g/cm ³ | Scott Volumeter |
| Tap density (g/cm ³) | 2.44 ± 0.16 g/cm ³ | Tap densitometer |
| O/U | 2.125 ± 0.037 | Gravimetry |
| F content | < 50 ppm | Pyrohydrolysis |

| Impurities (ppm) | ICP-MS |
|--|-----------------|
| Al | 119.5 |
| B, Cd, Cr, Co, Cu, Mo, Ta, Th, Ti, W, V | below detection |
| Mg | below detection |
| Ca | 58.2 |
| Fe | 47.2 |
| Pb | 0.13 |
| Mn | 0.26 |
| Ni | 0.13 |
| Rare Earths | < 1 |
| Si | 106.4 |
| Zn | below detection |

3.2. Modeling the UO₂ ex-ADU pellet process

The previous study [6] also shown that the densities of UO₂ ceramic pellet samples prepared from UO₂ ex-ADU powders at conversion temperatures of 973 K (700 °C) and 1023 K (750 °C) and at sintering temperature of 1973 K for 6 h were $10.21 \pm 0.27 \times 10^3 \text{ kg/m}^3$ and $10.14 \pm 0.17 \times 10^3 \text{ kg/m}^3$, respectively with the above conversion temperatures. Retesting sinterability of the UO₂ ex-ADU powders at conversion temperatures of 973 K, 1023 K and 1073 K (800 °C) was performed at a sintering temperature of 1973 K for 8 h, the average densities of the UO₂ ceramic pellet samples were $10.27 \pm 0.06 \times 10^3 \text{ kg/m}^3$, $10.26 \pm 0.09 \times 10^3 \text{ kg/m}^3$ and $10.58 \pm 0.06 \times 10^3 \text{ kg/m}^3$, respectively with the above conversion temperatures. On the other hand, testing sinterability of the UO₂ ex-ADU powder at conversion temperature of 1073 K was performed at sintering temperatures of 1923 K for 8 h and 2013 K for 4 h. The average densities of the UO₂ ceramic pellet samples were $10.23 \pm 0.12 \times 10^3 \text{ kg/m}^3$ and $10.46 \pm 0.11 \times 10^3 \text{ kg/m}^3$, respectively with the above sintering temperatures and sintering times.

From the above experimental results, region of the experimental planning was determined and coded on CCD type of CCF as follows: conversion temperatures (X₁) of 973 K (coded level of -1), 1023 K (coded level of 0) and 1073 K (coded level of 1); sintering temperatures (X₂) of 1923 K (coded level of -1), 1973 K (coded level of 0) and 2023 K (coded level of 1); and sintering time (X₃) of 4 h (coded level of -1), 6 h (coded level of 0) and 8 h (coded level of 1). Experimental studies on effect of the sintering temperature and time, and the conversion temperature on the UO₂ pellet density were performed based on the designed matrix under the defined conditions (as in Table 4) in order to obtain the good match data for modeling the UO₂ pellet process.

The effects of the sintering temperatures and times, and the conversion temperatures on the UO₂ pellet density were studied. The results of 17 experimental runs (as in Table 4) were entered into the MODDE 5.0 software in order to fit model by multiple linear regression. The results of 17 runs based on CCD type of CCF were also given in Table 4. The regression coefficients estimated by the software are: $b_0 = 10.30$, $b_1 = 0.31$, $b_2 = 0.16$, $b_3 = 0.06$, $b_{22} = -0.10$, $b_{33} = 0.05$, $b_{12} = -0.08$ and $b_{13} = -0.03$. The probability values (p-value) of b_{11} and b_{23} coefficients were greater than 0.05, indicating insignificant confidence levels; hence, they were rejected. The accuracy and variability of the above model could be evaluated by the coefficient of determination (R²). The R² for the UO₂ pellet process was calculated to be 0.996, explaining that the variability of response is at 99.6 % confidence level, and only 0.4 % of the total variations cannot be explained by the model. Moreover, the value of adjusted determination coefficient (adj. R²) of 0.992 was also close to 1. Thus, the calculated model for the UO₂ pellet process had a good agreement with the experimental data. Final calculated equation for the pellet density which incorporates the types of coded coefficients was shown in Eq. (16).

$$Y (\times 10^3 \text{ kg/m}^3) = 10.30 + 0.31X_1 + 0.16X_2 + 0.06X_3 - 0.10X_2^2 + 0.05X_3^2 - 0.08X_1X_2 - 0.03X_1X_3 \quad (16)$$

The calculated vs. experimental plot for the UO₂ pellet density was shown in Fig. 2 (a). It could be seen that the experimental results were distributed relatively near to a straight line with good agreement of the calculated (predicted) and experimental (actual) results. This demonstrates that the fitted regression coefficient to the equation (good fit of data) and the CCD model with an experimental design can be effectively applied for controlling the UO₂ pellet process.

The best way to visualize the influence of independent variables on the response is to draw surface response plots of the model. The shapes of three-dimensional response surfaces of the regression model constructed by MODDE 5.0 software show the nature and extent of the interactive relationships between independent variables and response, as in Fig. 2 (b).

It can be seen from Eq. (16) that b_1 (of X_1) and b_2 (of X_2) linear coefficients of regression model show positive effect on Y (UO₂ pellet density), therefore its response surface had a local maximum value.

Effect of the technological parameters on the UO₂ pellet process could be assessed through the coefficients of regression model, as in Eq. (16). The b_1 (of X_1) linear coefficient is much greater than the b_{12} (of X_1X_2) and b_{13} (of X_1X_3) interactive ones; contribution of X_1 on Y would be linear and could be quantitatively calculated by $b_0 \pm 0.26$. The b_3 (of X_3) linear coefficient was the same as the b_{33} (of X_3^2) quadratic one and their sum was greater than the b_{13} (of X_1X_3) interactive coefficient; so contribution of X_3 on Y would be half linear and half quadratic; this can be seen from Fig. 2 that the sintering temperature and time had a positive effect in conversion temperature range of 973 K to 1073 K, but a trend of inefficiency in low range from 973 K to 1023 K (the contour of the sintering temperature vs. the sintering time on the UO₂ pellet density at 973 K is similar to that at 1023 K) and efficiency in higher range from 1023 K to 1073 K was observed for the influence of the conversion temperature on the UO₂ pellet process; contribution of X_3 on Y could be quantitatively calculated by $b_0 \pm 0.10$. Contribution of X_2 on Y was small because positive effect of the b_3 (of X_3) linear coefficient would be eliminated by negative effects of the b_{22} (of X_2^2) quadratic and b_{12} (of X_1X_2) interactive ones; and the contribution could be quantitatively calculated by $b_0 \pm 0.02$. Thus, the contributions of X_1 , X_2 and X_3 to Y could be in order of $X_1 > X_3 > X_2$. The assessing of relationship between the X_i and Y would suggest controlling the UO₂ ex-ADU pellet process, that is necessary and important for nuclear fuel fabrication and design aspects of commercial nuclear reactors. One of characteristics of sintered UO₂ pellet products for nuclear fuel is the density achieving value of 10.30×10^3 kg/m³ to 10.70×10^3 kg/m³ [4]. From the proposed model, the technological parameters for the UO₂ pellet process would be calculated so that the UO₂ pellet product has a desirable density.

Otherwise, SSA of the UO₂ ex-ADU powders calculated from Eq. 15 at the conversion temperature of 973 K, 1023 K and 1073 K are 3.7 m²/g, 3.0 m²/g and 2.3 m²/g, respectively [6]. It could be seen that general UO₂ powder SSA of around 2.3 m²/g is of sinterability.

On the base of the experimental and modeling studies, a flow sheet for preparing the UO₂ ex-ADU pellet product of the density of 10.5×10^3 kg/m³ was proposed, as in Fig. 3, and could be described as follows: the ADU was converted into UO₂ powder in rotary furnace through calcination in atmosphere of stream and N₂ mixture and reduction in atmosphere of H₂ and N₂ mixture at temperature of 1073 K for 5 h, the UO₂ powder obtained would be of the sinterability; the UO₂ pellet preparing was carried out with the stages: blending with U₃O₈ (10 wt.%) as additive and ammonium oxalate (0.25 wt.%) as pore former, prepressing at 200 MPa pressure, granulating under 20 mesh, pressing at 350 to 400 MPa to form green pellet and sintering in high temperature furnace in H₂ and N₂ mixture at 1973 K for 7.0 h to 8.0 h; density of the UO₂ pellet product would be approximately 10.5×10^3 kg/m³.

Table 4
Central composite rotatable design arrangement and results.

| Run order | Independent variables | | | | | | Responses | | |
|-----------|-----------------------|----------------|----------------|-----------------------------|----------------------|------------------------------|---|----------|---|
| | Coded level | | | Real value | | | Experimental (Actual) | | Calculated (Predicted), in 10 ³ kg/m ³ |
| | X ₁ | X ₂ | X ₃ | Sintering temperature, in K | Sintering time, in h | Conversion temperature, in K | Density, in 10 ³ kg/m ³ | CV, in % | |
| 1 | -1 | -1 | -1 | 1923 | 4 | 773 | 9.59 ± 0.12 | 1.30 | 9.60 |
| 2 | 1 | -1 | -1 | 2023 | 4 | 773 | 10.46 ± 0.10 | 1.00 | 10.44 |
| 3 | -1 | 1 | -1 | 1923 | 8 | 773 | 10.08 ± 0.15 | 1.50 | 10.07 |
| 4 | 1 | 1 | -1 | 2023 | 8 | 773 | 10.60 ± 0.09 | 0.83 | 10.59 |
| 5 | -1 | -1 | 1 | 1923 | 4 | 873 | 9.77 ± 0.17 | 1.77 | 9.78 |
| 6 | 1 | -1 | 1 | 2023 | 4 | 873 | 10.48 ± 0.08 | 0.74 | 10.49 |
| 7 | -1 | 1 | 1 | 1923 | 8 | 873 | 10.23 ± 0.12 | 1.22 | 10.25 |
| 8 | 1 | 1 | 1 | 2023 | 8 | 873 | 10.65 ± 0.11 | 1.05 | 10.64 |
| 9 | -1 | 0 | 0 | 1923 | 6 | 823 | 10.00 ± 0.16 | 1.58 | 9.98 |
| 10 | 1 | 0 | 0 | 2023 | 6 | 823 | 10.58 ± 0.12 | 1.10 | 10.60 |
| 11 | 0 | -1 | 0 | 1973 | 4 | 823 | 10.06 ± 0.16 | 1.58 | 10.05 |
| 12 | 0 | 1 | 0 | 1973 | 8 | 823 | 10.35 ± 0.08 | 0.81 | 10.36 |
| 13 | 0 | 0 | -1 | 1973 | 6 | 773 | 10.26 ± 0.10 | 0.98 | 10.29 |
| 14 | 0 | 0 | 1 | 1973 | 6 | 873 | 10.44 ± 0.11 | 1.03 | 10.40 |
| 15 | 0 | 0 | 0 | 1973 | 6 | 823 | 10.29 ± 0.12 | 1.16 | 10.30 |
| 16 | 0 | 0 | 0 | 1973 | 6 | 823 | 10.28 ± 0.11 | 1.11 | 10.30 |
| 17 | 0 | 0 | 0 | 1973 | 6 | 823 | 10.31 ± 0.12 | 1.15 | 10.30 |

CV is coefficient of variation.

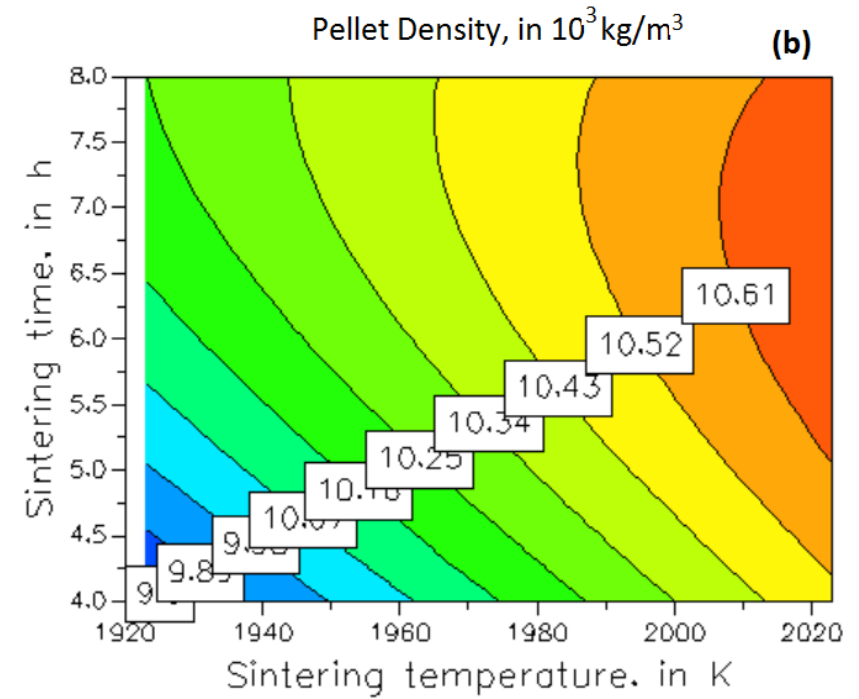
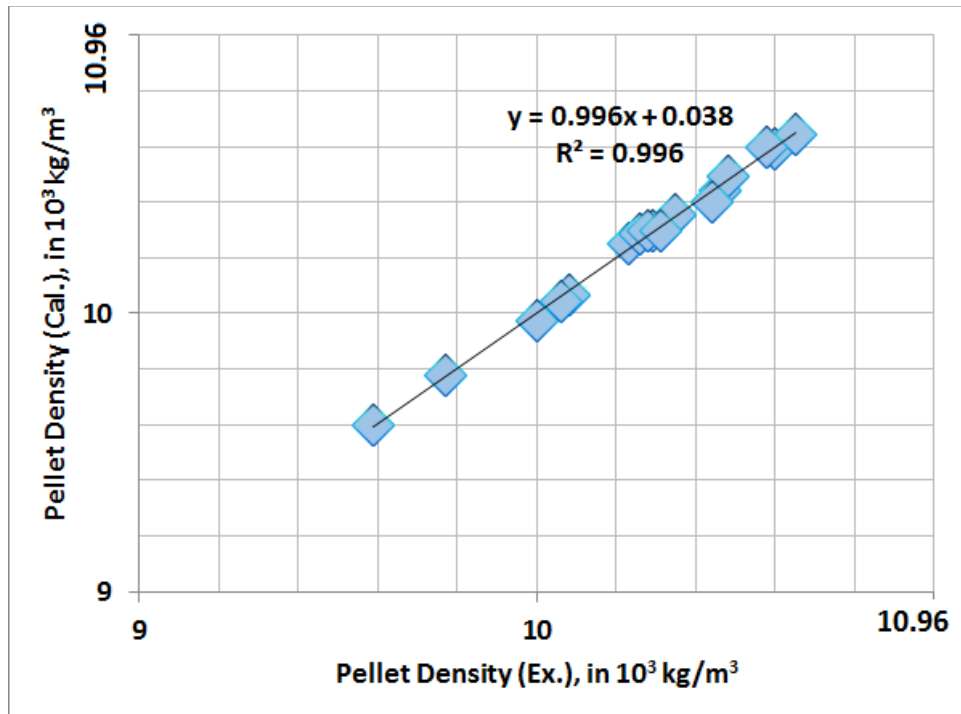


Fig. 2. Linear correlation between calculated and experimental values for the UO_2 pellet process (a) and contours of the sintering temperature vs. the sintering time on the UO_2 pellet density at 1073 K (b) levels of the conversion temperature (b).

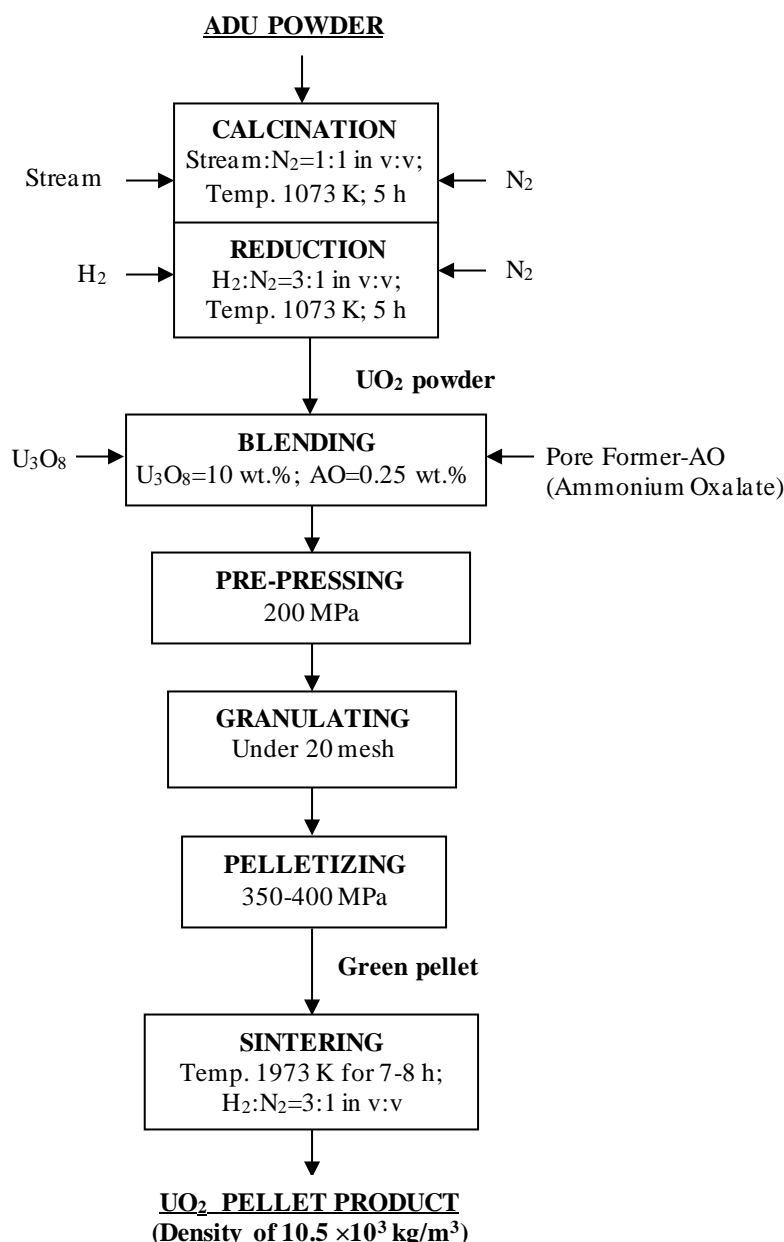


Fig. 3. Flow sheet of the UO₂ pellet process from the UO₂ ex-ADU powder.

Table 5 indicated various mechanical and physical characteristics of the pellet product and American Society for Testing and Materials (ASTM) international standards are used to determine some important characteristics of the UO₂ pellet products, including ratio of O/U, average grain size, porosity, resintering and etc.

Table 5

| Inspection items | The pellet | Methods |
|---|---------------|-------------------------------------|
| Density, in 10 ³ kg/m ³ | 10.52 – 10.58 | ASTM C373-88 (Hydrostatic) [9] |
| Ratio of O/U | 1.998 ± 0.003 | ASTM C696-99 (Gravimetry) [10] |
| Average grain size, in μm | 31.4 ± 2.3 | ASTM E 112-96 (Metallo-graphy) [11] |
| Hardness, in Hv | 749 ± 122 | Vicker |
| Porosity, in % (volume) | 3.96 ± 0.79 | ASTM C373-88 [9] |
| Resintering, in % | 0.53 ± 0.23 | [8] |
| Content of F, in ppm | 6 | ASTM C696-99 (Pyrohydrolysis) [10] |
| Content of Cl, in ppm | 20.5 | ASTM C696-99 (Pyrohydrolysis) [10] |

| | | |
|----------------------------|-----------------|----------------------------|
| Content of C, in ppm | 99 | ASTM C776-06 [10] |
| Impurities , in ppm | | ASTM C776-06 (ICP-MS) [10] |
| Al | 114.2 | |
| Ca+Mg | 54.5 | |
| Cr, Co, Th, B, Cd | below detection | |
| Fe | 44.9 | |
| Ni | 0.13 | |
| Si | 102.3 | |
| Rare Earths | < 1 | |

Conclusions

we proposed a mathematical model describing the effect of the fabrication parameters on SSA of UO_2 powders. To the best of our knowledge, the Brandon model as presented in equation (15) is used for the first time to describe the relationship between the essential fabrication parameters [(reduction temperature (T_R), calcination temperature (T_C), calcination time (t_C) and reduction time (t_R)] and SSA of the obtained UO_2 powder product. The proposed model was tested with Wilcoxon's rank sum test, showing a good agreement with the experimental parameters. The proposed model was well applied for roughly predicting SSA of UO_2 powders that is fabricated by means of calcination and reduction of ADU at our institution.

Modeling the UO_2 ex-ADU pellet process, using RSM based on CCD type of CCF was proposed. The quadratic mathematical model for the pellet density was shown a good agreement with the experimental data. The technological parameters for the UO_2 pellet process could be calculated from the proposed model so that the UO_2 pellet product has the desirable density level. And the flow sheet for preparing the UO_2 ex-ADU pellet product of the density of $10.5 \times 10^3 \text{ kg/m}^3$ was established.

Acknowledgments

Authors would like to acknowledge the financial support from the National Science and Technology Program, code KC.05-17/11-15, Vietnam Ministry of Science and Technology.

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