Anomaly Detection Based on Wavelet Domain GARCH Random Field Modeling

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Outline

What is Anomaly Detection?

Objective

Image anomaly detection is the process of distilling a small number of clustered pixels, which differ from the images general characteristics

Main Stages

- Selection of an appropriate feature space
- Selection of a statistical model for the feature space representing the image clutter
- Selection of a detection algorithm

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Feature Space

The detection process is generally performed with respect to an appropriate feature space in which a clear segregation between the anomalous elements and the rest of the background clutter in the scene is possible

The feature space is often derived using

- Single resolution spatial analysis
- Multi-resolution analysis
- Integration of both

Motivation for a Multiresolution Feature Space

(Yu et.al. 1992)

- Features of interest are generally present in different sizes
- Allows processing of different scales and orientations in parallel

(Strickland et.al. 1997)

Objects in imagery create a response over several scales in a multiresolution representation of an image, and therefor the wavelet transform can server as a means for computing a feature set for input to a detector

Motivation for a Multiresolution Feature Space (Cont.)

(Goldman and Cohen 2005)

Allows capturing of periodical patterns of various period length which often appear in natural clutter images

(Laine et.al. 1994)

Orientation and scale selectivity of the wavelet transform are related to biological mechanisms of the human visual system

Statistical Models

- The Gaussian distribution is a common basis for feature space statistical models due to its mathematical tractability
- Most random field models are based on the spatial interaction of pixels in local neighborhoods
  - The value of each pixel is predicted based on its neighboring pixels
  - The prediction error is the innovations process

Statistical Models (cont.)

Spatial Interaction of Pixels

$$y(s) = \sum_{k \in \Omega_{neighbor}} \alpha(k) y(s+k) + \epsilon(s)$$

$$\epsilon(s) \sim N(0, \rho^2)$$

Gauss Markov Random Field (GMRF)(Woods, 1972)

$$E\left\{\epsilon(s)\epsilon(s+k)\right\} = \begin{cases} \rho^2, & \text{if } k = (0,0) \\ -\alpha(k)\rho^2, & \text{if } k \in \Omega_{neighbor} \\ 0, & \text{otherwise} \end{cases}$$

Statistical Models (cont.)

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Detection Algorithms

Hypothesis Testing

H_0 - Target absent (clutter only)
H_1 - Target Present

Detectors

- Single Hypothesis Test (SHT)
- Matched Filter Detector
- Matched Subspace Detector (MSD)

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Single Hypothesis Test SHT

- Solves the problem of an undefined anomaly
- Measurs the distance from the clutter mean

$$d^2 = (\mathbf{y}(s) - \boldsymbol{\mu}_{\mathbf{y}})^T \Sigma_{\mathbf{y}}^{-1} (\mathbf{y}(s) - \boldsymbol{\mu}_{\mathbf{y}}) \underset{H_0}{\overset{H_1}{\gtrless}} \eta_d$$

$$H_0 : \mathbf{y} \sim N(\boldsymbol{\mu}_{\mathbf{y}}, \Sigma_{\mathbf{y}})$$

$$H_1 : \mathbf{y} \sim N(\boldsymbol{\mu}_?, \Sigma_?)$$

Drawbacks

- As the dimension of the data increases, the error of the SHT increases significantly
- If information about the anomalies is made available *a priory* it cannot be incorporated into the detection scheme

Matched Filter Detector

When a typical signature of the target is available

$$H_0 : \mathbf{y} \sim N(\boldsymbol{\mu}_{\mathbf{y}}, \Sigma_{\mathbf{y}})$$

$$H_1 : \mathbf{y} \sim N(\boldsymbol{\mu}_{\mathbf{t}}, \Sigma_{\mathbf{t}})$$

$$\mathcal{L} = \frac{P_{\mathbf{y}}(\mathbf{y}(s)|H_1)}{P_{\mathbf{y}}(\mathbf{y}(s)|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_d$$

The log likelihood ratio detector is given by the ratio of the conditional probability density functions of the two hypothesis:

$$\mathcal{L} = \frac{1}{2}(\mathbf{y} - \boldsymbol{\mu}_{\mathbf{y}})^T \Sigma_{\mathbf{y}}^{-1} (\mathbf{y} - \boldsymbol{\mu}_{\mathbf{y}}) - \frac{1}{2}(\mathbf{y} - \boldsymbol{\mu}_{\mathbf{t}})^T \Sigma_{\mathbf{t}}^{-1} (\mathbf{y} - \boldsymbol{\mu}_{\mathbf{t}})$$

Compares the Mahalanobis distances of the observed feature vector **y** from the centers of the two classes (Manolakis and Show, 2002)

Matched Filter Detector (Cont.)

Fisher's Linear Discriminant

If the target and background classes have the same covariance matrix, that is, $\Sigma_{\mathbf{y}} = \Sigma_{\mathbf{t}}$, the quadratic terms disappear, and the likelihood ration detector becomes:

$$\mathcal{L} = (\mu_{\mathbf{t}} - \mu_{\mathbf{y}})^T \Sigma_{\mathbf{y}}^{-1} \mathbf{y}$$

This is a linear detector:

$$\mathcal{L} = \mathbf{c}^T \mathbf{y} = \sum_{k \in \Omega_{image}} c_k y_k$$

The coefficient vector:

$$\mathbf{c} = \Sigma_{\mathbf{y}}^{-1} (\mu_{\mathbf{t}} - \mu_{\mathbf{y}})$$

The detector output is a linear combination of normal random variables and is therefor normally distributed (Manolakis and Show, 2002)

Matched Subspace Detector

- The anomaly signature is unknown and assumed to be in a subspace
- Detecting subspace signals in subspace interference and additive WGN

$$\begin{aligned} H_0 : \mathbf{y} &= S\phi + \epsilon \\ H_1 : \mathbf{y} &= H\psi + S\phi + \epsilon \end{aligned}$$

$$\epsilon \sim N(\mathbf{0}, \rho^2 \mathbf{I})$$

(Scharf and Friedlander, 1994)

$$P_S\mathbf{y}(s) = S(S^T S)^{-1} S^T \mathbf{y}(s)$$

$$\begin{aligned} \hat{\epsilon_{H_0}} &= (I - P_S)\mathbf{y} \\ \hat{\epsilon_{H_1}} &= (I - P_{HS})\mathbf{y} \end{aligned}$$

Matched Subspace Detector(Cont.)

GLRT

$$\mathcal{L}(s) = 2\log\left[\frac{P(\epsilon(s) \mid H_1)}{P(\epsilon(s) \mid H_0)}\right] = 2\log\left[\frac{\exp\left(\frac{[\epsilon\hat{H_1}]^2}{2\rho^2}\right)}{\exp\left(\frac{[\epsilon\hat{H_0}]^2}{2\rho^2}\right)}\right]$$

$$= \frac{1}{\rho^2}\mathbf{y}^T(P_{HS} - P_S)\mathbf{y} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_{\mathcal{L}}$$

(Scharf and Friedlander, 1994)

Matched Subspace Detector(Cont.)

SNR

We define the SNR as the second power of the ratio between the signal which does not lie in the interference subspace, and the standard deviation of the noise:

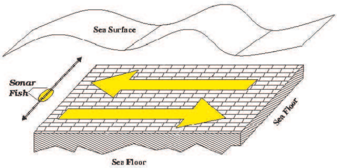
$$SNR = \frac{1}{\rho^2}[H\psi]^T[I - P_S][H\psi]$$

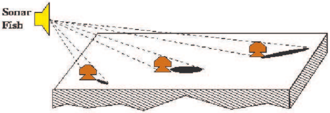
$$\mathcal{L} \sim \begin{cases} \chi_u^2(0), & \text{under } H_0 \\ \chi_u^2(SNR), & \text{under } H_1 \end{cases} \qquad \begin{aligned} P_{FA} &= 1 - P[\chi_u^2(0) \leq \eta_{\mathcal{L}}] \\ P_D &= 1 - P[\chi_u^2(SNR) \leq \eta_{\mathcal{L}}] \end{aligned}$$

Under hypothesis H_1, the non-centrality parameter of the chi-square distribution of $\mathcal{L}$ is equal to the SNR

Side-Scan Sonar Images

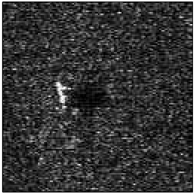
Image Acquisition

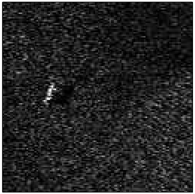




Chen et. al 1999

Examples

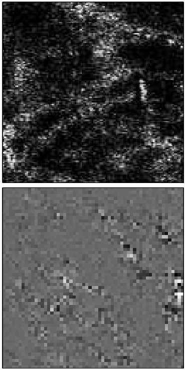




Motivation for a GARCH Model

Drawbacks of Gaussian Based Statistical Models

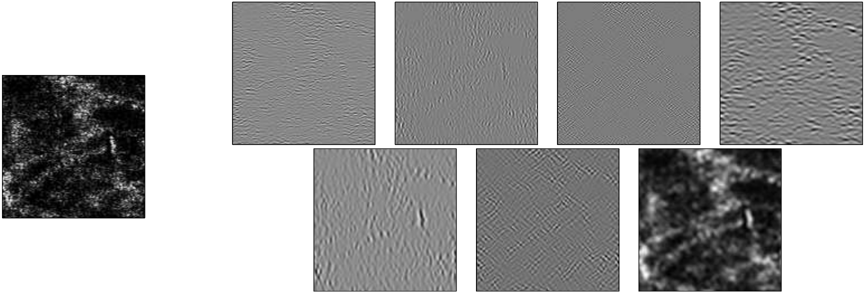
- Not appropriate for modeling two common phenomena of often used feature spaces:
    - **Heavy tails** of the probability density function of the features - known as excess kurtosis
    - **Volatility clustering** - large changes tend to follow large changes and small changes tend to follow small changes



Motivation for a GARCH Model (Cont.)

Wavelet Based Feature Space (Willsky, 2002)

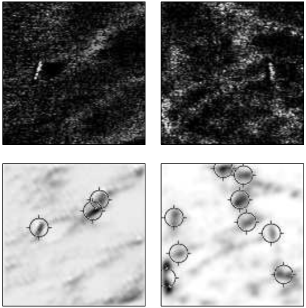
- Yields wavelet coefficients that show excess kurtosis
- Spatial and scale-to-scale statistical dependencies of wavelet coefficients exist



Motivation for a GARCH Model (Cont.)

(Goldman and Cohen 2005)

Anomaly detection algorithm for detecting regions which appear unlikely with respect to a multi-resolution GMRF model of the background image, using an MSD



The heavy tailed distribution and the clustering of innovations, cannot be accounted for by the GMRF model underlying the detection algorithm

Calls for:

An alternative statistical model

N-Dimensional GARCH Model Definition

1-D GARCH (Bollerslev, 1986)

- GRACH - Generalized Autoregressive Conditional Heteroscedasticity
- Often used as a statistical model for time series
- It is an extension to the ARCH model introduce by Engle 1982
- Creates a heavy tailed distribution characterized by clustering of innovations
- The 1-D GARCH has been shown to be useful in modeling different economic phenomena

N-Dimensional GARCH Model Definition (Cont.)

Model Order:

$$\mathbf{q} = (q_1, q_2, \ldots, q_N), \;\; q_i \geq 0, \;\; i = 1, \ldots, N$$

$$\mathbf{p} = (p_1, p_2, \ldots, p_N), \;\; p_i \geq 0, \;\; i = 1, \ldots, N$$

Neighborhood:

$$\Gamma_1 = \{\mathbf{k} \mid 0 \leq k_i \leq q_i, \; i = 1, \ldots, N \text{ and } \mathbf{k} \neq \mathbf{0}\}$$

$$\Gamma_2 = \{\mathbf{k} \mid 0 \leq k_i \leq p_i, \; i = 1, \ldots, N \text{ and } \mathbf{k} \neq \mathbf{0}\}$$

Random Fields and Variables:

$\mathbf{i} = (i_1, i_2, \ldots, i_N)$ is an N-D index vector

$\epsilon_{\mathbf{i}}$ is a random variable on an N-D lattice

$h_{\mathbf{i}}$ its variance conditioned upon the information set:

$\psi_{\mathbf{i}} = \left\{ \{\epsilon_{\mathbf{i-k}}\}_{\mathbf{k}\in\Gamma_1}, \{h_{\mathbf{i-k}}\}_{\mathbf{k}\in\Gamma_2} \right\}$

$\Gamma = \Gamma(\mathbf{i}) = \left\{\mathbf{k} \mid k_j \leq i_j, \; j = 1, \ldots, N\right\}$ is an N-D causal neighborhood of location $\mathbf{i}$

$\eta_{\mathbf{i}} \overset{iid}{\sim} N(0,1)$ is a random variable on an N-D lattice independent of $\{h_{\mathbf{k}}\}_{\mathbf{k}\in\Gamma}$

N-D $GARCH(\mathbf{p};\mathbf{q})$ Process Definition

$$\begin{aligned} \epsilon_{\mathbf{i}} &= \sqrt{h_{\mathbf{i}}}\,\eta_{\mathbf{i}} \\ h_{\mathbf{i}} &= \alpha_0 + \sum_{\mathbf{k}\in\Gamma_1} \alpha_{\mathbf{k}} \epsilon^2_{\mathbf{i-k}} + \sum_{\mathbf{k}\in\Gamma_2} \beta_{\mathbf{k}} h_{\mathbf{i-k}} \end{aligned}$$

and is therefore conditionally distributed as:

$$\epsilon_{\mathbf{i}} \mid \psi_{\mathbf{i}} \sim N(0, h_{\mathbf{i}})$$

Conditions for a non-negative conditional variance

$$\begin{aligned} \alpha_0 &> 0 \\ \alpha_{\mathbf{k}} &\geq 0, \quad \mathbf{k} \in \Gamma_1 \\ \beta_{\mathbf{k}} &\geq 0, \quad \mathbf{k} \in \Gamma_2 \end{aligned}$$

N-Dimensional GARCH Model Definition (Cont.)

N-D $GARCH(\mathbf{p};\mathbf{q})$ Process Definition

$$\epsilon_{\mathbf{i}} = \sqrt{h_{\mathbf{i}}}\,\eta_{\mathbf{i}}$$
$$h_{\mathbf{i}} = \alpha_0 + \sum_{\mathbf{k}\in\Gamma_1} \alpha_{\mathbf{k}}\epsilon^2_{\mathbf{i}-\mathbf{k}} + \sum_{\mathbf{k}\in\Gamma_2} \beta_{\mathbf{k}} h_{\mathbf{i}-\mathbf{k}}$$

and is therefore conditionally distributed as:

$$\epsilon_{\mathbf{i}} \mid \psi_{\mathbf{i}} \sim N(0, h_{\mathbf{i}})$$

WSS is a necessary condition for guarantying bounded variance for an infinite lattice

Theorem

The $GARCH(\mathbf{p};\mathbf{q})$ is wide-sense stationary with:

$$E(\epsilon_{\mathbf{i}}) = 0$$
$$\mathrm{var}(\epsilon_{\mathbf{i}}) = \alpha_0 \left[1 - \sum_{\mathbf{k}\in\Gamma_1} \alpha_{\mathbf{k}} - \sum_{\mathbf{k}\in\Gamma_2} \beta_{\mathbf{k}}\right]^{-1}$$
$$\mathrm{cov}(\epsilon_{\mathbf{i}}, \epsilon_{\mathbf{k}}) = 0,\ \forall \mathbf{i} \neq \mathbf{k}\,,$$

if and only if

$$\mathbf{1}^T(\boldsymbol{\alpha} + \boldsymbol{\beta}) < 1$$

Conditions for a non-negative conditional variance

$$\alpha_0 > 0$$
$$\alpha_{\mathbf{k}} \geq 0, \quad \mathbf{k} \in \Gamma_1$$
$$\beta_{\mathbf{k}} \geq 0, \quad \mathbf{k} \in \Gamma_2$$

N-Dimensional GARCH Model Definition (Cont.)

- At every location ($\mathbf{i}$), both the N-D neighboring squared field values and the N-D neighboring conditional variances play a role in the current conditional variance
- This yields clustering of variations, which is an important characteristic of the GARCH process

Private Cases

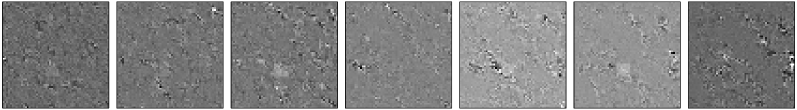
- When $\mathbf{q} = \mathbf{p} = \mathbf{0}$ $\epsilon_{\mathbf{i}}$ is WGN
- When $N = 1$, that is: $\mathbf{q} = q_1$ and $\mathbf{p} = p_1$ we have the 1-D GARCH model of Bollerslev 1986

N-D $GARCH(\mathbf{p};\mathbf{q})$ Process

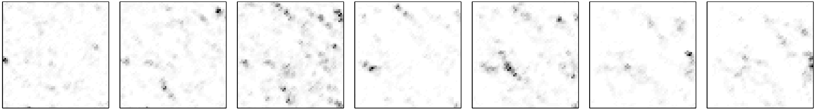
$$\begin{aligned} \epsilon_{\mathbf{i}} &= \sqrt{h_{\mathbf{i}}}\,\eta_{\mathbf{i}} \\ h_{\mathbf{i}} &= \alpha_0 + \sum_{\mathbf{k}\in\Gamma_1} \alpha_{\mathbf{k}}\,\epsilon^2_{\mathbf{i}-\mathbf{k}} + \sum_{\mathbf{k}\in\Gamma_2} \beta_{\mathbf{k}}\,h_{\mathbf{i}-\mathbf{k}} \end{aligned}$$

N-D GARCH Example

Seven layers of a GARCH synthetic image with a Gaussian shaped anomaly (Kurtosis = 26.87)



Seven layers of the conditional variance field of the synthetic GARCH data presented above. Darker areas represent higher conditional variance values



Undecimated Wavelet Transform Feature Space

Let Y be a 2-D image of size $K_1 \times K_2$. We use an undecimated wavelet transform into z levels to create a multiresolution representation of Y

Option 1

$$\epsilon_{i_1,i_2} = [d^1_{LH}, d^1_{HL}, d^1_{HH}, d^2_{LH}, d^2_{HL}, d^2_{HH}, \ldots, d^z_{LH}, d^z_{HL}, d^z_{HH}, s^z_{LL}]^T_{(i_1,i_2)}$$

Option 2 (Strickland 1997)

$$\epsilon_{i_1,i_2} = [d^1_{LH} + d^1_{HL}, d^1_{HH}, d^2_{LH} + d^2_{HL}, d^2_{HH}, \ldots, d^z_{LH} + d^z_{HL}, d^z_{HH}, s^z_{LL}]^T_{(i_1,i_2)}$$

Assumption

We assume there is a set of wavelet filters such that **Y** can be modeled as a 3-D GARCH process

The Need for a Modified MSD

The GAP

- The matched filter detector requires a typical signature of the targets
- Single hypothesis schemes make no use of *a priory* available information about the anomaly
- The MSD was developed for signal detection in subspace interference and WGN

Calls for:

A modified MSD

The Need for a Modified MSD

The GAP

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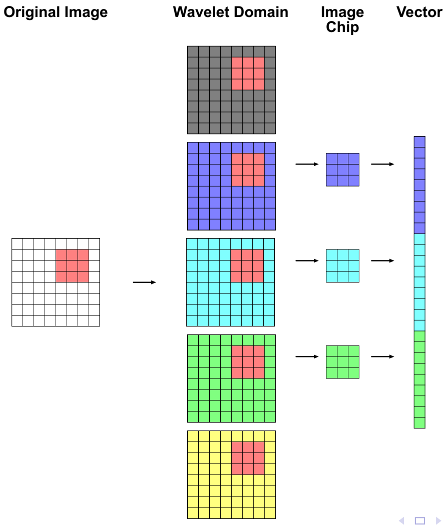
Calls for:

A modified MSD

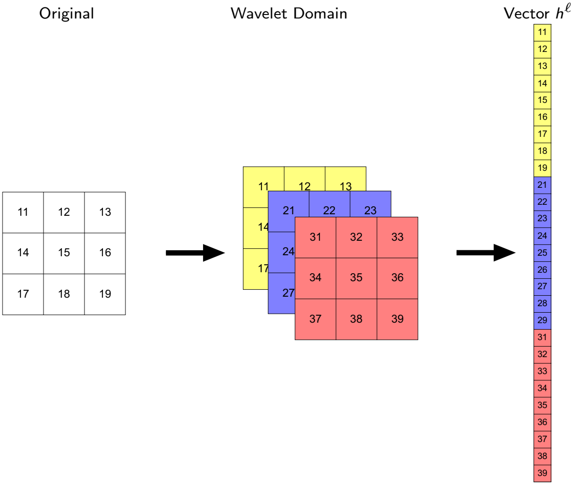
Anomaly Subspace of Layer ℓ

- Start with a training set of G_ℓ images containing anomalies at known image locations
- Training images are passed through the process of undecimated wavelet transform
- Cut an anomaly chip of size $L_1^\ell \times L_2^\ell \times L_3^\ell$ around the spatial center of the anomaly in layer ℓ
- Alternative: Create these anomaly chips synthetically by using prior knowledge
- Reshape each anomaly chip in a consistent order into a column vector
- Arrange the G_ℓ vectors associated with layer ℓ as columns in a matrix H_ℓ
- the columns or H_ℓ span the anomaly subspace for layer ℓ
- Repeat the procedure for every layer ℓ

Processing of a Training Image

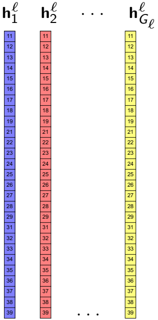


Processing of a Single Image Chip

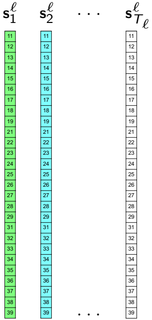


Anomaly and Interference Subspaces

$H_\ell =$



$S_\ell =$



Subspace Detection in GARCH Noise

Binary Hypothesis Test

$$H_0 : \mathbf{y}_{\ell,s} = S_\ell \phi_{\ell,s} + \mathbf{u}_{\ell,s} + \boldsymbol{\epsilon}_{\ell,s}$$
$$H_1 : \mathbf{y}_{\ell,s} = H_\ell \psi_{\ell,s} + S_\ell \phi_{\ell,s} + \mathbf{u}_{\ell,s} + \boldsymbol{\epsilon}_{\ell,s}$$

- Let $y_{\ell,s}$ represent a pixel at layer ℓ and spatial location s in the 3-D lattice $\mathbf{Y}$
- For each pixel $y_{\ell,s}$ we create a column vector $\mathbf{y}_{\ell,s}$ by row stacking an image chip of size $L_1^\ell \times L_2^\ell \times L_3^\ell$ centered around (ℓ, s)
- Let $\boldsymbol{\epsilon}_{\ell,s}$ be a result of row stacking a chip of a GARCH field of size $L_1^\ell \times L_2^\ell \times L_3^\ell$ centered around (ℓ, s)
- Let $\mathbf{u}_{\ell,s}$ be a vector representing an explanatory field

Subspace Detection in GARCH Noise (Cont.)

Projections:

$$P_{S_\ell} = S_\ell (S_\ell^T S_\ell)^{-1} S_\ell^T$$

$$P_{H_\ell S_\ell} = [H_\ell S_\ell] \left([H_\ell S_\ell]^T [H_\ell S_\ell] \right)^{-1} [H_\ell S_\ell]^T$$

GARCH Innovations Field:

$$H_0 : \epsilon^0_{\ell,s} = \mathbf{y}_{\ell,s} - \mathbf{u}_{\ell,s} - S_\ell \phi_{\ell,s} = (I - P_{S_\ell})[\mathbf{y}_{\ell,s} - \mathbf{u}_{\ell,s}]$$

$$H_1 : \epsilon^1_{\ell,s} = \mathbf{y}_{\ell,s} - \mathbf{u}_{\ell,s} - S_\ell \phi_{\ell,s} - H_\ell \psi_{\ell,s} = (I - P_{H_\ell S_\ell})[\mathbf{y}_{\ell,s} - \mathbf{u}_{\ell,s}]$$

Single Layer Detection

Conditional Likelihood Ratio

$$
\begin{aligned}
L_{\ell,s} &= 2\log\left[\frac{P(\epsilon_{\ell,s} \mid H_1)}{P(\epsilon_{\ell,s} \mid H_0)}\right] \\
&= {\epsilon^{0}_{\ell,s}}^{T}\Sigma^{-1}_{\ell,s}\epsilon^{0}_{\ell,s} - {\epsilon^{1}_{\ell,s}}^{T}\Sigma^{-1}_{\ell,s}\epsilon^{1}_{\ell,s} = \\
&= \left[\Sigma^{-1/2}_{\ell,s}(\mathbf{y}_{\ell,s} - \mathbf{u}_{\ell,s})\right]^{T}(P_{H_\ell S_\ell} - P_{S_\ell}) \\
&\quad \times \left[\Sigma^{-1/2}_{\ell,s}(\mathbf{y}_{\ell,s} - \mathbf{u}_{\ell,s})\right]
\end{aligned}
$$

$$
L_{\ell,s} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_\ell
$$

Single Layer Detection (Cont.)

$$H_0 \quad : \quad L_{\ell,s} \sim \chi^2_{\mu_\ell}(0)$$
$$H_1 \quad : \quad L_{\ell,s} \sim \chi^2_{\mu_\ell}(SNR_{\ell,s})$$

$$P_{FA} = 1 - P\left[\chi^2_{\mu_\ell}(0) \leq \eta_\ell\right]$$
$$P_D = 1 - P\left[\chi^2_{\mu_\ell}(SNR_\ell) \leq \eta\right]$$

$$SNR_{\ell,s} = \left[(H_\ell \psi_{\ell,s})(I - P_{S_\ell})\right]^T \Sigma^{-1}_{\ell,s} \left[(H_\ell \psi_{\ell,s})(I - P_{S_\ell})\right]$$

Multiple Layers

Define the selected subset of layers as: $\Omega \subset \{1, 2, \ldots, K3\}$ such that the final detection image is:

Detection

$$D_s = \sum_{k \in \Omega} L_{k,s} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

SNR

$$SNR = \sum_{k \in \Omega} SNR_k$$

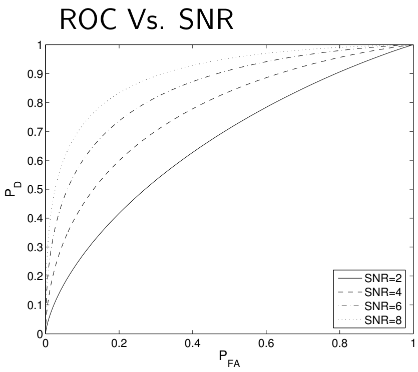
P_{FA} and P_D

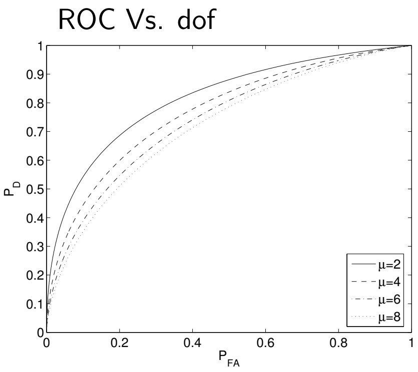
$$P_{FA} = 1 - P\left[\chi^2_{\sum_{k \in \Omega} \mu_k}(0) \leq \eta\right]$$

$$P_D = 1 - P\left[\chi^2_{\sum_{k \in \Omega} \mu_k}(SNR) \leq \eta\right]$$

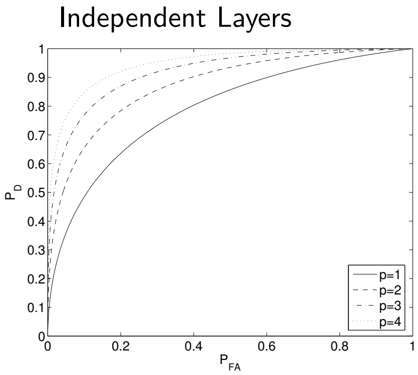
independent layers

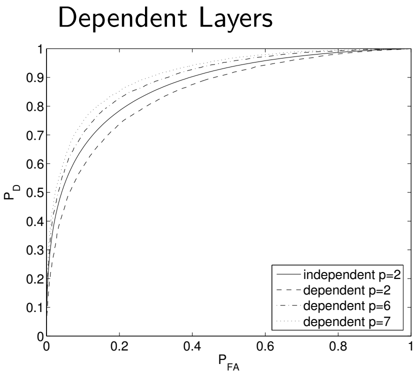
Single Layer Detection - Performance





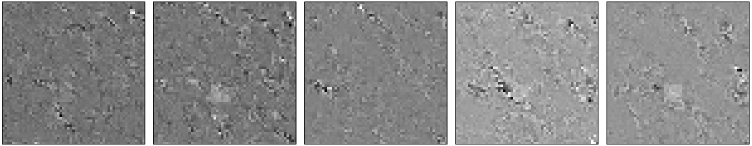
Multiple Layers - Performance



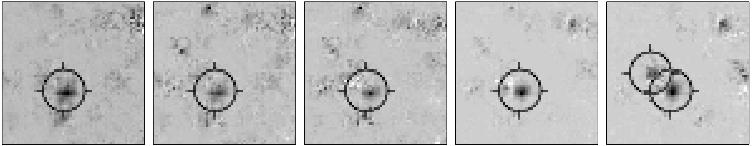


Single Layer Detection

Layers 2 — 6 of the GARCH synthetic image with a Gaussian shaped anomaly

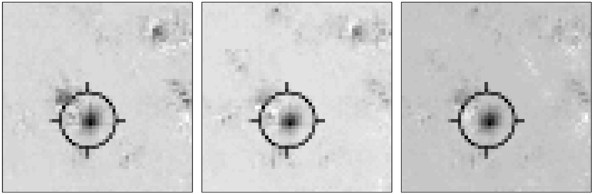


Single layer detection with $L_3^\ell = 3$



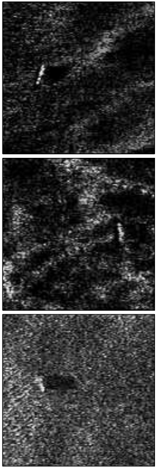
Multiple Layers Detection

layers 2, 6, $L_3^\ell = 3$ | layers $2-6$, $L_3^\ell = 3$ | Layer 4, $L_3^\ell = 7$

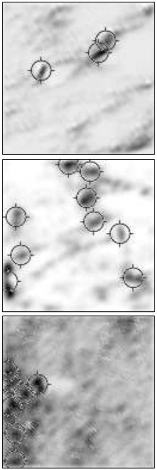


Detection in Side-Scan Sonar Imagery

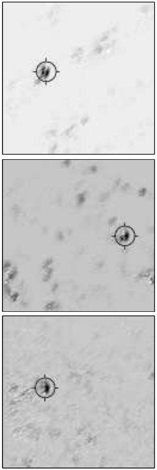
Original Images



Goldman and Cohen

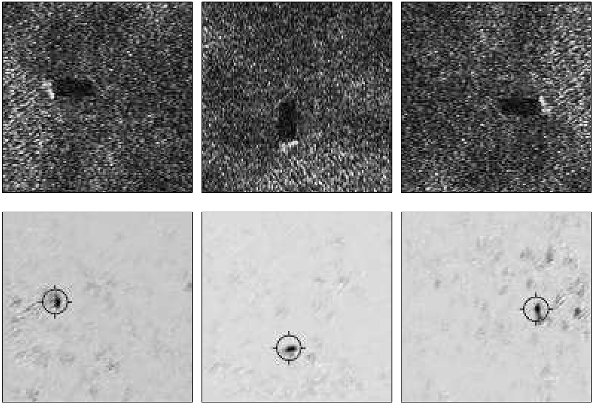


Proposed Method $\Omega = \{3\}, L_3 = 5$



Orientation

Original | Rotated 90^o | Rotated 180^o



Summary

- Introduced a multidimensional GARCH model characterized by heavy tails and clustering of innovations
- These characteristics are of interest since they are common in image multiresolution representations, and cannot be well modeled by Gaussian based statistical models
- Introduced a modified MSD Operating in GARCH noise.
- Demonstrated the performance of the proposed approach on synthetic and real data
- Compared with a GMRF based method, we presented improved performance