Policy Gradient Approaches for Multi-Objective Sequential Decision Making

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Outline

Motivations and Goals

GOALS:

- Analysis of advantages and limits of MOMDP techniques
- Explore **Policy Gradient** in MOMDPS

MOTIVATIONS:

- Policy Gradient techniques are widespread in RL
- Real–world applications are often multi–objectives
- Literature provides few MORL algorithms

Outline

Contributes

ALGORITHMIC: we propose two MORL policy gradient algorithms

- Radial Algorithm (RA)
- Pareto Following Algorithm (PFA)

EMPIRICAL: several test have been performed on different domains in order to evaluate proposed algorithms

- Linear–Quadratic–Gaussian regulator
- Deep Sea Treasure
- Water Reservoir

ANALYTIC:

- As far as we now, it is the first deep analysis of MORL Policy Gradient algorithms after Shelton (2001)
- and the first analysis on the use of **stochastic policies** in MORL

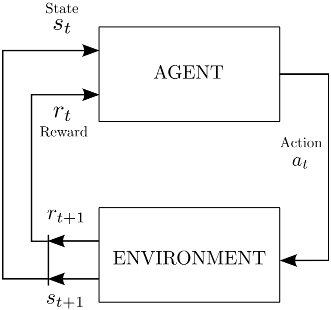
Outline

Markov Decision Process (MDP)

MDP:

$$\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma, D \rangle$$

$$J(\pi) = \mathbb{E}\left[\sum_{t=1}^{T} \gamma^{t-1} r_t | \pi, s_0 \sim D\right]$$
$$= \int_{s \in \mathcal{S}} d^{\pi}(s) \int_{a \in \mathcal{A}} \pi(a|s) \mathcal{R}(s,a) \mathrm{d}a\mathrm{d}s$$



MDP algorithms

- Dynamic Programming
- Linear Programming
- **Reinforcement Learning**

Algorithms for continuous MDPs

- **Policy Gradient**
- Genetic Algorithms
- Classification–based algorithms

MDP: Trajectory–based Policy Gradient

Parametric policy model: $\pi(a|s,\boldsymbol{\theta})$, e.g., **Gauss** or **Gibbs** policy models

Gradient Estimate:

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \mathbb{E}_{\tau \sim p(\cdot|\boldsymbol{\theta})} \left[\nabla_{\boldsymbol{\theta}} \log p(\tau|\boldsymbol{\theta}) R(\tau) \right]$$

$$R(\tau) = \sum_{t=1}^{T} \gamma^{t-1} r_t$$

Gradient Ascent:

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha_t \cdot \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$$

Advantages

- Continuous state and action space
- On–policy and "off-policy" learning
- Direct learning in the policy space

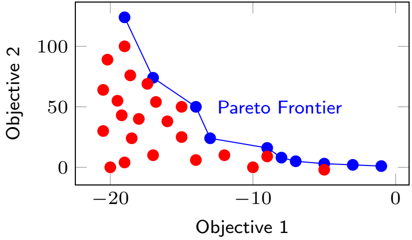
Drawbacks

- Local Optimum
- High–variance gradient estimate
- Tuning of the learning step α_t

Multi–Objective MDPs

VECTORIAL RETURN

$$\boldsymbol{J}(\pi) = [J_1(\pi), J_2(\pi), \dots, J_q(\pi)]^{\mathsf{T}}$$
$$= \mathbb{E}\left[\sum_{t=1}^{T} \gamma^{t-1} \mathbf{r}_t | \pi, s_0 \sim D\right]$$



SOLUTION CONCEPT: Pareto Dominance

$$\boldsymbol{J}(\pi) \succ \boldsymbol{J}(\bar{\pi}) \Leftrightarrow (\exists k \mid J_k(\pi) > J_k(\bar{\pi})) \wedge (\nexists h \mid J_h(\pi) < J_h(\bar{\pi})).$$

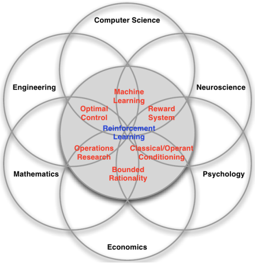
PARETO FRONTIER in policy space

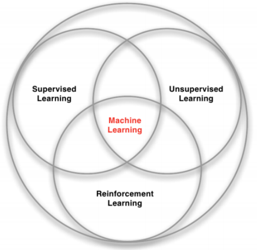
$$\Pi^* = \{\pi^* \in \Pi \;:\; \nexists \pi \in \Pi \mid \boldsymbol{J}(\pi) \succ \boldsymbol{J}(\pi^*)\}$$

Multi–Objective MDPs – 2

Solve MOMDPs:

- Reinforcement Learning
  - Single–policy vs Multiple–policies
  - Linear scalarization vs Non-linear scalarization
- Mathematical Optimization
  - Evolutionary Algorithms
  - Gradient Ascent





Multi–Objective Policy Gradient

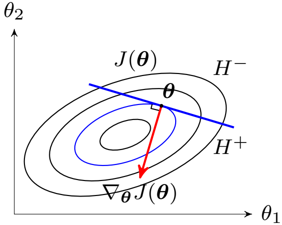
Concepts

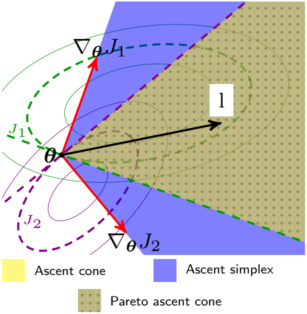
- Half Spaces
- Ascent Cone

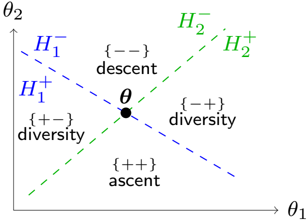
  $C(\boldsymbol{\theta}) = \{\boldsymbol{l} : \boldsymbol{l} \cdot \nabla_{\boldsymbol{\theta}} J_i(\boldsymbol{\theta}) \geq 0\}$
- Ascent Simplex

  $S(\boldsymbol{\lambda}, \boldsymbol{\theta}) = \sum_{i=1}^{q} \lambda_i \nabla_{\boldsymbol{\theta}} J_i(\boldsymbol{\theta})$
- **Pareto–Ascent Cone**

  $S(\boldsymbol{\lambda}, \boldsymbol{\theta}) \cap C(\boldsymbol{\theta})$





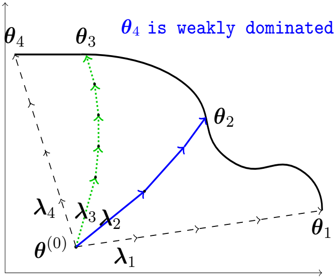


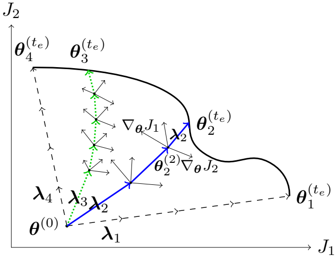
Null Pareto–Ascent Cone ⇒ (local) optimal solution

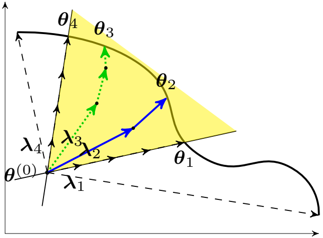
Radial Algorithm

Idea: p gradient ascent searches are performed, each one following a different, *uniformly spaced* direction in the **ascent simplex**

Problem: weak optimal solutions





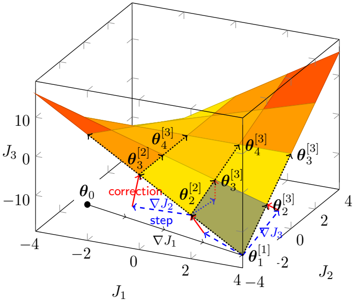


Pareto Following Algorithm

Phase 1: A solution on the Pareto frontier is reached by considering a single objective

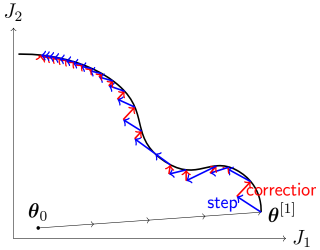
Phase 2: Exploration

- Improvement step: move the solution toward **one** objective at a time
- Correction step: improvement may lead the point outside the frontier. Correction **moves** the point again **on the frontier**



Problems:

- Can reach deterministic policies
- Need to **reintroduce stochasticity** (e.g., based on the entropy)
- Tuning of learning rate



Outline

Experiments

Multi–Objective learning difficulties

- More than two objectives
- **Continuous** state and action space
- **Stochastic** environments
- **Concave** Pareto frontiers

Domains

- Linear–Quadratic–Gaussian regulator
- `Deep Sea Treasure`
- `Water Reservoir`

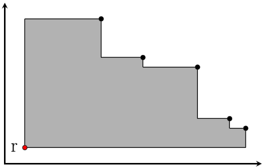
Evaluation Criteria

Loss – Castelletti et al. (2013)

$$l(\widehat{\mathcal{J}}^M, \mathcal{J}^*, W, p) = \int_{w \in W} \frac{J_w^* - \widehat{J}_w^{M,*}}{\Delta J_w^*} p(\mathrm{d}w)$$

$$\Delta J_w^* = \sum_{i=1}^{q} w_i \left(\max_{\bar{w} \in W} J_{\bar{w},i}^* - \min_{\bar{w} \in W} J_{\bar{w},i}^* \right)$$

Hyper volume – Zitzler et al. (2003)



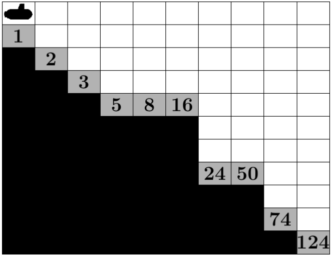
Comparison Algorithms

- Multi–Objective Fitted Q–iteration (MOFQI) (Castelletti et al., 2012)
- S–Metric Selection Evolutionary Multi–Objective Algorithm (SMS-EMOA) (Beume et al., 2007)

Deep Sea Treasure (Vamplew et al., 2008)

GOALS: `Vamplew et al. (2008)`

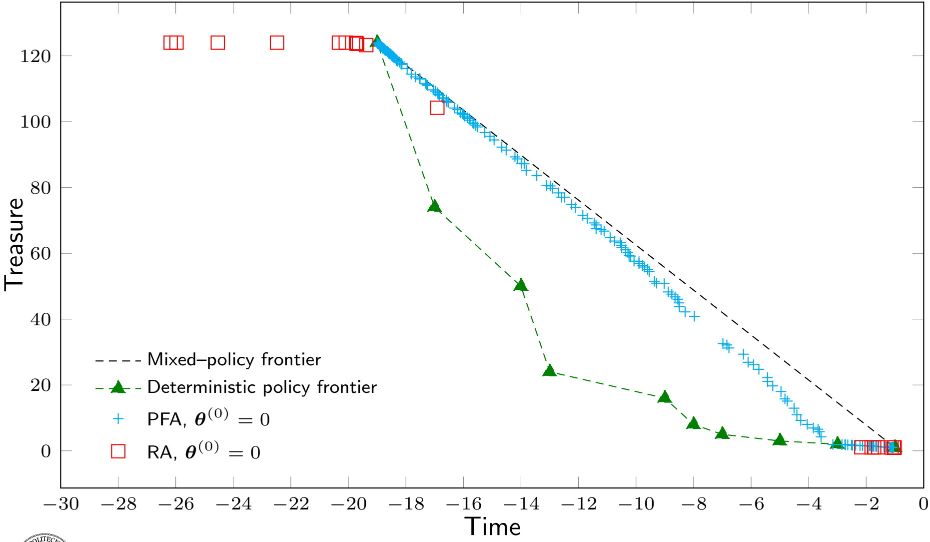
- Maximise treasure value
- Minimise time steps



FEATURES:

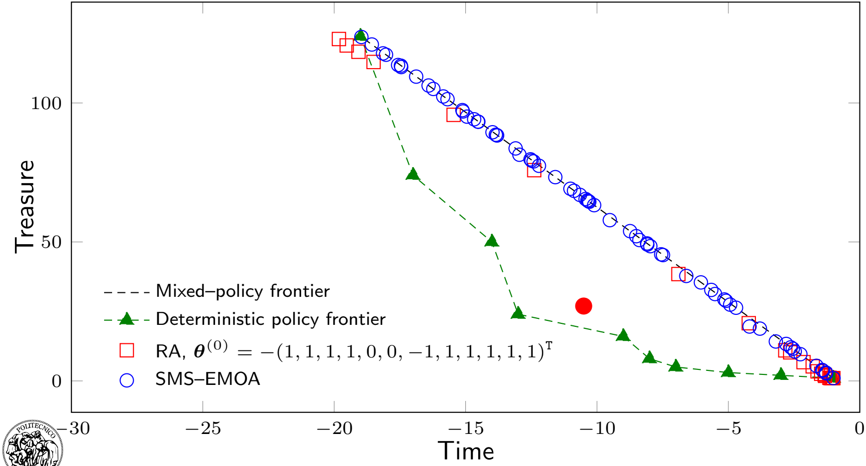
- Deterministic Pareto frontier is concave
- Optimal Frontier is obtained by **mixture–policy** that can be approximate by stochastic policies

Deep Sea Treasure – 2



Deep Sea Treasure – 3

Algorithm	Hyper–Volume	# Policies
PFA	0.4589	2,012
RA	0.3999	2,256
SMS–EMOA	0.4895	6,200



Water Reservoir (Castelletti et al., 2013)

MODEL: `Castelletti et al. (2013)`

$$s_{t+1} = s_t + \epsilon_{t+1} - \max(\underline{a}_t, \min(\bar{a}_t, u_t))$$

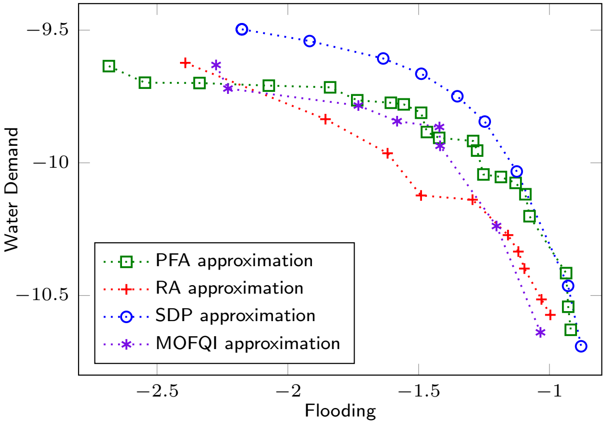
RESERVOIR INFLOW

$$e_{t+1} = \mathcal{N}(40, 100)$$

REWARD FUNCTIONS

$$\mathcal{R}_1(s_t, a_t, s_{t+1}) = -\max(h_{t+1} - \bar{h}, 0) \qquad \textit{flooding}$$
$$\mathcal{R}_2(s_t, a_t, s_{t+1}) = -\max(\bar{\rho} - \rho_t, 0) \qquad \textit{irrigation supply}$$
$$\mathcal{R}_3(s_t, a_t, s_{t+1}) = -\max(\bar{e}_t - e_{t+1}, 0) \qquad \textit{hydropower supply}$$

Water Reservoir – 2



Algorithm	Loss (2-obj.)	Loss (3-obj.)
Radial	0.0945	0.0123
Pareto following	0.0811	0.0162
MOFQI (Pianosi et al., 2013)	0.1870	0.0540
FQI (Ernst et al., 2005)	0.1910	0.0292

Outline

Conclusions

Advantages

Policy Gradient

- **Continuous** state and action space
- **Arbitrary** number of objectives
- **Stochastic** policies
- Works with **concave** Pareto frontiers

PFA and RA features

- **Scalable**
- **Parallel**

Drawbacks

RA:

- Lost of performances when there are weakly dominated solutions

PFA:

- Require randomization
- Highly sensible to learning parameters

Future Works

Algorithm related works

- **Radial Algorithm**
  - Different sample methods in order to avoid weakly dominated solutions
- **Pareto Path Following Algorithm**
  - Investigate policy randomization

Other branches

- Pareto Frontier Functional approximation `(Pirotta et al., 2014)`
- Off–Policy algorithms

Thank you for your attention

Slides and source code at:

http://home.dei.polimi.it/pirotta

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LQG

Single Objective

$$s_{t+1} = A\, s_t + B\, a_t, \qquad a_t \sim \mathcal{N}(K \cdot s_t, \Sigma)$$

$$r_t = -s_t^{\mathsf{T}} Q s_t - a_t^{\mathsf{T}} R a_t$$

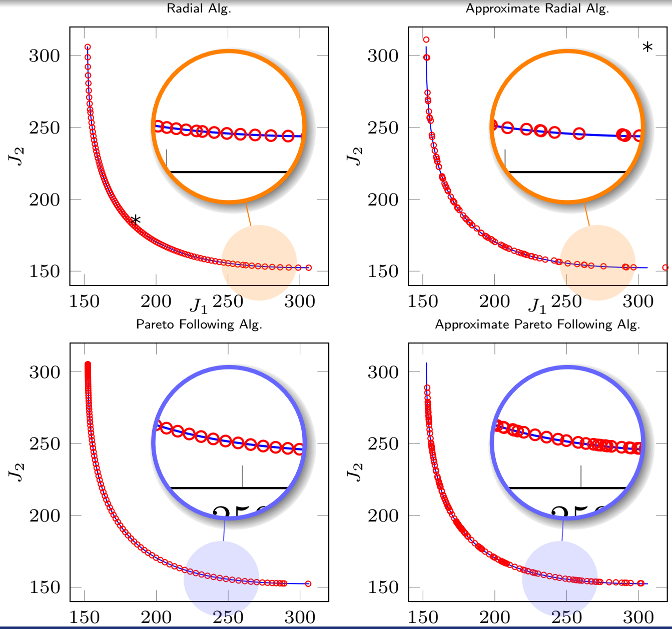
Multi Objective

$$\mathcal{R}_i(s,a) = -s_i^2 - \sum_{i \neq j} a_j^2$$

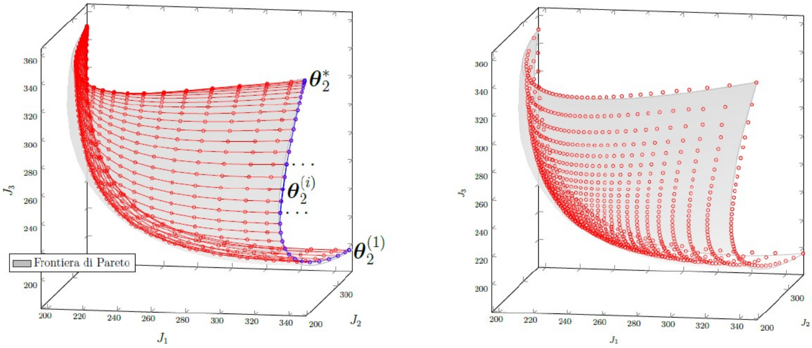
Why it is interesting?

- The Pareto frontier is known
- Closed–form for performance measure $\mathbf{J}$
- It is possible to evaluate algorithm behaviours in exact and approximate scenarios

LQG – 2


Radial Alg.
Approximate Radial Alg.
Pareto Following Alg.
Approximate Pareto Following Alg.
J_2
J_1
300
250
200
150
150
200
250
300

LQG – 3



Algorithm	α_s	α_c	ϵ_s	ϵ_c	Directions	Iterations	Time	Solutions	Loss
PFA	0.005	0.1	0.01	0.01	-	2,317	21s	943	$3.34e-04$
RA	0.5	-	0.01	-	989	3,840	17s	989	$9.45e-05$