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# Adaptive Iterative Soft-Input Soft-Output Parallel Decision-Feedback Detectors for Asynchronous Coded DS-CDMA Systems

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The optimum and many suboptimum iterative soft-input soft-output (SISO) multiuser detectors require a priori information about the multiuser system, such as the users' transmitted signature waveforms, relative delays, as well as the channel impulse response. In this paper, we employ adaptive algorithms in the SISO multiuser detector in order to avoid the need for this a priori information. First, we derive the optimum SISO parallel decision-feedback detector for asynchronous coded DS-CDMA systems. Then, we propose two adaptive versions of this SISO detector, which are based on the normalized least mean square (NLMS) and recursive least squares (RLS) algorithms. Our SISO adaptive detectors effectively exploit the a priori information of coded symbols, whose soft inputs are obtained from a bank of single-user decoders. Furthermore, we consider how to select practical finite feedforward and feedback filter lengths to obtain a good tradeoff between the performance and computational complexity of the receiver.

**Keywords and phrases:** soft-input soft-output multiuser detection, adaptive multiuser detection, parallel decision-feedback detection, adaptive soft-input soft-output parallel decision-feedback detection, asynchronous coded CDMA systems.

## 1. INTRODUCTION

Iterative soft-input soft-output (SISO) multiuser receivers for coded multiuser systems have received widespread attention since they can provide near single-user performance in a system with multiple-access interference (MAI) by iteratively combining multiuser detection and single-user decoding. The optimum SISO multiuser detector employs either the cross-entropy minimization [1] or the maximum *a posteriori* (MAP) algorithm [2]. The computational complexity of these techniques is exponentially proportional to the number of users which can be prohibitive for large systems. Therefore, much work has been done on reduced-complexity suboptimum SISO multiuser detectors.

SISO multiuser detection based on the reduced-complexity MAP algorithms which are applied to the trellis of the multiple-access channel is proposed in [3, 4]. The simplest SISO multiuser detector is the soft interference canceller proposed in [5, 6], which has a linear computational complexity in terms of the number of users. However, it slowly converges to the performance of the single-user system. Linear iterative SISO multiuser detectors, which employ a decorrelator [7] or a minimum mean square error (MMSE) filter [8] on the output of the soft interference cancellation, significantly improve the system performance. Moreover, their computational complexity is only a cubic function of the number

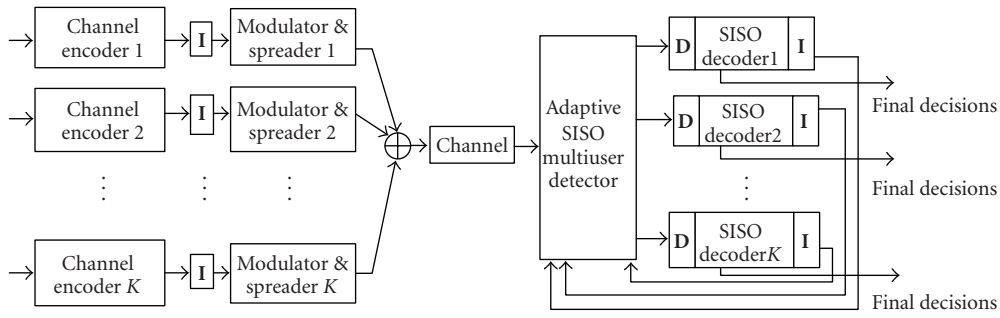


Figure 1: A general coded DS-CDMA system with an iterative receiver (**I** and **D** denote interleavers and deinterleavers, respectively).

of users. In [9, 10], nonlinear MMSE-based SISO decision-feedback detectors are investigated.

The above optimum and suboptimum SISO multiuser detectors require accurate a priori information about the multiuser system, such as all users' received signature waveforms which are functions of their transmitted signature waveforms, relative delays, and the channel impulse response. In practical situations, this information may not be easily obtainable for time-varying fading channels.

Fortunately, if the system parameters are constant or slowly varying, adaptive detectors (non-SISO) can successfully track these parameters from the received signal [11, 12, 13, 14, 15]. In [16], an adaptive SISO parallel decision-feedback detector for synchronous direct-sequence code-division multiple-access (DS-CDMA) systems with short spreading sequences is presented. By employing an approximate least squares algorithm and soft symbol estimates, the detector exploits the joint statistics of soft symbol estimates and transmitted symbols.

In this paper, we use adaptive algorithms in the iterative SISO parallel decision-feedback detector (PDFD) for asynchronous coded DS-CDMA systems in order to avoid the need for the a priori information about system parameters, such as multiple users' spreading codes and relative delays between users. First, we derive the optimum SISO parallel decision-feedback detector assuming the receiver knows the transmitted signature waveforms and relative delays between all the users. Then, we propose two adaptive versions of this SISO detector, which employ the normalized least mean square (NLMS) and recursive least squares (RLS) algorithms to estimate the filter coefficients of the detector. All users are assumed to employ short spreading codes. A training sequence is required for each user. Our adaptive SISO detectors effectively exploit the a priori information of coded symbols, which is obtained from the soft outputs of a bank of single-user decoders, to further improve their convergence performance.

Furthermore, for adaptive implementation of the SISO PDFD for asynchronous DS-CDMA systems, we select practical finite feedforward and feedback filter lengths to obtain a good tradeoff between the system performance and computational complexity of the receiver. We employ a feedforward filter which covers a two-symbol duration for each user and we consider several options for the feedback filter length. Monte-Carlo simulation results for these adaptive SISO detectors are presented and compared.

The outline of the rest of this paper is as follows. A system model of asynchronous coded DS-CDMA systems is introduced in Section 2. The optimum SISO PDFD with a general processing window for asynchronous coded DS-CDMA systems is derived in Section 3. Adaptive SISO PDFDs are proposed in Section 4, which are based on the NLMS and RLS algorithms. Monte-Carlo simulation results are presented and compared in Section 5. Finally in Section 6, the conclusions are given.

## 2. SYSTEM MODEL AND NOTATION

Throughout the paper, matrices and vectors are denoted as boldface uppercases and lowercases, respectively. Notations $(\cdot)^*$, $(\cdot)^H$, and $(\cdot)^T$ denote the complex conjugate, Hermitian transpose, and transpose, respectively.

A general coded DS-CDMA system with an iterative receiver is shown in Figure 1. There are $K$ active users in the system. The information bits of each user are first encoded, then interleaved, modulated, and spread before they are transmitted over the channel. The iterative receiver consists of two parts, an adaptive soft-input soft-output multiuser detector and a bank of SISO single-user decoders, which are separated by deinterleavers and interleavers. These two parts cooperate iteratively by transferring updated extrinsic soft information of coded symbols between them.

In our paper, we consider an asynchronous coded DS-CDMA system over the additive white Gaussian noise (AWGN) channel. The equivalent baseband received multiuser signal is

$$r(t) = \sum_{k=1}^{K} \sum_{i=1}^{N_b} b_k(i) s_k(t - iT - \tau_k) + n(t), \tag{1}$$

where $K$ is the number of active users, $N_b$ is the number of symbols transmitted by each user, $b_k(i)$ is the $i$th coded symbol of the $k$th user, $s_k(t)$ is its transmitted signature waveform, $\tau_k$ and $T$ are the delay of user $k$ and the symbol interval, respectively, and $n(t)$ is an additive white Gaussian noise process with double-sided power spectral density $N_0/2$. Each user's information bits are encoded and then BPSK modulated, that is, $b_k(i) \in \{+1, -1\}$.

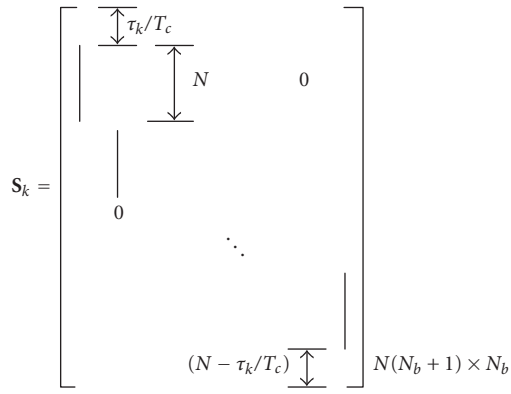


FIGURE 2: System signature matrix $\mathbf{S}_k$ of user $k$, where the nonzero part of each column is the signature vector $\mathbf{s}_k$ of user $k$.

For simple implementation, we consider a chip-synchronous and symbol-asynchronous DS-CDMA system. All users' delays are uniformly distributed in $[0, T]$ and are multiples of $T_c$, which is the chip interval. In the receiver, first we employ a chip-matched filter on the received signal $r(t)$ and then sample its output at frequency $1/T_c$. If the system is chip-asynchronous, we can oversample the output of the chip-matched filter and design a fractionally spaced feedforward filter instead. Without loss of generality and for simplicity of notation, we assume the delays of multiple users satisfy the following inequality:

$$0 \le \tau_1 \le \tau_2 \le \cdots \le \tau_K \le T. \tag{2}$$

The symbol vector consisting of the transmitted symbols of all users is denoted as

$$\mathbf{b} = \left[\mathbf{b}_1^T, \ldots, \mathbf{b}_k^T, \ldots, \mathbf{b}_K^T\right]^T_{KN_b \times 1}, \tag{3}$$

where

$$\mathbf{b}_k = \left[b_k(1), b_k(2), \ldots, b_k(N_b)\right]^T. \tag{4}$$

The received signal vector $\mathbf{r}$ at the output of the chip-matched filter during the whole symbol transmission interval can be expressed as follows:

$$\mathbf{r} = \mathbf{S}\mathbf{b} + \mathbf{n}, \tag{5}$$

where $\mathbf{S}$ is the system signature matrix and can be expressed as

$$\mathbf{S} = \left[\mathbf{S}_1, \ldots, \mathbf{S}_k, \ldots, \mathbf{S}_K\right]_{N(N_b+1) \times KN_b}. \tag{6}$$

The construction of $\mathbf{S}_k$ in (6) is shown in Figure 2, where the nonzero part of each column is the signature vector $\mathbf{s}_k$ of user $k$ and $N$ is the number of chips per coded symbol. The vector $\mathbf{n}$ in (5) is an $N(N_b + 1) \times 1$ column vector which represents the output noise component of the chip-matched filter. It has zero mean and covariance matrix $\sigma_n^2\mathbf{I}$, where $\sigma_n^2$ is the variance of the output noise component.

## 3. OPTIMUM SISO PDFD FOR ASYNCHRONOUS DS-CDMA SYSTEMS

In general, the optimum SISO PDFD filters for asynchronous DS-CDMA systems have infinite lengths [17]. For implementation purposes, we consider finite-length feedforward and feedback filters. Furthermore, these filters are suitable for use in adaptive applications. The use of these filters in our adaptive detectors will be discussed in detail in Section 4.

In the receiver, we assume that the processing window length is $N_p$, which is measured in chips and is much less than $N_b \times N$. In each processing window, the received signal vector is denoted as $\overline{\mathbf{r}}_{N_p \times 1}$, which consists of $N_p$ rows of $\mathbf{r}$ falling to this processing window. The windowed system signature matrix $\overline{\mathbf{S}}_{N_p \times KN_b}$ and noise vector $\overline{\mathbf{n}}_{N_p \times 1}$ consist of $N_p$ corresponding rows of $\mathbf{S}$ and $\mathbf{n}$, respectively. Therefore, we have the following equation:

$$\overline{\mathbf{r}} = \overline{\mathbf{S}}\mathbf{b} + \overline{\mathbf{n}}. \tag{7}$$

We can write $\mathbf{b}$ as the following sum:

$$\mathbf{b} = \mathbf{b}_U + \mathbf{b}_D, \tag{8}$$

where $\mathbf{b}_U$ consists of the symbols which are not fedback and its other elements are zeros. The nonzero elements of $\mathbf{b}_D$ consist of the fedback symbols. They have no common elements. In the same way by which we construct $\mathbf{b}_U$ and $\mathbf{b}_D$, we extract columns of $\overline{\mathbf{S}}$ and construct the corresponding signature matrices $\overline{\mathbf{S}}_U$ and $\overline{\mathbf{S}}_D$. Therefore, the windowed received signal vector $\overline{\mathbf{r}}$ can also be expressed as

$$\overline{\mathbf{r}} = \overline{\mathbf{S}}_U\mathbf{b}_U + \overline{\mathbf{S}}_D\mathbf{b}_D + \overline{\mathbf{n}}. \tag{9}$$

The feedforward filter of user $k$ has $N_p$ taps and is denoted by a column vector $\overline{\mathbf{m}}_{fk}$. The feedback filter $\overline{\mathbf{m}}_{bk}$ of user $k$ has the size $KN_b \times 1$, whose nonzero elements are corresponding to fedback symbols. That is, its effective number of taps is determined by the number of fedback symbols. The optimum filters satisfy the following minimum mean square error (MMSE) criterion:

$$\min_{\overline{\mathbf{m}}_{fk}, \overline{\mathbf{m}}_{bk}} E\left[b_k(i) - \overline{\mathbf{m}}_{fk}^H \cdot \overline{\mathbf{r}} - \overline{\mathbf{m}}_{bk}^H \cdot \hat{\mathbf{b}}_D\right]^2. \tag{10}$$

Nonzero elements of $\hat{\mathbf{b}}_D$ are soft symbol estimates of those elements of $\mathbf{b}_D$, respectively. We will introduce the soft symbol estimate of each coded symbol in the following.

The soft inputs of a SISO multiuser detector, $\{\lambda_{\text{in}}[b_k(j)]$, $1 \le k \le K$, $1 \le j \le N_b\}$, are extrinsic log-likelihood ratios (LLRs) of $\{b_k(j)\}$ provided by a bank of $K$ single-user decoders. Based on these inputs, we can obtain the soft symbol estimate of $\{b_k(j)\}$:

$$\hat{b}_k(j) = E\left[b_k(j) \,|\, \lambda_{\text{in}}[b_k(j)]\right] = \tanh\left(\frac{\lambda_{\text{in}}[b_k(j)]}{2}\right). \tag{11}$$

Furthermore, we have the following a priori statistics (12) for nonzero elements of $\mathbf{b}_U$ and $\mathbf{b}_D$. For fedback symbols, their mean values are their soft symbol estimates, while nonfedback symbols have zero mean. Note that $b_k(i)$ in (10) belongs

to nonfedback symbols. Denote $u$ and $v$ as one of the nonzero elements of $\mathbf{b}_U$ and $\mathbf{b}_D$, respectively. The soft symbol estimate of $v$ is denoted as $\hat{v}$. Thus, we have

$$\begin{aligned} E[u] &= 0, \\ E[u^2] &= 1, \\ E[v] &= \hat{v}, \\ E[v^2] &= 1 - (\hat{v})^2. \end{aligned} \tag{12}$$

We also assume that all users' transmitted symbols are independent of one another and of the background noise vector $\overline{\mathbf{n}}$ as well.

Employing the above statistics about the coded symbols, we can get the optimum feedforward and feedback filters of user $k$ which satisfy the MMSE criterion in (10):

$$\overline{\mathbf{m}}_{fk} = \left(\overline{\mathbf{R}}_U + \overline{\mathbf{R}}_D + \sigma_n^2\mathbf{I}\right)^{-1} \cdot \mathbf{s}_{b_k(i)}, \tag{13}$$

$$\overline{\mathbf{m}}_{bk} = -\overline{\mathbf{S}}_D^H \cdot \overline{\mathbf{m}}_{fk}, \tag{14}$$

where

$$\begin{aligned} \overline{\mathbf{R}}_U &= \overline{\mathbf{S}}_U \overline{\mathbf{S}}_U^H, \\ \overline{\mathbf{R}}_D &= \overline{\mathbf{S}}_D \left[\mathbf{I} - \operatorname{diag}\left(\hat{\mathbf{b}}_D \hat{\mathbf{b}}_D^H\right)\right] \overline{\mathbf{S}}_D^H, \end{aligned} \tag{15}$$

and $\mathbf{s}_{b_k(i)}$ is a one column of $\overline{\mathbf{S}}_U$, whose column index is the same as the row index of $b_k(i)$ in $\mathbf{b}_U$. The feedforward filter in (13) is actually a linear MMSE filter which suppresses the interference from non-fedback symbols, as well as the residual interference after canceling the fedback symbols and the background Gaussian noise.

From (15), we can see that the optimum feedforward and feedback filters require the knowledge of all users' signature vectors and delays. In order to avoid the need for this information, we can adaptively implement the SISO PDFD, which will be discussed in the next section.

## 4. ADAPTIVE SISO PDFD FOR ASYNCHRONOUS DS-CDMA SYSTEMS

In this section, we assume that both short spreading codes and delays of all users are unknown to the receiver. We design and employ adaptive SISO PDFDs to track these parameters from the received signal directly.

It is well known that the asynchronous system performance can be improved by using detection filters with an increased number of taps. However, increasing the number of taps increases the computational complexity of the detector. Moreover, this will have an adverse effect on the convergence speed. Therefore, we need to select suitable filter lengths to achieve a good tradeoff among the system performance, detector complexity, and system overhead.

In the parallel decision-feedback detector, the feedforward and feedback filters cooperate to suppress the multiple-access interference. Specifically, the feedback filter tries to cancel some interfering symbols, while the feedforward filter suppresses the remaining MAI, as well as the residual interference due to imperfect cancellation by the feedback filter and the background Gaussian noise. Therefore, if the feedback filter effectively cancels most of the interference caused by the interfering symbols, the remaining interference to be suppressed by the feedforward filter is reduced.

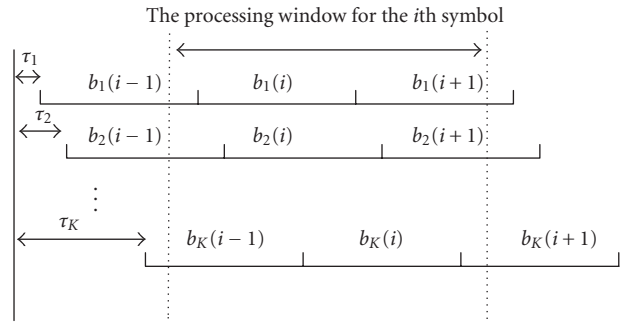


Figure 3: An asynchronous system.

On each iteration except for the first one, the SISO PDFD can obtain soft symbol estimates of all symbols from soft inputs. Thus, we have both causal and noncausal soft symbol decisions of interfering symbols for the interested symbol. We may cancel part or all of them by the feedback filter.

In this paper, we employ a feedforward filter which covers a two-symbol duration and consider several options for the feedback filter length. The length of the observation interval is $2T$, which is the minimum length such that one complete symbol of each user falls in this interval regardless of its relative delay. Figure 3 shows the processing window of the detector in the $i$th signaling interval. The output vector $\mathbf{r}(i)$ of the chip-matched filter in this processing window is

$$\mathbf{r}(i) = \begin{bmatrix} \mathbf{P}^- & \mathbf{P}^0 & \mathbf{P}^+ \end{bmatrix} \begin{bmatrix} \mathbf{b}(i-1) \\ \mathbf{b}(i) \\ \mathbf{b}(i+1) \end{bmatrix} + \mathbf{n}(i), \tag{16}$$

where $\mathbf{b}(i) = [b_1(i) \;\; b_2(i) \;\; \cdots \;\; b_K(i)]^T$ and $\mathbf{n}(i)$ is a Gaussian random vector with zero mean and covariance matrix $\sigma_n^2\mathbf{I}_{(2N\times 2N)}$. We define the punctured signature vectors of user $k$ as

$$\begin{aligned} \mathbf{p}_k^- &= \left[ \left(\mathbf{s}_k^r\right)^H \;\; \mathbf{0}^H \right]^H_{(2N\times 1)}, \\ \mathbf{p}_k^0 &= \left[ \mathbf{0}^H_{(1\times N_k^r)} \;\; \mathbf{s}_k^H \;\; \mathbf{0}^H_{(1\times N_k^l)} \right]^H_{(2N\times 1)}, \\ \mathbf{p}_k^+ &= \left[ \mathbf{0}^H \;\; \left(\mathbf{s}_k^l\right)^H \right]^H_{(2N\times 1)}, \end{aligned} \tag{17}$$

where $\mathbf{0}$ is a column vector. $\mathbf{s}_k^l$ and $\mathbf{s}_k^r$ are denoted in Figure 4 and are parts of $\mathbf{s}_k$:

$$\mathbf{s}_k = \left[ \left(\mathbf{s}_k^l\right)^H \;\; \left(\mathbf{s}_k^r\right)^H \right]^H. \tag{18}$$

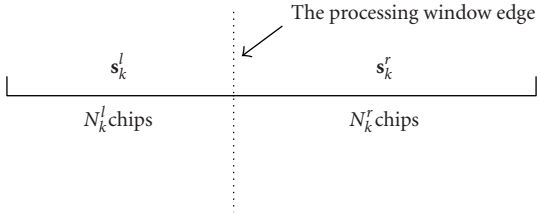


FIGURE 4: Punctured signatures of the $k$th user in the asynchronous system.

The matrices $\mathbf{P}^-$, $\mathbf{P}^0$, and $\mathbf{P}^+$ in (16) are constructed as follows:

$$\begin{aligned}\mathbf{P}^- &= [\mathbf{p}_1^- \ \ \mathbf{p}_2^- \ \ \cdots \ \ \mathbf{p}_K^-],\\ \mathbf{P}^0 &= [\mathbf{p}_1^0 \ \ \mathbf{p}_2^0 \ \ \cdots \ \ \mathbf{p}_K^0],\\ \mathbf{P}^+ &= [\mathbf{p}_1^+ \ \ \mathbf{p}_2^+ \ \ \cdots \ \ \mathbf{p}_K^+].\end{aligned} \tag{19}$$

Thus, when multiple users' delays are unknown to the receiver, for the symbol of interest $b_k(i)$ of user $k$, it has at most $(3K-1)$ interfering symbols. For implementation of the adaptive SISO multiuser detector in Figure 1, we consider three adaptive SISO PDFDs with the same feedforward filter length, that is, $2N$ taps. The feedback filter of the first detector (labeled as detector1) has $(K-1)$ taps which tries to cancel the current $(K-1)$ interfering symbols for the desired symbol. Detector2 has a feedback filter with $(2K-1)$ taps which tries to cancel the current $(K-1)$ and previous $K$ interfering symbols. The feedback filter of detector3 has $(3K-1)$ taps and tries to cancel all possible previous, current, and future interfering symbols.

In the following, we employ the NLMS and RLS algorithms in adaptive SISO PDFDs to update the feedforward filter $\overline{\mathbf{m}}_{fk}$ and feedback filter $\overline{\mathbf{m}}_{bk}$. Moreover, the a priori information of coded symbols is employed efficiently to improve the performance of the adaptive detector. The adaptive SISO PDFD requires only a training sequence for each user to estimate all filter coefficients.

The adaptive detector employing the NLMS algorithm to resolve the MMSE criterion in (10) updates the feedforward and feedback filters of user $k$ as follows for $m = 0, 1, 2, \ldots$:

$$\begin{aligned}\overline{\mathbf{m}}_{fk}(m+1) &= \overline{\mathbf{m}}_{fk}(m) - \frac{\tilde{\mu}_f}{a + \left\|\overline{\mathbf{r}}(m)\right\|^2}\left|\tilde{b}_k(m)\right| e_k^*(m)\overline{\mathbf{r}}(m),\\ \overline{\mathbf{m}}_{bk}(m+1) &= \overline{\mathbf{m}}_{bk}(m) - \frac{\tilde{\mu}_b}{a + \left\|\tilde{\mathbf{b}}_D(m)\right\|^2}\left|\tilde{b}_k(m)\right| e_k^*(m)\tilde{\mathbf{b}}_D(m),\end{aligned} \tag{20}$$

where $m$ is the recursive index and also the time index, $\tilde{\mu}_f$ and $\tilde{\mu}_b \in (0, 2)$ and are step sizes for the feedforward and feedback filters, respectively. $a$ is a small positive constant. The error signal for the $m$th recursion is

$$e_k(m) = \tilde{b}_k(m) - \overline{\mathbf{m}}_{fk}^H(m)\cdot\overline{\mathbf{r}}(m) - \overline{\mathbf{m}}_{bk}^H(m)\cdot\tilde{\mathbf{b}}_D(m), \tag{21}$$

where $\tilde{b}_k(m) = b_k(m)$ and $\tilde{\mathbf{b}}_D(m) = \mathbf{b}_D(m)$ in the training mode, $\tilde{b}_k(m) = \hat{b}_k(m)$ and $\tilde{\mathbf{b}}_D(m) = \hat{\mathbf{b}}_D(m)$ in the decision-directed mode. Furthermore, in the decision-directed mode, $|\hat{b}_k(m)|$ is used as the reliability of the error signal $e_k(m)$ in (20). Both filters are updated per symbol and their initial states are $\overline{\mathbf{m}}_{fk}(0) = \mathbf{0}$ and $\overline{\mathbf{m}}_{bk}(0) = \mathbf{0}$.

When the detector employs the RLS algorithm, we denote $\mathbf{w}_k(m) = [\overline{\mathbf{m}}_{fk}^H(m) \ \ \overline{\mathbf{m}}_{bk}^H(m)]^H$ and $\mathbf{u}(m) = [\overline{\mathbf{r}}^H(m) \ \ \tilde{\mathbf{b}}_D^H(m)]^H$. Then the filters are updated for $m = 0, 1, 2, \ldots$:

$$\begin{aligned}\mathbf{g}_k(m+1) &= \frac{\lambda^{-1}\mathbf{P}_k(m)\mathbf{u}(m+1)}{1+\lambda^{-1}\mathbf{u}^H(m+1)\mathbf{P}_k(m)\mathbf{u}(m+1)},\\ \xi_k(m+1) &= \tilde{b}_k(m+1) - \mathbf{w}_k^H(m)\mathbf{u}(m+1),\\ \mathbf{w}_k(m+1) &= \mathbf{w}_k(m) + \mathbf{g}_k(m+1)\left|\tilde{b}_k(m+1)\right|\xi_k^*(m+1),\\ \mathbf{P}_k(m+1) &= \lambda^{-1}\mathbf{P}_k(m) - \lambda^{-1}\mathbf{g}_k(m+1)\mathbf{u}^H(m+1)\mathbf{P}_k(m).\end{aligned} \tag{22}$$

The algorithm is initialized with $\mathbf{P}_k(0) = \delta^{-1}\mathbf{I}$, where $\delta$ is a small positive number and $\mathbf{w}_k(0) = \mathbf{0}$.

Both of the adaptive detectors described above try to exploit the joint statistics of the received signal vector $\overline{\mathbf{r}}$, the transmitted symbol $b_k$ or its soft estimate $\hat{b}_k$, and the soft symbol estimates $\hat{\mathbf{b}}_D$ which are fedback. In the first iteration, since there is no fedback information of coded symbols, we only employ a linear MMSE feedforward filter and set the feedback filter coefficients to zeros for each user.

The output of the adaptive SISO PDFD is

$$y_k(m) = \overline{\mathbf{m}}_{fk}^H(m)\cdot\overline{\mathbf{r}}(m) + \overline{\mathbf{m}}_{bk}^H(m)\cdot\hat{\mathbf{b}}_D(m). \tag{23}$$

Applying the Gaussian assumption to the output in (23), we can calculate the soft outputs of the SISO PDFD. For the $m$th symbol of the $k$th user, the output $y_k(m)$ can be expressed as

$$y_k(m) = \mu_k b_k(m) + \eta_k, \tag{24}$$

where $\mu_k$ is a constant and $\eta_k$ is a Gaussian random variable with zero mean and variance $\sigma_{\eta_k}^2$:

$$\begin{aligned}\mu_k &= E\left[b_k^*(m)y_k(m)\right],\\ \sigma_{\eta_k}^2 &= E\left[y_k(m) - \mu_k b_k(m)\right]^2.\end{aligned} \tag{25}$$

Estimates of (25) can be obtained by the corresponding sample averages in (26), respectively, where we replace $b_k(m)$ by $\tilde{b}_k(m)$ in these equations:

$$\begin{aligned}\hat{\mu}_k &= \frac{1}{N_b}\sum_{m=1}^{N_b}\tilde{b}_k^*(m)y_k(m),\\ \hat{\sigma}_{\eta_k}^2 &= \frac{1}{N_b}\sum_{m=1}^{N_b}\left[y_k(m) - \hat{\mu}_k\tilde{b}_k(m)\right]^2.\end{aligned} \tag{26}$$

The soft output, that is, the extrinsic log-likelihood ratio, of $b_k(m)$ is

$$\lambda_k^o(m) = \log\frac{P\left[y_k(m)\,|\,b_k(m)=+1\right]}{P\left[y_k(m)\,|\,b_k(m)=-1\right]} = \frac{2\mu_k y_k(m)}{\sigma_{\eta_k}^2}. \tag{27}$$

## 5. SIMULATION RESULTS

The DS-CDMA system which we simulate in this section has 12 active users. All users employ the same convolutional code with rate 1/2, constraint length 7, and generators [1011011], [1111001]. Each user has a randomly selected short spreading code. The spreading factor is 16 chips per information bit. The system load is 12/16 ($K$/spreading factor). Multiple users' delays are randomly selected and fixed during simulation.

There are 300 training symbols which are randomly selected and inserted at the beginning of coded symbol frames of each user. SISO single-user decoders are based on the log-MAP algorithm in [18]. Noise random variables at the output of the chip-matched filter are identical independent Gaussian random variables with zero mean and $N_0/2$ variance.

At the first iteration, since there are no soft inputs from single-user decoders, only a feedforward filter is employed for each user. That is, at this time, a linear minimum mean square error filter is used instead. It is initially trained by the training symbols, and then is used for the following transmitted coded symbols. For the later iterations, both the feedforward and feedback filters are employed. After the training mode, they are updated by fedback symbol decisions. In the first two iterations, the filter coefficients are initialized to zeros before the adaptive algorithm is employed. In each of the following iterations, the filter coefficients are set to the values obtained at the end of the previous iteration.

We consider an asynchronous DS-CDMA system over the additive white Gaussian noise (AWGN) channel. It is assumed that the receiver has no knowledge of the short spreading codes used by the users and their delays. Three adaptive SISO PDFDs proposed in Section 4 are simulated. Figures 5 and 6 show average bit error rates of all users in the first, second, and tenth iterations provided by three adaptive detectors based on the NLMS and RLS algorithms, respectively. In (20) of the NLMS algorithm, we use $a = 0.00001$, and step sizes $\tilde{\mu}_f = \tilde{\mu}_b = 0.2$ in the training mode and $\tilde{\mu}_f = \tilde{\mu}_b = 0.05$ in the decision-directed mode. Parameters in (22) of the RLS algorithm are $\lambda = 1$ and $\delta = 0.04$. For comparison, we also show the bit error rate performance of the single-user system in these two figures, where the user's spreading code and delay are known to the receiver. In Figures 5 and 6, we observe that after the first iteration, all three detectors have similar performances and their curves appear to overlap. A similar behaviour is observed for the second iteration of detector1 and detector2 in Figure 5 and all three detectors in Figure 6.

We can see that with our adaptive SISO detectors, the system performance is improved with the increased number of iterations. Furthermore, Figure 6 shows that the performance provided by the adaptive RLS receiver approaches the performance of the single-user system after a few iterations at high signal-to-noise ratios. Among the three adaptive SISO PDFDs proposed in Section 4, detector3 provides the best performance, though it has the highest computational complexity, since its feedback filter has the maximum number of taps compared with the other two detectors.

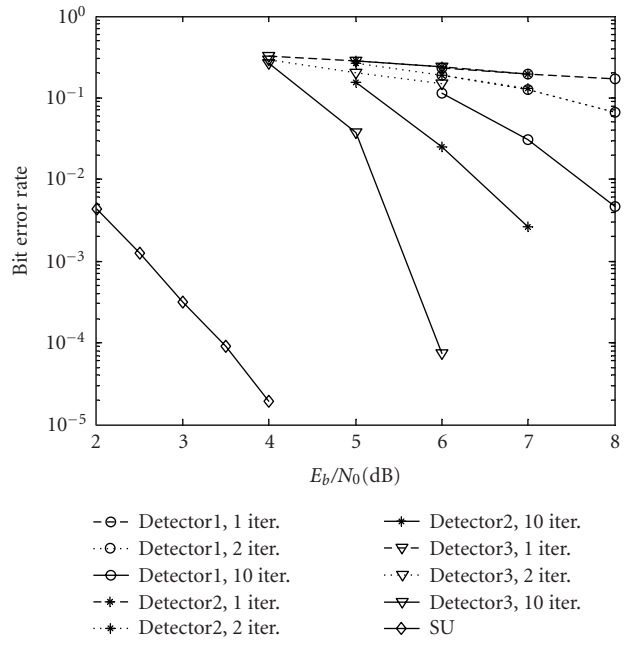


Figure 5: Bit error rate performance provided by three NLMS adaptive SISO PDFDs for the asynchronous DS-CDMA system at the first, second, and tenth iterations, and that of the single-user system (SU).

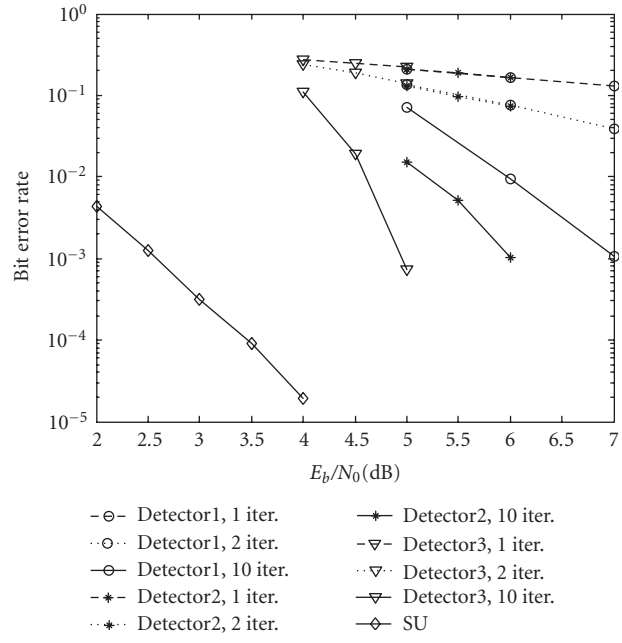


Figure 6: Bit error rate performance provided by three RLS adaptive SISO PDFDs for the asynchronous DS-CDMA system at the first, second, and tenth iterations, and that of the single-user system (SU).

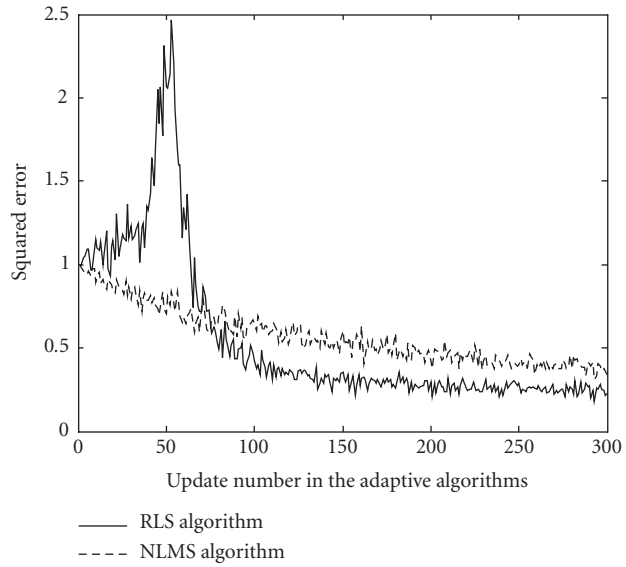


FIGURE 7: Comparison between the experimental learning curves of the adaptive SISO PDFD detector3 based on the NLMS and RLS algorithms after the second iteration during the training mode at SNR = 6 dB.

By comparing average bit error rates of all the users provided by the adaptive detector based on the RLS algorithm in Figure 6 and those obtained by the NLMS algorithm in Figure 5, we can see that the bit error rate performance provided by the adaptive SISO PDFD based on the RLS algorithm is better than the one provided by the detector based on the NLMS algorithm. For example, at a bit error rate $10^{-3}$, detector3 based on the RLS algorithm has about 0.7 dB gain with respect to detector3 based on the NLMS algorithm. This is due to the faster convergence property of the RLS algorithm, which is shown by Figure 7. The averaged squared errors $e_k^2(m)$ and $\xi_k^2(m)$ after the second iteration of the adaptive detector3 during the training mode versus the number of updates in the NLMS and RLS algorithms, respectively, are shown and compared in Figure 7. We set the signal-to-noise (SNR) ratio of each user to 6 dB. Each curve of the squared error is averaged over 200 independent trials of the experiment. However, the RLS algorithm has a greater computational complexity. Denote the length of the adaptive filter as $L$. The computational complexity of the RLS and the NLMS algorithms are $\sim O(L^2)$ and $\sim O(L)$ per update, respectively.

## 6. CONCLUSIONS

In this paper, first we presented an optimum SISO parallel decision-feedback detector for asynchronous coded DS-CDMA systems, and then proposed an adaptive implementation of it when all users' signature waveforms and relative delays were unknown to the receiver. All users were assumed to employ short spreading codes. A chip-synchronous and symbol-asynchronous DS-CDMA system was considered. A training sequence was required by each user. We showed that the resulting system performance provided by adaptive SISO PDFDs approaches that of the single-user system after a few iterations at high signal-to-noise ratios. Moreover, the adaptive detector employing the RLS algorithm provides a better bit error rate performance than the adaptive detector based on the NLMS algorithm, though at the expense of higher computational complexity. For asynchronous coded DS-CDMA systems, we further showed that the adaptive detector with more feedback filter taps gives a better bit error rate performance.

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