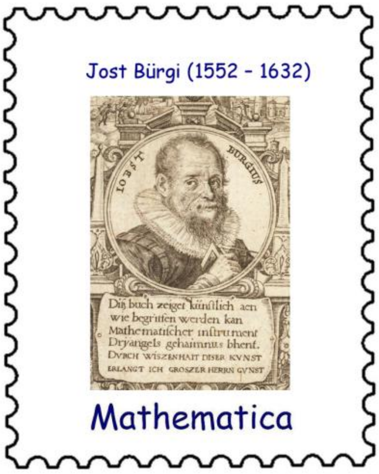
JOST BÜRGI (February 28, 1552 – January 31, 1632)

by HEINZ KLAUS STRICK, Germany



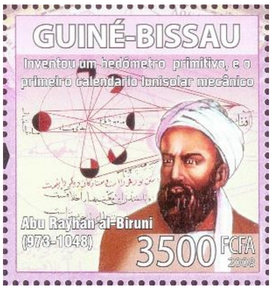
JOST BÜRGI grew up in Lichtensteig, a village with 400 inhabitants in Toggenburg (Canton St Gallen, Switzerland). Since the Reformation, the population in the village had been divided, after half of the inhabitants converted to the Protestant faith. At the village school JOST BÜRGI learnt to read, write and do basic arithmetic.

What further training the inquisitive boy received after he left his village and what places he visited can no longer be said with certainty. The fact that he was employed as an instrument maker at the court of Landgrave WILLIAM IV of Hesse-Kassel in the summer of 1579 suggests that he must have been apprenticed to outstanding masters.

WILLIAM IV (called THE WISE) had an observatory built in Kassel and the accuracy of his measurements of the fixed stars stood comparison with those of TYCHO BRAHE. He commissioned BÜRGI to build astronomical instruments, sextants, celestial globes and precision clocks, for the Prince had set himself the ambitious goal of proving COPERNICUS's heliocentric model. BÜRGI was to support him in his astronomical observations.

In 1586, WILLIAM IV enthusiastically informed TYCHO BRAHE that BÜRGI ("a second ARCHIMEDES") had constructed a clock in a new way, which deviated from the actual time by less than one minute in the course of 24 hours. This was the very first clock on which seconds could also be read.

The time unit second as the 60th part of a minute was introduced around the year 1000 by the universal scholar AL-BIRUNI. From the 13th century onwards, it was referred to in Europe as *pars minuta secunda* (second diminished part).



In 1590, when the mathematician CHRISTOPH ROTHMANN, who had been specially employed for the astronomical calculations, left the observatory, BÜRGI took on this work as well – and also showed exceptional talent here.

The year 1591 was important in Bürgi's life. He married the daughter of a pastor from a neighbouring village, though the marriage would remain childless. When his father-in-law died in the same year, Bürgi adopted his wife's 3-year-old brother. The boy developed into a respected mathematician and astronomer thanks to his adoptive father's schooling.

In the same year, Bürgi finished his work on an astronomical clock illustrating the Copernican system. For this purpose, he had intensively studied a German translation (the *Graz manuscript*) of the writing of Copernicus's *De revolutionibus orbium coelestium* (for he himself had never learned Latin).

In order to be able to carry out multiplications more quickly, he used the method of *prosthaphaeresis* (*prosthesis* = addition, *aphaeresis* = subtraction). He calculated the sine tables needed for this himself (*Canon Sinuum*).

With the help of the relation

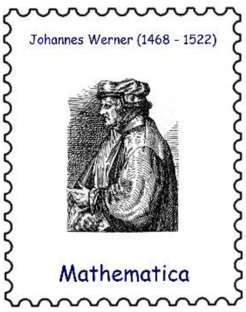
$$\sin(\alpha)\cdot\sin(\beta) \;=\; \tfrac{1}{2}\,[\cos(\alpha-\beta)-\cos(\alpha+\beta)],$$

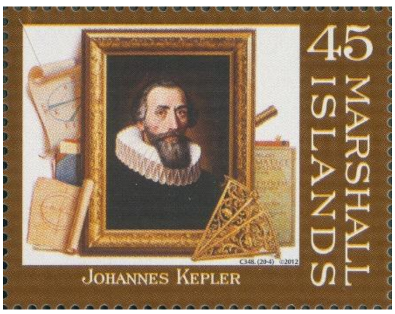
one can, for example, calculate the product 0.876 · 0.439 (= 0.384564) surprisingly accurately:

To the values $\sin(\alpha)=0.876$ and $\sin(\beta)=0.439$ correspond the angles $\alpha \approx 61°\ 9'\ 48''$ and $\beta \approx 26°\ 2'\ 24''$, and thus $\alpha - \beta \approx 35°\ 7'\ 24''$ and $\alpha + \beta \approx 87°\ 12'\ 12''$, and further

$$0.876\cdot 0.439 \approx \tfrac{1}{2}\cdot(0.817914-0.048786)=0.384564\,.$$

The method of *prosthaphaeresis*, i.e. calculating products with the help of trigonometric functions, was discovered by Johannes Werner (1468-1522) and further developed by Christopher Clavius and Bürgi, among others. The proof of the above-mentioned relationship came from Bürgi. The formulae were already known to the Egyptian astronomer Ibn Yunus (951-1009).



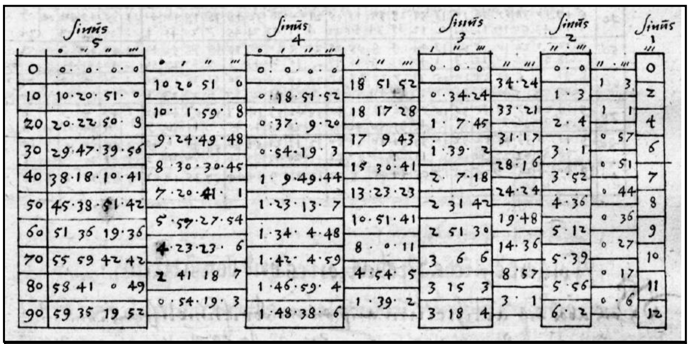


Bürgi succeeded in keeping secret the method he used to create the sine tables. Johannes Kepler and others later tried in vain to deduce the method from the hints in the writings of his pupil Nicolaus Reimers Ursus. As we know today, Bürgi presented a work entitled *Fundamentum Astronomiae* to Emperor Rudolf II in 1592. This work was discovered and made accessible in 2013 by Menso Folkerts in the archives of the University Library in Wrocław.

The work consists of two books, the second part of which deals exclusively with calculations of plane and spherical trigonometry.

The book begins with basic arithmetic (including root extraction), arithmetic in the hexagesimal system and the method of *prosthaphaeresis*. Then follows an explanation of the method by which the values of trigonometric functions were usually determined:

In regular n-gons (n = 3, 4, 5, 6, 10) one can exactly determine the sine values of 18°, 30°, 36°, 45° and 60° and from this, with the help of the half-angle formulae and the addition theorems, arrive at a sequence of angles with distances of 1½°, and by approximate calculation also at a distances of 1°. After describing this tedious method, BÜRGI turns to his simpler and more pleasant method, which he calls *the artificial method*.



Source: Menso Folkerts, Dieter Launert, Andreas Thom: Jost Bürgi's Method for Calculating Sines, https://arxiv.org/abs/1510.03180

Using the example of a table to be created for the angles 0°, 10°, 20°, ..., 90°, he starts with approximate values (natural numbers) which are to be set in relation to the last number in the rightmost column, i.e.

$\sin(90°) = 12/12,\ \sin(80°) = 11/12, \ldots,$
$\sin(10°) = 2/12$

Then half of the last number is entered in the lowest cell of the preceding column, and the numbers in the last column are continuously added (in the hexadecimal system). The next column is then created by entering 0 in the uppermost cell and continuously adding the values from the last but one column. The cells created in this way now contain considerably better values for the sine than the column filled in first. In the example, BÜRGI carried out five iterations of the algorithm and thus obtained the sine values with 6-digit accuracy, cf. the following EXCEL table, where the exact values are listed in the first row for comparison.

Angle	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Exact value	0	0.173648	0.342020	0.5	0.642788	0.766044	0.866025	0.939693	0.984808	1
Step 1	0	0.166667	0.333333	0.5	0.583333	0.666667	0.750000	0.833333	0.916667	1
Step 2	0	0.174033	0.342541	0.5	0.640884	0.762431	0.861878	0.936464	0.983425	1
Step 3	0	0.173679	0.342057	0.5	0.642713	0,765904	0.865870	0.939583	0.984769	1
Step 4	0	0.173650	0.342022	0.5	0.642784	0.766039	0.866019	0.939688	0.984806	1
Step 5	0	0.173648	0.342020	0.5	0.642787	0.766044	0.866025	0.939692	0.984808	1

After finding the lost script, ANDREAS THOM was able to prove that the algorithm is indeed excellently suited to determine the values of the sine function with arbitrary accuracy.

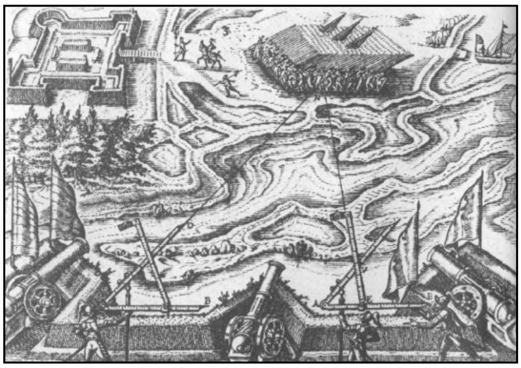
It is unclear how BÜRGI came up with the idea for his algorithm. He obviously experimented with the method of finding out and reproducing underlying regularities through continuous difference formation.

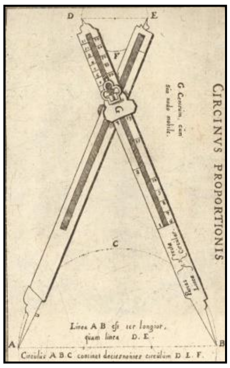
BÜRGI's sine table from 1592 contained 5400 values (from 0° to 90° with step size 1'). Later he made another table (81,000 values from 0° to 45° with a step size of 2''), which, however, has been lost.

BÜRGI's reputation as an ingenious and extremely precise instrument maker also reached the imperial court in Prague. In 1592, RUDOLF II ordered a mechanical globe from BÜRGI through his uncle, Prince WILLIAM IV, on which the movements of the planets could be read.

Barely five months later BÜRGI presented his new masterpiece to the Emperor in a personal audience, and a few weeks later also the explanations for the operation of this apparatus as well as the *Fundamentum Astronomiae*.

Returning to Kassel, he learnt of the death of his employer. However, he was able to continue his work with the same conditions under his WILLIAM IV's son MAURICE (called THE LEARNED). At the request of the Emperor, who even elevated him to the peerage, BÜRGI took over a workshop in Prague Castle at the end of 1604. There he worked closely with JOHANNES KEPLER. But he also returned again and again to the court in Kassel, where he died in January 1632.

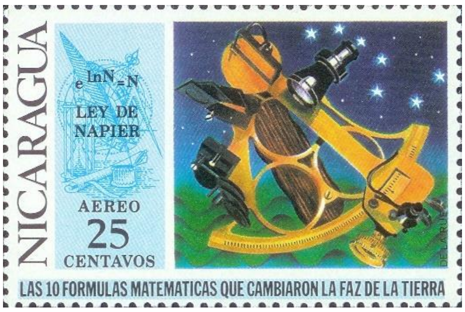


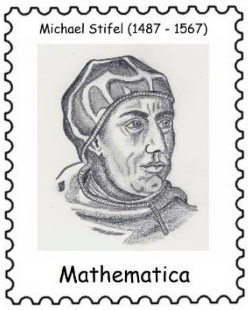


With great creativity and extreme precision, BÜRGI developed a series of tools for measuring and drawing over the years: He even received a patent for his *triangular instrument* for measuring inaccessible points, while the *proportional compass* he invented with adjustable scales and verniers was copied everywhere without a licence. With the help of the compass, maps can be enlarged or reduced; special divisions of distances are also possible, for example, according to the golden section.

Some of his mechanical celestial globes, clocks, sextants and armillary spheres can still be admired today in museums in Kassel, Prague, Dresden, Zurich, Paris, Weimar, Vienna and Upsala.

BÜRGI's discovery of logarithms in 1588 and the first logarithm tables he produced, unlike his contributions to astronomy, attracted little attention. On the one hand, he was reluctant to present his discoveries in scientific circles because of his lack of knowledge of Latin, and on the other hand, he was hesitant because of the negative experiences with how his inventions had been dealt with. It was not until 1620, after NAPIER's writings had already been widely disseminated, that he had his *Arithmetical and Geometrical Progress Tables* printed – at the urging of KEPLER, who himself had been using NAPIER's logarithmic tables since 1617.





As in MICHAEL STIFEL's *Arithmetica integra* from 1544, BÜRGI showed the connections between the arithmetical sequence 0, 1, 2, 3, ... (he does not invent his own name for this, but calls them *roseate* numbers, because they are printed in red in the book) and the geometrical sequence $2^0,\ 2^1,\ 2^2,\ 2^3,\ \ldots$ (*black* numbers).

However, since the powers grow too quickly to the base q = 2, he chose q = 1.0001 as the base, then recursively calculated the elements of the sequence with $a^{n+1} = a^n \cdot 1.0001$ and the large starting value $a_0 = 10^8$ (in order to avoid decimal points).

In contrast to the tables used later, BÜRGI's 58-page table work with a total of 23030 entries is an *antilogarithm* table, this means that the logarithms are written in the margin and one has to search inside for the appropriate numerals to be used for the calculation (multiplication, division, taking roots).

Only one complete copy of BÜRGI's logarithmic tables (with detailed explanations and 26 example calculations) still exists today. The publication came too late (only after NAPIER's tables had been disseminated), at an inconvenient place and at an unfortunate time: the 30-year war broke out in Prague in 1618, in the turmoil of which a large part of the edition that had just been published was destroyed.

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https://www.spektrum.de/wissen/jost-buergi-1552-1632/1433860

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