Non-equilibrium perturbations of the vertically unstable mode in tokamaks

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THIRD IAEA TECHNICAL MEETING ON PLASMA DISRUPTIONS AND THEIR MITIGATION

3 SEPTEMBER 2024

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SPECIAL ACKNOWLEDGEMENT:

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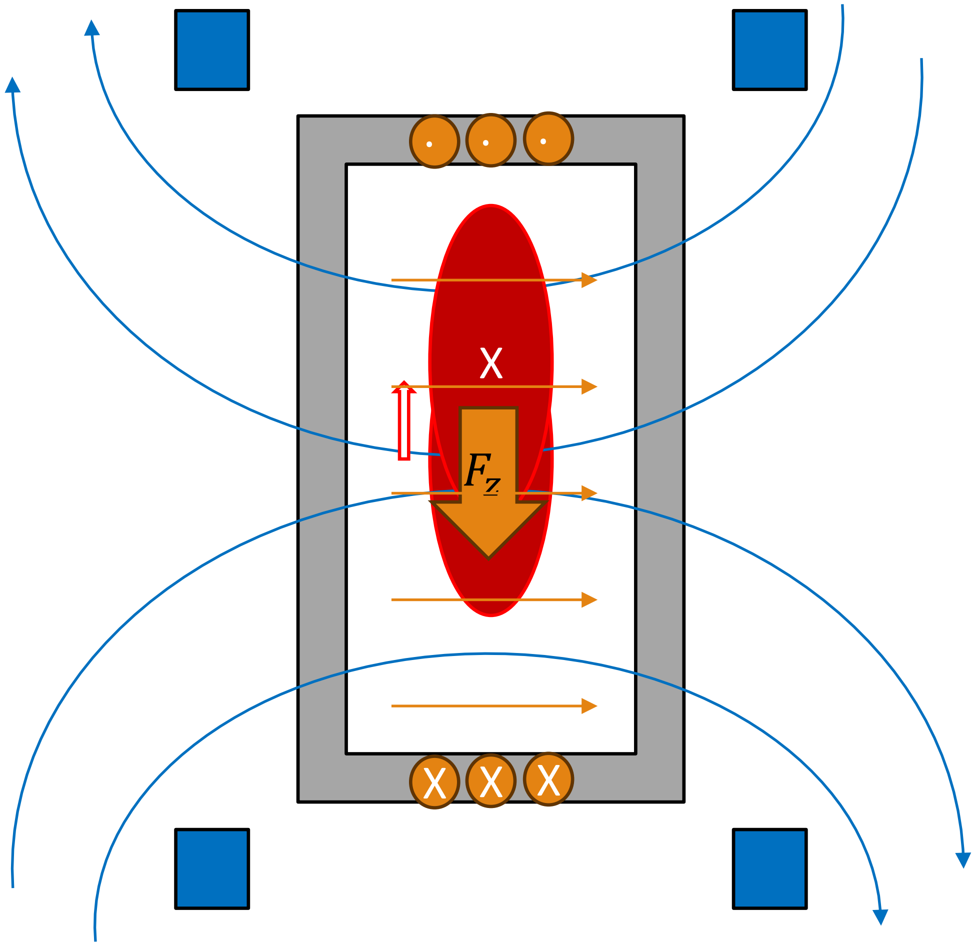
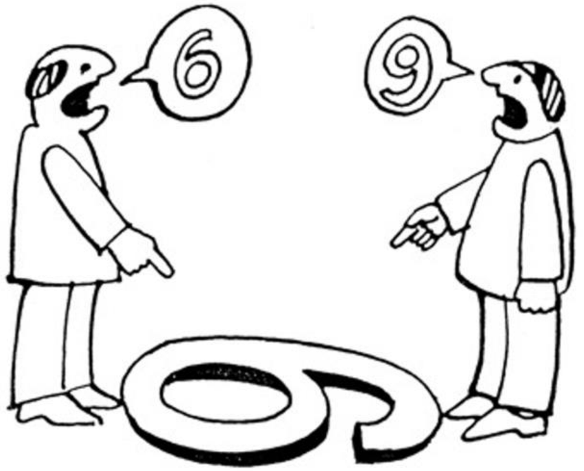
Outline

- Motivation: translating physics requirements to control engineers
- The rigid filament model: asymptotic solution and phase portrait
- Mapping dynamic perturbations to quasi-static models
- Validation of the theory on a TCV-like geometry
- Comparison of Thermal Quench perturbation in dynamic and massless MHD models [preliminary!]



Motivation

- **Maximum allowable vertical displacement**: a mass-less definition
- Comparing **massy models** with engineering oriented **mass-less models**
- Make physicists and engineers **agree on models and definitions!**



During the ITER Design Review (2007–2008), the plasma VS system was revised, in particular from the point of view of the margin in the plasma VS required for reliable plasma operation [7, 8]. Major results of the analysis of VS performance in existing devices were reported in [8]. To characterise the performance of the VS systems, a parameter $\max(Z_0)$ was defined as the maximum value of plasma vertical displacement due to free drift (VS system is switched off) that can be reversed, if the VS system is switched on when $Z = Z_0$ ('Minor VDE'). The conclusions reached in relation to the requirements for the VS capability were that the performance of the VS system in the tokamaks studied could be described in terms of the parameter max (Z_0) normalised to a, the (horizontal) minor radius of the plasma:

(1) 'reliable' operation corresponds to $\max(Z_0)\,/\,a > 5\%$ (for ITER, $\max(Z_0) > 100$ mm);
(2) typical 'robust' operation corresponds to $\max(Z_0)/a \approx 10\%$ (for ITER, $\max(Z_0) \approx 200$ mm)

Y. Gribov *et al* 2015 *Nucl. Fusion* **55** 073021

Rigid Filament Model

ASSUMPTIONS

- plasma = rigid, axisymmetric, current-carrying ring
- Only vertical displacements allowed
- State variables:
  - vertical position z_p
  - vertical velocity v_p
  - wall currents $\boldsymbol{I_w}$
- Eventual input:
  - active coil currents $\boldsymbol{I_a}$

WALL SINGLE-MODE MODEL

$$\frac{d}{dt}\begin{bmatrix} i_u \\ v_p \\ z_p \end{bmatrix} = \begin{bmatrix} -1/\tau_u & -F_I^u/L_u & 0 \\ F_I^u/m_p & 0 & F_z^{a0}/m_p \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_u \\ v_p \\ z_p \end{bmatrix}$$

- plasma mass: m_p
- inductance and resistance wall: L_u, R_u
- time constant wall single mode: $\tau_u = L_u/R_u$
- stabilising force per unit wall current: F_I^u
- destabilising force per unit displace: F_z^{a0}

Rigid Filament Model

ASYMPTOTIC SOLUTION VIA SINGULAR PERTURBATION METHOD*:

Unstable mode

$$z_0(t) = \boxed{\left[\Delta z_0 + \frac{1}{m_u F_Z^{a0}}\left(F_I^u\,\Delta i_{u,0} + F_Z^{a0}\Delta z_0\right)\right]\exp(\gamma_u t)} +$$

$$\boxed{\begin{aligned} &-\frac{1}{m_u F_Z^{a0}}\left[F_I^u\,\Delta i_{u,0} + F_Z^{a0}\Delta z_0\right]\cos\left(\frac{\sqrt{m_u}}{\tau_A}t\right)\exp\left[-\frac{m_u+1}{2}\gamma_u t\right] + \\ &+\frac{\tau_A}{\sqrt{m_u}}\Delta v_0 \sin\left(\frac{\sqrt{m_u}}{\tau_A}t\right)\exp\left[-\frac{m_u+1}{2}\gamma_u t\right]\end{aligned}}$$

Damped oscillatory modes*

Growth rate: $\gamma_u = \frac{1}{m_u \tau_u}$

Hypothesis:

$$\frac{\tau_A}{\tau_u} \ll 1$$

$m_u > 0 \rightarrow$ the instability is brought to the electromagnetic time scale!

Rigid Filament Model

ASYMPTOTIC SOLUTION

Unstable mode

$$z_0(t) = \boxed{\left[\Delta z_0 + \frac{1}{m_u F_z^{a0}}\left(F_I^u\,\Delta i_{u,0} + F_z^{a0}\Delta z_0\right)\right]\exp(\gamma_u t)} +$$

$$\boxed{\begin{gathered}-\frac{1}{m_u F_z^{a0}}\left[F_I^u\,\Delta i_{u,0} + F_z^{a0}\Delta z_0\right]\cos\left(\frac{\sqrt{m_u}}{\tau_A}t\right)\exp\left[-\frac{m_u+1}{2}\gamma_u t\right] + \\ + \frac{\tau_A}{\sqrt{m_u}}\Delta v_0 \sin\left(\frac{\sqrt{m_u}}{\tau_A}t\right)\exp\left[-\frac{m_u+1}{2}\gamma_u t\right]\end{gathered}}$$

Damped oscillatory modes*

MAIN PROPERTIES

- Direction of unstable motion is determined both by Δz_0 and $\Delta i_{u,0}$
- The unstable motion is not solicited if the wall response is:

$$\Delta i_{u,0} + \frac{F_I^u}{L_u}\Delta z_0 = 0$$

Ideal wall

- Oscillatory modes are not solicited if the perturbation is quasi-static:
  - $\Delta v_0 = 0$
  - $F_I^u\,\Delta i_{u,0} + F_z^{a0}\Delta z_0 = 0$

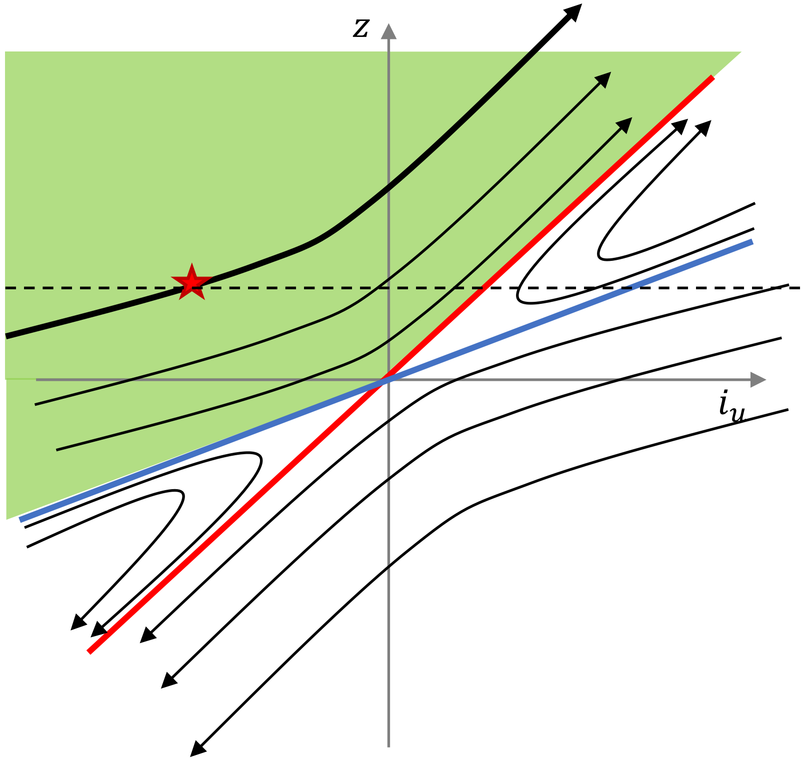
Mech. Equilibrium

* See also Barberis Tet al *Journal of Plasma Physics*, 88:905880511 (2022)

Rigid Filament Model

PHASE PORTRAIT (APPROXIMATE)

#1 weaker wall reaction than in the quasi-static limit



Unstable direction
(**equilibrium**!)

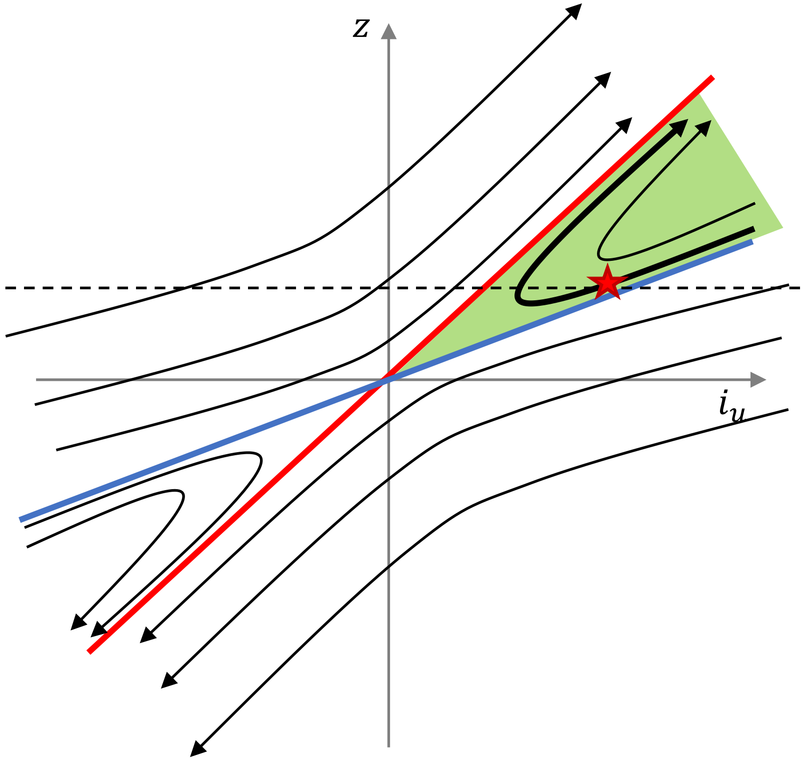
Trace in the (z, i_u) plane of damped oscillatory mode plane
(**ideal wall** response!)

MAIN PROPERTIES

- Unstable direction:
$$e_1 = [F_I^u \quad 0 \quad F_z^{a0}]'$$
- Plane damped oscillatory modes:
$$e_2 = [(F_I^u)^2/L_u \quad 0 \quad F_I^u]$$
$$e_3 = [0 \quad 1 \quad 0]'$$
- Initial electro-mechanical equilibrium point is a saddle!

Rigid Filament Model

PHASE PORTRAIT (APPROXIMATE)



#2 stronger wall reaction than in the quasi-static limit
weaker than ideal wall limit

Unstable direction (**equilibrium**!)

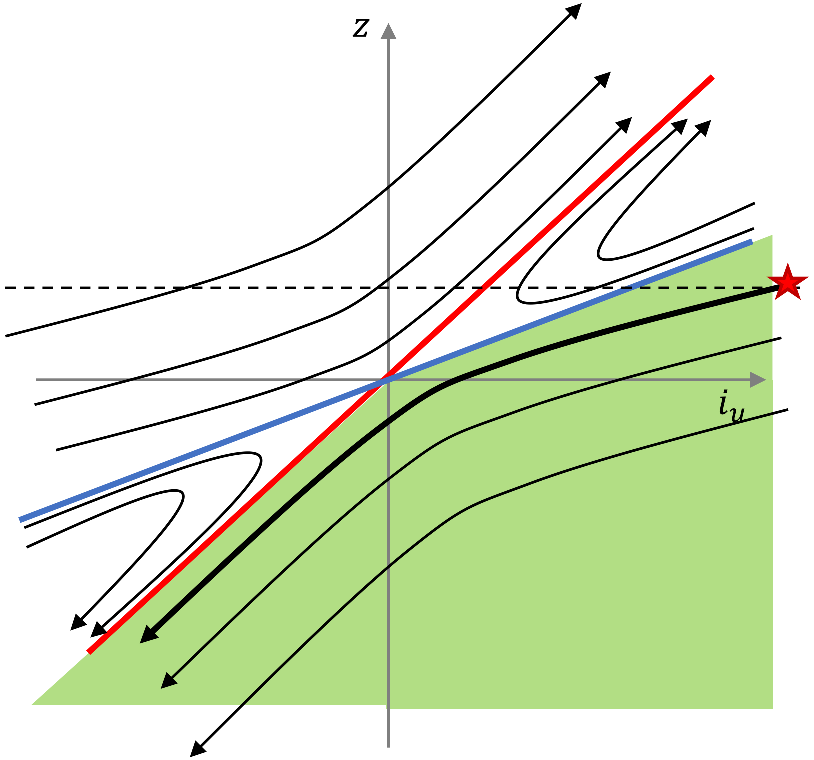
Trace in the (z, i_u) plane of damped oscillatory mode plane (**ideal wall** response!)

MAIN PROPERTIES

- Unstable direction:
$$e_1 = [F_I^u \quad 0 \quad F_z^{a0}]'$$
- Plane damped oscillatory modes:
$$e_2 = [(F_I^u)^2/L_u \quad 0 \quad F_I^u]$$
$$e_3 = [0 \quad 1 \quad 0]'$$
- Initial electro-mechanical equilibrium point is a saddle!

Rigid Filament Model

PHASE PORTRAIT (APPROXIMATE)



#3 stronger wall reaction than in the quasi-static limit **even stronger than ideal wall limit**

Unstable direction (**equilibrium**!)

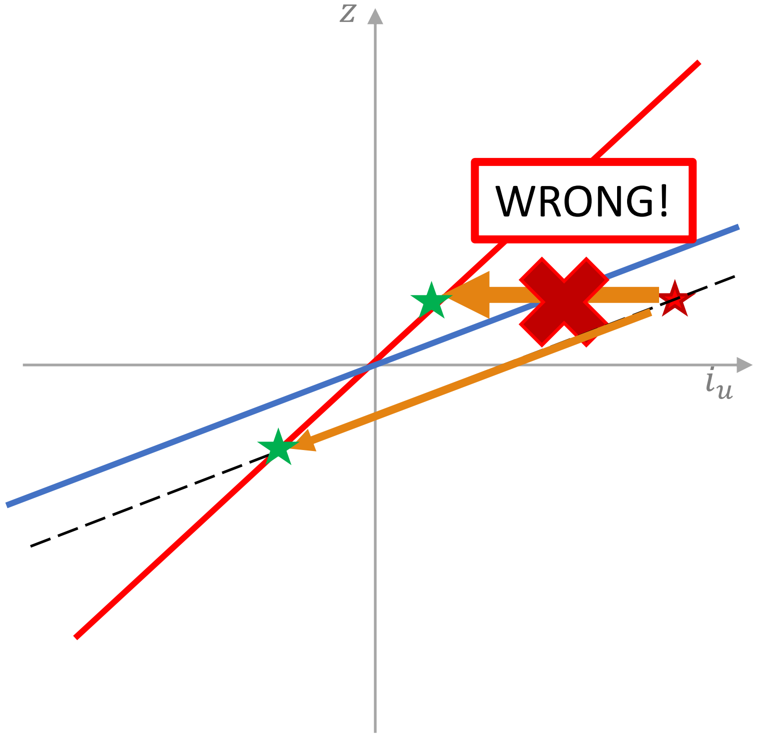
Trace in the (z, i_u) plane of damped oscillatory mode plane (**ideal wall** response!)

MAIN PROPERTIES

- Unstable direction:
  $$e_1 = [F_I^u \quad 0 \quad F_z^{a0}]'$$
- Plane damped oscillatory modes:
  $$e_2 = [(F_I^u)^2/L_u \quad 0 \quad F_I^u]$$
  $$e_3 = [0 \quad 1 \quad 0]'$$
- Initial electro-mechanical equilibrium point is a saddle!

Dynamic to massless map

PROJECTION IN PHASE SPACE



- Projection to the massless model should be «parallel» to the **ideal wall response** (hyper-)plane

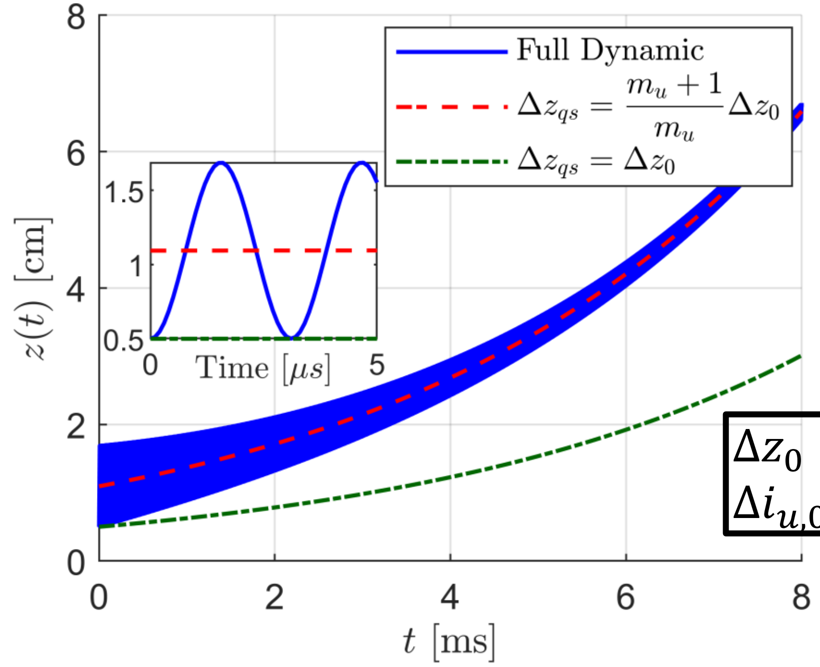
$$\Delta z_{qs,0} = \frac{m_u + 1}{m_u}\Delta z_0 + \frac{F_I^u}{m_u F_z^{a0}}\Delta i_{u,0}$$

- Highlighting the force-imbalance correction:

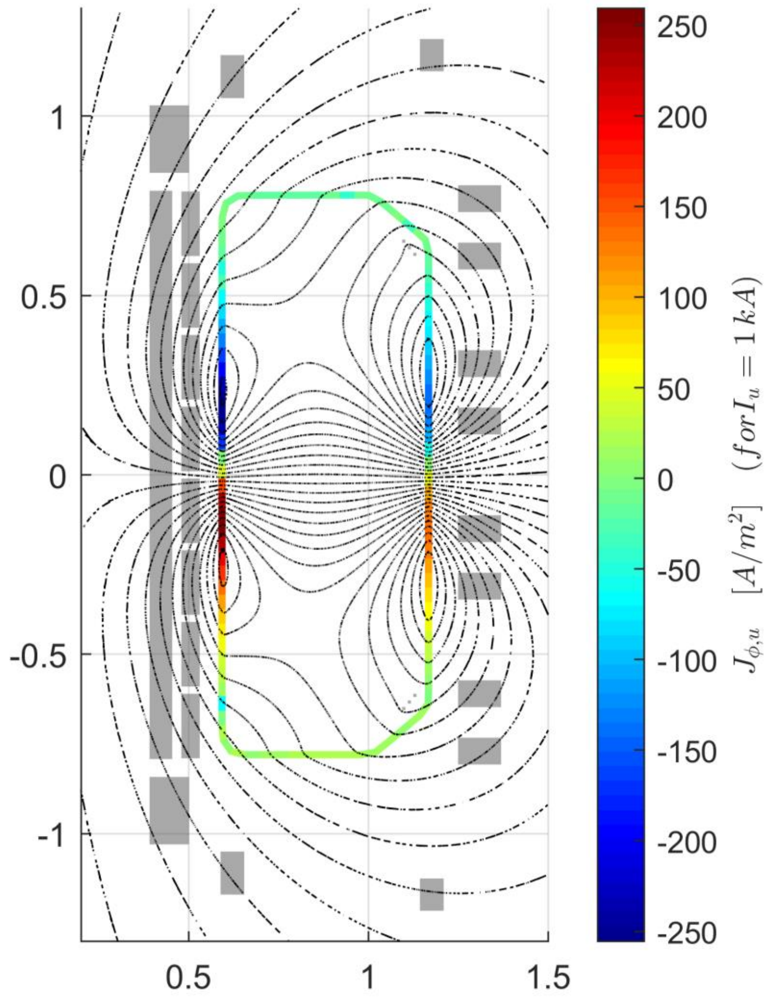
$$\Delta z_{qs,0} = \Delta z_0 + \frac{1}{m_u F_z^{a0}}\left(F_Z^{a0}\Delta z_0 + F_I^u \Delta i_{u,0}\right)$$

TCV-like example

- electromagnetic time inst: $\tau_u = 5.3$ ms
- stability margin: $m_u = 0.842$
- time constants ratio: $\tau_A/\tau_u \simeq 10^{-5}$



$\Delta z_0 = 5\ mm$
$\Delta i_{u,0} = 0\ A$



TCV-like example

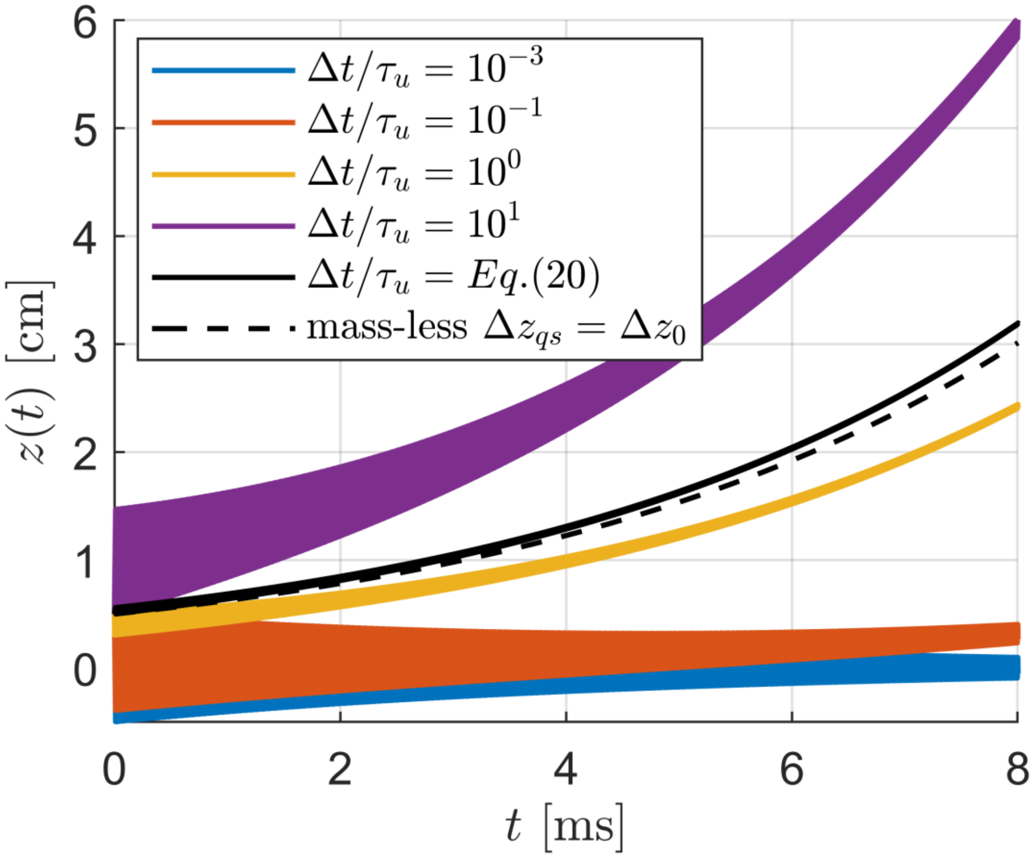
- «Virtual» experiment: plasma ring moved with constat velocity in a specified time Δt
- $\rightarrow$ scan in the normalized time $\Delta t/\tau_u$
- The final stabilizing current is given by:

$$\Delta i_{u,0} = -\frac{F_I^u}{L_u}\left[\frac{\Delta z_0}{\Delta t/\tau_u}\cdot\left(1-e^{-\Delta t/\tau_u}\right)\right]$$

$$F_I^u \Delta i_u + F_z \Delta z = 0$$

$$\frac{\Delta t}{\tau_u} \simeq 1$$

$$\frac{\Delta t}{\tau_u} \simeq (e-2)\frac{m_u+1}{e-m_u-1} \quad (20)$$



MHD models comparison

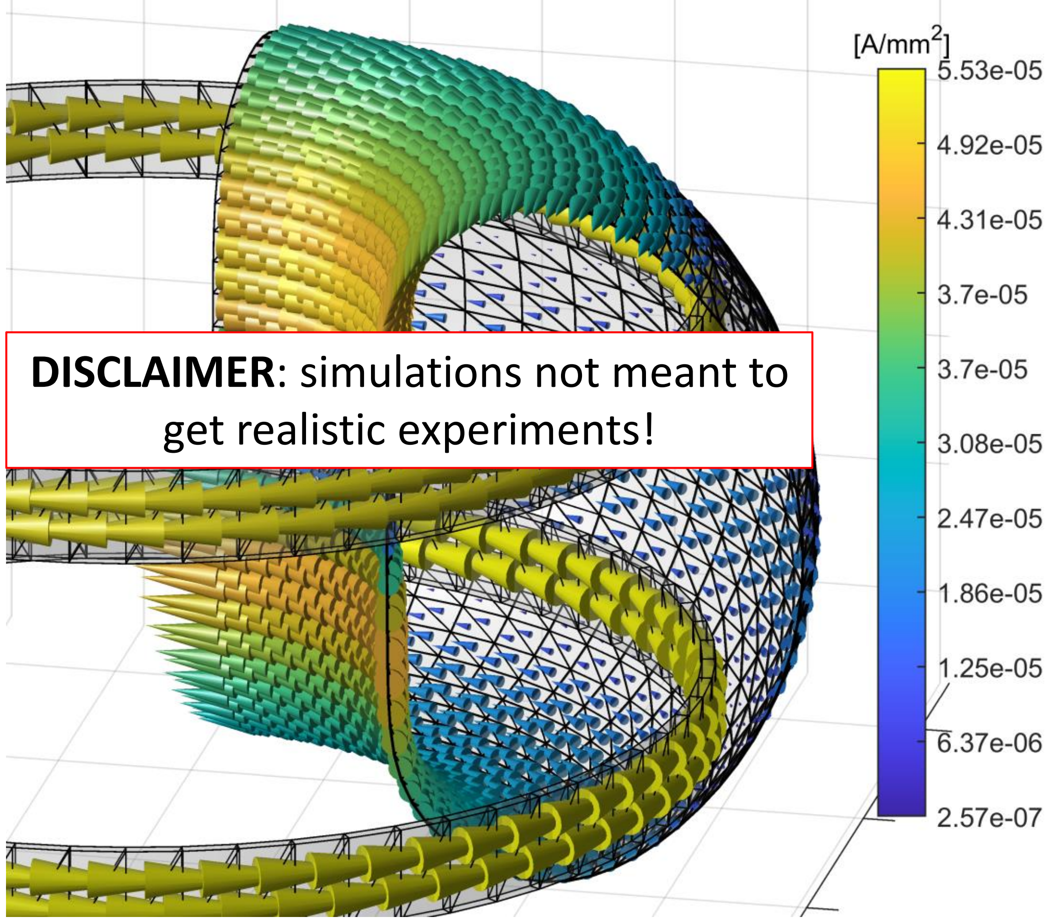
JOREK-CARIDDI

- Reduced Magneto-Hydro-Dynamic 3D
- Single temperature, no neutrals, axisymmetric, without $v_{\parallel}$

Identical wall model!

CARMA0NL

- Magneto-Hydro-Static 2D with 3D wall
- Equilibrium parameters are an input



DISCLAIMER: simulations not meant to get realistic experiments!

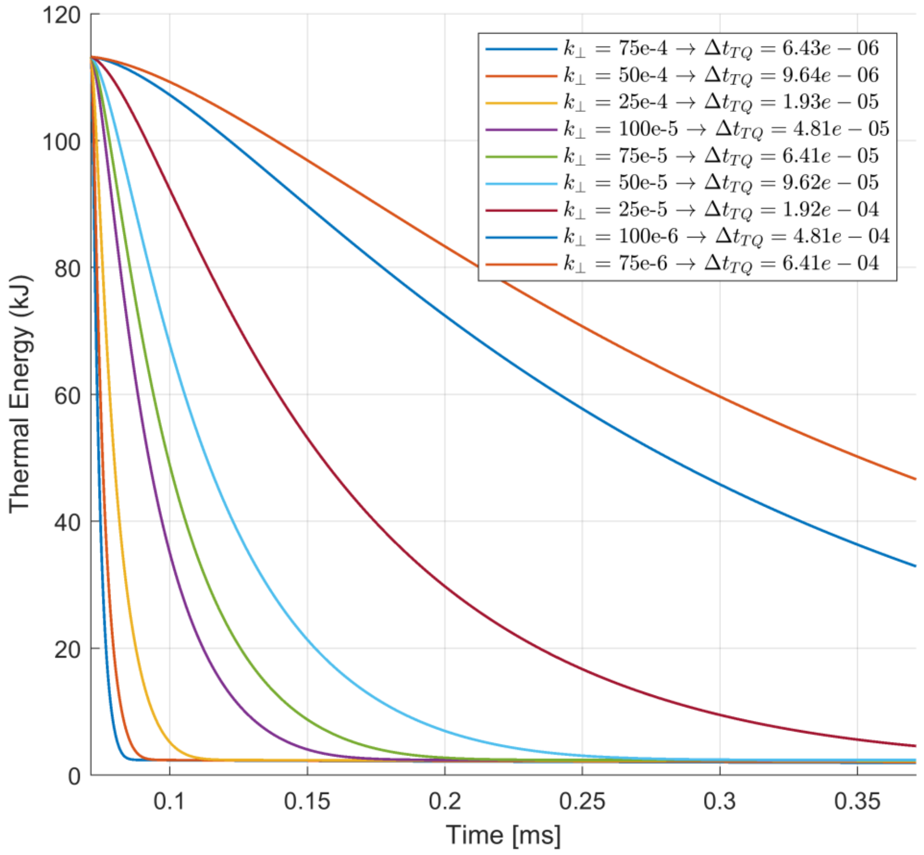
JOREK-CARIDDI

- ASDEX-U like initial MHD equilibrium
- Scan simulations with $k_\perp$ perturbation, *i.e.* with different Thermal Quench times
- Diffusion coefficients scaled properly so that the **ratio among diffusion times*** is the same for all simulations (η, $k_\parallel$, $D_\perp$, $D_\parallel$)
- Alfvén time and wall time constants are the same for all simulations

Thermal quench time:**

$$\Delta t_{TQ} = \frac{t_{90\%} - t_{20\%}}{0.7}$$

$t_{X\,\%}$ = time when the thermal energy reaches the X % of the pre-disruption value



$\Delta t_{TQ} \in [6,600]\ \mu s$

* See details in: F J Artola et al 2024 Plasma Phys. Control. Fusion 66 055015
** Definition used in: G. Arnoux et al Nucl. Fusion 49 085038 (2009)

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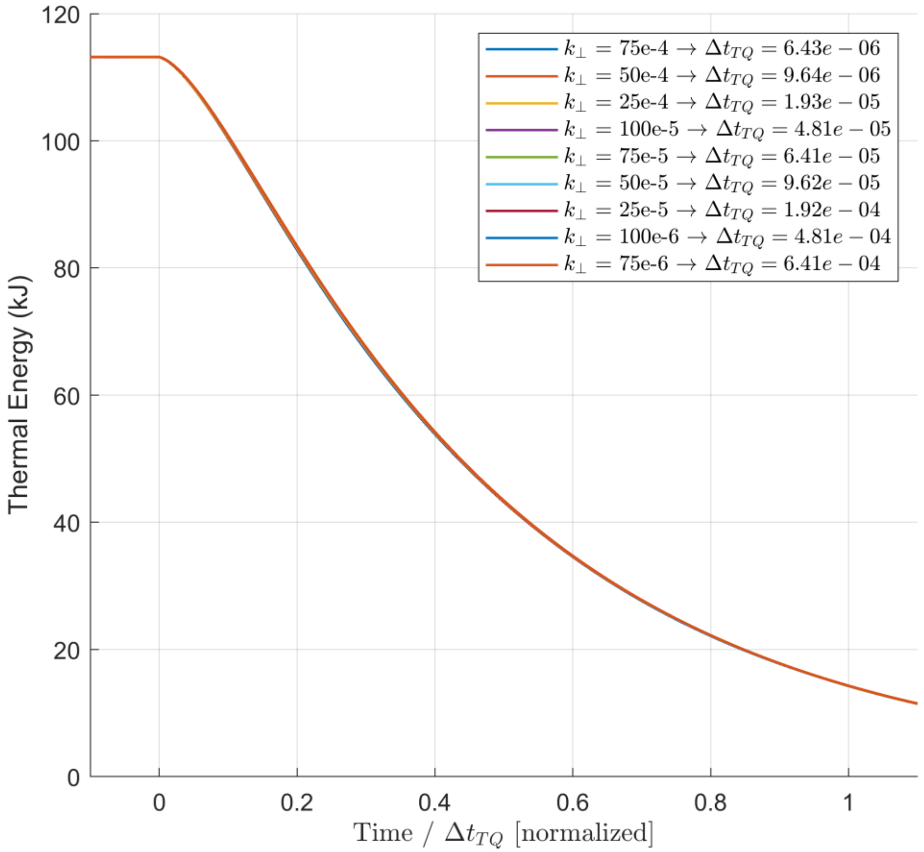

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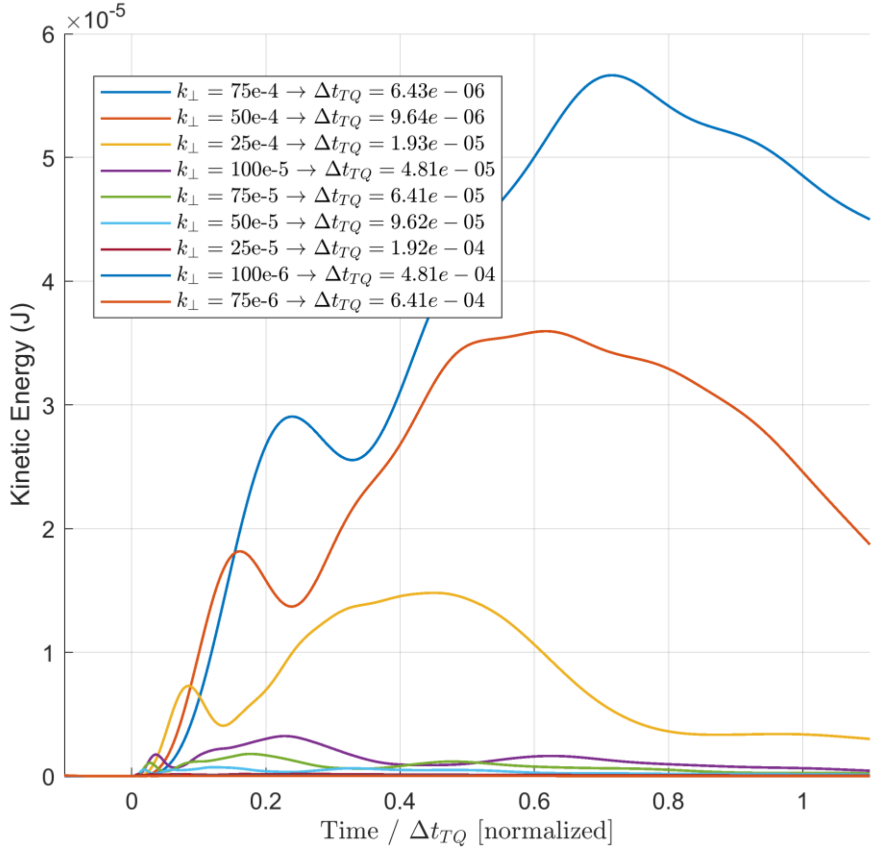
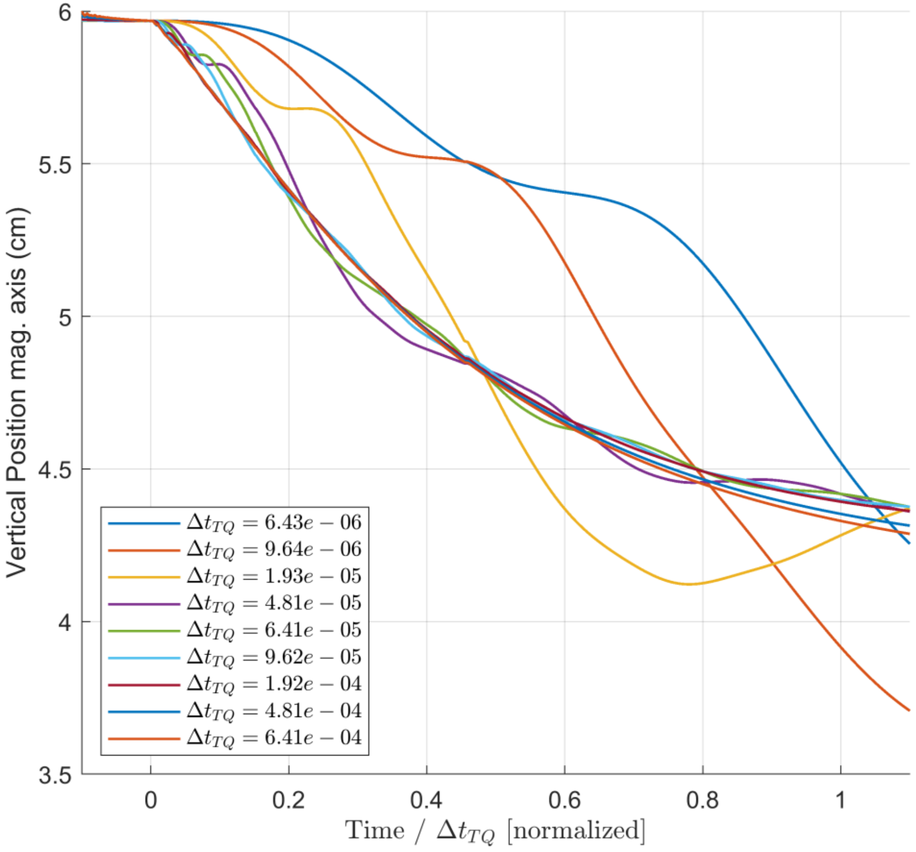
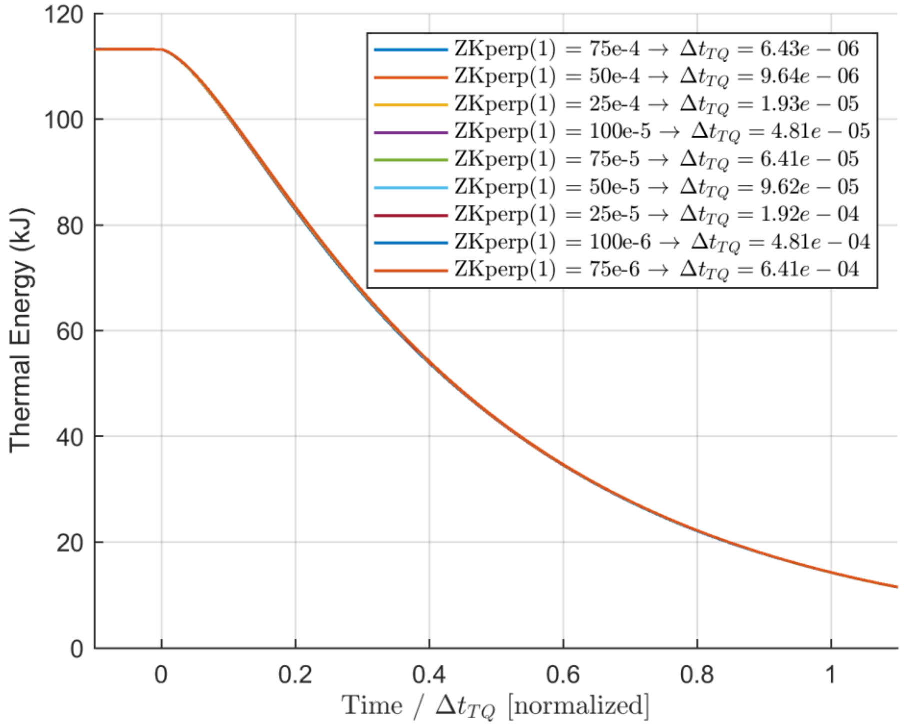
JOREK-CARIDDI





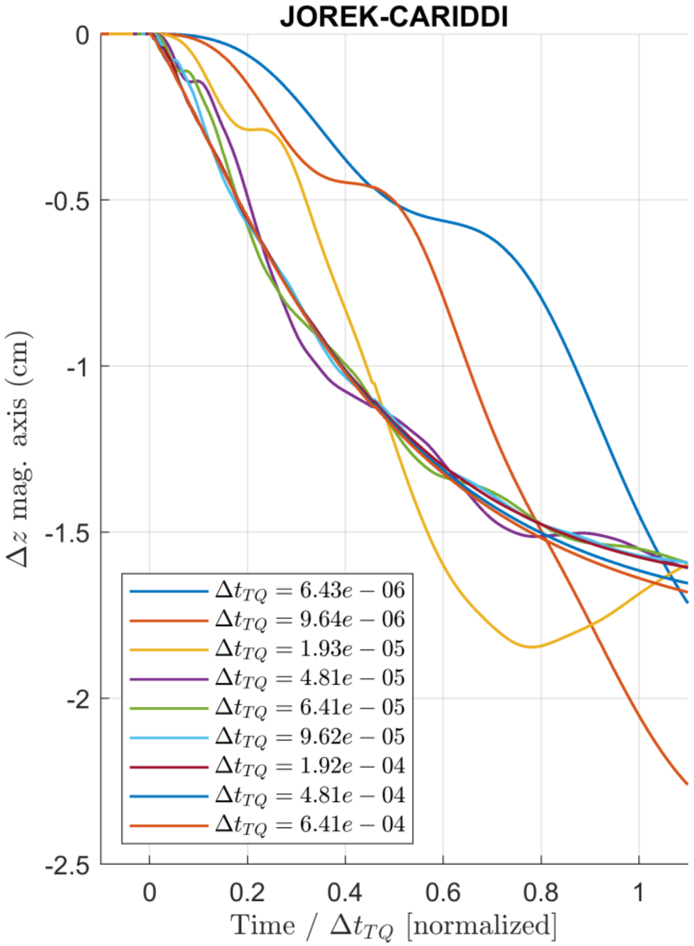
CarMa0NL

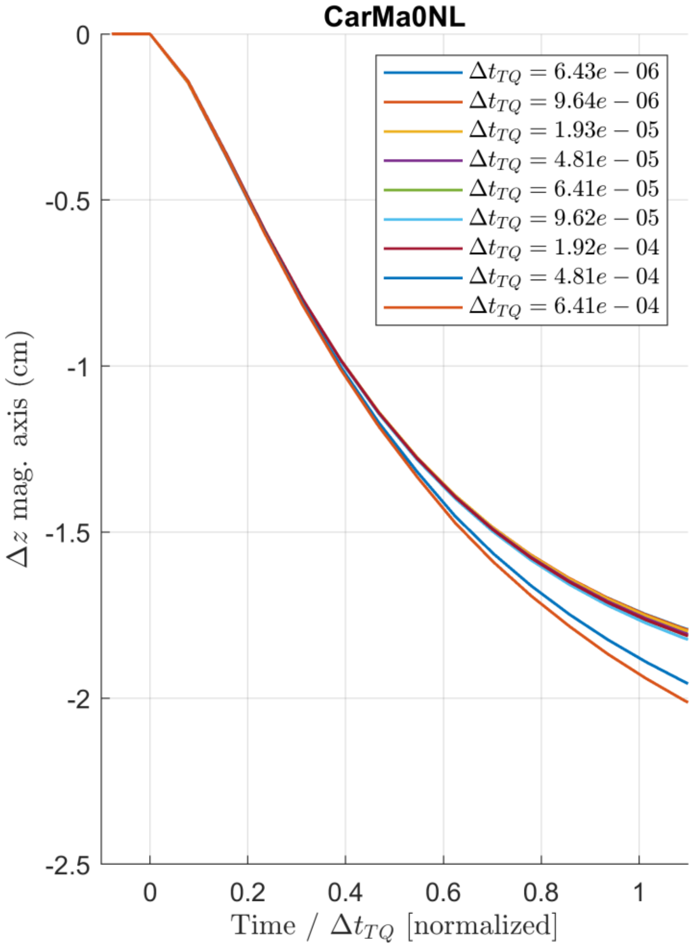
- CarMa0NL is run imposing a $\boldsymbol{\beta_0}$ **evolution** consistent with the JOREK-CARIDDI thermal energy;
- The plasma current is either kept fixed or varied as in JOREK-CARIDDI
- Remainder equilibrium parameters are kept fixed
- Time step scaled in each simulation according to the TQ time



$$j_\varphi(\tilde{\psi}, R) = R\left[\lambda \frac{\beta_0}{R_0}\left(1-\tilde{\psi}^{\alpha_{m,p}}\right)^{\alpha_{n,p}}\right] + \frac{1}{R}\left[\lambda\, R_0\, (1-\beta_0)\left(1-\tilde{\psi}^{\alpha_{m,f}}\right)^{\alpha_{n,f}}\right]$$

Comparison Z_p





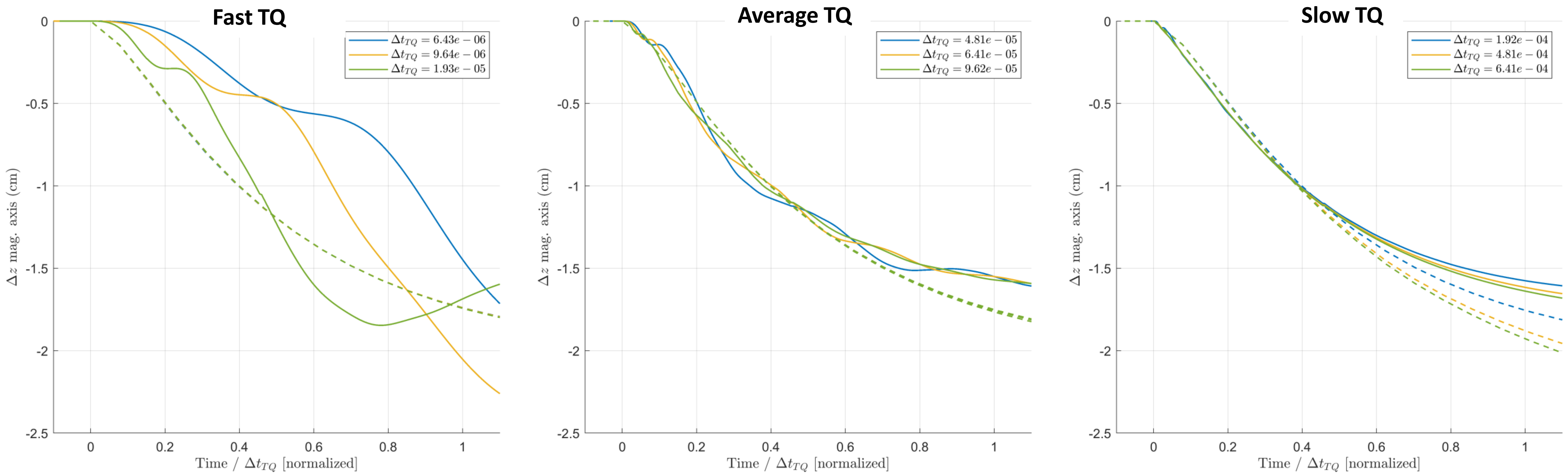
- The mass-less model predicts nearly the same Δz at the end of TQ for all $\Delta t_{TQ} \leq 200\ \mu s$
- For longer TQ times the wall is not anymore “ideal”
- The different vertical position variation for small Δt_{TQ} in JOREK-CARIDDI could be indeed related to plasma inertia
- **Open question**: can we map the initial conditions between dynamic and massless also for MHD models?

Comparison Z_p

$$\tau_A = \sqrt{\frac{m_p}{F_z^{a0}}} \simeq 2\mu s$$

$m_p \simeq 2.2\ \mu g$

$F_z^{a0} \simeq 602\ kN/m$

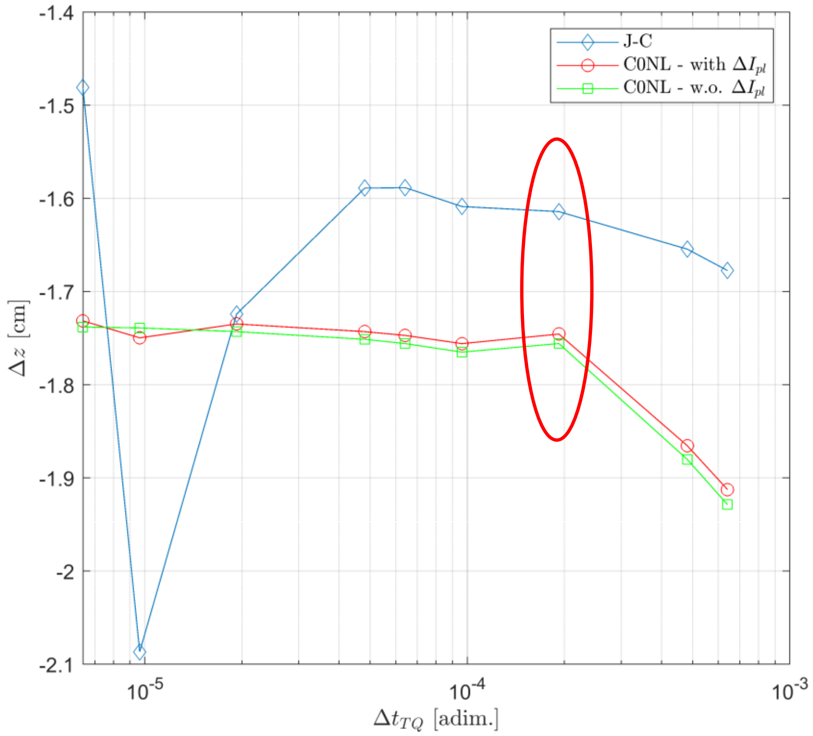


$\boldsymbol{\tau_A} \simeq 2\ \mu s$ | $20\ \mu s$ | $\boldsymbol{\tau_{iw}} = 200\ \mu s$ | $2\ ms$ | $20\ ms$ | $\boldsymbol{\tau_u} \simeq 200\ ms$

Test region TQ time

Comparison Z_p

- **Interpretation #1**: we find oscillations at the end of TQ when TQ time gets closer to the Alfvén time
- **Interpretation #2**: oscillations persist at the end of TQ when we are in the “ideal wall limit” region
- How to discriminate? Either scan in τ_A **or in** $\boldsymbol{\tau_u}$, TQ time fixed at the threshold between persistent/damped oscillation



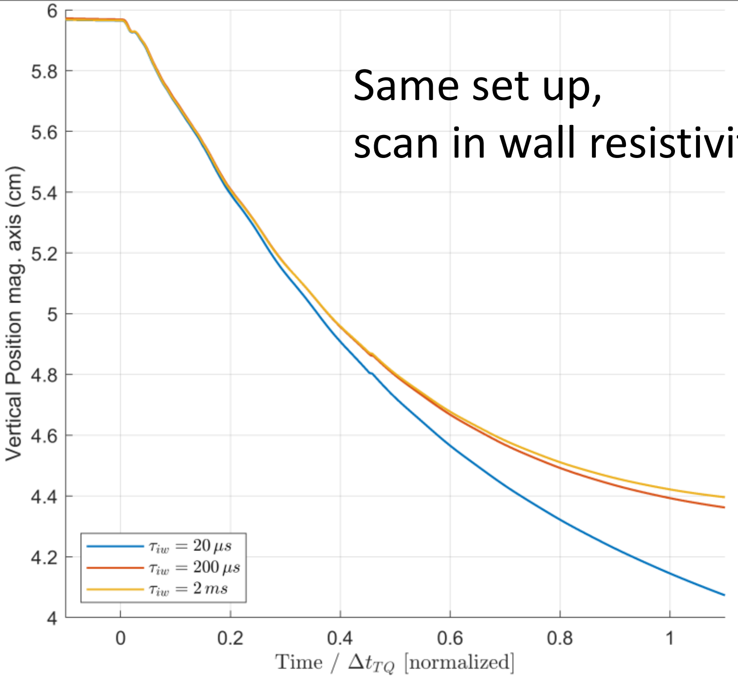
Ideal wall limit region

$\boldsymbol{\tau_A} \simeq 2\ \mu s$ | $20\ \mu s$ | $\boldsymbol{\tau_{iw}} = 200\ \mu s$ | $2\ ms$ | $20\ ms$ | $\boldsymbol{\tau_u} \simeq 200\ ms$

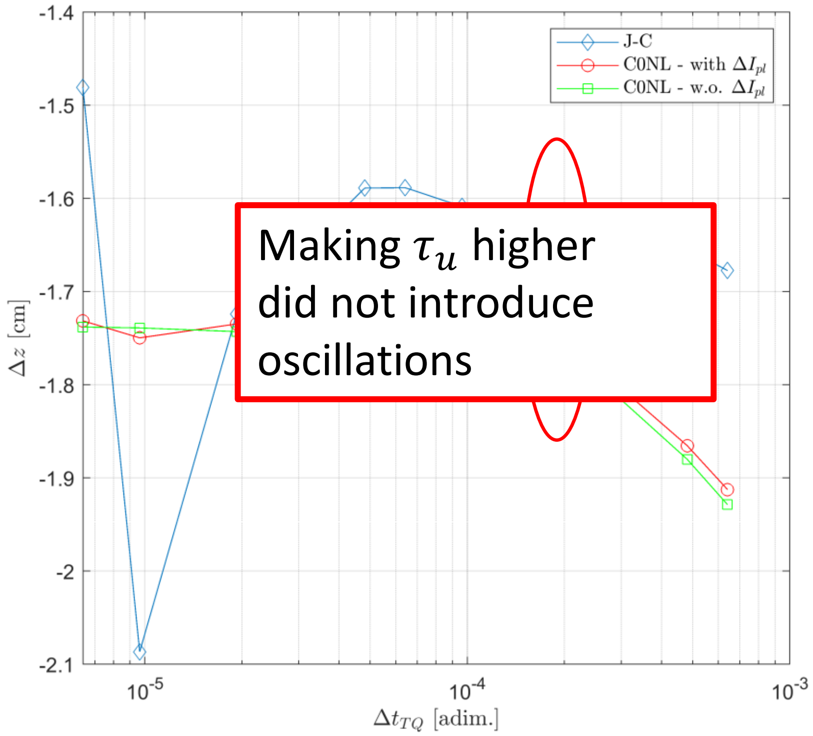
Test region TQ time

Comparison Z_p

- How to discriminate? Either scan in τ_A **or in** $\boldsymbol{\tau_u}$, for the TQ time which seems to represent the threshold between persistent/damped oscillation



Same set up,
scan in wall resistivity.



Making τ_u higher did not introduce oscillations

Ideal wall limit region

$\boldsymbol{\tau_A} \simeq 2\ \mu s$ | $20\ \mu s$ | $\boldsymbol{\tau_{iw}} = 200\ \mu s$ | $2\ ms$ | $20\ ms$ | $\boldsymbol{\tau_u} \simeq 200\ ms$

Test region TQ time

Discussion

CONCLUSIONS

- Comparison of dynamic and massless models require careful **mapping of initial conditions** (along the «ideal wall reaction» hyper-plane)
- Simple $\boldsymbol{\beta_p}$ **drop** of massless models may be not so accurate for fast TQ times (and eventually conservative)
- For $\boldsymbol{\Delta t_{TQ}} \to \boldsymbol{\tau_A}$ we may observe inertial phenomena on the magnetic axis vertical position during/after the TQ

POSSIBLE FUTURE WORK

- Investigate the role of **plasma current diffusion** in simple rigid filament models;
- The role of other possible **damping factors** for the oscillation shall be studied (both in rigid and fluid models)
- Translate the **mapping** between dynamic and mass-less models from the rigid to the **fluid context**

References

[1] Nicola Isernia and Fabio Villone 2023 *Plasma Phys. Control. Fusion* **65** 105007

[2] F. J. Artola *et al* 2024 *Plasma Phys. Control. Fusion* **66** 055015

[3] Y. Gribov *et al* 2015 *Nucl. Fusion* **55** 073021

[4] T. Barberis *et al* 2022 *Journal of Plasma Physics* **88** 905880511

[5] G. Arnoux *et al* 2009 *Nucl. Fusion* **49** 085038

[6] E. A. Lazarus, J. B. Lister, and G. H. Neilson 1990 *Nucl. Fusion* **30** 111

[7] N. Isernia *et al* 2023 *Phys. Plasmas* **30** 113901

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[9] F. Villone *et al* 2013 *Plasma Phys. Control. Fusion* **55** 095008

BACKUP slides

Rigid Filament Model

ASSUMPTIONS

- plasma = rigid, axisymmetric, current-carrying ring
- Only vertical displacements allowed
- State variables:
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- Eventual input:
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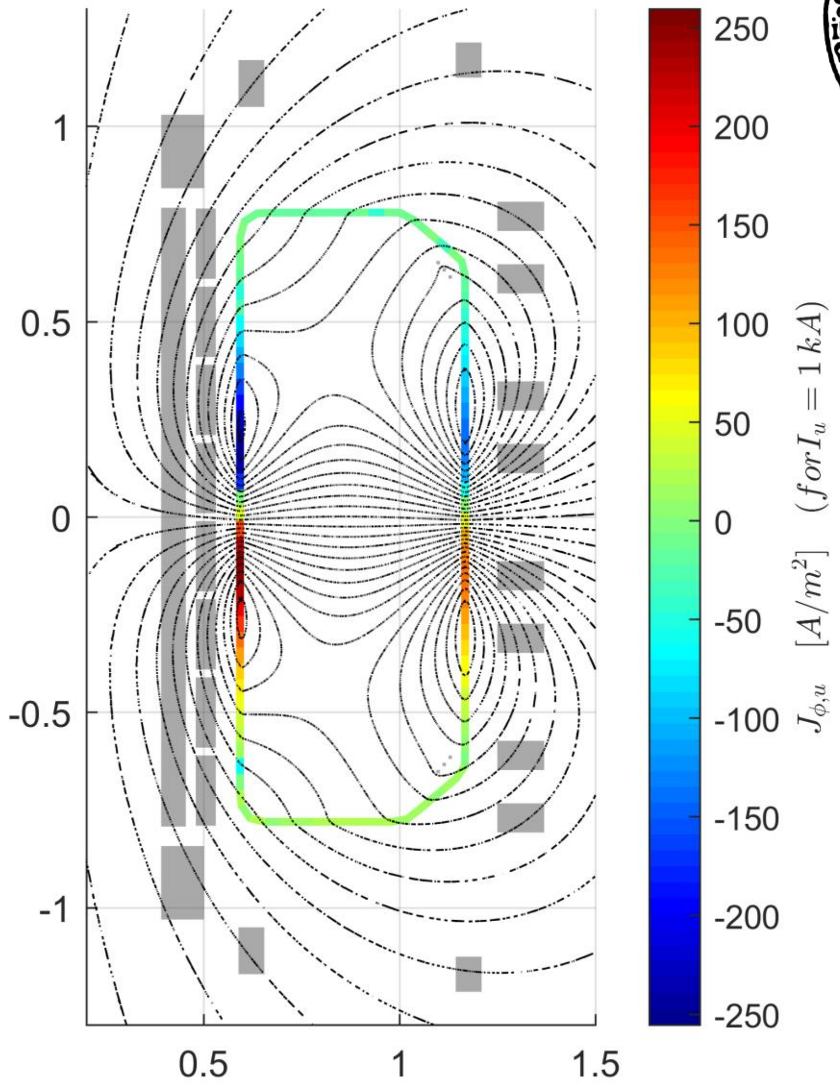


Rigid Filament Model

STRATEGY OF SOLUTION

- Stability margin: $\boxed{m_u = \left[\frac{(F_I^u)^2}{L_u} - F_z^{a0}\right] / F_z^{a0}}$
- Alfvèn time: $\tau_A = \sqrt{m_p / F_z^{a0}}$
- "Small" parameter: $\varepsilon = \tau_A / \tau_u$
- Truncated expansion:

$$z_p = z_0 + \varepsilon\, z_1 + \ \dots$$

- Two time scale separation:

$$\frac{d}{dt} = \frac{1}{\varepsilon}\frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_2}$$

WALL SINGLE-MODE MODEL

$$\left(\frac{\tau_A}{\tau_u}\right)^2 \left[\dddot{z}_p + \frac{1}{\tau_u}\ddot{z}_p\right] + \frac{m_u}{\tau_u^2}\dot{z}_p - \frac{1}{\tau_u^3} z_p = 0$$

- plasma mass: m_p
- inductance and resistance wall: L_u, R_u
- time constant wall single mode: $\tau_u = L_u / R_u$
- stabilising force per unit wall current: F_I^u
- destabilising force per unit displace: F_z^{a0}

$m_u > 0 \rightarrow$ the instability is brough to the electromagnetic time scale!

Rigid Filament Model

ASYMPTOTIC SOLUTION

Unstable mode

$$z_0(t) = \boxed{\underbrace{\left[\Delta z_0 + \frac{1}{m_u F_z^{a0}}\left(F_I^u\,\Delta i_{u,0} + F_z^{a0}\Delta z_0\right)\right]}\exp(\gamma_u t)} +$$

$$z_u \geq 0 \leftrightarrow (1+m_u)F_z^{a0}\Delta z_0 + \; F_I^u\,\Delta i_{u,0} \geq 0$$

$$z_u \geq 0 \leftrightarrow F_I^u \cdot \left(\boxed{\frac{F_I^u}{L_u}\Delta z_0} + \Delta i_{u,0}\right) \geq 0$$

$$F_I^u = I_{pl}\left.\frac{\partial M_{u,p}}{\partial z}\right|_{I_u,V_p,Z_p}$$

Opposite of the electric current inductively induced by a plasma ring displacement Δz_0!

MAIN PROPERTIES

- Direction of unstable motion is determined solely by initial wall current and initial displacement

Upward/downward direction determined only by the sign of Δz_0 when:

$$|\Delta \mathrm{i}_{u,0}| \leq \frac{F_I^u}{L_u}|\Delta z_0|$$

Rigid Filament Model

ASYMPTOTIC SOLUTION

Unstable mode

$$z_0(t) = \boxed{\left[\Delta z_0 + \frac{1}{m_u F_z^{a0}}\left(F_I^u \,\Delta i_{u,0} + F_z^{a0}\Delta z_0\right)\right]\exp(\gamma_u t)} +$$

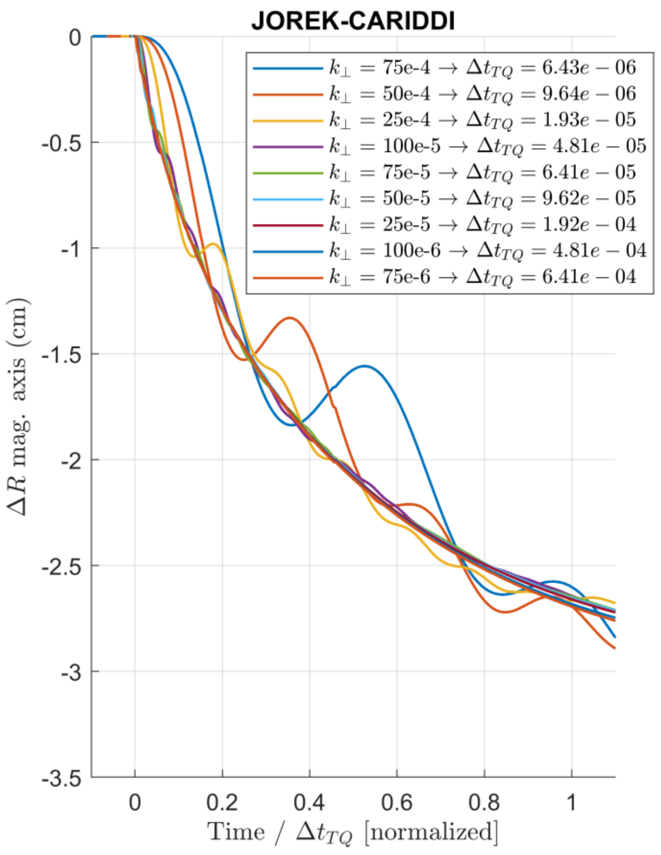
$$\boxed{\begin{aligned} &-\frac{1}{m_u F_z^{a0}}\left[F_I^u \,\Delta i_{u,0} + F_z^{a0}\Delta z_0\right]\cos\left(\frac{\sqrt{m_u}}{\tau_A}t\right)\exp\left[-\frac{m_u+1}{2}\gamma_u t\right] + \\ &+\frac{\tau_A}{\sqrt{m_u}}\Delta v_0 \sin\left(\frac{\sqrt{m_u}}{\tau_A}t\right)\exp\left[-\frac{m_u+1}{2}\gamma_u t\right] \end{aligned}}$$

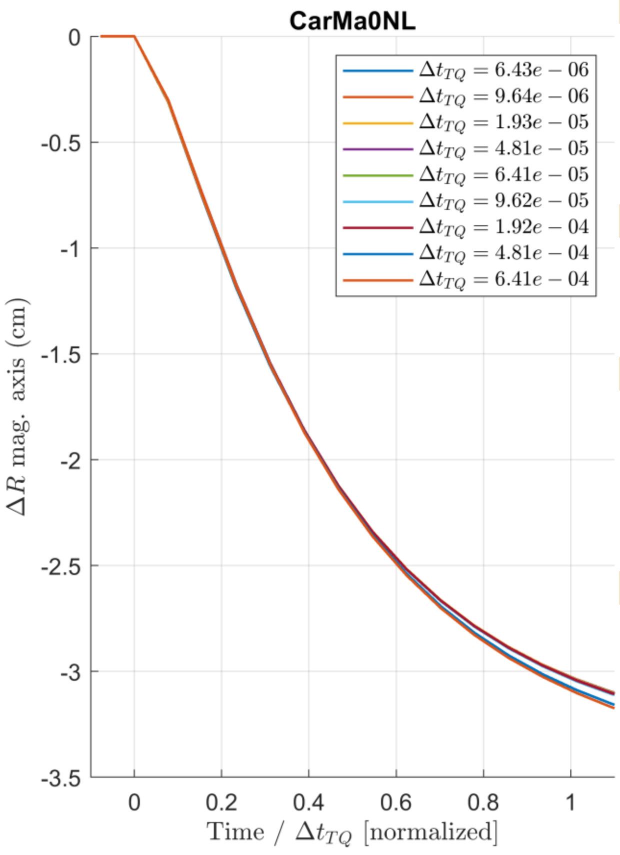
Damped oscillatory modes*

MAIN PROPERTIES

- Unstable direction:
  $$e_1 = [F_I^u \quad 0 \quad F_z^{a0}]'$$
- Plane damped oscillatory modes:
  $$e_2 = [(F_I^u)^2/L_u \quad 0 \quad F_I^u]$$
  $$e_3 = [0 \quad 1 \quad 0]'$$
- Initial electro-mechanical equilibrium point is a saddle!

Comparison R_p





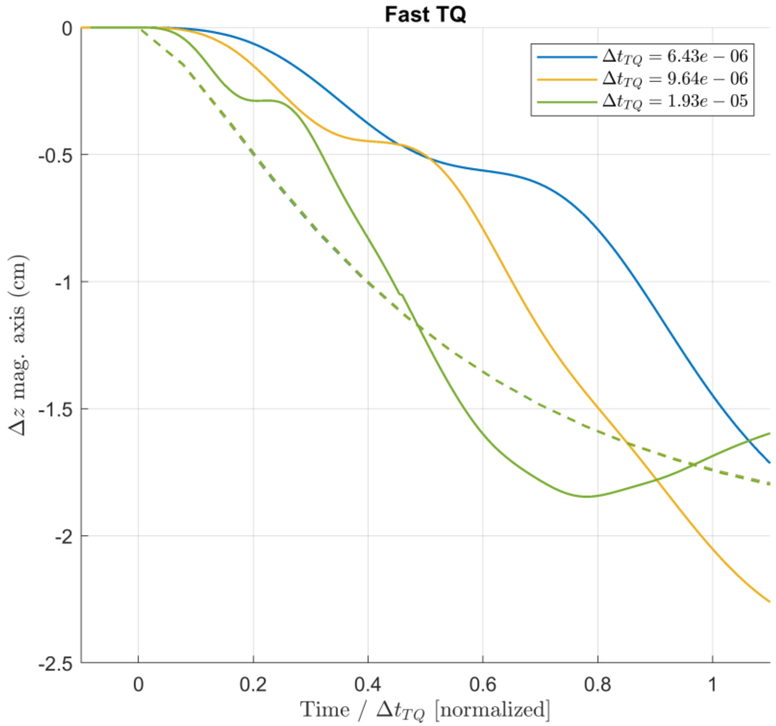
- The mass-less model predicts nearly the same ΔR at the end of TQ for all
$$\Delta t_{TQ} \le 200\ \mu s$$
- For longer TQ times the wall is not anymore "ideal"
- The different radial position variation for small Δt_{TQ} in JOREK-CARIDDI should be indeed related to plasma inertia
- **Open question**: can we map the initial conditions between dynamic and massless also for MHD models?

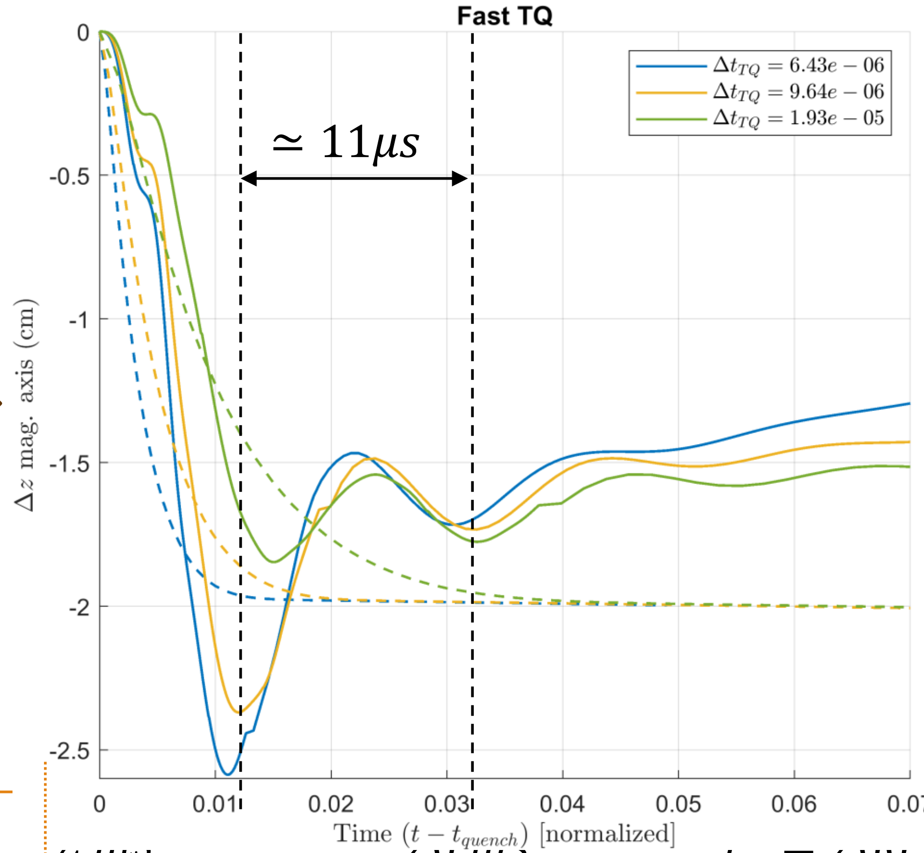
Comparison Z_p

$$\tau_A = \sqrt{\frac{m_p}{F_z^{a0}}} \simeq 2\mu s$$

$m_p \simeq 2.2\ \mu g$

$F_z^{a0} \simeq 602\ kN/m$





$\tau_A \simeq 2\ \mu s$ 20 μs 200 μs

Test region TQ time