Automated Termination Proofs for Java Programs with Cyclic Data

M. Brockschmidt, R. Musiol, C. Otto, J. Giesl

LuFG Informatik 2, RWTH Aachen University, Germany

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Automated Termination Analysis for Imperative Programs

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- Synthesis of Linear Ranking Functions
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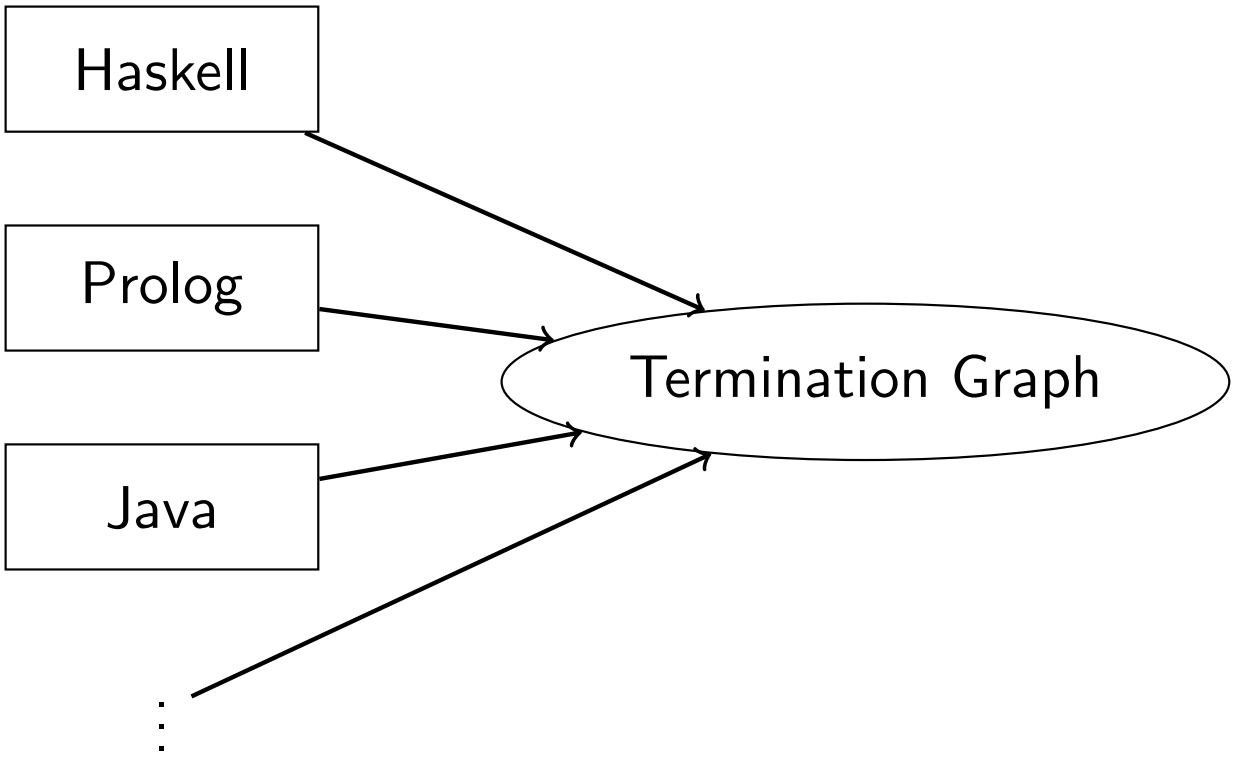

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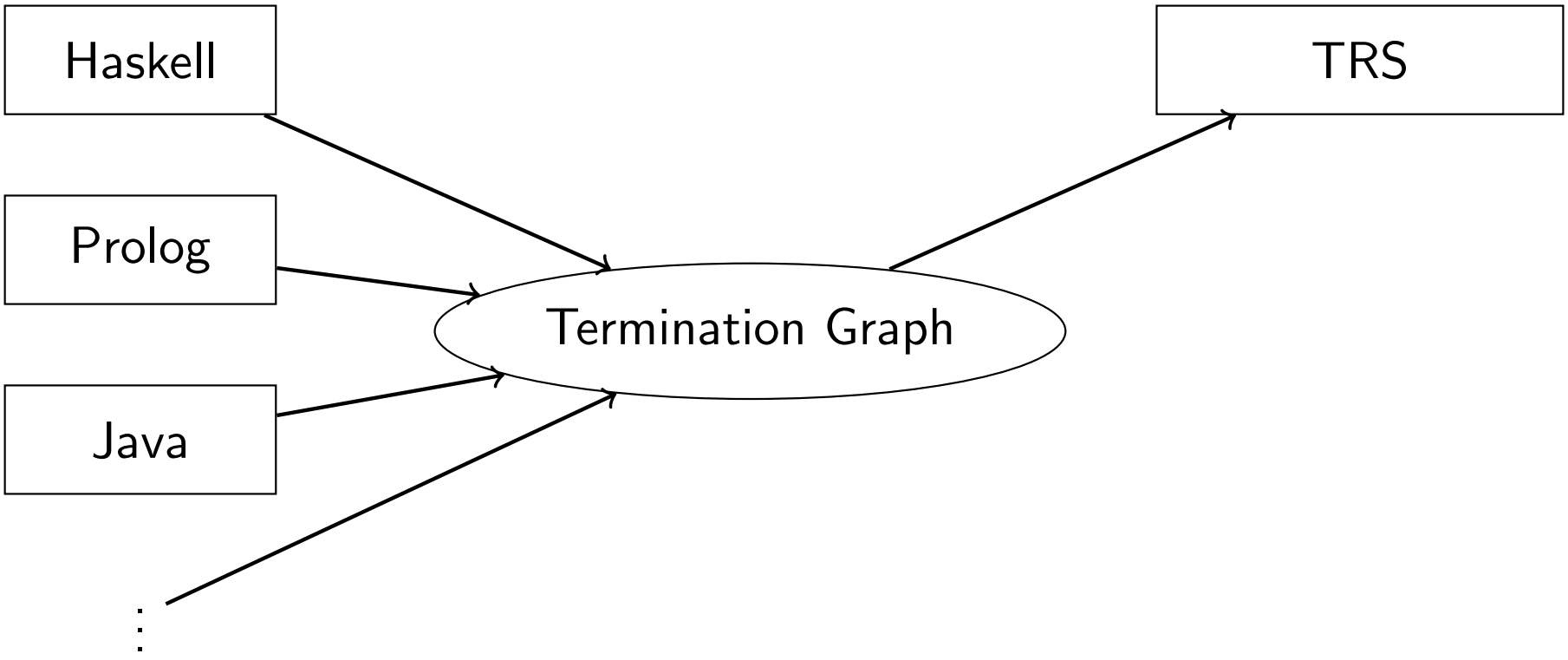

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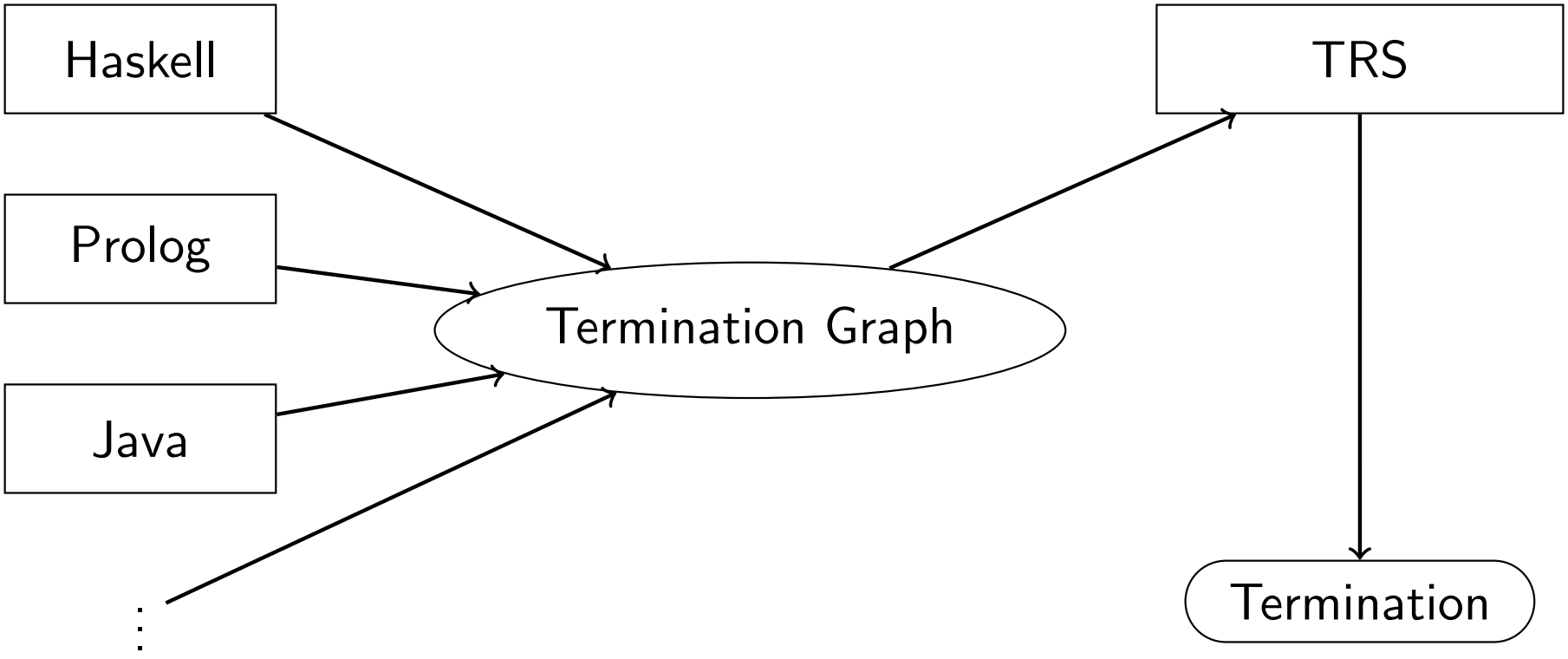

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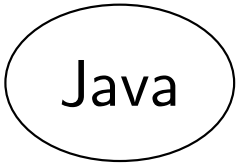

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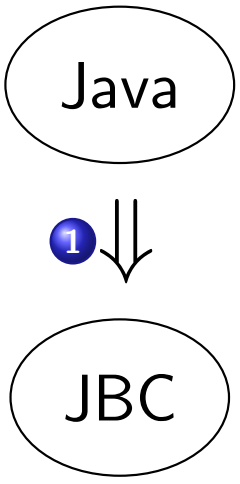

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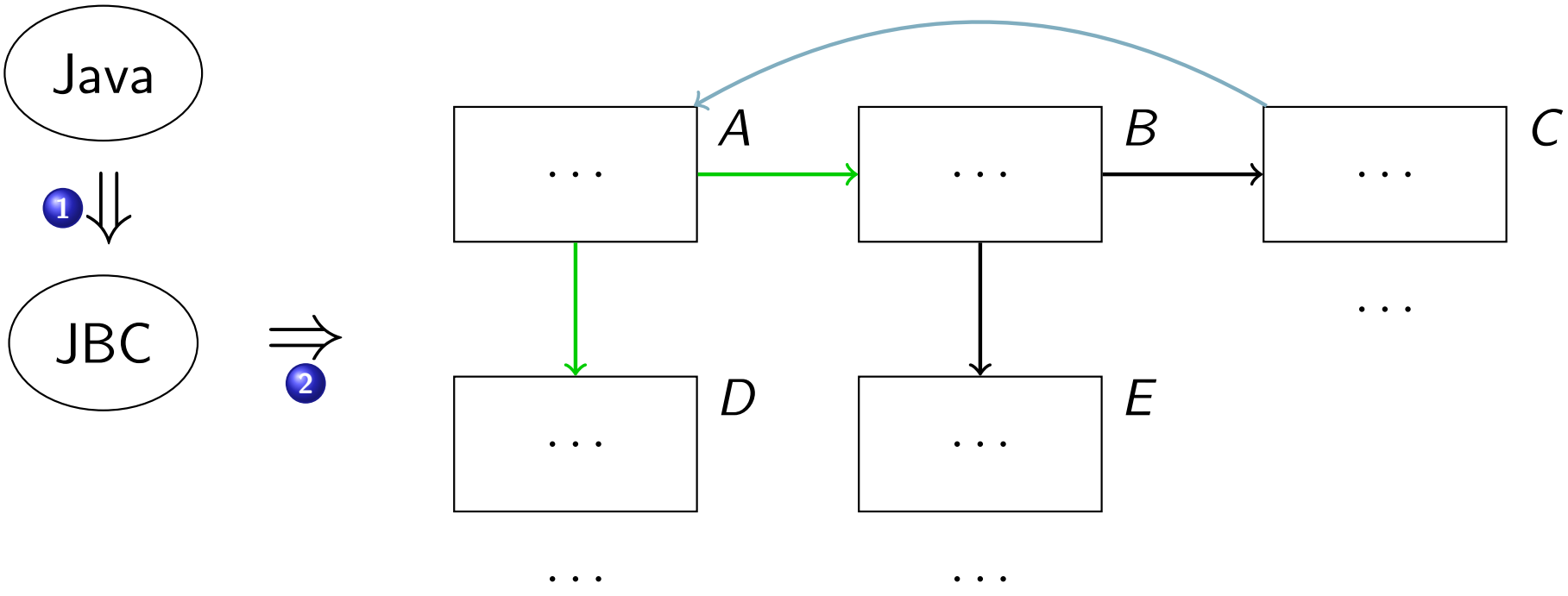

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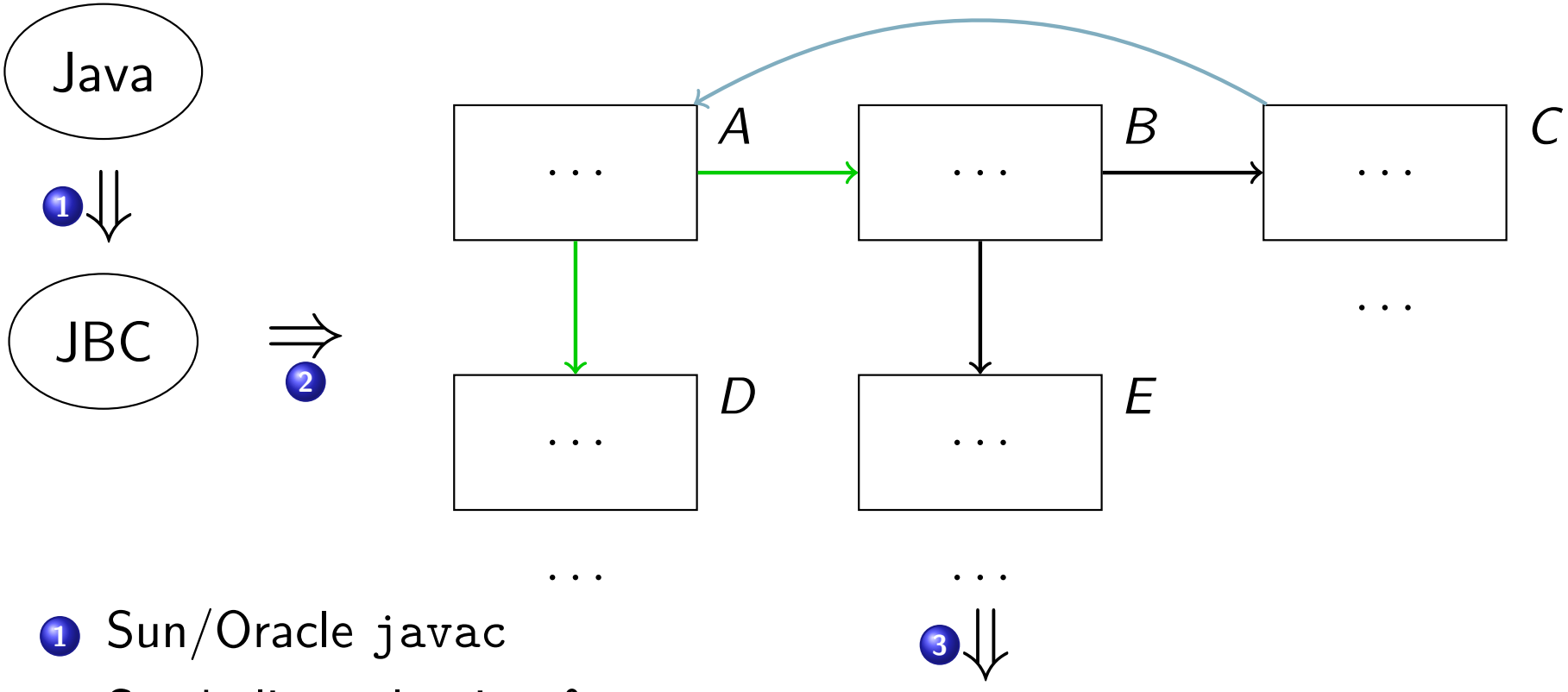

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3. Post-processing & Translation to TRS

$$f_A(\ldots) \longrightarrow f_B(\ldots) \qquad f_B(\ldots) \longrightarrow f_C(\ldots)$$

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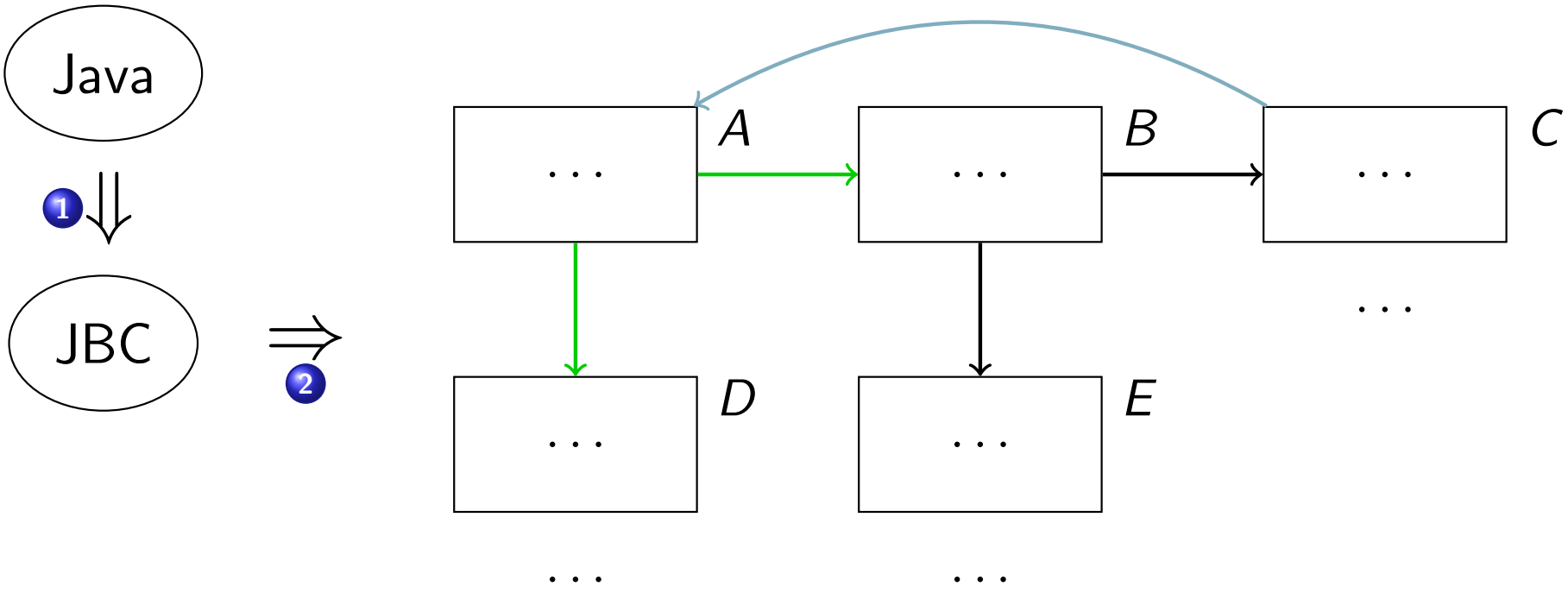

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Terminates

Rewriting-backed approach: Advantages

Handling of user-defined data structures:

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public class List {
  int value;
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- **TRS techniques** search for well-founded orders automatically

$\Rightarrow$ Complex orders for user-defined data structures possible

Rewriting-backed approach: Challenges

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- Solution:
  1. Find suitable measures on Termination Graph level
  2. Encode (numeric) measures into TRS

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  L1 p, n;
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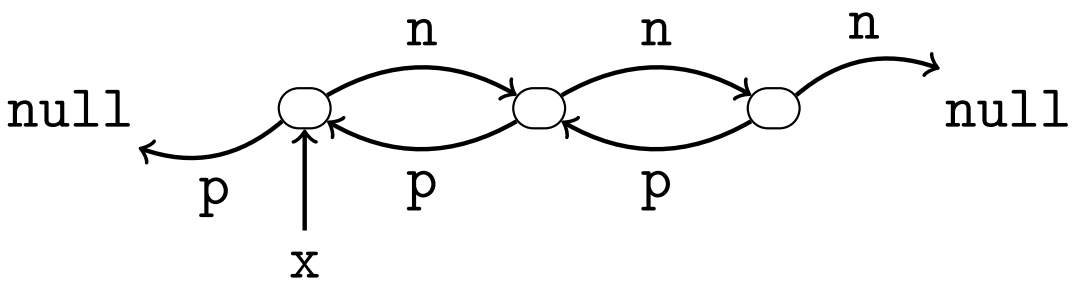

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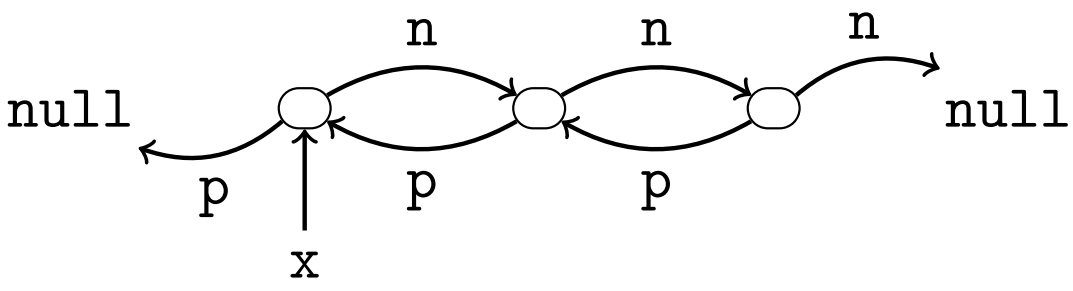

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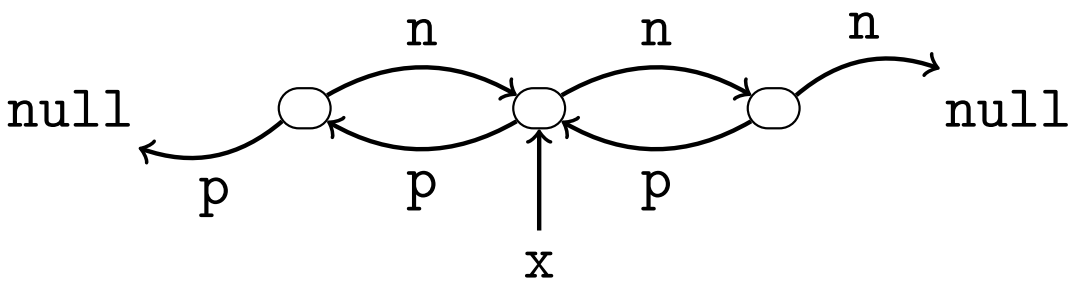

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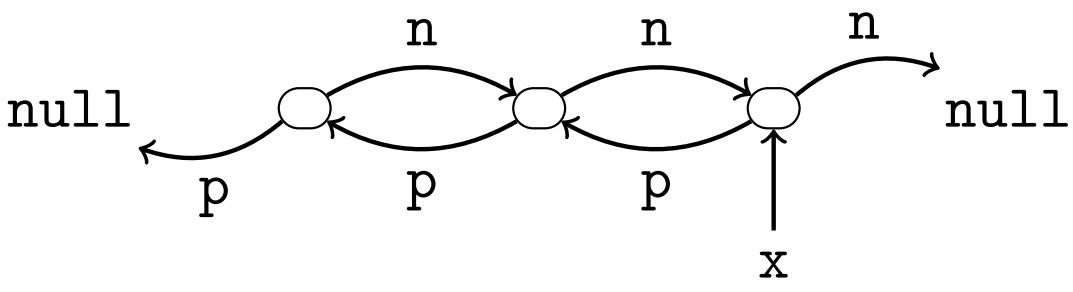

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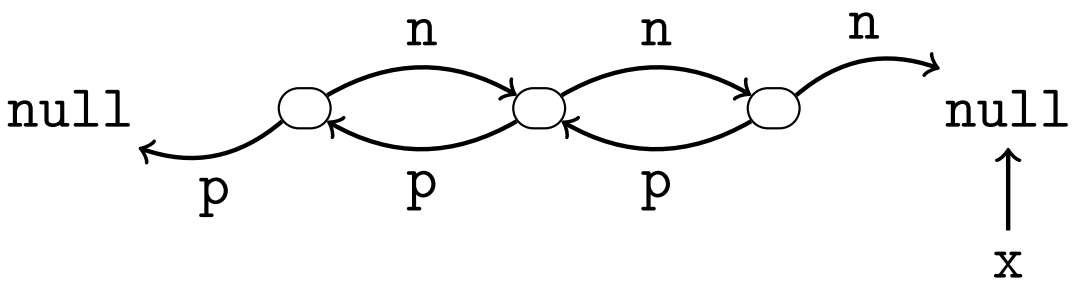

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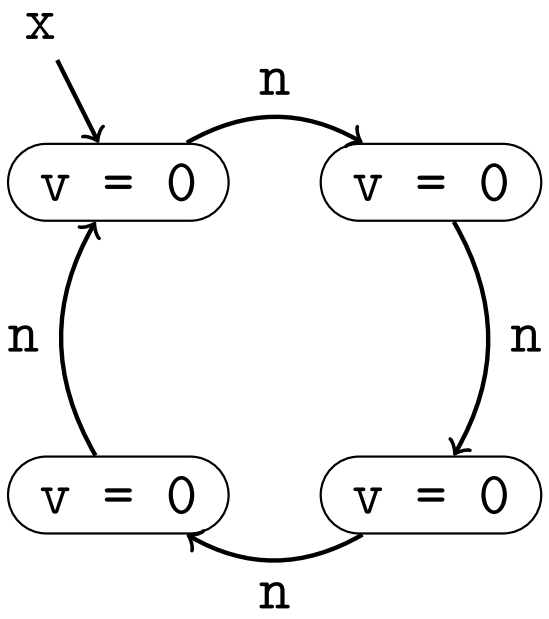

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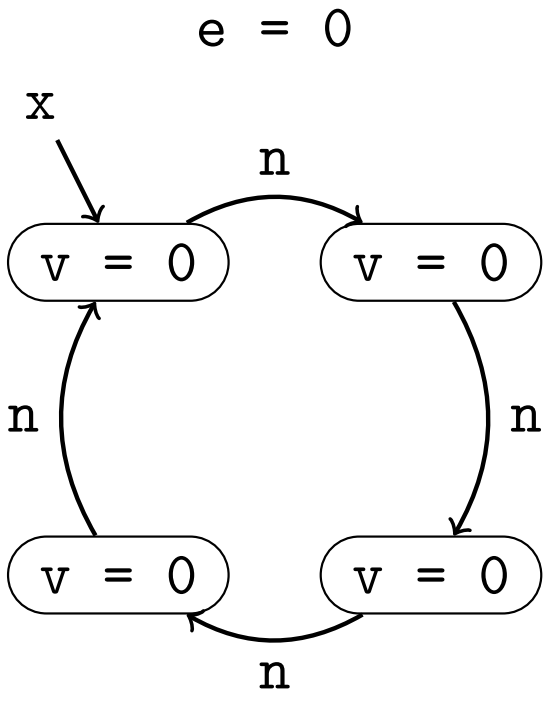

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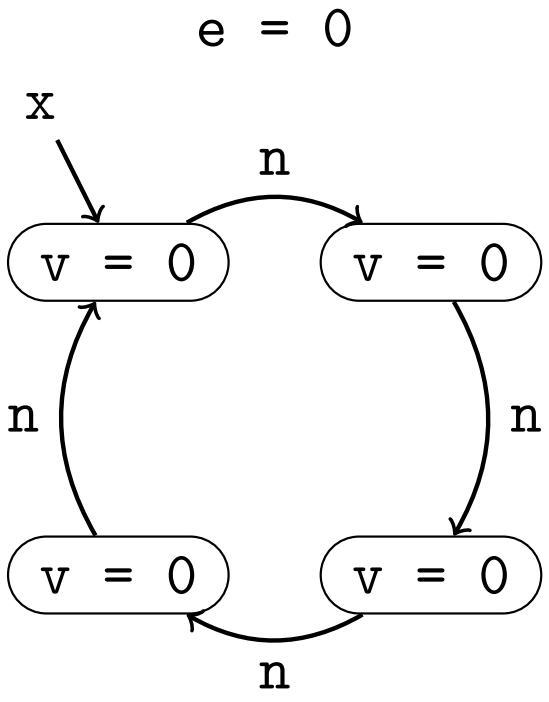

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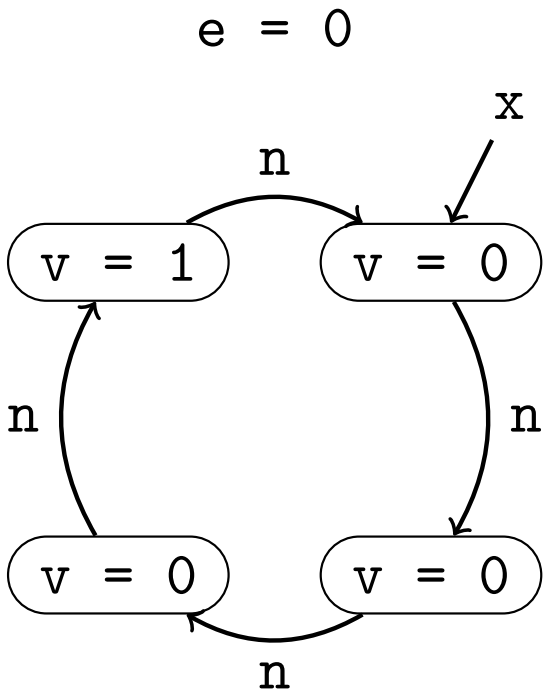

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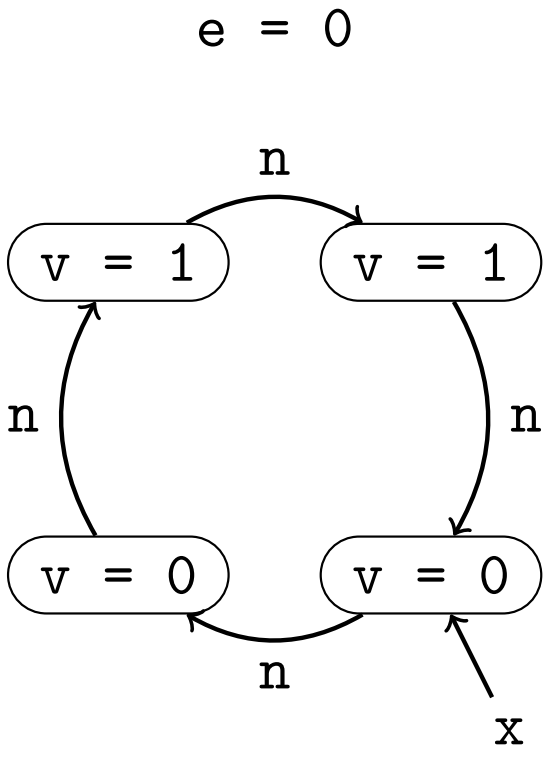

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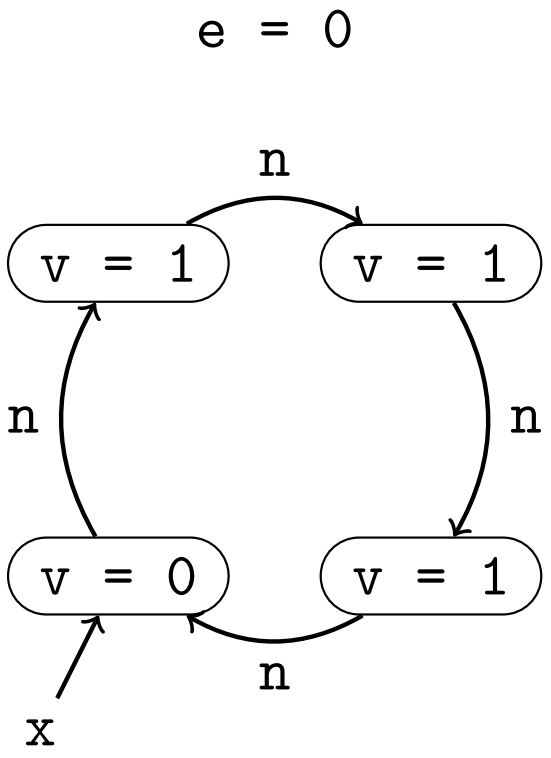

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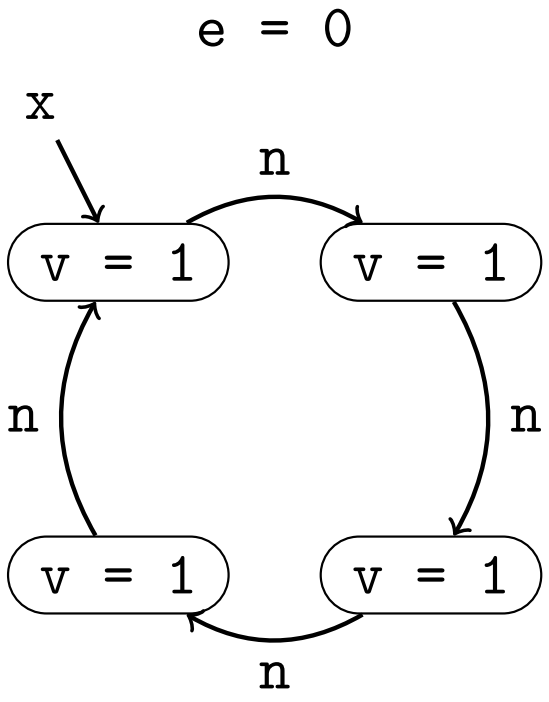

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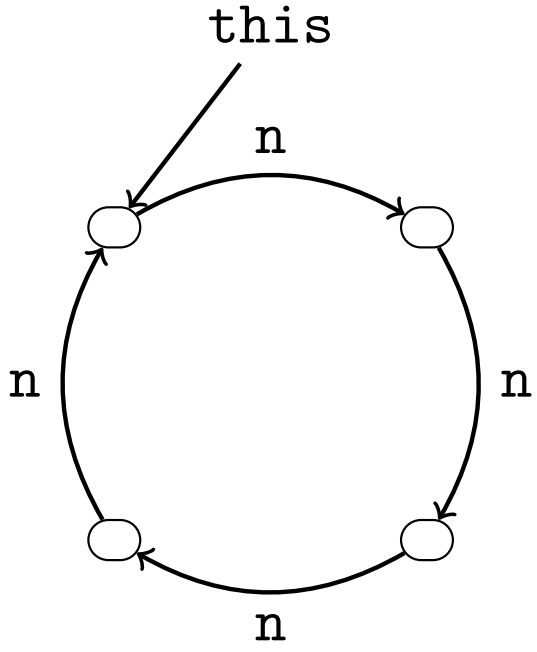

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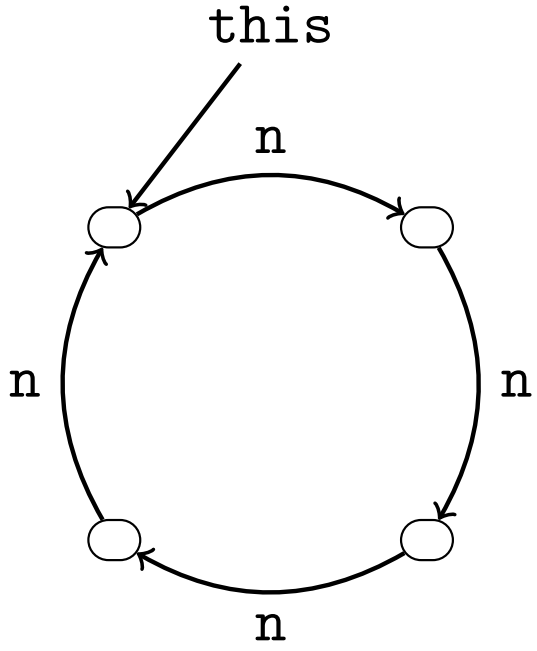

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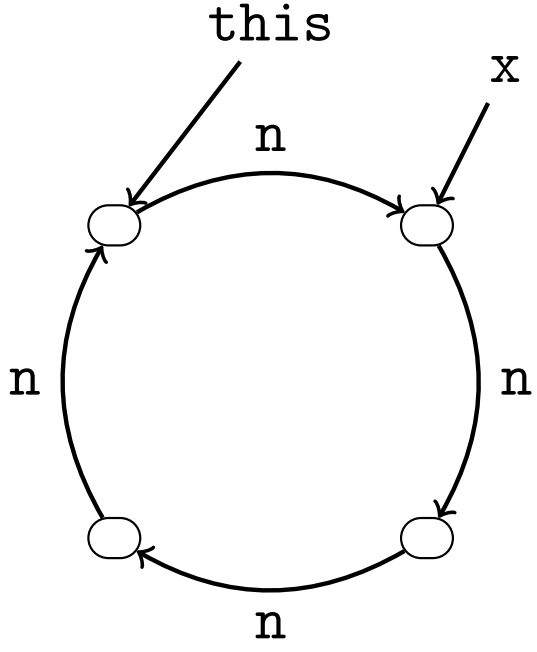

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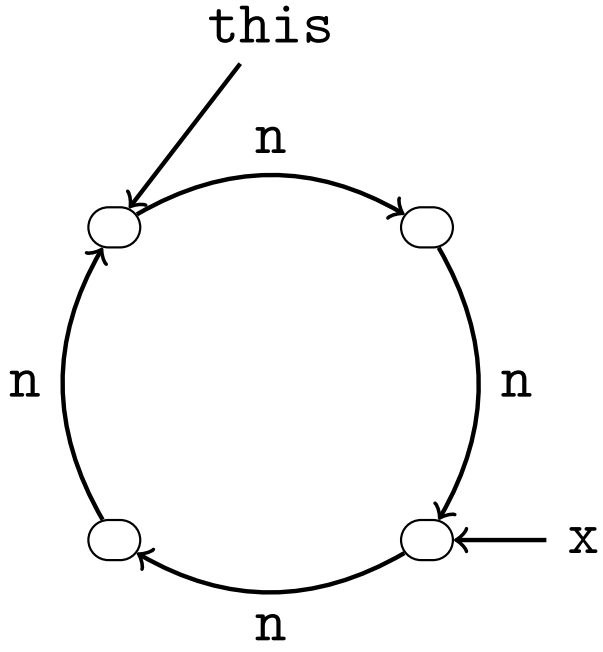

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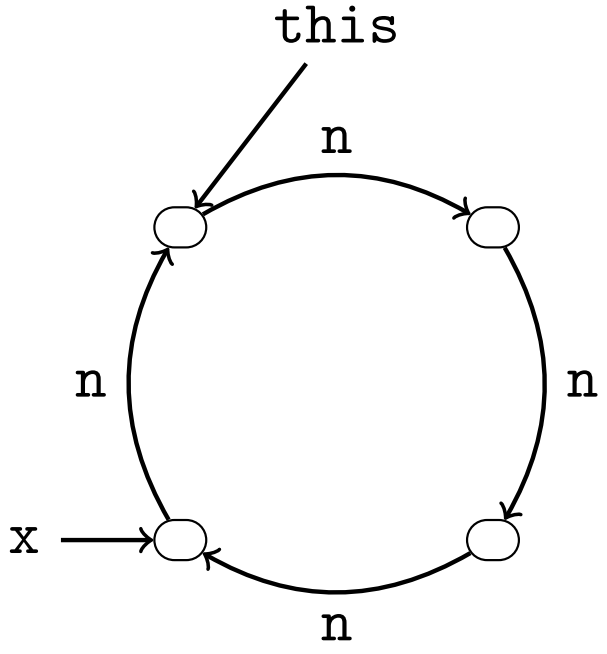

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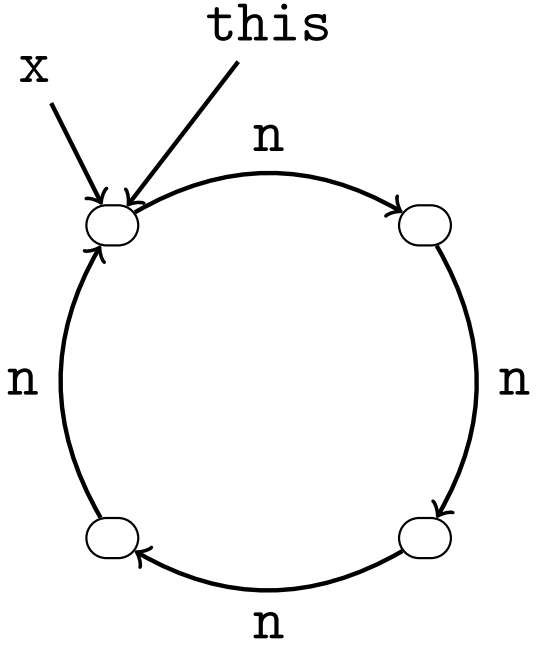

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```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{\begin{array}{ll} o_1:\mathrm{L}(\mathtt{n}=o_2) & o_2:\mathrm{L}(?) \\ o_1,o_2\circlearrowleft & o_1 =^{?} o_2 \\ o_1 \diagdown\!\!\diagup o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}}$$

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
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06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{\begin{array}{ll} o_1:\mathrm{L}(\mathtt{n}=o_2) & o_2:\mathrm{L}(?) \\ o_1,o_2\circlearrowleft & o_1 =^{?} o_2 \\ o_1 \not\perp o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}}$$

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\frac{05 \mid t : o_1, x : o_2 \mid \varepsilon}{\begin{array}{ll} o_1 : \mathrm{L}(\mathtt{n} = o_2) & o_2 : \mathrm{L}(?) \\ o_1, o_2 \circlearrowleft & o_1 =^{?} o_2 \\ o_1 \diagdown\!\!\diagup o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}}$$

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
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05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

Heap information:

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{\begin{array}{ll} o_1:\mathrm{L}(\mathtt{n}=o_2) & o_2:\mathrm{L}(?) \\ o_1,o_2\circlearrowleft & o_1 =^{?} o_2 \\ o_1 \diagdown\!\!\diagup o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}}$$

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

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00: aload_0
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06: aload_0
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10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{\begin{array}{ll} o_1:\mathrm{L}(\mathtt{n}=o_2) & o_2:\mathrm{L}(?) \\ o_1,o_2\circlearrowleft & o_1 =^{?} o_2 \\ o_1 \diagdown\!\!\diagup o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}}$$

Heap information:

- At o_1 is known L object

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\begin{array}{l|l|l} 05 & \mathtt{t}:o_1,\mathtt{x}:o_2 & \varepsilon \end{array}$$

$$\begin{array}{ll} o_1:\mathtt{L}(\mathtt{n}=o_2) & o_2:\mathtt{L}(?) \\ o_1,o_2\circlearrowleft & o_1 =^{?} o_2 \\ o_1 \diagdown\!\!\diagup o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}$$

Heap information:

- At o_1 is known L object
- At o_2 is unknown L object or null

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\begin{array}{l} 05 \mid \mathtt{t}: o_1, \mathtt{x}: o_2 \mid \varepsilon \\ \hline o_1 : \mathtt{L}(\mathtt{n} = o_2) \quad o_2 : \mathtt{L}(?) \\ o_1, o_2 \circlearrowleft \quad o_1 =^{?} o_2 \\ o_1 \backslash\!/ o_2 \quad o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}$$

Heap information:

- At o_1 is known L object
- At o_2 is unknown L object or null
- Integers: $i_1 : \mathbb{Z}$, $i_2 : [2, \infty)$

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{\begin{array}{ll} o_1:\mathtt{L}(\mathtt{n}=o_2) & o_2:\mathtt{L}(?) \\ o_1,o_2\circlearrowleft & o_1 =^{?} o_2 \\ o_1 \diagdown\!\!\diagup o_2 & o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1 \end{array}}$$

Heap information:

- At o_1 is known `L` object
- At o_2 is unknown `L` object or `null`
- Integers: $i_1 : \mathbb{Z}$, $i_2 : [2, \infty)$

Only explicit sharing

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{o_1:\mathtt{L}(\mathtt{n}=o_2) \quad o_2:\mathtt{L}(?) \\ o_1,o_2\circlearrowleft \quad o_1 =^{?} o_2 \\ o_1 \backslash\!/ o_2 \quad o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1}$$

Heap information:

- At o_1 is known `L` object
- At o_2 is unknown `L` object or `null`
- Integers: $i_1 : \mathbb{Z}$, $i_2 : [2, \infty)$

Heap predicates: **Only explicit sharing**

- References o_1, o_2 may be cyclic

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
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10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{o_1{:}\mathtt{L}(\mathtt{n}=o_2) \quad o_2{:}\mathtt{L}(?) \quad o_1,o_2\circlearrowleft \quad o_1 =^{?} o_2 \quad o_1 \diagdown\!\!\diagup o_2 \quad o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1}$$

Heap information:

- At o_1 is known L object
- At o_2 is unknown L object or null
- Integers: $i_1 : \mathbb{Z}$, $i_2 : [2, \infty)$

Heap predicates: **Only explicit sharing**

- References o_1, o_2 may be cyclic
- References o_1, o_2 may be equal

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
    L x = this.n;
    while (x != this)
      x = x.n; }}
```

```
00: aload_0
01: getfield n
04: astore_1      #x = this.n
05: aload_1
06: aload_0
07: if_acmpeq 18 #jump if x == this
10: aload_1
11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon$$

$o_1{:}\mathtt{L}(\mathtt{n}=o_2)$ $\quad o_2{:}\mathtt{L}(?)$

$o_1, o_2 \circlearrowleft$ $\quad o_1 =^{?} o_2$

$o_1 \not\downarrow\!\!\downarrow o_2$ $\quad o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1$

Heap information:

- At o_1 is known L object
- At o_2 is unknown L object or null
- Integers: $i_1 : \mathbb{Z}$, $i_2 : [2, \infty)$

Heap predicates: **Only explicit sharing**

- References o_1, o_2 may be cyclic
- References o_1, o_2 may be equal
- References o_1, o_2 may share

Abstract Java virtual machine states

```
class L {
  L n;
  void iterate() {
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```
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11: getfield n
14: astore_1      #x = x.n
15: goto 05
18: return
```

Stack frame:

- Next instruction
- Local variables
- Operand stack

$$\frac{05 \mid \mathtt{t}:o_1,\mathtt{x}:o_2 \mid \varepsilon}{o_1{:}\mathtt{L}(\mathtt{n}=o_2) \quad o_2{:}\mathtt{L}(?) \quad o_1,o_2\circlearrowleft \quad o_1 =^{?} o_2 \quad o_1 \diagdown\!\!\diagup o_2 \quad o_2 \xrightarrow{\{\mathtt{n}\}}{}^{!} o_1}$$

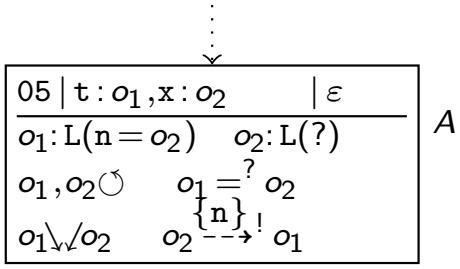
Heap information:

- At o_1 is known `L` object
- At o_2 is unknown `L` object or `null`
- Integers: $i_1 : \mathbb{Z}$, $i_2 : [2, \infty)$

Heap predicates: **Only explicit sharing**

- References o_1, o_2 may be cyclic
- References o_1, o_2 may be equal
- References o_1, o_2 may share
- Reference o_2 definitely reaches o_1 when following the field `n`

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```

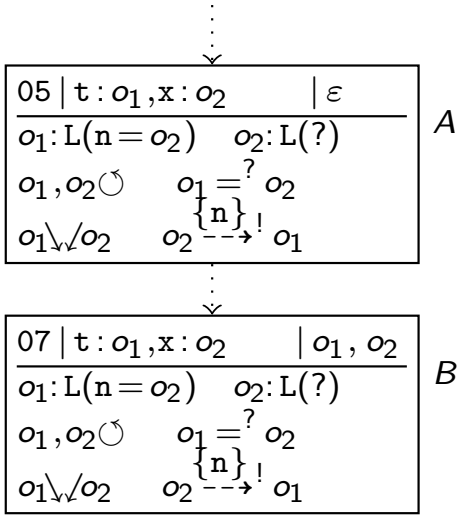


State *A*:

- t some definitely cyclic list
- x second element in list

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



State *A*:

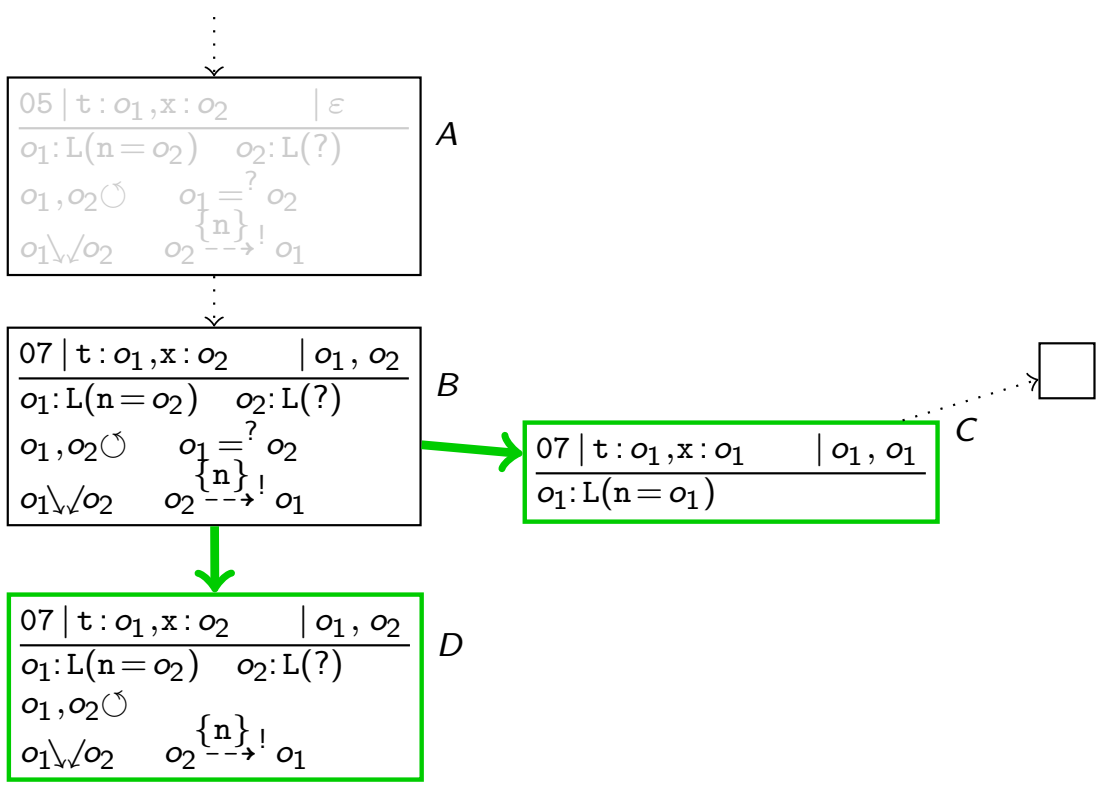
- `t` some definitely cyclic list
- `x` second element in list

State *B*:

- First equals second element?

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



State *A*:

- t some definitely cyclic list
- x second element in list

State *B*:

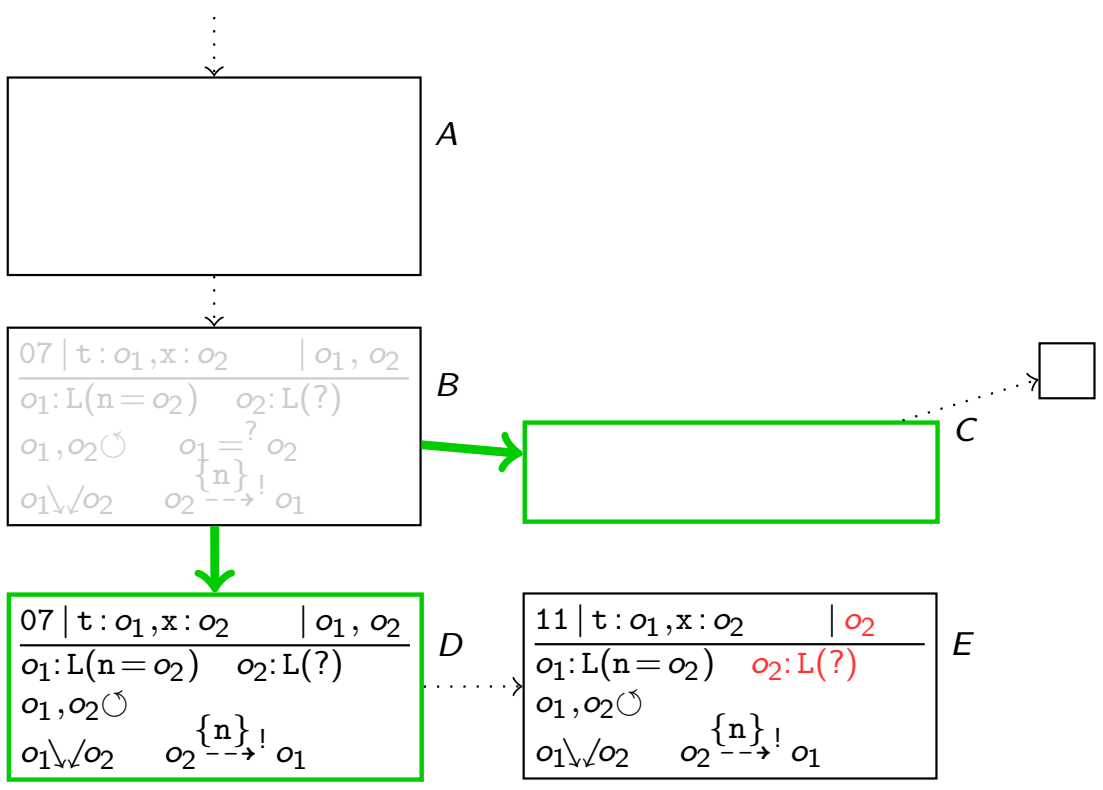
- First equals second element?

⇒ Refinement

- In *C*: References equal (↷ program ends)
- In *D*: References not equal

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



State E:

- Access to unknown object o_2

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
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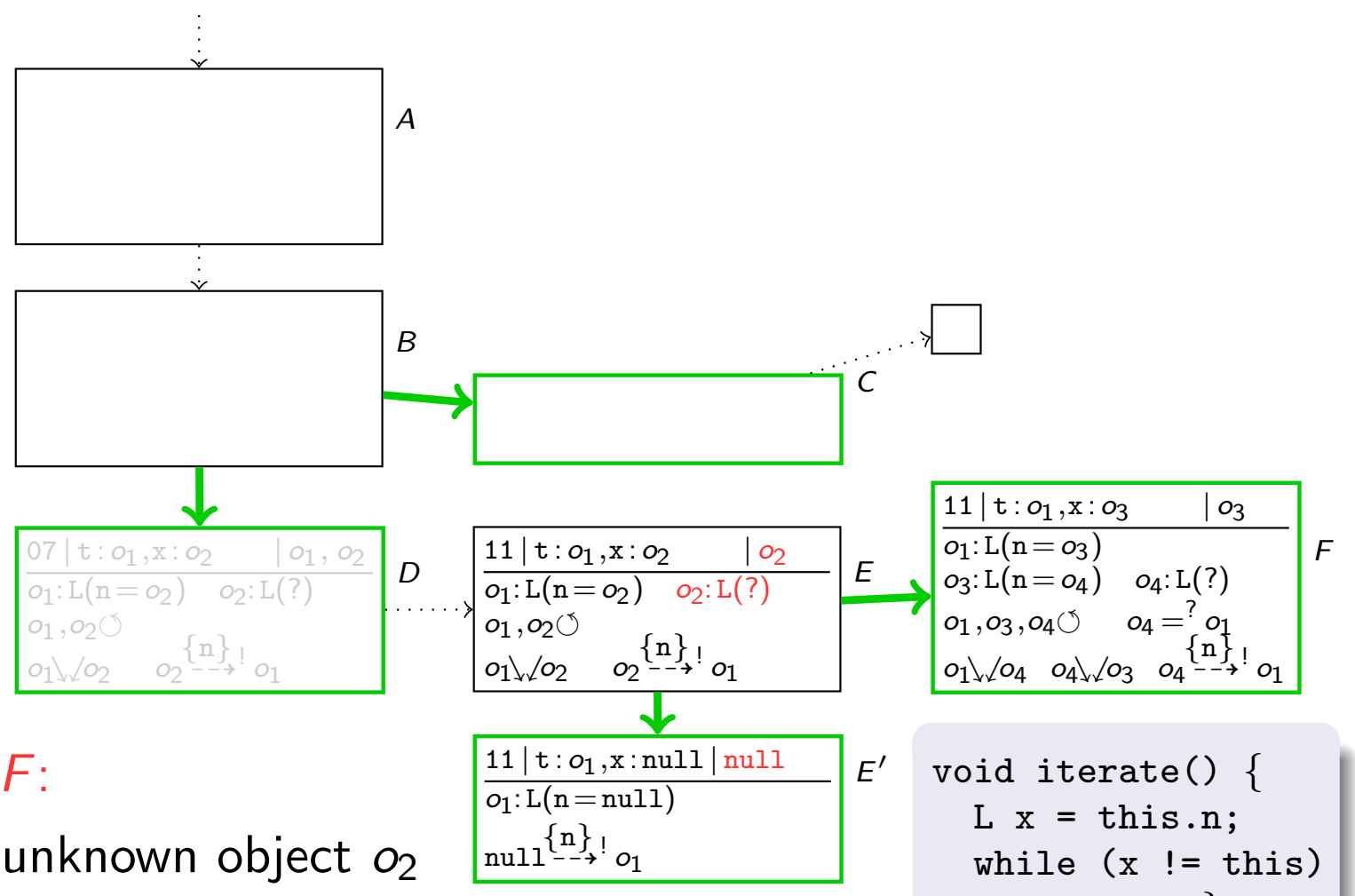
```

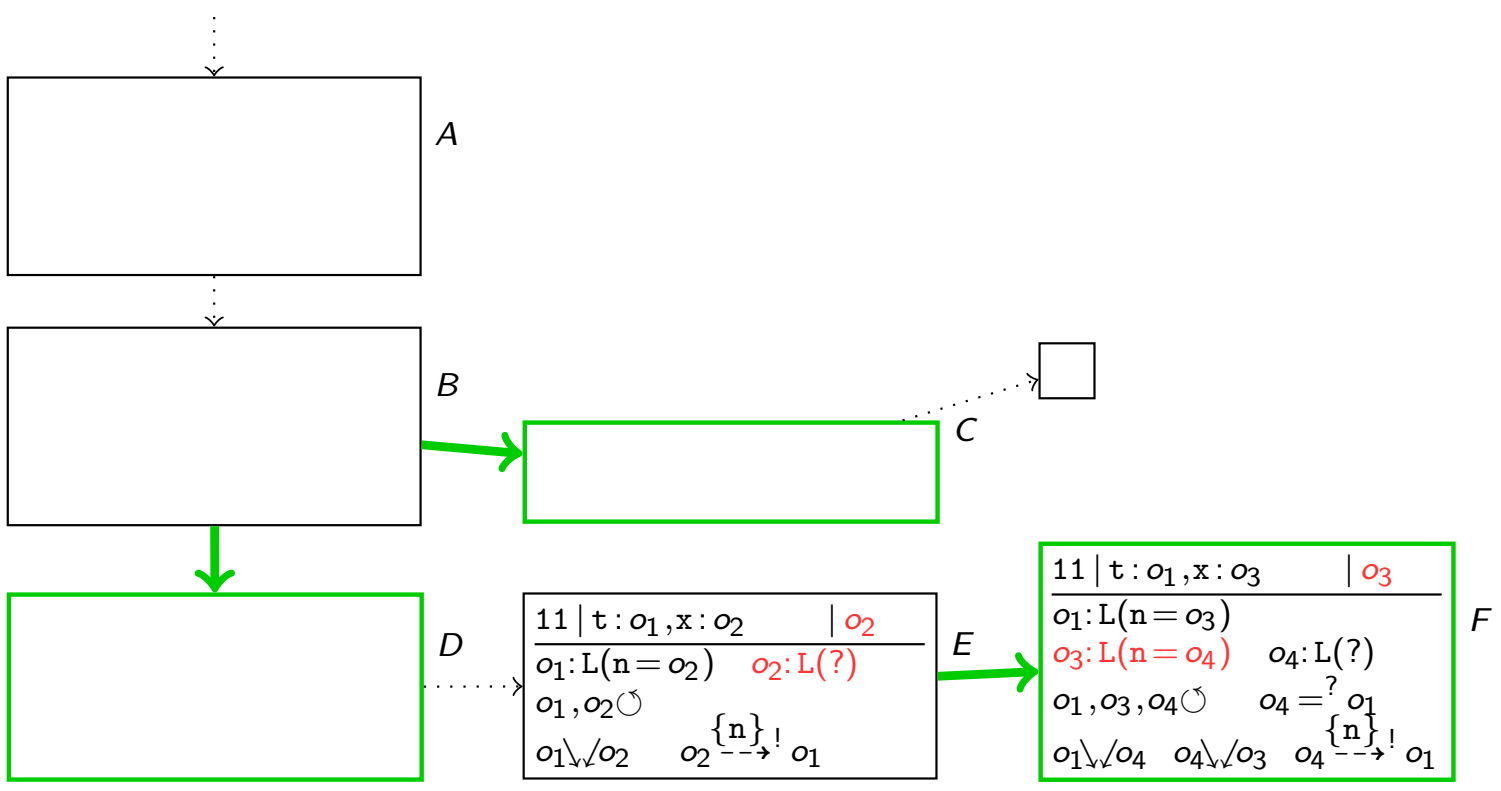


States E, F:

- Access to unknown object o_2
  $\Rightarrow$ Refinement
- In E': Case $o_2 = \texttt{null}$ not possible (implies o_2 not reaching o_1)

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



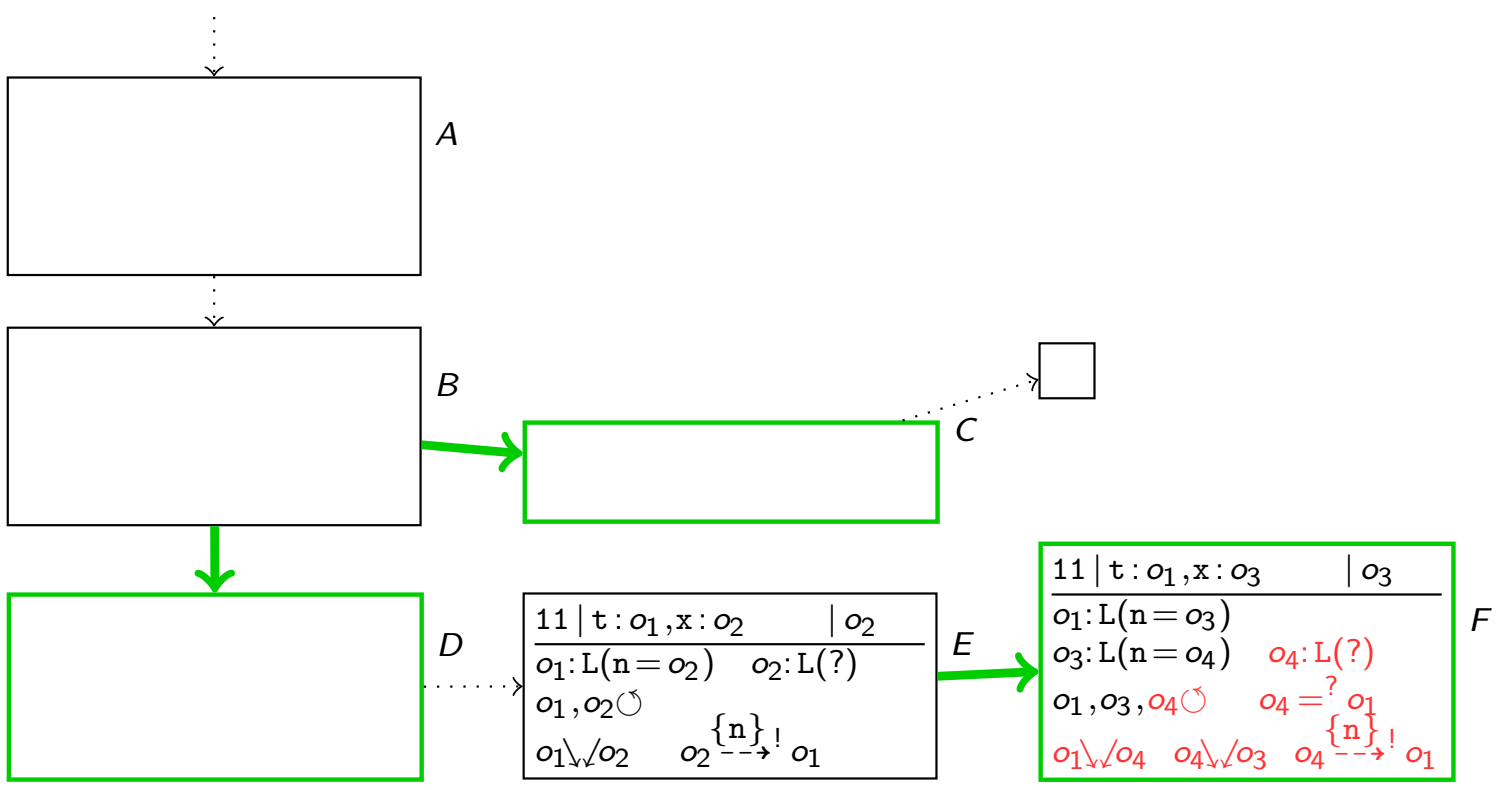
States E, F:

- Access to unknown object o_2
  $\Rightarrow$ Refinement

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

- In E': Case $o_2 = \texttt{null}$ not possible (implies o_2 not reaching o_1)
- In F: o_2 renamed to o_3, pointing to L-object with successor o_4:

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



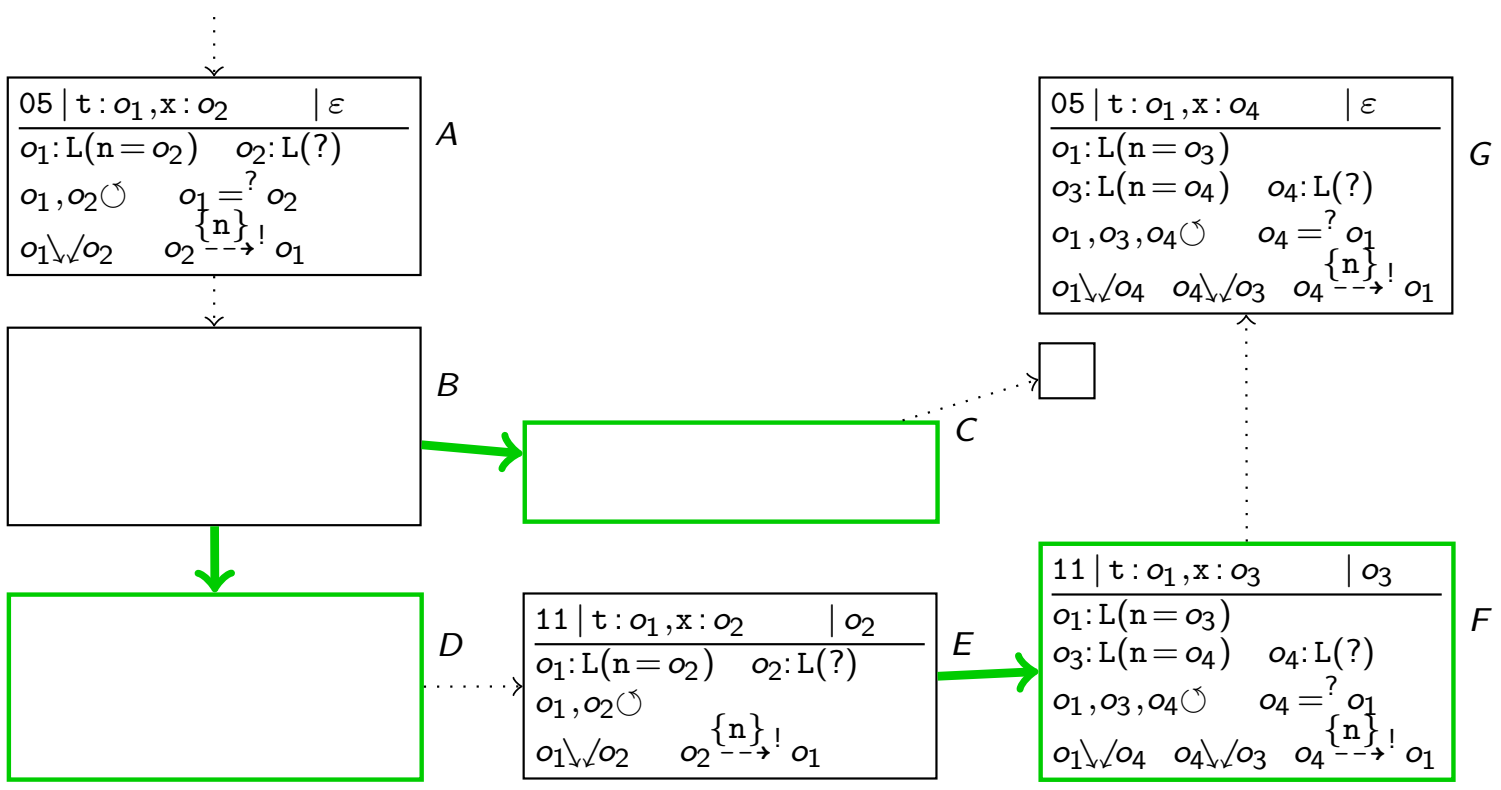
States E, F:

- Access to unknown object o_2
  $\Rightarrow$ Refinement

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

- In E': Case $o_2 = \texttt{null}$ not possible (implies o_2 not reaching o_1)
- In F: o_2 renamed to o_3, pointing to L-object with successor o_4:
  - o_4 possibly cyclic
  - o_4 possibly equal to o_1 and may reach o_1, o_3
  - o_4 definitely reaches o_1 via field n

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```




```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

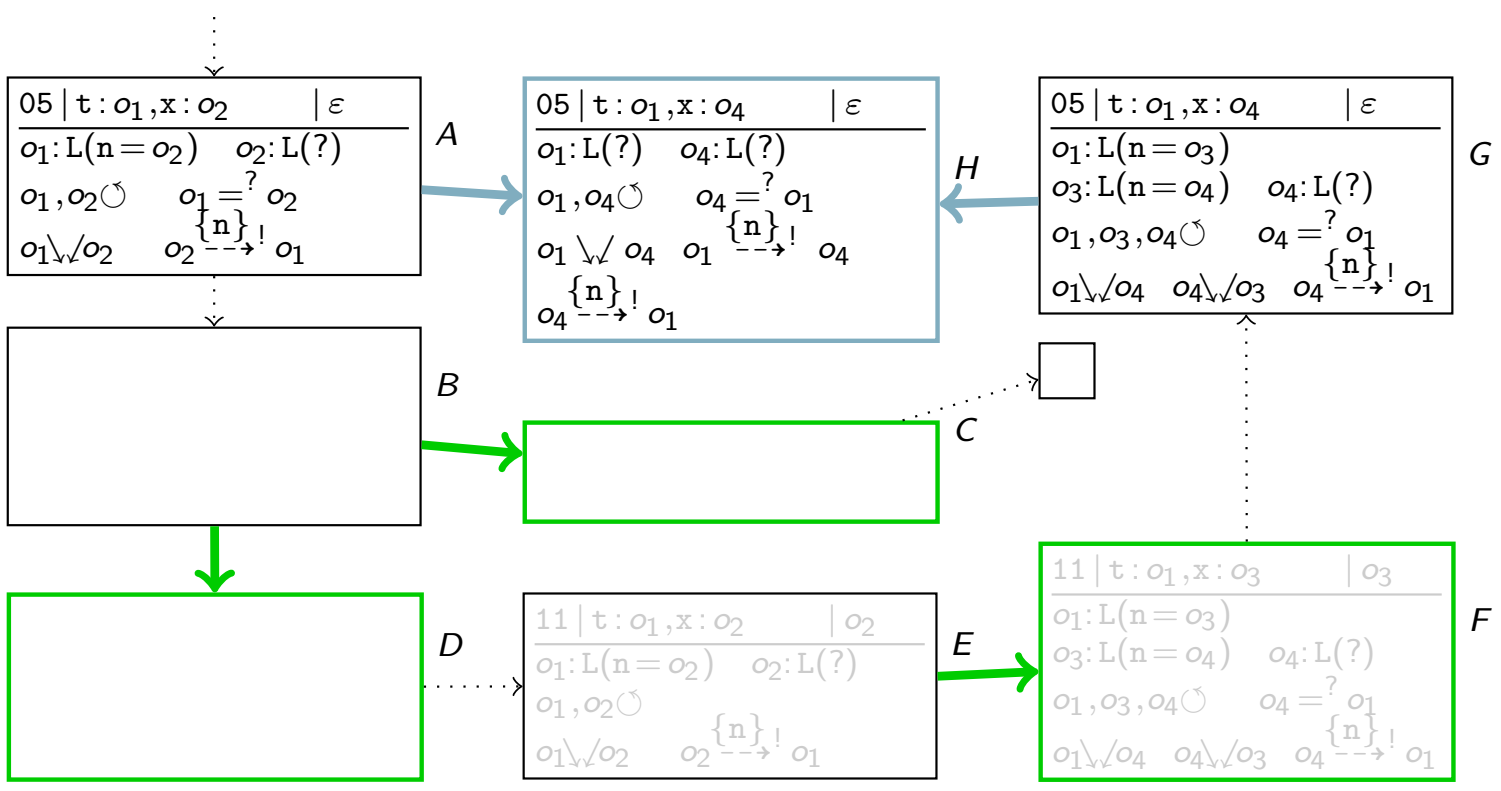
States G, H:

- Same program position as A $\Rightarrow$ Generalize

  In A: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_2 = \texttt{x}$

  In G: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_3 \xrightarrow{\texttt{n}} o_4 = \texttt{x}$

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



States G, H:

- Same program position as A $\Rightarrow$ Generalize

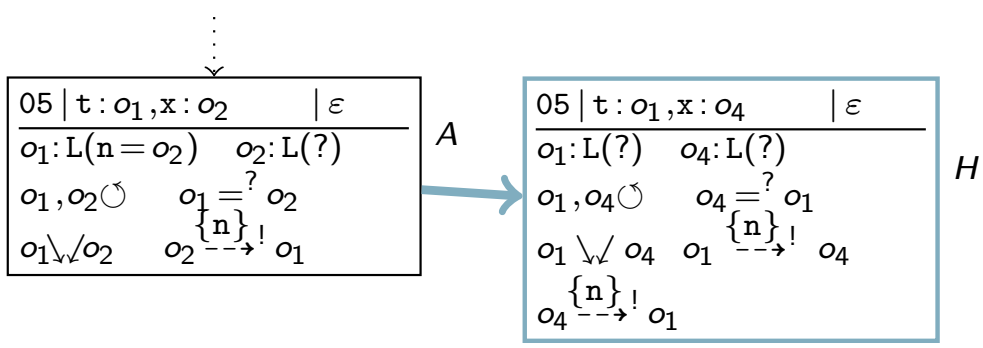
  In A: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_2 = \texttt{x}$

  In G: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_3 \xrightarrow{\texttt{n}} o_4 = \texttt{x}$

$\Rightarrow$ In H: Abstract to $\texttt{this} = o_1 \xrightarrow{\{\texttt{n}\}!} o_4 = \texttt{x}$

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



States G, H:

- Same program position as A $\Rightarrow$ Generalize

  In A: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_2 = \texttt{x}$

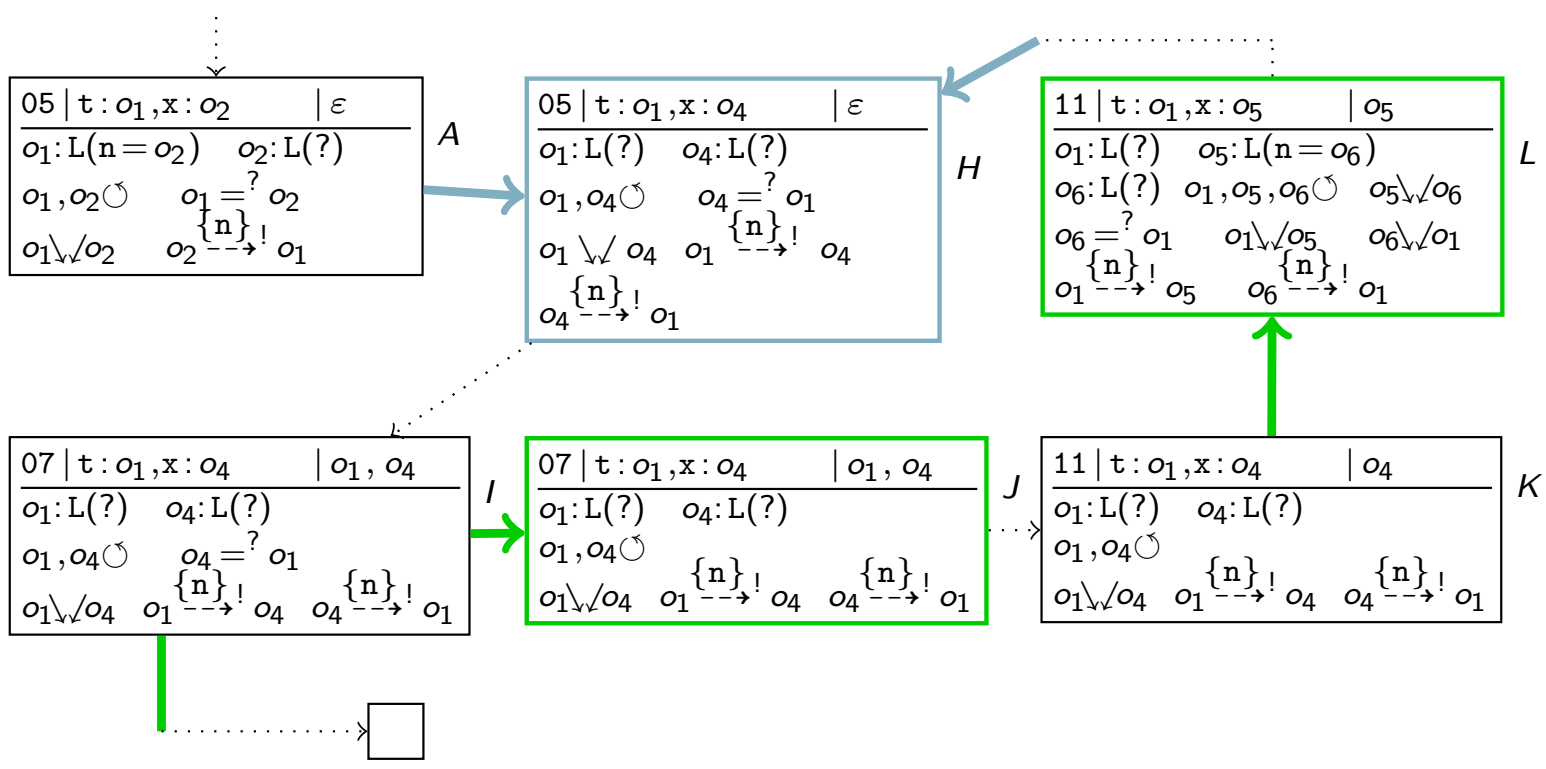
  In G: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_3 \xrightarrow{\texttt{n}} o_4 = \texttt{x}$

$\Rightarrow$ In H: Abstract to $\texttt{this} = o_1 \xrightarrow{\{\texttt{n}\}}{}^{!} o_4 = \texttt{x}$

- Restart construction from more general state

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
11: getfield n
14: astore_1
15: goto 05
18: return
```



States G, H:

- Same program position as A $\Rightarrow$ Generalize

  In A: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_2 = \texttt{x}$

  In G: $\texttt{this} = o_1 \xrightarrow{\texttt{n}} o_3 \xrightarrow{\texttt{n}} o_4 = \texttt{x}$

  $\Rightarrow$ In H: Abstract to $\texttt{this} = o_1 \xrightarrow{\{\texttt{n}\}!} o_4 = \texttt{x}$

- Restart construction from more general state

States I, J, K, L: As before

```
void iterate() {
  L x = this.n;
  while (x != this)
    x = x.n; }
```

```
00: aload_0
01: getfield n
04: astore_1
05: aload_1
06: aload_0
07: if_acmpeq 18
10: aload_1
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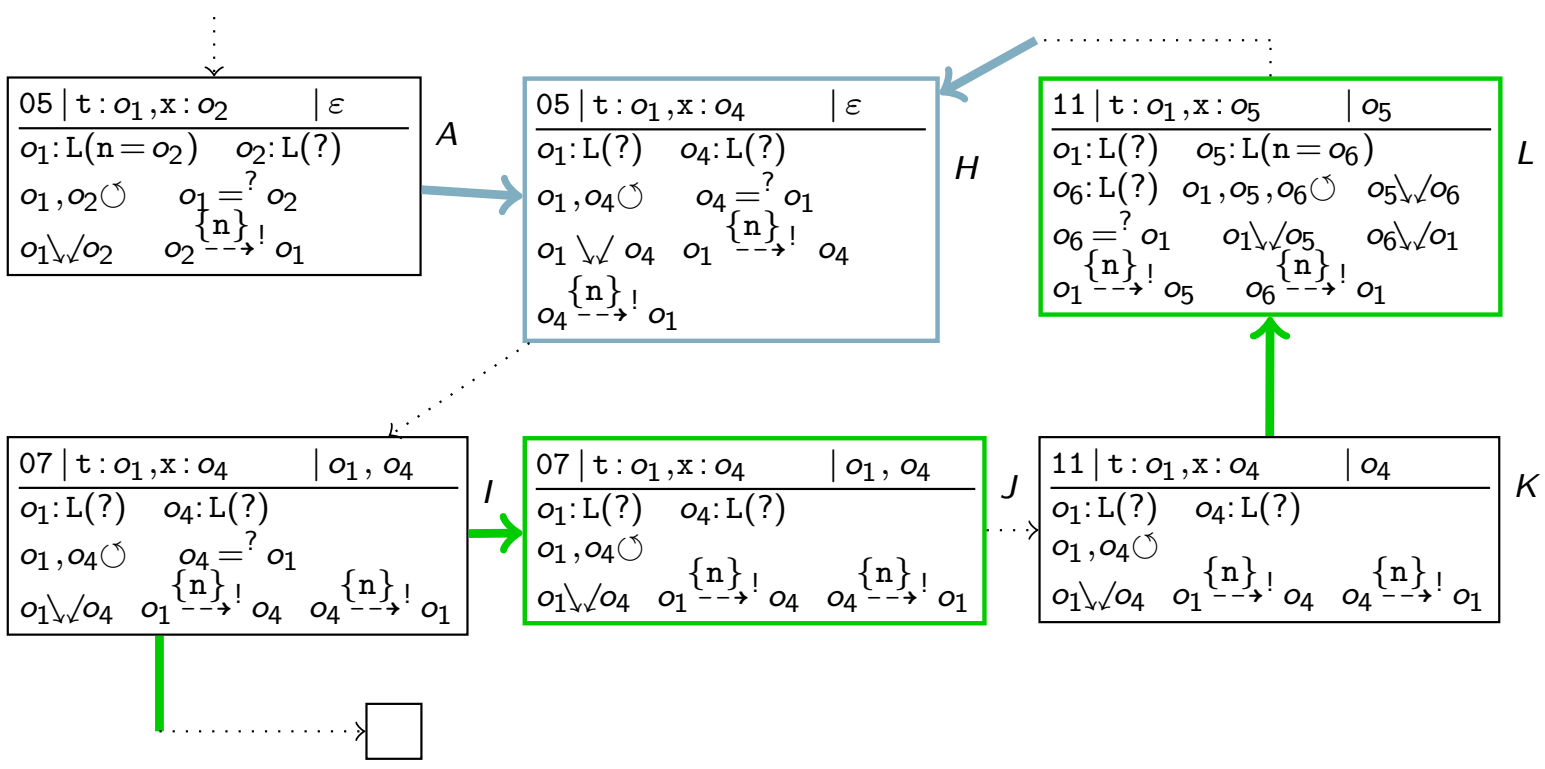

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07: if_acmpeq 18
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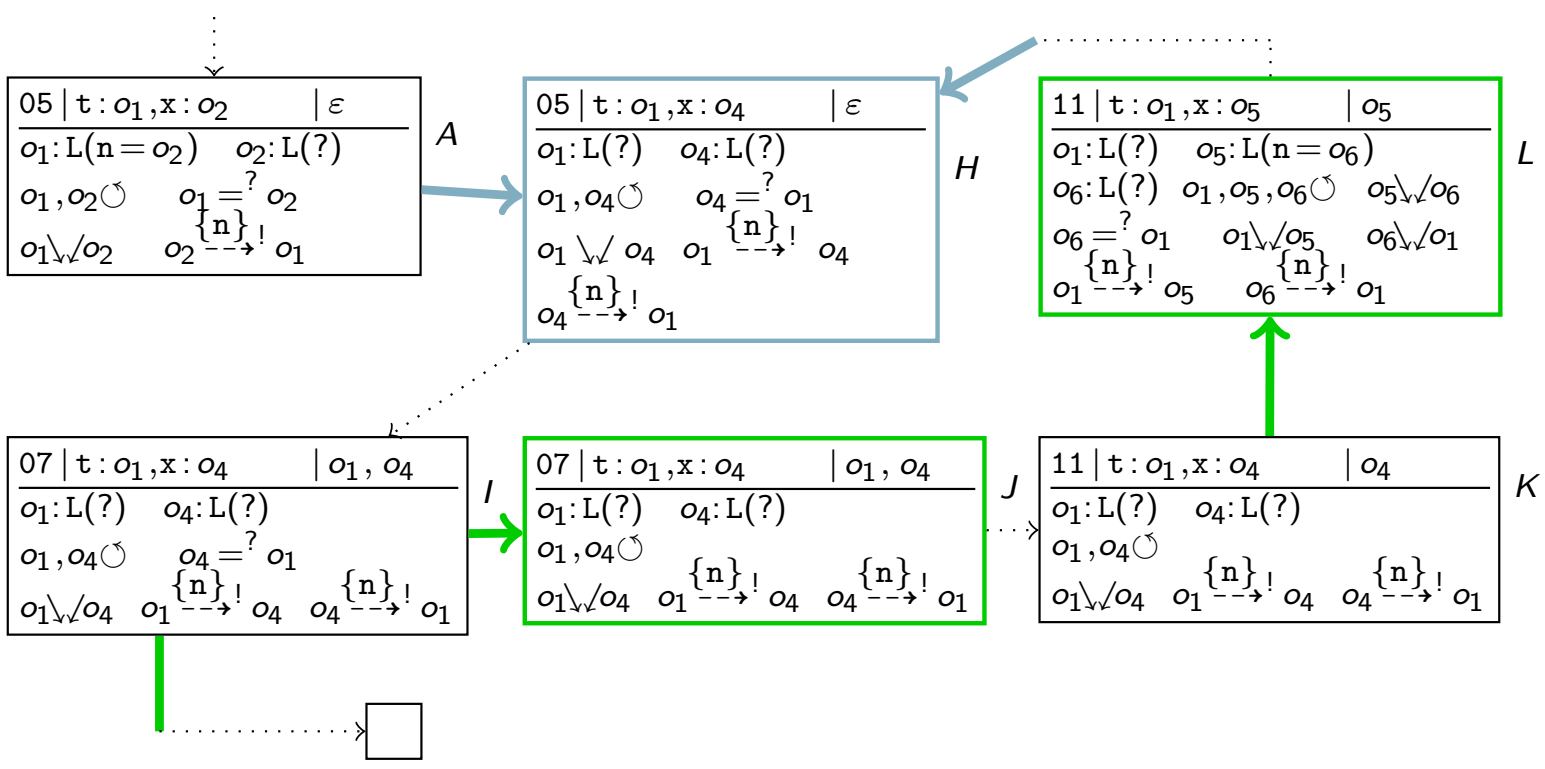

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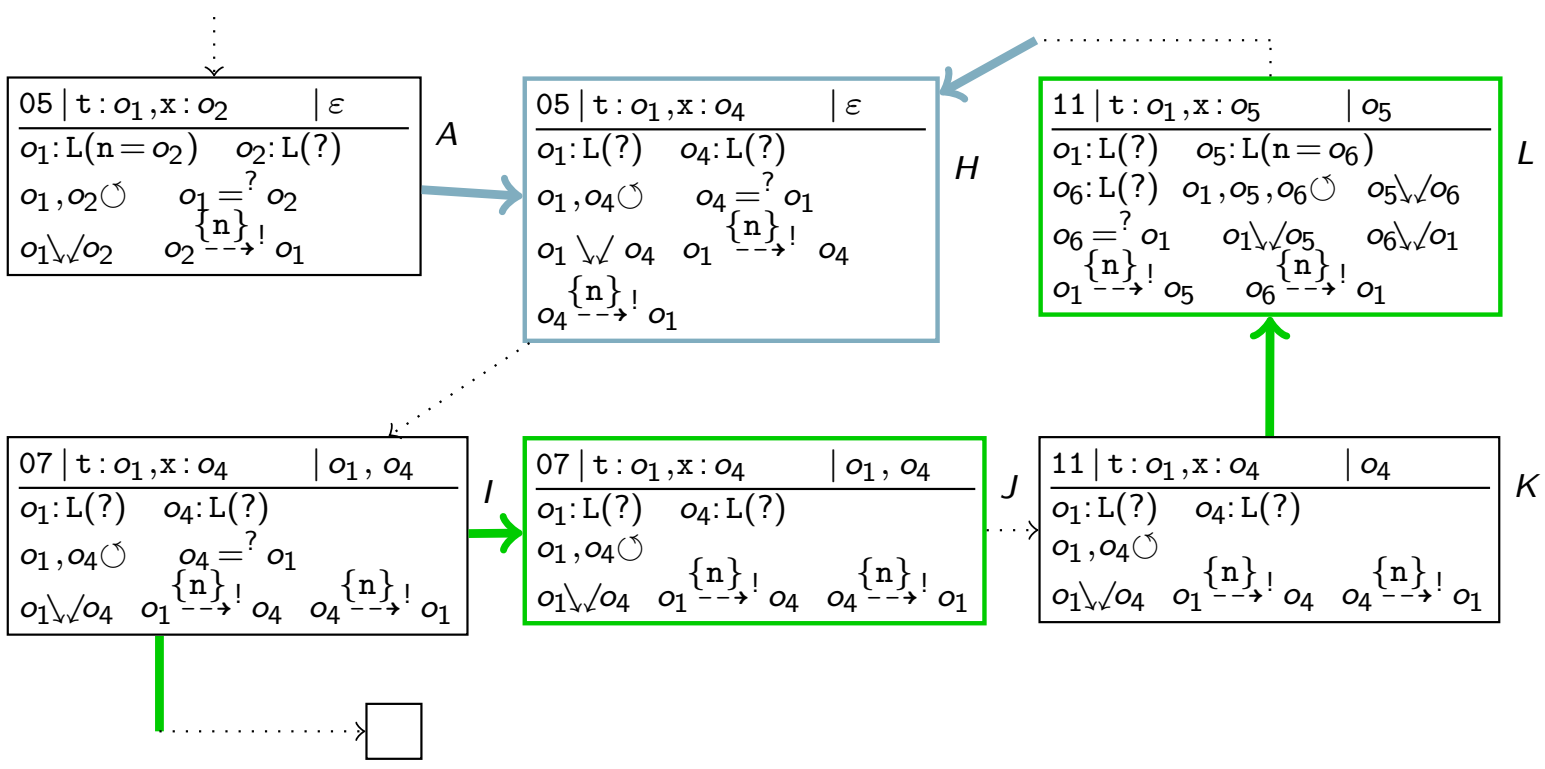

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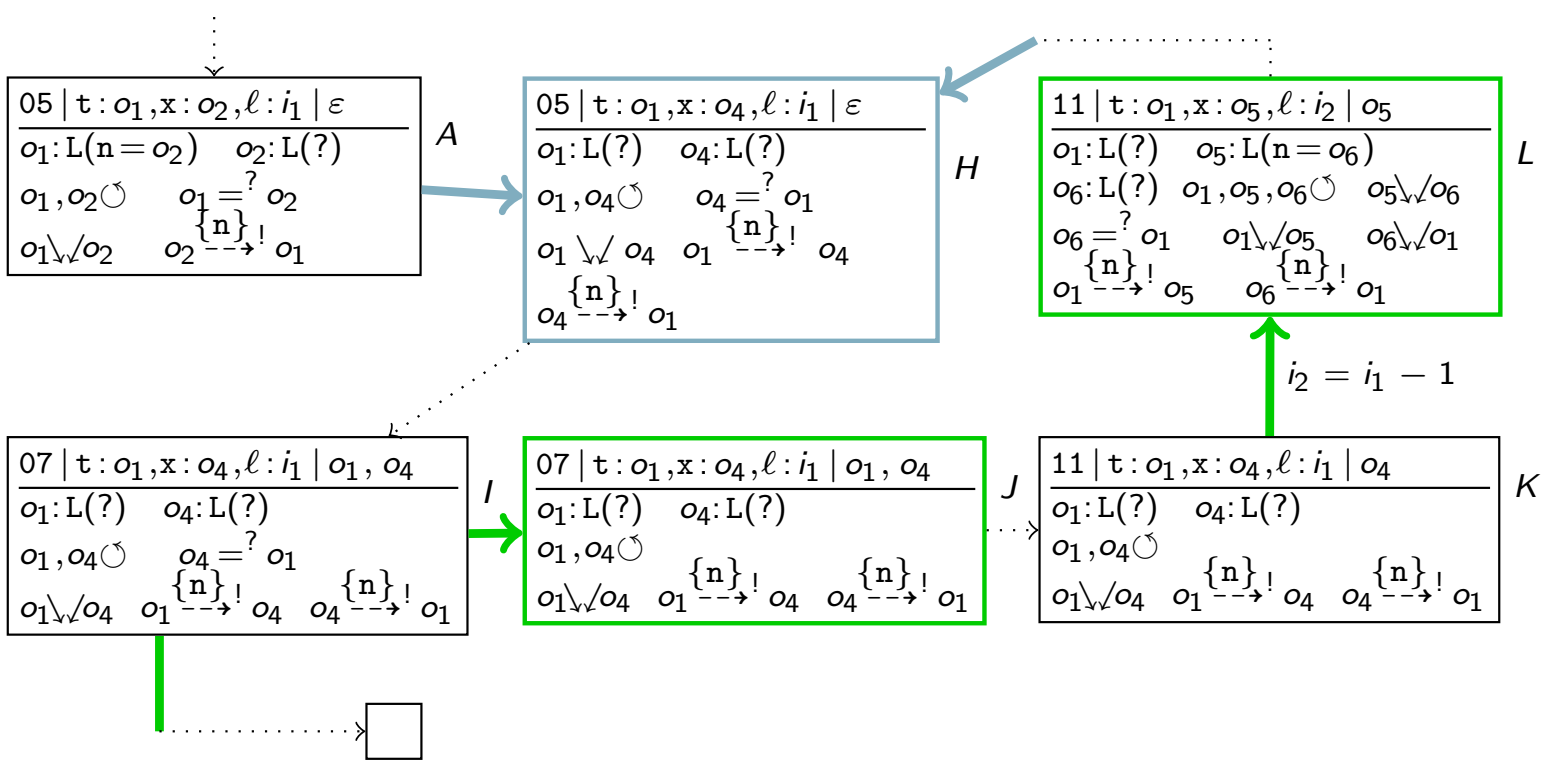

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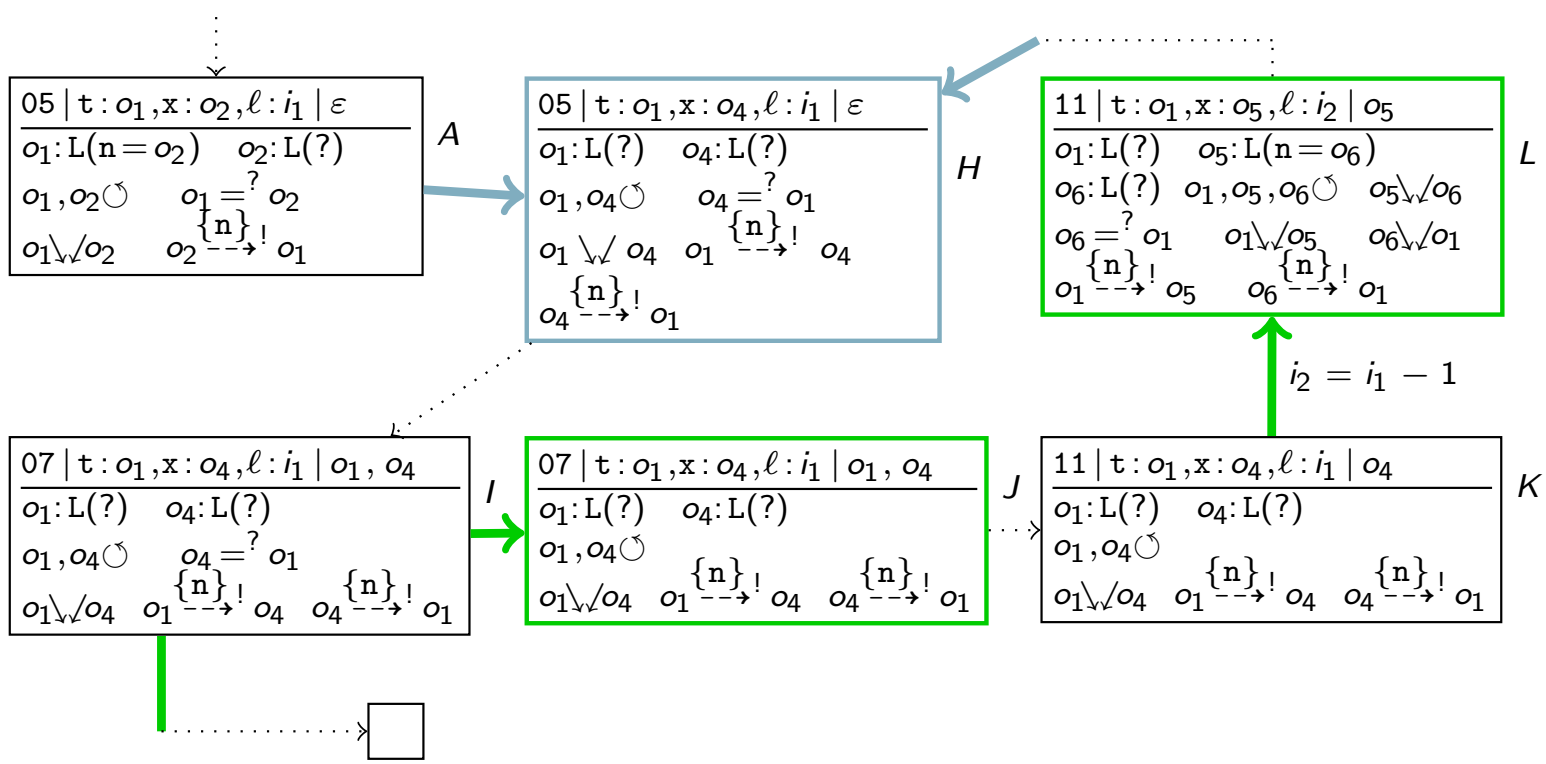

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Automatically created TRS:

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	Term	**NonT**	**Fail**	**t** (s)	**Term**	**NonT**	**Fail**	**t** (s)
AProVE '12	267	81	39	9.5	51	0	9	15.8
AProVE '11	225	81	81	11.4	23	0	37	18.3
Julia	191	22	174	4.7	32	0	28	8.2
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- http://aprove.informatik.rwth-aachen.de/eval/JBC-Cyclic