**Leonardo of Pisa**, also known as **Fibonacci** (1170–1250)


by Heinz Klaus Strick, Germany

In the year 1999, the Commonwealth of Dominica, a Caribbean island nation in the Lesser Antilles (80,000 residents), issued a number of postage stamps in honour of the new millennium. Representing the early thirteenth century can be found, in addition to St. Francis of Assisi and Genghis Khan, a stamp bearing the portrait of a young man named Leonardo Fibonacci and the year 1202. The name "Fibonacci" comes from the Italian, as an abbreviation of "figlio di Bonaccio", or "son of Bonaccio".

In 1202, Leonardo of Pisa, known as Fibonacci, published a book with the title *Liber Abaci*, which may be freely translated as "book of calculation". This publication was an event of particular significance for the history of mathematics in Europe.

Leonardo's father, Giulielmo, belonged to the Bonacci family. He worked as a notary and agent for merchants of the republic of Pisa in the harbour city Bugia (today Béjaïa), in Algeria. There and during his travels in other Mediterranean lands, Leonardo became familiar with the mathematics of the Arabs, including calculation in the decimal system using the "nine digits" of the Indians and the zero (Islamic mathematicians had taken over what we know as "Arabic numerals" from Indian mathematicians in the eighth century). He also studied the writings of the Greeks (which, thanks to the translations by Islamic scholars, were not lost to posterity).

Leonardo returned as an independent scholar to his home city of Pisa, and there he wrote his *Liber Abaci*. The original manuscript is lost. However, an exemplar of the edition of 1228 survives. In 1220, Leonardo wrote his *Practica Geometriae* (a collection of geometry problems) and in 1225, the *Liber Quadratorum* (book of perfect squares), along with smaller works such as *Flos* (Flower).

The book contains problems that were posed to him, in his capacity as an eminent mathematician, at the court of the Hohenstaufen emperor Frederick II and whose solutions he presented to the emperor. From 1228 on, Fibonacci was granted an annual stipend by the city of Pisa for his services as arithmetician and tax assessor.

Many of the problems posed in Fibonacci's books come from Islamic or Greek sources. But Fibonacci further developed the methods of solution. And his books inspired other writers.

Luca Pacioli (1445–1517), for instance, used many of Fibonacci's ideas and problems in his 1494 *Summa*. Indeed, even in Leonhard Euler's *Complete Introduction to Algebra* of 1770, one can find some problems that had appeared centuries earlier in the *Liber Abaci*.



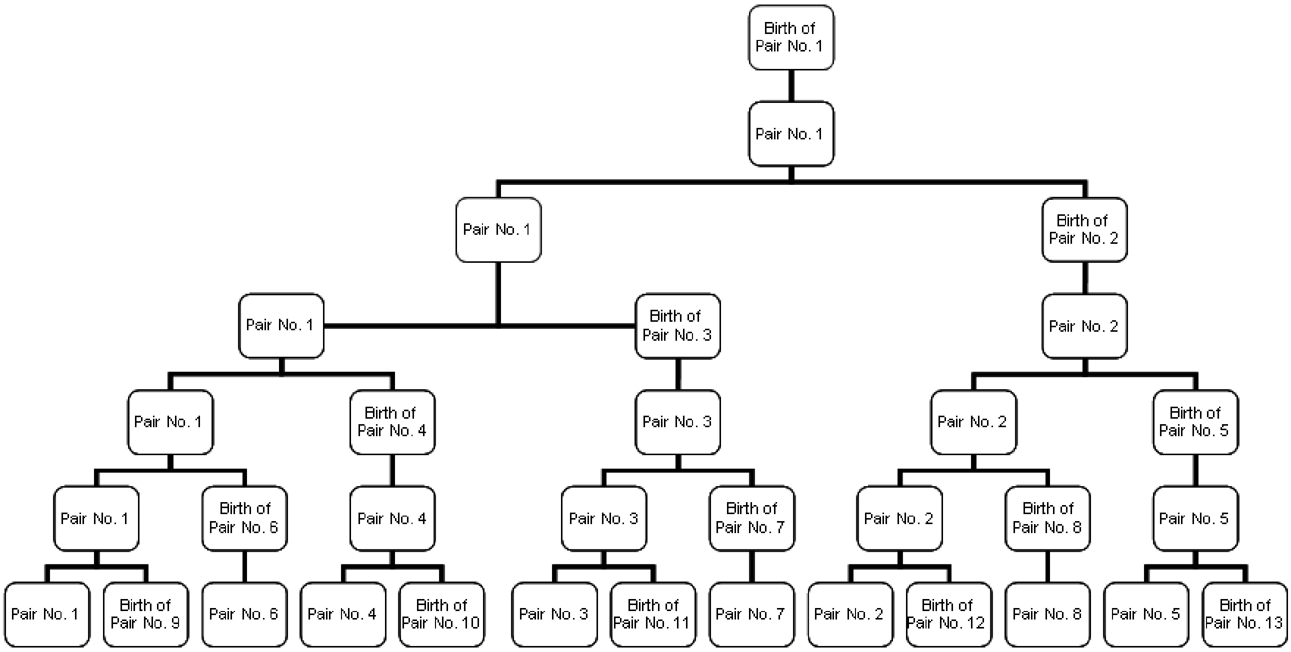
The introductory chapters of the *Liber Abaci* are concerned with calculating with natural numbers and common fractions (Fibonacci was the first to use a horizontal bar to indicate a fraction). Here can also be found methods for determining a common denominator as well as rules for when one number is divisible by another (divisibility by 2, 3, 5, 9 as well as divisibility rules for 7, 9, and 11).

Then come problems that can be solved with the "rule of three" and the conversion of currencies and measurements, as well as other problems of great importance to merchants (for example, the distribution of profits among shareholders). A further chapter deals with problems related to mixtures and compounds.


The book contains a great number of problems on a diverse variety of subjects that one could consider recreational mathematics in the broadest sense; arithmetic sequences are discussed as well as systems of linear equations. Then in Chapter 12 there appears, more or less in passing, the most famous problem of the *Liber Abaci*, namely the "Rabbit Problem":

A newly born pair of rabbits becomes capable of reproduction at age one month. After a gestation period of an additional month, their first offspring are born. This additional pair reproduces in the same fashion as the first pair. How many rabbits will there be in the nth month?

Assuming that a rabbit, once born, lives forever and that every pair of rabbits, once mature, produces a new pair every month (which, in turn, reproduce after one month), we obtain the following result:




After one month, there exists only one pair, which becomes fertile, so that at the end of the second month, the first pair of offspring (pair number 2) comes into existence. So now two pairs exist. The first pair produces another set of offspring in the next month (pair 3), and therefore, at the end of the third month, there are three pairs of rabbits. But now pair 2 has become fertile, and at the end of the fourth month, pair 2 produces a new pair of rabbits (pair 4), and pair 1 likewise produces a new pair (pair 5). Thus there are five pairs of rabbits at the end of the fourth month. And so on.

The term "FIBONACCI sequence" for the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, … is due to the French mathematician EDOUARD LUCAS (1842–1891), author of *Récréations mathématiques* and inventor of the game "Towers of Hanoi" (also known as Tower of Brahma and LUCAS's Tower). The terms of the Fibonacci sequence can be calculated according to the recurrence formula $f_{n+1} = f_n + f_{n-1}$ .

One of the properties of the FIBONACCI numbers is that the quotients of each pair of adjacent terms of the sequence form another sequence whose values alternate above and below a limiting value. This limit is the so-called golden ratio $\Phi = \frac{1+\sqrt{5}}{2} = 1{,}61803398...$


For solving quadratic equations, FIBONACCI makes use of the methods of AL-KHWARIZMI (780–850), but now with zero permitted as a solution.

FIBONACCI was the first to accept negative numbers as solutions to equations (he interpreted them as debit quantities).

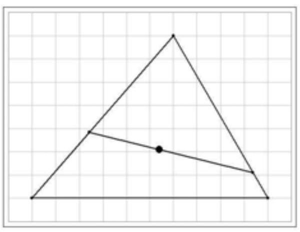
Here is an example: Suppose that three men are travelling. If the first man finds a purse and takes its contents, he now has twice as much money as the second and third together; if the second had found it, he would have had three times as much as the third and fourth together; if the third had found it, he would have had four times as much as the fourth and first together; if the fourth had found it, he would have had five times as much as the first and second together. How much does each man have?

In his *Liber Quadratorum*, FIBONACCI, among other things, looks at methods for finding *Pythagorean triples*. He notes that if one takes an odd perfect square and adds to it all the odd numbers less than this number, then this sum is itself a perfect square, and the sum of the two perfect squares is again a perfect square. For example: 25 + (1 + 3 + 5 + … + 23) = 25 + 144 = 169.

He proves the following: there exist no natural numbers *x*, *y* such that both $x^2 + y^2$ and $x^2 - y^2$ are perfect squares.

FIBONACCI also poses the problem of finding a (rational) square such that if one adds or subtracts 5, one again obtains a square. Another problem is to find three integers *x*, *y*, *z* such that $x + y + z + x^2$, $x + y + z + x^2 + y^2$, $x + y + z + x^2 + y^2 + z^2$ are all squares.


In *Flos*, one finds, among other things, the cubic equation $x^3 + 2x^2 + 10x = 20$ (*… ut inveniretur quidam cubus numerus, qui cum suis duobus quadratis et decem radicibus in unum collectis essent viginti*). This equation had been solved by the Persian poet, philosopher, and mathematician OMAR KHAYYAM (1048–1131) using geometric methods involving the intersection of a circle by a hyperbola. FIBONACCI proved that the solution cannot be expressed as a rational number nor as the square root of a rational number, and he calculated an approximate value to nine decimal places.



FIBONACCI's *Practica Geometriae* contains a number of measurement and calculational problems involving lengths, areas, and volumes, but also, for example, questions about how a triangle can be divided into two equal parts by a line passing through an internal point of the triangle (a problem solved already by EUCLID).

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First published 2006 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/leonardo-von-pisa-1170-1250/850434

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English version first published by the *European Mathematical Society* 2014

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