An innovative pricing strategy for electronic products at a Dutch online retailer using discrete choice modelling and game theory

van Oosten, Mardi R.

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THESIS

RESEARCH REPORT

AN INNOVATIVE PRICING STRATEGY FOR ELECTRONIC PRODUCTS AT A DUTCH ONLINE RETAILER USING DISCRETE CHOICE MODELLING AND GAME THEORY

Mardi van Oosten
1006873

First supervisor: L.P.J. Schlicher
Second supervisor: Z. Atan

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Abstract

The e-commerce sector is under pressure due to the high inflation rates. Consumers spend less money in online retail than during the pandemic and costs are rising for the online retailers. In order to respond to these changes, and the competitive nature of the online market, companies are constantly seeking innovative pricing strategies to ensure more profit while keeping prices low enough to attract customers. Online retailer Circulus [1] is currently experimenting with new pricing strategies, such as up-pricing (i.e., charging a higher price). The company recently discovered that up-pricing might be an interesting new strategy as competitors sometimes seem to follow. This higher price could allow for more revenue for online retailer Circulus, under the assumption that the amount of sales remains (more or less) the same. The current experiments with up-pricing are based on trial and error and have not been validated by scientific research. Therefore, online retailer Circulus wants a better understanding of the conditions under which up-pricing is beneficial. In order to understand this, it is important to know how consumers select a product based on price and how competitors react to price changes. In literature, choice behavior is typically modeled with discrete choice modelling and competitor behavior is typically modeled with game theory. It was investigated how these approaches could be applied, in order to determine whether up-pricing is a beneficial strategy for online retailer Circulus. It was found that a discrete choice model could be composed to determine the relative importance of influencing factors for the behavior of a consumer when selecting an online retailer. Additionally, the discrete choice model was used as input for the online retailer game. This game modelled the situation where online retailers set a price based on the developed discrete choice model and considering the influencing factors. It was shown that there exists a Nash equilibrium for the game with a continuous and a discrete strategy set. For the online retailer game with a continuous strategy set, the uniqueness of the Nash equilibrium could not be proven or disproven due to time constraints. However, for the online retailer game with a discrete strategy set a unique Nash equilibrium could be identified when reducing the step size of the strategy set. The discrete online retailer game was used to determine the Nash equilibrium prices for several online retailers. Based on these Nash equilibrium prices, it could be determined that up-pricing is a beneficial strategy for online retailer Circulus as long as the current prices of the online retailers are lower than their Nash equilibrium prices and under the condition that each online retailer sets its Nash equilibrium price.

Keywords: discrete choice modelling, non-cooperative game theory, Nash equilibrium, pricing, revenue management

[1] Circulus is not the real name of the company where this study was performed, but will be used throughout this report to respect the data privacy of the company.

Management summary

Introduction

E-commerce companies are constantly exploring innovative pricing strategies to increase revenue. The current increase in inflation rates caused these innovative strategies to become even more important. Online retailer Circulus, one of the biggest online retailers of the Netherlands, faced a decrease in sales due to the high inflation rates. The company is calling for cost-cutting measures and therefore recently started reconsidering its pricing strategy regarding the electronics department. This department sells domestic appliances like airfryers and fridges, but also products like phones and laptops. The current pricing strategy of the electronics department is focused on matching the lowest price of competitors in the market. Although company Circulus currently applies this pricing strategy, the company recently discovered that increasing the price (up-pricing) might be an interesting new strategy as competitors sometimes seem to follow. This higher price would allow for more revenue for company Circulus if the amount of sales remains the same. The current experiments with up-pricing are based on trial and error and have not been validated by scientific research. Therefore, company Circulus wants a better understanding of the conditions under which up-pricing is beneficial. This could help company Circulus to deal with the high level of competition and the uncertain conditions of the market. In order to understand the conditions under which up-pricing is beneficial, company Circulus needs to know how consumers select a product based on price and how competitors react to price change. The following main research question was used to guide this research:

Main research question: *Under which conditions is up-pricing a beneficial strategy for online retailer Circulus?*

Research approach and results

To understand whether up-pricing is a beneficial strategy, it should be investigated how consumers select an online retailer and how competitors respond to price changes. Therefore, the first step of this research focused on identifying the factors that influence consumer behavior when selecting an online retailer. This was done through interviews and a literature review. Subsequently, the impact of these factors on the behavior of the consumer was estimated through the composition of a discrete choice model. Discrete choice models are a common approach in literature to analyze consumer behavior (Bierlaire, 1998). It was investigated in literature what type of discrete choice models exist and which input data is needed to estimate discrete choice models. Based on the findings it was decided to perform a stated preference survey to measure the effect of the influencing factors. For the stated preference survey, four influencing factors were selected: good experience, price, shipping cost and user-friendly website. Furthermore, the survey was focused on one electronic product, an electronic toothbrush, to ensure the feasibility of the survey for the respondent.

Using the survey data, three different discrete choice models were composed that each considered a different distribution of the terms in the utility functions. The utility function describes the value that a consumer assigns to choosing a specific online retailer. The base model assumed that all terms were linearly distributed for the utility function. The quadratic model extended the base model, through adding two quadratic parameters for the four-level attributes. Lastly, the logarithmic model assumed linear distributions for the binary attributes and a logarithmic distribution for the four-level attributes. Using statistical tests, it was found that the base model, with a linear distribution for all attributes in the utility function, fitted the survey data the best. The models were estimated with the PandasBiogeme package from Bierlaire (1998).

The selected discrete choice model was used to estimate the parameter values for the attributes good experience, price, shipping cost and user-friendly website. It was found that good experience and user-friendly website play the biggest role in the decision of the respondent and have a positive effect. In contrast, the attributes price and shipping cost had a smaller, negative effect on the utility function.

Subsequently, in order to study competitor behavior a game theoretical approach was applied. A non-cooperative game was composed, using the output of the discrete choice model as input for the payoff functions of the online retailers in the game. Each online retailer had the goal to maximize their revenue (payoff) in the game, where the revenue is composed by the probability to be selected by the consumer multiplied by the difference between the price and the cost of the online retailer. For every online retailer $i \in N$, the payoff function $f_i : P \to \mathbb{R}_+$ is defined by the following formula:

$$f_i(p) = \frac{e^{\beta_0^i - \beta_p \cdot p_i}}{\sum_{j \in N} e^{\beta_0^j - \beta_p \cdot p_j} + 1} \cdot (p_i - c_i), \ \forall p \in P \tag{1}$$

In this formula β_0^i represents the initial preference of the consumer for online retailer i, β_p represents the price parameter, p_i represents the price set by online retailer i and c_i represents the cost that online retailer i is facing. The initial preference parameter β_0^i was composed for each online retailer i with the influencing factors from the stated preference survey regarding good experience, shipping cost and user-friendly website.

A solution concept that is often used to analyze a non-cooperative game is the Nash equilibrium. A Nash equilibrium is a vector of strategies that represents the state where no player has an incentive to unilaterally deviate from their selected strategy, assuming that the other players adhere to their chosen strategy (Osborne et al., 2004). A game may contain a unique Nash equilibrium, multiple Nash equilibria or no Nash equilibrium (Basar et al., 2010). The existence of a Nash equilibrium was investigated for the online retailer game with a continuous strategy set $P_i \in [0, \overline{p_i}]$ with $\overline{p_i} \in \mathbb{R}_+$ and with a discrete strategy set $P_i \in \{0, 1, ..., 40\}, \forall i \in N$.

For the online retailer game with a continuous strategy set, the existence of a Nash equilibrium could be proven using the theorem of Arrow & Debreu (1954). All three conditions of the theorem were met for the online retailer game with a continuous strategy set, such that it could be concluded that at least one pure strategy Nash equilibrium existed. Proving or disproving the uniqueness of the Nash equilibrium could not be accomplished for the continuous game in this research due to time constraints.

For the online retailer game with a discrete strategy set, a python code was composed in order to identify all Nash equilibria. Additionally, in order to evaluate the uniqueness of the Nash equilibrium of the discrete game, a test framework was applied, where the parameter values regarding initial preference, price and cost were varied. It was found that the games with 3 or 4 players were not robust to these parameter changes and instances occurred with two Nash equilibria. To determine which Nash equilibrium to select, the payoff for each player was evaluated. It was found that a Nash equilibrium existed that resulted in a higher payoff for each player than the other Nash equilibrium. The Nash equilibrium with the higher payoff was defined as dominant and it was expected that the online retailers would select this Nash equilibrium. Additionally, the discrete game was composed with an altered discrete strategy set using a smaller step size, such that the continuous game could be approached. This resulted in an unique Nash equilibrium for the discrete game.

With the identified unique Nash equilibrium the effects of up-pricing for online retailer Circulus could be determined. It should be noted that this effect could only be determined for the electric toothbrush under consideration. Currently, competitor A and online retailer Circulus have a price of 20 euros for this product, competitor B has a price of 26 euros and competitor C has a price of 23 euros. According to the unique Nash equilibrium of the discrete online retailer game competitor A should set a price of 20 euros, online retailer Circulus a price of 24.25 euros, competitor B a price of 25.5 euros and competitor C a price of 24.5 euros. This would result in an increased payoff for each player. Therefore, according to the findings of the game, it is expected that when online retailer Circulus applies up-pricing and increases its price to 24.25 euros, competitor C will follow this action and set a higher price. competitor B will decrease its price from 26 to 25.5 euros, but maintain a higher price than online retailer Circulus. In contrast, competitor A is expected not to follow and will maintain a price of 20 euros. It is expected that up-pricing is a beneficial strategy for online retailer Circulus in the modelled situation when all competitors set their Nash equilibrium price. However, the benefits of up-pricing were found to be strongly dependent on the parameter values. It was observed that as the cost parameter increased the Nash equilibrium prices increased for the online retailers to compensate, such that up-pricing may no longer be beneficial. Additionally, the Nash equilibrium prices decreased as the price parameter increased, which reduced the possibilities for up-pricing. Furthermore, an increase in the initial preference parameter caused an increase in the Nash equilibrium prices, allowing for increased revenue when applying up-pricing.

It was shown that up-pricing is only a beneficial strategy when the current prices of the online retailers are lower than the prices of the identified Nash equilibrium. Additionally, it was found that up-pricing is beneficial until a certain price level, which constrains the online retailers from setting a too high unattractive price. Another identified risk to the beneficial effects of up-pricing, considered the possibility of competitors deviating from their Nash equilibrium price to remain competitive.

Lastly, a unique Nash equilibrium was found for the game where the online retailers varied in their goal. In this altered game three players had the goal to maximize revenue, while the fourth player had the goal to set its price equal to the lowest price in the market. Up-pricing could then still be applied through setting a price of 23.75 euros for online retailer Circulus. However, competitors with a different goal, such as setting the lowest price in the market, will not follow.

Conclusion

It depends on the parameter values whether up-pricing causes an increase in revenue, since it was observed that parameter changes cause alterations in the prices of the Nash equilibrium. For the studied situation with online retailer Circulus the estimated parameter values caused the Nash equilibrium prices to be higher than the current prices of the online retailers, such that up-pricing could be a beneficial strategy. On the other hand, if the parameters for cost, initial preference and price would have been varied such that the Nash equilibrium prices were lower than the current prices of the competitors, down-pricing would have been the most beneficial strategy. To summarize, the success of up-pricing depends on the parameter values of the considered online retailer game. It is concluded that up-pricing is a beneficial strategy under the condition that the parameter values result in a Nash equilibrium with higher prices than the current prices of the online retailers and under the condition that each competitor selects its Nash equilibrium price.

Scientific contribution

This research provided insight into the factors that influence consumer behavior in the selection of an online retailer. Additionally, a non-cooperative game was composed to gain insight in competitor behavior. It was proven that there exists a Nash equilibrium for the online retailer game with a continuous and a discrete strategy set. Online retailer Circulus may use these insights to optimize their up-pricing strategy for the studied product. Unfortunately, the results cannot be generalized to the other product groups of online retailer Circulus. However, the proposed framework can be used to generate results for other product groups.

Preface

With this thesis my Master's degree in Operations Management and Logistics at Eindhoven University of Technology will be completed. Firstly, I would like to thank Loe Schlicher, my first assessor, for the guidance during this process. The provided feedback has surely helped me to improve the quality of this thesis project. Secondly, I would like to thank the second assessor Zumbul Atan for the constructive feedback. Thirdly, I would like to thank Annejet van der Vegte, my supervisor at the company. She has been a great support and helped me to navigate through the challenges. She learned me a lot about the company and the possibilities within the field of pricing. Lastly, I would like to thank my family and friends for their support during this thesis project.

Mardi van Oosten

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Symbol	**Definition**	**Example**
$\{\}$	Set of elements	$N = \{1, 2, 3, 4\}$
$i \in N$	i is an element of N	$2 \in N$
$N \setminus \{i\}$	Set N without i	$N = \setminus\{4\} = \{1, 2, 3\}$
$\mathbb{R}$	Set of real numbers	$\mathbb{R} = (-\infty, \infty)$
$\mathbb{R}_+$	Set of all non-negative real numbers	$\mathbb{R}_+ = (0, \infty)$
$\mathbb{N}_0$	Set of all natural numbers, including zero	$\mathbb{N}_0 = \{0, 1, 2, \dots\}$

Table 1: Symbol definition

1 Introduction

E-commerce companies provide services and products to consumers via the internet (Ho et al., 2007). Examples of such products and services are electronics, fashion, books, online education or online entertainment. The revenues generated in the e-commerce market are substantial; Osman (2021) shows that the revenue of e-commerce sales in the Netherlands was 4.5 billion US dollars in 2021. Currently, inflation rates are rising in the Netherlands due to the war in Ukraine (Statistics Netherlands (CBS), 2022). In the year 2022, consumer goods and services had a price increase of 10 percent compared to the prices in 2021. At the same time, wages have not grown as rapidly, leaving consumers with less purchasing power (Statistics Netherlands (CBS), 2023). Additionally, costs are increasing for the e-commerce companies due to the high inflation rates.

An example of an e-commerce company, that has to deal with the consequences of the increasing inflation rates, is online retailer Circulus [2]. This e-commerce company is located in the Netherlands and Belgium and offers a broad assortment to its customers, counting 41 million products. Examples of product groups that online retailer Circulus offers are furniture, books, electronics, fashion and toys. The company has a customer base of 13 million people across the Netherlands and Belgium. The revenue of online retailer Circulus was 3.3 billion dollars in the year 2020. This rapidly grew to a revenue of 4.1 billion in 2021. However, in 2022, online retailer Circulus faced some challenges. The high inflation rates caused a decrease in the amount of purchases, calling for cost-cutting measures.

1.1 Problem definition

As a consequence of the need for cost-cutting measures, online retailer Circulus recently started reconsidering its pricing strategy regarding the electronics department. This department sells domestic appliances like airfryers and fridges, but also products like phones and laptops. The current pricing strategy of the electronics department is focused on matching the lowest price of competitors in the market. Although online retailer Circulus currently applies this pricing strategy, the company recently discovered that increasing the price (up-pricing) might be an interesting new strategy as competitors sometimes seem to follow. This higher price would allow for more revenue for online retailer Circulus if the amount of sales remains the same, which is a favorable outcome considering the increasing costs that online retailer Circulus is facing due to the current inflation rates. The current experiments with up-pricing are based on trial and error and have not been validated by scientific research. Therefore, online retailer Circulus wants a better understanding of the conditions under which up-pricing is beneficial. This could help online retailer Circulus to deal with the high level of competition and the uncertain conditions of the market.

In order to understand the conditions under which up-pricing is beneficial, online retailer Circulus needs to know how consumers select a product based on price and how competitors react to price change. The problem that online retailer Circulus is facing with this, is formulated as:

> *It is unclear under which conditions up-pricing is a beneficial strategy for online retailer Circulus considering consumer behavior and competitor behavior.*

1.2 Research questions

In order to understand the effects of up-pricing, insight should be gained in two aspects. Firstly, it should be investigated how consumers select a product. It should be investigated whether this choice solely depends on price or whether other factors, like popularity of the online retailer, might play a role. Additionally, it should be investigated how competitors react to price change. Online retailer Circulus wants to be able to predict whether competitors will follow when they apply up-pricing or whether competitors will keep a lower price to attract customers for instance. When more insight is gained regarding these two aspects, the effect of up-pricing on revenue can be determined for online retailer Circulus. Based on whether this effect is positive or negative, it can

[2] Circulus is not the real name of the company where this study was performed, but will be used throughout this report to respect the data privacy of the company.

be determined whether up-pricing is a beneficial strategy. Therefore, the following main research question is used to guide this research:

Main research question: *Under which conditions is up-pricing a beneficial strategy for online retailer Circulus?*

To answer this question, the first step should be to investigate how consumers select a certain product. It should be investigated which factors influence this choice. This leads to the first sub research question:

Sub research question 1: *Which factors may influence the choice of a consumer to select an online retailer?*

Subsequently, it should be investigated what methods exist to predict how consumers make a choice based on the factors for sub research question 1. An approach that is often used in literature is discrete choice modelling (Bierlaire, 1998). Discrete choice models have been of great use in the e-commerce sector, for instance, to determine the probability that a consumer will select a certain online retailer (Danaf et al., 2019). Typically, there are several types of discrete choice models that may be applicable in this research. Which discrete choice models exist in literature will be explored in sub research question 2:

Sub research question 2: *Which discrete choice models exist in literature to model consumer behavior?*

The next step will be to select suitable discrete choice models. For each of these models, it should be investigated what type of data these models require as input to allow for an estimation of the models. This leads to sub research questions 3.a:

Sub research question 3.a: *Which input data can be used to estimate the discrete choice models?*

Furthermore, it will be determined how well the models fit the acquired data through the use of statistical tests. This leads to sub research questions 3.b:

Sub research question 3.b: *Which discrete choice model represents consumer behavior the best?*

After selecting the best fitting discrete choice model, the next step will be to investigate competitor behavior. In order to study competitor behavior, a game theoretic approach will be applied. Game theory is a mathematical discipline that has often been applied to economical contexts (Peters, 2015). This discipline focuses on decision making when various players are involved. A game will be developed to simulate the situation at online retailer Circulus, where the players of the game concern online retailer Circulus and its competitors. This game will be referred to as the online retailer game. A concept that is often used to analyze the game and to predict the outcome is the Nash equilibrium. A Nash equilibrium is a vector of strategies that represents the state where no player has an incentive to unilaterally deviate from their selected strategy, assuming that the other players adhere to their chosen strategy (Osborne et al., 2004). In this research it will firstly be investigated whether a Nash equilibrium exists for the composed online retailer game. This leads to the following sub research question:

Sub research question 4.a: *Does a Nash equilibrium exist for the online retailer game?*

A game may contain multiple Nash equilibria. An important step in this research is therefore to check for the existence of multiple Nash equilibria.

Sub research question 4.b: *If there always exists a Nash equilibrium in the online retailer game, is it unique?*

When multiple Nash equilibria exist, it should be determined which equilibrium should be used for the situation at online retailer Circulus. Approaches on how to determine this will be described in the Research method section.

Sub research question 4.c: *If multiple Nash equilibria exist in the online retailer game, which Nash equilibrium should be selected to determine the optimal pricing strategy for online retailer Circulus in the game?*

Based on the results from sub research questions 4.a, b, c and d it will be determined what the effects of up-pricing will be for online retailer Circulus. This will be done through comparing the current market prices with the prices for the (selected) Nash equilibrium. The difference between these prices shall be evaluated for each player in the online retailer game. Subsequently, it can be decided what the effect of up-pricing will be for each player. In case there is no Nash equilibrium, the underlying cause of this outcome shall be investigated to determine the effects of up-pricing. This leads to the last sub research question:

Sub research question 5: *What will be the effects of up-pricing for online retailer Circulus considering the innovative pricing strategy from this research?*

1.3 Deliverables

The expected output of this research will be an extensive report describing the effects of an innovative pricing strategy for online retailer Circulus, namely up-pricing. A mathematical model will be developed to determine the effects of up-pricing for online retailer Circulus. This mathematical model will be an integration of a discrete choice model and a game theoretic approach. The results will be valuable for online retailer Circulus to improve their strategy for up-pricing. Additionally, the results will contribute to academic insights in the field of pricing.

2 Research method

In this section it will be determined for each sub research question what research methods should be applied in order to answer the sub research question.

Sub research question 1: *Which factors may influence the choice of a consumer to select an online retailer?*

This question will be addressed through an interview with a survey data specialist at online retailer Circulus. The survey data specialist works for the department within online retailer Circulus that focuses on gaining insight in consumer behavior. These insights are gained through conducting surveys with the consumers of online retailer Circulus. The specialist will be interviewed to get an overview of the influencing factors that he/ she feels play an important role in how consumers choose to buy from a specific online retailer. The interview will be of an unstructured nature to allow for a free flow of the conversation (Zhang & Wildemuth, 2009). Additionally, it will be investigated in literature what factors may play a role in consumer decision making. The aim of this literature review is to identify factors that may not have come forward during the interviews, allowing for a complete set of influencing factors. The literature review will be performed according to the rapid review method. This method involves extracting data from relevant sources through the use of search strings. The rapid review method is less rigorous than a systematic literature review, but an advantage is that it provides insight in a short amount of time (Tricco et al., 2015). The databases that will be used are Google Scholar, Scopus and Web of Science as these databases are recognized for their reliability and scientific validity. The findings from the interviews and the literature review will be integrated in order to compose an answer to the first sub research question.

Sub research question 2: *Which discrete choice models exist in literature to model consumer behavior?*

A literature study concerning discrete choice modelling will be performed. The different types of discrete choice models and their application in an e-commerce context will be investigated in literature. This literature review will use a snowballing approach, also called pearl growing. With this research method the reference list of an article or citations to the article are used to identify new papers that may contribute to the literature review (Wohlin, 2014). When the reference list is used this is called backward snowballing, when the citations are used this is called forward snowballing. The databases that will be used are Google Scholar, Scopus and Web of Science.

Sub research question 3.a: *Which input data can be used to estimate the discrete choice model?*

In the discrete choice models the influencing factors from sub research question 1, such as price and popularity, will be represented by parameters. These parameters need to be estimated. One popular approach to estimate these parameters of the discrete choice models, is through the use of a survey (Muller & MacLehose, 2014). It will be investigated in literature how to compose a suitable survey to estimate the parameters of this study. Based on these findings a survey will be composed and conducted. The survey data can then be used to estimate the parameters of the discrete choice models.

Sub research question 3.b: *Which discrete choice model represents consumer behavior the best?*

In this step it will be tested which discrete choice model is the most appropriate in this situation. Using the insights from the literature review on discrete choice modelling it will be determined what form of discrete choice model best suits the survey data and the research question. This may result in multiple suitable models, such as the multinomial logit and mixed logit. The different models will be estimated using the survey data from sub research question 3.a and a Python script. The goodness-of-fit of the different models will be tested using statistical tests such as R-squared, Chi-square and likelihood ratio statistic (Blizzard & Hosmer, 2006).

Additionally, multiple variants of the multinomial logit model will be tested, which differ in their utility distributions. The utility function describes the value that a consumer assigns to choosing an online retailer. The utility functions may be linear, but may also contain quadratic or logarithmic components. Different forms of the utility functions will be tested and the corresponding multinomial logit models will be assessed through statistical tests.

The best fitting model will be selected and will be used to calculate the probability that a single customer will select an online retailer when setting a certain price. This will be used as input for the composition of the game.

Sub research question 4.a: *Does a Nash equilibrium exist for the online retailer game?*

Firstly, the online retailer game will be composed. The players of the game are online retailer Circulus and its benchmark A competitors. The payoff function for each player will be composed based on the outcome of the discrete choice model. To prove the existence of the Nash equilibrium, the payoff function shall be evaluated using theories in literature regarding the existence of a Nash equilibrium (Arrow & Debreu, 1954). When it is not possible to prove this, the game shall be solved through numerical experiments.

Sub research question 4.b: *If there always exists a Nash equilibrium in the online retailer game, is it unique?*

To determine whether the Nash equilibrium is unique, the output from the method of sub research question 4.a will be investigated. The initial approach will be to prove the uniqueness of the Nash equilibrium using theories from literature. Such theories include the evaluation of the best response function, for instance (Fujiwara-Greve, 2015). When the uniqueness of the Nash equilibrium cannot be proven, numerical experiments shall be applied.

Sub research question 4.c: *If multiple Nash equilibria exist in the online retailer game, which Nash equilibrium should be selected to determine the optimal pricing strategy for online retailer Circulus in the game?*

In case there turn out to be multiple Nash equilibria, several concepts could be applied to determine which equilibrium should be selected for the situation at online retailer Circulus. An example of such a concept is the dominant strategy equilibrium. If there are two Nash equilibria, one of the Nash equilibria is defined to be dominant when the strategies from this Nash equilibrium result in a higher payoff for each player than the other Nash equilibrium (Khan & Sun, 2002). For the game under consideration, the dominant strategy Nash equilibrium shall be investigated when multiple Nash equilibria are identified.

Sub research question 5: *What will be the effects of up-pricing for online retailer Circulus considering the innovative pricing strategy from this research?*

In case a Nash equilibrium exists, the prices of the Nash equilibrium will be compared with the current prices of the selected products. The interpretation of how these prices differ from each other, can be used to determine whether up-pricing is a beneficial pricing strategy. The difference in revenue for the situation with and without up-pricing will be compared in order to quantify the effect of up-pricing. Additionally, the effect of the influencing factors will be assessed to gain insight in the effect of up-pricing. In case there is no Nash equilibrium, the underlying cause of this outcome shall be investigated. The aim of this investigation will be to determine whether the absence of a Nash equilibrium has any consequences for the effects of up-pricing, since it might be an indication of drawbacks associated with up-pricing.

3 Influencing factors consumer behavior

In this chapter the first sub-research question, regarding the factors that may influence consumer behavior, will be answered. In order to identify the influencing factors an unstructured interview was performed with a single survey data specialist at online retailer Circulus. During the interview the survey data specialist indicated that several influencing factors have been identified in previous survey studies of online retailer Circulus. Additionally, the specialist noted that the effect size of the different factors might differ per product group. This will be discussed later on, in section 5.1. The influencing factors that came forward during the interview, have been listed in the table below:

Influencing factor	**Explanation**
Good price (lowest price)	Customers buy their products at online retailer Circulus, because they know that online retailer Circulus overall has the lowest price in the market.
Experience / trust	When customers have had previous good experiences with buying products at online retailer Circulus, they will be more likely to buy their products in the future at online retailer Circulus as well.
Big collection	Customers like to be able to choose from different products.
No / low delivery cost	Customers prefer an online retailer that has no delivery cost.
Flexibility of delivery (fast and delivery options)	Customers tend to prefer online retailers that offer fast delivery and multiple delivery options, such as at home or at a delivery point.
User-friendly website	It is important that customers can easily find the products that they are looking for through easy search terms.
Easy to compare prices	Customers like to have the option to compare prices of similar products that online retailer Circulus sells.
Easy to compare products	Customers like to have the option to compare products with similar features that online retailer Circulus offers.
Reviews of other customers	Customers like to have insight in the customer reviews of the products that they are interested in.
Insight in what is popular	Customers like to know what products are sold frequently at online retailer Circulus.

Table 2: Influencing factors from the interview with the survey data specialist at online retailer Circulus

Additionally, a rapid literature review was performed to see whether the influencing factors of online retailer Circulus were in line with the influencing factors mentioned in literature or whether there were additional factors that were not mentioned during the interview, but might play an important role. The following influencing factors, that consumers consider when selecting an online retailer, were identified in literature:

Influencing factor	**Explanation**
Price	According to Dhawan (2008) the cost of the product is one of the most important influencing factors that customers consider when buying a product online. Customers desire the best value for their money. A lower price can be a significant attraction for customers, but it is not always the only factor that the customers consider (Cho & Sagynov, 2015).
Brand reputation / Trust	A strong brand reputation is an important factor in a customer's retailer selection, as it provides assurance of the quality of the service of the online retailer (Gefen, 2002). Trust is an essential factor for a customer to consider when selecting an online retailer. The reputation of a online retailer for providing reliable and secure transactions can build trust and confidence in the brand. Privacy and security also play a role in this (Shergill & Chen, 2005).
Product variety / assortment	Customers want a wide range of products to choose from. An online retailer that offers a diverse range of products is more likely to attract and retain customers (Rohm & Swaminathan, 2004).
Shipping cost and time	Shipping cost and the time it takes for a product to arrive are important factors for customers to consider when selecting an online retailer. Low shipping cost and fast delivery time can have a positive influence on the overall shopping experience (Vakulenko et al., 2019).
User-friendly website	Customers prefer a convenient site, that allows for easy search terms to find the desired product (Rohm & Swaminathan, 2004).
Possibility to compare prices or products	Another important influencing factor for consumer in the selection of an online retailer concerns the ability to compare prices and products. The customer desires to have this information to allow for an informed decision to buy a certain product (Jiang, 2002).
Customer reviews	Customer reviews can play a significant role in a customer's decision-making process, as they offer an insight into the quality, functionality, and reliability of the product. Customers like to have access to these reviews to be able to make an informed decision (Chen et al., 2004).
Product availability	Customers want to be sure that the product they are looking for are available. Out-of-stock products can lead to a loss of trust in the online retailer (Srinivasan et al., 2002).
Payment options	The availability of various payment options, such as credit card or PayPal can influence a customer's decision to purchase from a certain online retailer. Overall more options increase the chance of the customer to select the online retailer (Lee, 2002).
Return policy	A flexible return policy can be a positive influence in the online retailer selection of a customer as it may reduce the perceived risk of buying a product from the online retailer (Wang et al., 2017).
Personal preferences	Loyalty to a specific brand or retailer can be a significant influence for customers. A customer's personal preferences and past experiences with an onilne retailer can influence their decision to purchase from that retailer again (Gefen, 2002).

Table 3: Influencing factors from the rapid literature review

When comparing the resulting factors from the interview and the literature review the influencing factors of price, trust, collection, delivery cost, comparing prices and products and customer reviews came forward in both studies. During the interview it was mentioned that insight in what is popular is also an important factor for a consumer to consider when selecting an online retailer. However, this factor was not identified during the literature review. An explanation for this may be the use of a rapid literature review approach. When using a different approach, such as a systematic literature review, other possible factors may have come forward. Furthermore, the literature review identified the influencing factors of product availability, payment options, return policy and personal preferences, that were not mentioned during the interview with the survey data specialist of online retailer Circulus. Due to the unstructured nature of the interview only the influencing factors that were present in the survey data from online retailer Circulus were identified by the survey data specialist. Summing the resulting factors from the interview and the literature review, results in the following 12 influencing factors:

Total list of influencing factors
Price
Trust
Assortment
Shipping cost and time
User-friendly website
Compare prices or products
Customer reviews
Insight in what is popular
Product availability
Payment options
Return policy
Personal preferences

Table 4: Total list of resulting influencing factors

4 Literature review discrete choice modelling

In this chapter the second sub-research question, regarding which discrete choice models exist in literature to model consumer behavior, will be answered. A snowballing approach was used in order to find relevant scientific articles describing these models.

Discrete choice models have often been applied in literature to mimic scenarios of the real world such that consumer behavior can be analyzed (Bierlaire, 1998). Also in the the e-commerce sector choice models have been of great use (Danaf et al., 2019). For instance, to determine the probability that a consumer will select a certain online retailer. Typically, several modelling assumptions should be considered when composing a choice model (Bierlaire, 1998).

First of all, it should be decided who makes the decisions. Is this an individual or group? What are the characteristics of the decision-maker(s)? Characteristics like gender, age and educational attainment could be factors to consider for this step. From now on, this research will be using the following definitions to represent these differences: an individual will be defined as a decision-maker. A homogeneous group, where all individuals have the same characteristics, will be defined as a decision-maker group. A heterogeneous group, where each individual has different characteristics, will be defined as decision-makers. These definitions are summarized in the table below.

Decision-maker	**Term**
Individual	*decision-maker*
Homogenous group: each individual in the group has the same characteristics	*decision-maker group*
Heterogenous group: the individuals in the group have different characteristics	*decision-makers*

Table 5: terms decision-maker

For simplicity, the term decision-maker shall be used in this chapter to explain the modelling assumptions.

The second assumption that should be made for composing the choice model concerns the choice set. This set defines the alternatives that the decision-maker can choose from (Bierlaire, 1998). Typically, this set is represented by C. The choice set may be similar for every decision-maker or vary per decision-maker (Bierlaire & Lurkin, 2017). Additionally, a choice set may either be continuous or discrete, this will be explained further in the next section. Furthermore, the alternatives (i.e., other options or choices) may contain attributes. For instance, when considering different e-commerce retailers as alternatives, an attribute of each alternative may be their delivery speed. Attributes can be specific for an alternative or may apply to all alternatives in the choice set.

The third assumption concerns the decision rules. These define the process that should be used by the decision-maker in order to arrive at a decision (Bierlaire, 1998). In the current research the random utility maximization model (RUM) will be used, where the decision-maker selects the alternative with the highest utility (Schlicher & Lurkin, 2022). The utility is a function of the observable and unobservable attributes. This function will be further explained later in this section. In order to compose a random utility model, there are several distinctions to be made. Three distinctions will be discussed in the section below. It should be noted that from now on, if a choice model is mentioned it builds on a random utility model.

Continuous and discrete choice models

There is the difference between continuous choice models and discrete choice models. Hanemann (1984) describes that continuous choice models are applicable in contexts where there is no distinct set of alternatives. A continuous choice is a set of alternatives that are defined by constraints and cannot be enumerated (Bierlaire, 1998). For instance, if a decision-maker would have a time period of 2 hours for online shopping and wants to visit the sites of online retailer 1 and online retailer 2, the continuous choice set C would be defined in the following matter:

$$C = \{(t_1, t_2) \mid t_1 + t_2 \leq 2, t_1 \geq 0, t_2 \geq 0\}. \tag{2}$$

In contrast, discrete choice models concern a choice from a non-empty finite set of distinct alternatives. To illustrate, the online retailer that the decision-maker decides to buy from after investigating both sites, could be defined as a discrete choice set as it involves a finite number of alternatives. The discrete choice set C is then defined as follows, where 1 represents choosing online retailer 1 and 2 represents choosing online retailer 2:

$$C = \{1, 2\}. \tag{3}$$

To summarize, the time decision-maker spends at the site of each online retailer is defined as a continuous choice and the online retailer that the decision-maker selects to buy from is defined as a discrete choice. As the subject under study for this thesis concerns the selection of an e-commerce retailer, it can be concluded that a discrete choice model is applicable in this setting. Therefore this literature review will focus on discrete choice models from now on.

Dynamic and static discrete choice models

When considering discrete choice models there is yet another distinction to be made, namely the use of dynamic or static discrete choice models. According to Arcidiacono & Ellickson (2011), dynamic discrete choice models take into account future outcomes, while this is not considered in a static discrete choice model. In a dynamic discrete choice model the decision-maker has a sequence of choices in several time periods, while considering the impact of their current decision on their future decisions. In the static discrete choice model the decision-maker chooses an alternative from a distinctive set and only has one choice to make, future choices are not considered.

This research will be focused on static discrete choice models as the decision under consideration concerns the selection of an e-commerce retailer. This decision does not consist of a sequence of choices in time, but concerns a single choice at a specific moment in time.

Aggregate and hierarchical Bayes disaggregate discrete choice models

A static discrete choice model may be of an aggregate or disaggregate nature. In an aggregate model the heterogeneity of the decision-maker sample is not accounted for, individuals and their choices are regarded to be similar (Renken, 1997). In this research this is defined as a (homogeneous) decision-maker group. This population is evaluated in its integrity. Daly (1982) state that aggregated models are often more practical when it is desired to predict the number of people that will make a certain choice instead of predicting the choice of an individual decision-maker. Furthermore, according to Bierlaire & Lurkin (2017) aggregate modeling approaches are typically used in choice modelling literature, but this method may lack in capturing causal mechanisms. To summarize, in an aggregate model choices are considered at a group level, where the group is homogeneous. This method allows for a simple representation of reality and has often been applied in literature.

In a disaggregate model, decision-makers are allowed to have their own preferences (Borghi, 2009). In this research this is defined as (heterogeneous) decision-makers. In a disaggregate model the socioeconomic characteristics of decision-makers are taken into consideration. One method that allows for the inclusion of decision-makers' heterogeneity is the use of Bayesian methods. Bayesian methods may be used to consider the impact of decision-makers' brand preference or the ways that social networks impact demand (Allenby & Rossi, 2006). The disaggregate discrete choice model is then defined as a hierarchical Bayesian discrete choice model. Bierlaire & Lurkin (2017) state that discrete choice models are a suitable approach for disaggregate models. In this way the causality between the explanatory variables and the choice can be observed. To conclude, in a disaggregate model choices are considered at the individual level. This causes the disaggregate model to have a greater level of complexity than aggregate models, but this also allow for a more accurate representation of the real-life situation. In order to compose a disaggregate model information about the characteristics of the decision-maker is required. In the e-commerce sector this is often an issue due to privacy concerns (Qiu et al., 2015). As this data privacy issue also concerns the situation at online retailer Circulus, an aggregate discrete choice model should be selected. Although an aggregate model is of a more simplistic nature, it may provide valuable insights. This has been observed in literature (Renken, 1997; Bierlaire & Lurkin, 2017).

The distinctions that were selected for the online retailer Circulus case are summarized in the table below, where the options in bold are selected for online retailer Circulus. The remaining research will focus on static aggregate discrete choice models. The author feels that this model is the most suitable approach to find a method that is applicable for the situation of online retailer Circulus.

Distinctions RUM model	
Continous	**Discrete**
Dynamic	**Static**
Aggregate	Disaggregate

Table 6: distinctions RUM model

Static aggregate discrete choice models

As mentioned before, static aggregate discrete choice models rely on the RUM model. The theory assumes that the decision-maker has perfect discrimination ability and will choose the alternative with the highest utility (Schlicher & Lurkin, 2022). The utility is a function of the observable and unobservable attributes of the alternatives in choice set C. Through accounting for the unobservable attributes, the model captures uncertainty. According to Manski (1977) uncertainty may occur due to unobserved attributes of the alternatives, unobserved attributes of the decision-maker or measurement errors.

Utility is modeled as a random variable that consists of a deterministic part (V) and a stochastic part (ϵ) (Bierlaire, 1998). The deterministic part represents the observable attributes and the stochastic part represents the unobservable attributes. The decision-maker can choose from choice set C. For each option $i \in C$ a utility function U_i can be composed. This function is then given by:

$$U_i(x_i) = V_i(x_i; \beta) + \epsilon_i, \ \forall x_i \in \mathbb{R}^n \tag{4}$$

In this function $V_i(x_i; \beta)$ is the deterministic term and $x_i \in \mathbb{R}^n$ represents a vector for the attributes of the alternative for the individual, where n defines the number of attributes (Bierlaire & Lurkin (2017). These alternatives may have attributes like delivery speed or price. Attributes of each decision-maker are the same as this research focuses on an aggregate model. The alternative-specific coefficient β is a vector of unknown parameters that has to be estimated for each alternative i (So & Kuhfeld, 1995). An example for which the β parameter could be used is to express the willingness of a customer to pay (Schlicher & Lurkin, 2022). ϵ_i is the error term, which captures the uncertainty of the model. The decision-maker will select the option i with the highest utility. As the utility function of option i is uncertain, it is desired to know the probability that the utility function of i is the highest (Schlicher & Lurkin, 2022):

$$P(i) = P(U_i(x_i) \geq U_j(x_j), \ \forall j \in C) \tag{5}$$

Varying assumptions about the deterministic and stochastic terms of this formula result in specific models (Walker & Ben-Akiva, 2002). The deterministic terms $V_i(x_i; \beta)$, or the systematic utility, are considered as a function of all attributes for the alternatives and for the decision maker and can be related to observable factors. The systematic utility function is in common practice assumed to be linear (Bierlaire, 1998), which simplifies the estimation of the model.

With the utility terms it is desired to determine whether the decision-maker will choose option i. It is assumed that the decision-maker chooses the alternative with the highest utility function. This probability should be expressed in the following manner (Bortolomiol et al., 2021):

$$P(i) = P(U_i(x_i) \geq U_j(x_j), \ \forall j \in C) \tag{6}$$

$$= P(V_i + \epsilon_i \geq V_j + \epsilon_j, \ \forall j \in C) \tag{7}$$

$$= P(\epsilon_j - \epsilon_i \leq V_i - V_j, \ \forall j \in C) \tag{8}$$

It should be noted that V_i represents $V_i(x_i;\beta)$ in the above function. The random vector $\epsilon_n = [\epsilon_1, \epsilon_2..., \epsilon_n]$ follows a continuous distribution with density function $f(\epsilon_n)$ (Train, 2009). Subsequently, the probability that the difference between the two random terms $\epsilon_j - \epsilon_i$, with $i, j \in N$ and $i \neq j$, is below the difference between $V_i - V_j$, can be rewritten through the use of the density function:

$$P(i) = P(\epsilon_j - \epsilon_i \leq V_i - V_j,\ \forall j \in C) \tag{9}$$

$$= \int_\epsilon I(\epsilon_j - \epsilon_i \leq V_i - V_j,\ \forall j \in C) f(\epsilon_n)\ d\epsilon_n \tag{10}$$

In this function I is an indicator function, which is equal to 1 when the expression within I is true and 0 when it is not true. Different assumptions about the error terms will lead to different specifications of the density function $f(\epsilon_n)$. Varying assumptions for the error term $\epsilon_i \in \epsilon_n$ have been applied in practice, which results in three different models (Bierlaire, 1998):

- When it is assumed that ϵ_i follows a linear distribution, this will lead to a **linear model**. However, as this model is not easily applicable for real situations, it has not often been applied in literature.

- When it is assumed that ϵ_i follows a normal distribution, this leads to a normal **probit model**. The probit model builds on the central limit theorem and assumes that the error terms are a function of independent observed quantities. However, a disadvantage of this model is that the practical use is limited as the probability function does not have a closed analytical form. Nonetheless this method has often been applied before (Bierlaire, 1998).

- When it is assumed that every ϵ_i follows and independent identically Gumbel distribution, this will lead to a **logit model**. Each ϵ_i is independent identically Gumbel distributed in random vector ϵ_n. This is the most commonly used model in literature and has the best practical implications. Bierlaire & Lurkin (2017) state that the logit model has been used most due to the simplicity of the closed-form probability expression of the logit model. Furthermore, the logit model is characterized by its independence from irrelevant alternatives. This states that the probability of choosing alternative 1 over alternative 2, will remain the same when a third alternative is added or removed (Benson et al., 2016).

It should be noted that as these distributions hold for ϵ_i, the same distributions will hold for vector ϵ_n. Logit models contain a closed-form expression for the integral in the expression (10) (Train, 2009). With the probit model the integral from the expression (10) will not have a closed form and should be evaluated through the use of simulation (Train, 2009). As the logit model has the best practical implications from the three aforementioned models, it shall be further explored in the following section.

4.1 Logit model

As mentioned before, the logit model assumes that the error terms from the utility functions follow an independent and identically Gumbel distribution (Bierlaire, 1998). The independence of the error terms indicates that the error term of the utility for one alternative is unrelated to the error term of the utility for another alternative (Train, 2009). To derive the logit model the error term in the utility function $U_i = V_i + \epsilon_i$ is assumed to be independent and identically Gumbel distributed. The density for each error term is expressed as the following (Train, 2009), with $\epsilon_i \in \mathbb{R}$:

$$f(\epsilon_i) = e^{-\epsilon_i} * e^{-e^{-\epsilon_i}} \tag{11}$$

The density function of the aforementioned vector of error terms ϵ_n can then be defined in the following manner:

$$f(\epsilon_n) = \prod_{i \in C} f_i(\epsilon_i) \tag{12}$$

Additionally, the cumulative distribution of ϵ_i is expressed as:

$$F(\epsilon_i) = e^{-e^{-\epsilon_i}} \tag{13}$$

According to Train (2009) the variance of the above distribution function is assumed to be $\pi^2/6$. Additionally, Train (2009) states that the difference between two independent identically Gumbel distributed variables is logistically distributed. This means that the $\epsilon_j - \epsilon_i$ from the probability function in the previous chapter will follow a logistic distribution when the error terms are assumed to be independent identically Gumbel distributed. In order to derive the choice probabilities for the logit model, the general probability function for utility maximization, which is represented in equation 7 in the previous section, shall be rewritten:

$$= P(V_i + \epsilon_i \geq V_j + \epsilon_j,\ \forall j \in C) \tag{14}$$

$$= P(V_i + \epsilon_i > V_j + \epsilon_j,\ \forall j \in C \setminus \{i\}) \tag{15}$$

$$= P(\epsilon_j < \epsilon_i + V_i - V_j,\ \forall j \in C \setminus \{i\}) \tag{16}$$

The cumulative distribution of ϵ_j can be composed given $\epsilon_i + V_i - V_j$ and ϵ_i. So, for a given $_i$, this cumulative distribution is then expressed as:

$$F(\epsilon_j) = e^{-e^{-(\epsilon_i + V_i - V_j)}},\quad \epsilon_j \in \epsilon_n \tag{17}$$

Given that the error terms are independent, the cumulative distribution for all j, where $j \in C \setminus \{i\}$ is the product of the cumulative distribution for each j:

$$P(i \mid \epsilon_i) = \prod_{\forall j \in C \setminus \{i\}} e^{-e^{-(\epsilon_i + V_i - V_j)}} \tag{18}$$

However, in reality ϵ_i is not given. Therefore, the integral should be taken of $P(i \mid \epsilon_i)$ over each value ϵ_i weighted by density:

$$P(i) = \int_{\epsilon_i = -\infty}^{\infty} \left(\prod_{\forall j \in C \setminus \{i\}} e^{-e^{-(\epsilon_i + V_i - V_j)}} \right) e^{-\epsilon_i} e^{-e^{-\epsilon_i}} d\epsilon_i \tag{19}$$

Rewriting the above formula results in the closed-form expression for the choice probability expressed as below. The derivations to arrive at this formula can be found in Appendix A.1.

$$P(i) = \frac{e^{V_i}}{\sum_{j \in C} e^{V_j}} \tag{20}$$

The deterministic term is often specified as linear in parameters, which leads to an arbitrarily close approximation (Train, 2009). In the above functions the simplified term V_i represents $V_i(x_i; \beta)$. Rewriting the equation with the formal notation, gives:

$$P(i) = \frac{e^{V_i(x_i;\beta)}}{\sum_{j \in C} e^{V_j(x_j;\beta)}} \tag{21}$$

This general framework for the logit model can be applied to the binary logit model and the multinomial logit model. With a binary logit model there are two alternatives to be considered. With the multinomial logit model three or more alternatives are considered. To illustrate the binary logit model, the previous example will be extended. A single decision-maker will be considered who can choose between the online retailer 1 and online retailer 2. The choice set can be expressed as $C = \{1, 2\}$. For each choice from set C a utility function is introduced, which represents the value that the decision-maker assigns to that choice:

$$U_1(p_1; \beta) = \beta_0^1 - \beta_1^1 \cdot p_1 + \epsilon_1 \tag{22}$$

$$U_2(p_2; \beta) = \beta_0^2 - \beta_1^2 \cdot p_2 + \epsilon_2 \tag{23}$$

In these formulas β_0^1 and β_0^2 represent alternative-specific constants. In this example the attribute of the alternatives that is considered is price, where β_1 represents a price-sensitivity parameter. The following assumptions are made in order to develop a numerical example of a logit model: $\beta_0^1 = 1$, $\beta_0^2 = 1.5$ and $\beta_1^1, \beta_1^2 = 0.6$. Subsequently, following the above derivations, the probability that the decision-maker selects alternative 1 is expressed as:

$$P(1) = P(\epsilon_2 - \epsilon_1 < V(x_1;\beta) - V(x_2;\beta)) = \frac{e^{1-0.6\cdot p_1}}{e^{1-0.6\cdot p_1} + e^{1.5-0.6\cdot p_2}} \tag{24}$$

Additionally, the probability that the decision-maker selects alternative 2 is expressed as:

$$P(2) = P(\epsilon_2 - \epsilon_1 > V(x_1;\beta) - V(x_2;\beta)) = \frac{e^{1.5-0.6\cdot p_2}}{e^{1-0.6\cdot p_1} + e^{1.5-0.6\cdot p_2}} \tag{25}$$

To conclude, the binary logit model is suitable to predict choice probabilities when the decision-maker can choose between two alternatives.

The multinomial logit model is an extension of the binary logit model, as it considers more than two alternatives. The choice set will therefore contain three or more alternatives. To illustrate the multinomial logit model, our previous example with online retailer 1 and online retailer 2 will be extended. The decision-maker can now choose from 3 options: buy from 1, buy from 2, do not buy the product. The option of not buying anything is given a value of zero. The choice set is then expressed as $C = \{1, 2, 0\}$. It is assumed that the choice set is the same for all decision-makers. The utility that the decision-maker assigns to each alternative, can be expressed with the following formulas:

$$U_A(x_1;\beta) = \beta_0^1 - \beta_1^1 \cdot x_1 + \epsilon_1 \tag{26}$$

$$U_B(x_2;\beta) = \beta_0^2 - \beta_1^2 \cdot x_2 + \epsilon_2 \tag{27}$$

$$U_0(x_0;\beta) = \beta_0^0 - \beta_1^0 \cdot x_0 + \epsilon_0 \tag{28}$$

Subsequently, the probability that the decision-maker will choose alternative 1 from choice set C can be expressed as:

$$P(1) = \frac{e^{\beta_0^1-\beta_1^1\cdot x_1}}{e^{\beta_0^1-\beta_1^1\cdot x_1} + e^{\beta_0^2-\beta_1^2\cdot x_2} + e^{\beta_0^0-\beta_1^0\cdot x_0}} \tag{29}$$

Multinomial logit model forms

The multinomial logit model may take different forms considering the characteristics of the decision-maker or the alternatives. When the choice of the consumer is defined as a function of the characteristics of the alternatives, this is called a conditional logit model. When the choice is a function of the characteristics of the consumer, it is a generalized logit model. When both the characteristics of the consumer and the alternatives are taken into account for determining the choice function, it is a mixed logit model (So & Kuhfeld, 1995). These different forms of the multinomial logit model will be illustrated with an example. Consider that a consumer is asked to choose a transportation mode to work; train, bike or car. For the transportation modes the variables train time, bike time and car time are considered. For the consumer the variable age is considered. When investigating the choice as a function of the age, a generalized logit model is used. This generalized model investigates a heterogeneous group of decision-makers. When investigating the choice as a function of the travel time, a conditional logit model is used. For the conditional logit model a homogeneous decision-maker group is considered. When considering all variables for the choice function, a mixed logit model is used. The mixed logit model thus focuses on a heterogeneous group of decision-makers.

It should be noted that these terms for different forms of the models are used for specifically logit models. However, they are closely related to the aforementioned terms aggregate and hierarchical Bayes disaggregate choice models, which can be used for choice models in general. So & Kuhfeld (1995) state that the terminology in literature is sometimes inconsistent, which leads to confusion. In some cases the multinomial logit model is used to describe a generalized logit model, while other articles consider the multinomial logit model as a mixed logit model. Therefore, it is of great importance to clearly state the variables under consideration for the choice function when working with logit models.

As mentioned before, an important assumption of the multinomial logit model is the independence of the error terms of utilities (Bierlaire, 1998). The model assumes that alternatives are independent. This leads to the situation that every alternative may end up with the same probability to be selected, when the parameters have the same value for every alternative. However, this may not always be the case in the real world. The nested logit model partly overcomes this limiting assumption of the multinomial logit model. The nested logit model is an extension of the multinomial logit model, where the assumption regarding the independence of irrelevant alternatives is relaxed. As mentioned before, this assumption states that the probability of choosing one alternative over the other, does not depend on a third alternative. In the nested logit model this assumption is violated, such that dependence among alternatives is captured. The choice set is divided into several nests that reflect the dependence of the alternatives (Bierlaire, 1998). A limitation of the nested logit model is that alternatives within the nests can be dependent, but that dependencies across nests are not considered (Bierlaire, 1998). Therefore, the model is not applicable when alternatives cannot be separated into nests nicely.

4.1.1 Parameter estimation

In the previous section logit models have been introduced that include parameters β_0^i and β_1^i, which are included in vector β. These parameters should be estimated, which has often been done in literature through maximum likelihood estimation (Bierlaire & Lurkin, 2017). In this section a technique to apply maximum likelihood estimation for the estimation of the parameters will be explained. Firstly, the estimation procedure will be applied to a binary logit model. Secondly, maximum likelihood estimation for a multinomial logit model will be explained.

In the binary logit model the choice set contains two alternatives, option i and j. In the previous section it was mentioned that the probability that a decision-maker chooses option i can then be expressed in the following manner:

$$P(i) = \frac{e^{\beta_0^i + \beta_1^i x_i}}{\sum_{j \in C} e^{\beta_0^j + \beta_1^j x_j}} \tag{30}$$

Maximum likelihood estimation can be used to estimate β_0 and β_1 from the above formula. This method considers a group of N homogeneous decision-makers. For this sample there are several observations, where decision-maker n chooses an alternative. For the maximum likelihood estimation, we are interested in the likelihood of N decisions from the decision-makers to be independently drawn from the population (Febrianti et al., 2021). The likelihood (or probability) of the sample is equal to the product of the probabilities of the individual decisions. For the estimation approach two indicator variables are introduced:

$$y_{in} = \begin{cases} 1, & \text{if the decision-maker n chooses alternative i} \\ 0, & \text{if the decision-maker n chooses alternative j} \end{cases} \tag{31}$$

$$y_{jn} = \begin{cases} 1, & \text{if the decision-maker n chooses alternative j} \\ 0, & \text{if the decision-maker n chooses alternative i} \end{cases} \tag{32}$$

The probability that the decision-maker n selects alternative i, is represented by $P_n(i)$ and the probability that decision-maker n selects alternative j, is represented by $P_n(j)$. For the product of these probabilities the following holds:

$$P_n(i)^{y_{in}} P_n(j)^{y_{jn}} = \begin{cases} P_n(i), & \text{if } y_{in} = 1 \text{ and } y_{jn} = 0 \\ P_n(j), & \text{if } y_{in} = 0 \text{ and } y_{jn} = 1 \end{cases} \tag{33}$$

In the above function, $P_n(i)$ and $P_n(j)$ are functions of β. β is a vector of parameters β_0^i and β_1^i. When there are more attributes involved, the vector of parameters β describes β_1, ... β_K, where K indicates the number of observable variables in the model. To illustrate, when there are three observable variables included, like price (variable 1), delivery speed (variable 2) and sustainability (variable 3), the vector will contain $\beta_1, \beta_2, \beta_3$. The likelihood function can then be written as:

$$\mathcal{L}(\beta_1, ..., \beta_K) = \prod_{n=1}^{N} P_n(i)^{y_{in}} P_n(j)^{y_{jn}} \tag{34}$$

Taking the log of function (34), results in the following formula:

$$\mathcal{L}(\beta_1, ..., \beta_K) = \sum_{n=1}^{N} (y_{in}\, ln(P_n(i)) + y_{jn}\, ln(P_n(j))) \tag{35}$$

Given that $y_{jn} = 1 - y_{in}$ and $P_n(j) = 1 - P_n(i)$, the above formula is rewritten as:

$$\mathcal{L}(\beta_1, ..., \beta_K) = \sum_{n=1}^{N} y_{in}\, ln(P_n(i)) + (1 - y_{in})\, ln(1 - P_n(i)) \tag{36}$$

The goal of maximum likelihood estimation is to find a vector of values for β that maximizes the resulting log-likelihood (Train, 2009). Taking the partial derivative of the above function over β_k and setting this equal to zero, results in a local maximum (Train, 2009):

$$\frac{\partial \mathcal{L}(\beta)}{\partial \beta} = 0 \tag{37}$$

This function should be solved simultaneously for each β_k in order determine the vector of parameters β (Czepiel, 2002). There are several statistical programs available to solve the defined model, like Excel Solver or RStudio. In this way, the probability that the model predicts the observed choice correctly is optimized (Bierlaire & Lurkin, 2017).

For the multinomial logit model a similar approach to the binary logit model can be applied for the maximum likelihood estimation. The indicator variable is then expressed in the following manner:

$$y_{in} = \begin{cases} 1, & \text{if the decision-maker n chooses alternative i} \\ 0, & \text{otherwise} \end{cases} \tag{38}$$

For the multinomial logit model there is no explicit solution for identifying the local maximum such as with the binary logit model. Therefore, a solution could be numerically approximated with methods such as the Newton-Raphson (NR) method (Croissant et al., 2012). Packages for the NR-approach of solving the maximum likelihood estimation for multinomial logit models are available in RStudio and Python.

In this chapter it was observed that discrete choice models, which rely on random utility models, have been used in literature to predict the choice of a consumer. The logit model has often been applied in practice to predict the choice of the consumer. The next step in this literature review will be to investigate the different applications of discrete choice modeling in the e-commerce sector.

4.2 Applications of discrete choice modelling in an e-commerce context

In the e-commerce context discrete choice modelling has been broadly used for revenue optimization (Bierlaire & Lurkin, 2017). In the article of Bierlaire & Lurkin (2017) a competitive market is considered, where advantage can be gained with price. The objective of this problem is to alter the price of a product such that the generated revenue is maximized. The model may be altered through taking into consideration the production price of a product. Assuming that the prices of competitors are fixed, the revenue can be determined through a logit model. The probability that the consumer selects a certain retailer is composed with the utility function, which depends on the price and specified parameters. Subsequently, the revenue can be calculated through multiplying the probability with the price, such that a maximum revenue can be determined. The resulting profit function is often non-concave, leading to a local optimum (Hanson & Martin, 2015). The local optimum represents the maximum possible value for the revenue and the corresponding price.

In addition to price-based competition, discrete choice modelling has also been used in e-commerce literature to study the effects of website quality and awareness (Zo & Ramamurthy, 2009). It was found that not only the prices of different online retailers matter, but that the quality of the website of the online retailer may also play a significant role in the choice behavior of the consumer.

Furthermore, Danaf et al. (2019) used discrete choice modelling in order to develop a recommendation system for online retailers. These recommendation systems generate personalized recommendations. The discrete choice model that was used for this research concerned a mixed logit model that accounted for customer heterogeneity. This model estimated the preferences of each consumer, which was used as input for the recommendation system.

Lastly, Qiu et al. (2015) applied discrete choice modelling in an e-commerce context in order to develop a prediction model. This model predicts which products are most likely to be bought from a specific online retailer based on customer behavior. Customer behavior was modelled using a hierarchical Bayesian discrete choice model.

5 Input data estimation discrete choice model

In this research a discrete choice model will be composed to analyze how consumers select a certain online retailer or decide buy the product in a physical store when the offers from retailers are unattractive. To compose this model input data is needed. In this chapter it will be investigated what method should be selected to generate input data for the estimation of the discrete choice model.

Two main approaches have been found in literature that are often used to acquire the required data for the estimation of a discrete choice model: revealed preference and stated preference surveys. Both methods have been broadly applied in literature to study consumer preferences and buying behavior. The main difference between the two approaches is that stated preference surveys ask individuals to indicate their choices, while revealed preference surveys gather preferences based on observed choices (Huang et al., 1997).

Revealed preference surveys build on the belief that the behavior of individuals will reveal the underlying preferences. In a revealed preference survey, the choices that an individual makes in real-life situations are observed and that information is used to determine the preferences of the individual (Boyle, 2003). To illustrate, if an individual consistently chooses to buy from online retailer Circulus, it can be inferred that they prefer online retailer Circulus over other online retailers. An advantage of the revealed preference survey is that it is based on actual behavior, which is generally considered to be a more accurate reflection of preferences than stated preferences. Additionally, a revealed preference survey tends to be less time-consuming and less expensive than a stated preference survey, as it does not require time from the individual to complete a survey (Houthakker, 1950). A disadvantage of the revealed preference survey is that it may not accurately reflect the true preferences of the individual when individuals are constrained in their choices. Additionally, the individual may not fully be aware of the consequences of their choices in a revealed preference survey (Boyle, 2003).

In a stated preference survey, an individual is presented with a set of scenarios and is asked to choose the alternative that he/she prefers. This allows for a more direct measure of the preferences, as the individual is explicitly asked to indicate his/her preferences (Kroes & Sheldon, 1988). An advantage of a stated preference survey is that it can be used to investigate preferences in hypothetical scenarios that would not be possible to observe in a revealed preference survey (DeShazo & Fermo, 2002). To illustrate, a stated preference survey could be used to ask an individual about his/her preferences for a certain product, even if the product has not been launched yet. A disadvantage of the stated preference survey is that it may not accurately reflect the true preferences of the individual if the presented alternatives are unclear or when they try to fill out the survey in a socially desirable manner. Furthermore, a stated preference survey is time-consuming as it requires time from the individual to fill out the survey (Kroes & Sheldon, 1988).

To conclude, both the revealed preference and the stated preference survey have advantages and disadvantages, and the choice of method depends on the topic under consideration and the research context. For the situation at online retailer Circulus there is no data available on how many consumers choose for online retailer Circulus or its competitors, when buying a certain product. This rules out the feasibility of a revealed preference survey. Therefore, a stated preference survey with hypothetical scenarios will be conducted. In this survey, the respondent selects a certain online retailer, or decides not to buy anything, based on given attribute values.

5.1 Stated preference survey

5.1.1 Survey composition

The goal of the stated preference survey is to get an insight in how the previously identified influencing factors play in a role in the consumer's decision to select an online retailer to buy from. The stated preference survey focuses on one specific product such that the survey is understandable and will not take too long to fill out for the respondent. The product that was selected is an electric toothbrush, which is depicted in the figure below.

Figure 1: Selected product for stated preference survey

The reason for selecting this specific toothbrush is that it is a basic product that people need in daily life, it is a necessary product. Additionally, the product is not gender specific, allowing for both men and women to fill out the survey. Furthermore, the price range of this product in the market is narrow (between 20 and 40 euros), such that the full price range can be covered in the survey.

Another important reason for selecting this specific product is that it is a perfect candidate for up-pricing. Currently, online retailer Circulus is selling this product for 20 euros and another competitor has this same low price, while the other competitors in the market maintain much higher prices ranging from 25 to 40 euros. This creates potential for online retailer Circulus to increase in price.

Attributes and attribute levels

The attributes for the survey were composed using the resulting influencing factors from Chapter 3. To maintain a focus on efficiency and simplicity and to limit the burden on the respondents, it was decided to test only four attributes in the survey. This decision was guided by the principle of parsimony, which emphasizes the importance of using the most straightforward and simplest methods to achieve the desired outcome. In order to select these four attributes the influencing factors of Table 4 from section 3 were used as a starting point. This list included the resulting influencing factors from the literature review and the interview with the survey data specialist of online retailer Circulus. For the product group of electric toothbrushes, which is small domestic appliances (SDA), the survey data specialist was asked to rank the influencing factors of the integrated list from high to low importance:

Total list of influencing factors
Price
Experience / trust
No / low delivery cost
Flexibility of delivery
Big collection
User-friendly website
Reviews of other customers
Easy to compare prices
Easy to compare products
Insight in what is popular

Table 7: Rating influencing factors

It should be noted that the influencing factors regarding product availability, payment options, return policy and personal preferences, are not included in the above rating. The survey data specialist of online retailer Circulus expected that the magnitude of the effects of the factors would be relatively small and that they should be excluded.

Subsequently, the rating from Table 7 were discussed with online retailer Circulus in order to come to a final selection. As the stated preference survey of this research considers a predefined product, namely the electric toothbrush, it was decided that the influencing factors regarding assortment, comparing prices or products, customer reviews, insight in what is popular and product availability are not suitable to be assessed with this survey. The following four influencing factors turned out to be the most important for online retailer Circulus: price, trust, shipping cost, user-friendly website.

The selected influencing factors will be included as attributes in the stated preference survey. Different attribute levels should be created for each attribute. These attribute levels will be manipulated to present different scenarios to the respondent. The respondent should indicate his/her preference for the different scenarios and the results will be analyzed to see how changes in the attributes influence consumer behavior. It is important that the attribute levels are designed such that they are representative of the scenarios that consumers face in the real world. This will help ensure that the results of the survey are accurate and can be used to inform decision making. The following attribute levels were created, in consolation with online retailer Circulus, for the selected influencing factors:

Attribute	**Number of levels**	**Level definition**	**Level coding**
Price	4	20 25 30 35	20 25 30 35
Good experience	2	No Yes	0 1
Shipping cost	2	No Yes	0 1
User-friendly website	4	1 2 3 4	1 2 3 4

Table 8: Attributes and attribute levels

According to Rose & Bliemer (2007) it is important to strive for the same number of levels for each attribute to ensure attribute level balance. Attribute level balance means that the levels of each attribute appear an equal number of times throughout the stated preference survey. However, this research aimed to keep the number of choice scenarios to a minimum to reduce the burden on the respondents. Therefore, it was decided to use a mix of 2 or 4 levels for the considered attributes. In this way no perfect balance is achieved in the number of appearances of each level, but a reasonable level of balance can still be obtained. This reasonable balance can be ensured using techniques such as orthogonal or fractional factorial designs. This concept will be explained in further detail later on in this section.

In order to ensure that the effect of each attribute level can be measured independently, the levels of each attribute were equally spaced, meaning that the difference between levels is consistent throughout the attribute range. The equally spaced attribute levels enable the respondent to understand and evaluate the attribute range. This allows for meaningful comparisons and increases the validity of the results of the survey.

Lastly, the range of the attribute levels for the attribute of price were considered. If the price range is too large, the attribute price may overpower the effect of the other influencing variables in the model. On the other hand, if the price is too small, price will not affect the choices of the respondents and the effect of price cannot be measured. Additionally, the price levels should make sense to the respondent considering the product, otherwise respondents may decide to fill out the survey randomly. It was decided that a reasonable price range was between 20 and 35 euros, as the lowest current price in the market is 20 euros for this toothbrush. The highest price in the market is currently 40 euros, but 5 levels would disrupt the attribute level balance of the model. There are only a few competitors in the market that set this high price, so it was decided to exclude this level from the experiment.

Choice sets

There are different approaches possible for the composition of the choice sets for the survey. Firstly, there is the full factorial design, which tests all possible combinations of the attributes. An advantage of this approach is that it allows for the identification of the main effects and interactions between attributes. However, for the current research when using two alternatives this would result in $(4 * 2 * 2 * 4) * (4 * 2 * 2 * 4) = 4096$ alternatives, which would require too much time from the respondent. Another approach is an efficient design, which uses information form literature about initial values of the parameters (Rose & Bliemer, 2007). The research could then be focused on the parameters that are expected to have the largest effect on consumer behavior. Lastly, there is the approach of the orthogonal fractional factorial design. This experimental design tests a subset of the possible combinations of the attribute levels. The subset is composed such that the main effects and interactions between the attributes are orthogonal, indicating that the effects of the attributes on the outcome are uncorrelated and statistically independent (Rose & Bliemer, 2007). Additionally, an orthogonal design ensures attribute level balance, such that the choice sets contain all attribute levels an equal number of times. With an orthogonal design all parameters can be estimated independently. This approach is less time consuming than the full factorial design.

In the current research an orthogonal fractional factorial design will be applied as there are no initial parameter values from literature that allow for an efficient design and a full factorial design is not possible due to time constraints. Further explanation on the characteristics and the applicability of an orthogonal fractional factorial design can be found in Appendix A.2. For the orthogonal fractional factorial design it was decided to use choice sets consisting out of two alternatives. As this research focuses on multinomial logit model and mixed logit models, a choice set of two alternatives is sufficient for the stated preference survey. If another discrete choice model such as the random regret minimization model would be tested, where the decision-maker minimizes the expected regret, at least three alternatives would be required per choice set (Chorus, 2012). To allow for the sequential choice method, where the respondent chooses between two alternatives every time, it was better to use unlabeled alternatives (online retailer 1, online retailer 2) instead of labeled alternatives (Competitor A, online retailer Circulus). Labeled alternatives are more fitting for the simultaneous choice method, where the respondent is presented with all alternatives at once and has to choose one (Louviere & Flynn, 2010). However, the researcher felt that labeled alternatives and simultaneous choice were not appropriate as this research focuses on the influence of specific aspects of online retailers and not on different types of retailers. Another reason to not use labeled alternatives was to prevent bias. With labeled alternatives bias could arise when customers prefer popular online retailers for unspecified reasons.

In order to compose a set of choice sets of the orthogonal fractional factorial design Ngene software was used. The Ngene manual contained clear instructions on the definition of the syntax for the experimental design. This resulted in an orthogonal design with 16 choice sets. There were minor correlations between the choice sets. The syntax and output of Ngene can be found in Appendix A.3. The output design of Ngene meets the conditions of an orthogonal fractional factorial design described in Appendix A.2. An example of a choice set is depicted in the figure below. The numerical values of the attributes 'experience' and 'shipping cost' have been replaced by words to clarify the differences for the respondent. Also the value of the attribute 'user-friendly website' is represented by a number of stars to make the survey more comprehensible for the respondent.

Attribute	Online retailer 1	Online retailer 2
Price	20	25
Experience	Bad	Bad
Shipping cost	No	No
User-friendly website	★☆☆☆	★★★★

Figure 2: Example choice set, 2 unlabeled alternatives

It should be noted that the 16 choice sets only allow for the measurement of the main effects. In order to test the interaction effects of the different attributes a foldover design should be applied. However, this would mean that there would have to be two different surveys and that the number of choice sets would increase. As it is not expected that interaction effects will play a significant role in the current situation and due to time constraints, it was chosen to focus this research solely on the measurement of the main effects of the selected attributes. This excludes the possibility to estimate a nested logit model with the survey data. Therefore, from now on, this research will focus on the estimation of multinomial logit models.

In the final survey a third alternative was added to each choice set, namely the opt-out alternative. The opt-out alternative is included to simulate the situation where the respondent decides to buy the product at a physical store and not from an online retailer. This may happen when alternative 1 and 2 do not appeal to the respondent. It was chosen to present the opt-out option in a separate question for each choice scenario instead of presenting the two alternatives and opt-out option simultaneously. The reason for this was to prevent respondents from choosing the opt-out option just because they find it hard to make a choice. This would cause the estimation of the parameters to be impossible when everyone selects the opt-out and there is not enough choice data available for alternative 1 and 2.

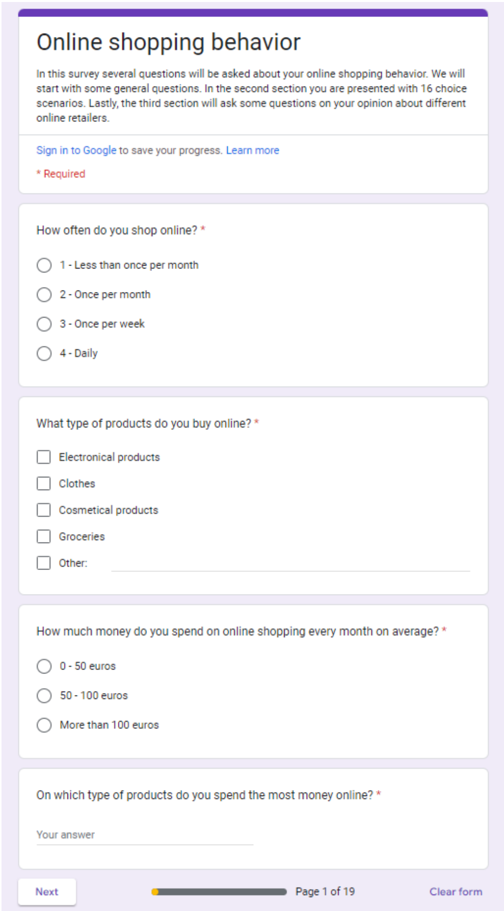
Background attributes

Background variables may influence people's behavior as well. These variables are not varied across the choice sets, but differ across individuals. The background variables that were included in this research concern the online spending habit of respondents. A section will be added at the beginning of the survey where the respondents are asked about their frequency of online shopping, what type of products they buy online and how much money they spend on online shopping on average every month.

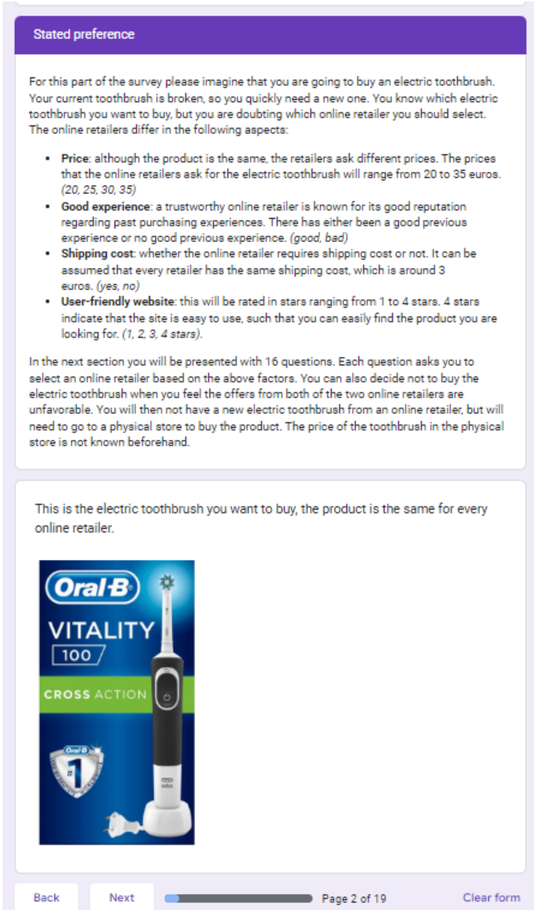
It was decided not to include any sociodemogrpahic variables in the survey. Such variables describe the social and economic characteristics of a population such as age, gender and income. Including the measurement of such variables would make the survey too long. Furthermore, it is not expected that the sociodemographic factors will have a significant influence on the choice behavior of respondents as a necessary, inexpensive, and gender-neutral product was selected. By excluding sociodemographic variables from the survey, the research can focus on understanding the preferences of respondents regarding the online retailer, rather than on how sociodemographic factors may impact the decision-making.

5.1.2 Survey design

The survey consists out of three sections. The first section focuses on the background variables regarding the online spending habit of the respondent. A short introduction to the survey is provided to the respondent and general questions about the spending habit of the respondent follow. The second part of the survey focuses on the 16 choice sets. Firstly, the situation and the influencing factors are explained to the respondent . Subsequently, the 16 choice sets are presented and the respondent is asked to select the preferred option.



(a) General introduction and questions measuring spending habit



(b) Explanation choice scenarios

Figure 3: First part survey

The last part of the survey focuses on the opinion of the respondent regarding specific online retailers. This section was added to gain insight in the initial preference of customers for certain online retailers and the reason of this preference. It should be noted that the names of the specific online retailers were provided to the respondents during the survey, but have been coded in this research to guide the data privacy of online retailer Circulus and its competitors.

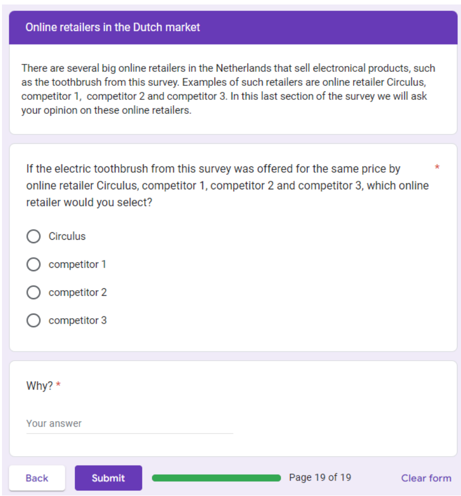


Figure 4: Questions specific online retailer

Testing the survey

To see whether the survey was comprehensible for the respondent, an initial version of the survey was tested using a small number of respondents. Five respondents were asked to fill out the pilot survey and to provide feedback afterwards. The following alterations were made based on the feedback:

- The initial version contained only numbers for the attribute levels. This turned out to confuse respondents. For the new version of the survey, it was chosen to use words 'good/bad' for the attribute levels of 'good experience' and 'yes/no' for the attribute levels of 'shipping cost'. For the attribute 'user-friendly website' it was decided to use stars to represent the levels.
- In the explanatory section before the choice tasks, no value was included for the shipping cost in the pilot version of the survey. In the new version of the survey a value of 3 euros was included for the shipping cost, such that the respondent could take a more informed decision. It should be noted that this value of 3 is smaller than the steps of 5 with which the price of the product differs in the survey. This was done to overcome interference.
- In the pilot version of the survey it was not clarified what the opt-out alternative meant. This lead to confusion among the respondents. Some respondents thought it meant you did not buy the product at all, while others interpreted it as buying it at another online retailer than the provided 2 options. In the new version of the survey, the opt-out version was defined as having to buy the product in a physical store and the price is then not known beforehand. This information was added to the explanatory section before the choice tasks.
- In the first version of the survey all 16 choice sets were presented on the same page. It turned out that the values from the previous choice set confused the respondent when filling out the next question. To solve this problem, the 16 choice sets were each presented on a separate page in the new version of the survey.

5.1.3 Survey distribution

The survey was distributed with a convenience sampling approach. The survey was distributed within the direct network of the researcher, which allowed for an efficient and cost-effective sampling. Additionally, employees from online retailer Circulus filled out the survey, which was possible due to the unlabeled alternatives that were used. Employees of online retailer Circulus would not be biased in this way.

It should be noted that this approach may limit the generalizability of the results as the direct network might not be an accurate representative of the larger population in terms of gender, age, income or education. However, as aforementioned, it was not expected that demographic variables would have a significant influence in this research. Convenience sampling was therefore reviewed as a sufficient method in this case.

The survey took about 10 minutes to fill out for the respondent, which could be done via a smartphone or computer. The survey was of an anonymous nature, such that no e-mail address or any other personal information about the respondent was required.

Required sample size

The minimum sample size was determined through the formula from Johnson & Orme (2010). This formula states the following:

$$\frac{n * t * a}{l} \geq 500 \tag{39}$$

Where n represents the number of respondents required, t represents the number of choice tasks, a represents the number of alternatives and l represents the highest number of levels in the choice set. Filling out this formula with the values from this research, results in:

$$n \geq \frac{500 * l}{a * t} = \frac{500 * 4}{2 * 16} = 63 \tag{40}$$

The formula suggests a minimum of 63 respondents in order to get a reliable estimate of the parameters. However, the larger the sample size the smaller the error of the estimates will be (Rose & Bliemer, 2007). Therefore, this research aimed to get as many respondents as possible.

6 Estimation discrete choice model

With the survey data, the multinomial logit model will be estimated to determine how the respondents choose between online retailer 1, online retailer 2 or a physical store based on the four attributes. Firstly, the survey data will be evaluated in the next section.

6.1 Sample characteristics

In total 97 people filled out the survey. The survey started with some questions about the general shopping behavior of the respondents. The first question concerned the online shopping frequency of the respondent and provided the respondent with four options: less than once per month, once per month, once per week or daily. Most respondents reported a shopping frequency of once per week, as can be seen in the chart below.

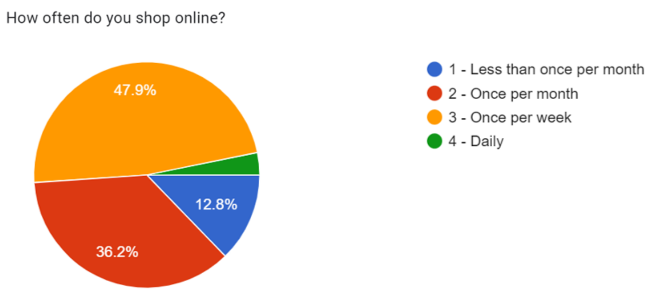


Figure 5: Frequency online shopping

The respondents were also asked about the type of products that they bought online. The question provided options that the respondent could select such as electronic products or clothes, but also included an 'other' option, such that respondents could write down a specific answer. The respondent was allowed to select more than one option. This led to a great variety of answers ranging from sporting gear to cleaning supplies and study tools. The most frequently mentioned product groups concerned electronic products and clothes. More than 70 percent of the respondents indicated to buy these products online. Also, cosmetic products and groceries were often mentioned by the respondents as products that they bought online.

Furthermore, the respondents were asked about the average amount of money they spent on online shopping every month. The respondents were provided with three options: between 0 and 50 euros, between 50 and 100 euros or more than 100 euros. The results of the survey indicated that the respondents were more or less evenly distributed among all three options. Approximately a third of the respondents selected each choice. This indicates a diverse set of spending amount among the respondents.

Figure 6: Average amount of money spend online

Lastly, the respondents were asked on which type of products they spend the most money online. This question was open-ended to gain a deeper understanding of the online shopping behavior of respondents. A wide variety of product types was mentioned, such as aids for studying, diapers, puzzles, vintage clothes, gifts and household items.

In the final section of the survey the respondents were asked to select their preferred online retailer and why they preferred this retailer. Many respondents indicated they preferred online retailer Circulus, which was not surprising as the survey was also send to employees of online retailer Circulus. However, the exact percentage of respondents that consisted out of online retailer Circulus employees is unknown as the survey was anonymous.

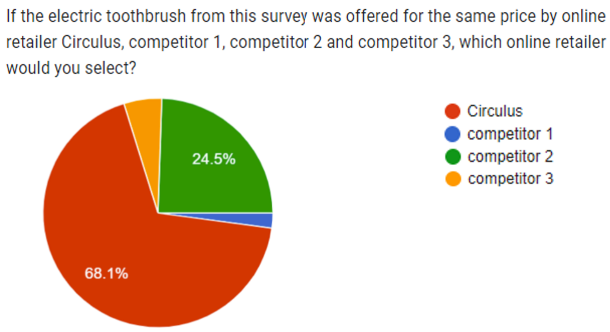


Figure 7: Preferred online retailers

The reasons that were indicated for selecting online retailer Circulus, described its big assortment, often the cheapest price, good reputation, user-friendly website, trustworthy and the benefits as a special member of online retailer Circulus. A special member of online retailer Circulus pays a yearly fee in exchange for benefits such as no shipping cost, extra discounts and free returns.

Competitor 1 was not often preferred by the respondents. The small number of respondents that did select competitor 1, indicated that they preferred competitor 1 for its broad assortment and fast delivery. However, other respondents mentioned they had bad experiences with competitor 1 in the past and therefore preferred online retailer Circulus or competitor 2.

The reasons indicated for selecting competitor 2 concerned their favorable warranty policy, good customer service and their guidance in choosing the suitable product.

Competitor 3 was not often selected, but an advantage that was mentioned for this competitor was the ability to go to a physical store with a defect product, even if bought at the online platform from competitor 3. Also the option to pick up an online order in the store was mentioned as an advantage.

6.2 Method model estimation

A tool that is often used to estimate the multinomial logit model is the PandasBiogeme package from Bierlaire (1998). This package is a library in python which provides methods for the estimation of different forms of discrete choice models, such as the multinomial logit model and the nested logit model. The package can be used to estimate the parameters of these models, to perform statistical tests and to generate predictions. The package estimates the parameters through maximum likelihood estimation as discussed in section 4.1.1. In order to use the package, the survey data was converted into the appropriate format in Microsoft Excel.

6.3 Parameter expectation

In this paragraph, the expectation of the contribution of the four attributes to the choice of the respondent will be discussed. It was expected that the attributes price and shipping cost would have a negative sign, since an increase in costs will probably decrease the attractiveness of the online retailer and thus its utility. On the contrary, it was expected that the attribute good experience would have positive sign, as it was expected that consumers will prefer an online retailer that has provided them with good service in the past. Additionally, the attribute user-friendly website was expected to have a positive sign, since it was expected that the consumer would be more attracted to an online retailer when the product can easily be found on its website.

6.4 Different models

The multinomial logit model contains utility functions for the three alternatives. In order to test the possible distributions for the attributes in the utility functions, different models will be tested. In this paragraph, the different models and their systematic utilities will be discussed in order to get an overview. For each multinomial logit model there is a set $N = \{1, 2, 3\}$, which represents the three alternatives from the stated preference survey: online retailer 1 (1), online retailer 2 (2), opt-out alternative (3). For this set it holds that alternative $i \in N$.

Linear multinomial logit model

The base model (b) assumes a linear distribution for the four attributes and captures the main effect of each attribute. In a linear model it is assumed that the marginal utility is constant, indicating that the increase in utility when improving from rate 1 to 2 is equal to the increase in utility when improving from rate 3 to 4. Considering equation 21 for the systematic utility from the previous section, the utility function for online retailer i in the base model is then defined as follows:

$$V_i^b((p_i, ge_i, sc_i, uw_i); \beta) = ASC_{ON} + \beta_p \cdot p_i + \beta_{ge} \cdot ge_i + \beta_{sc} \cdot sc_i + \beta_{uw} \cdot uw_i \quad (41)$$

The attribute values of good experience (ge_i) and shipping cost (sc_i) are equal to 0 or 1. The attribute value of price (p_i) can be $20, 25, 30$ or 35. The attribute value of user-friendly website (uw_i) can be $1, 2, 3$ or 4. β is a vector representing the parameter value of each attribute, where $\beta \in \mathbb{R}$. The parameter value indicates the importance of each attribute for the choice of the respondent. Furthermore, ASC_{ON} represents the alternative-specific constant, which is the initial intention of a consumer to buy a product online (for online retailer 1 and 2) . The utility function of the opt-out option, which is going to a physical store, is equal to the alternative-specific constant ASC_{OUT}, as the attribute values for this alternative are all equal to zero.

Quadratic multinomial logit model

In the real world, it is often observed that the positive effect on the utility is often bigger when improving from rating 1 to 2 than when moving from rating 3 to 4 (Molin et al., 2017). In economics, this effect is called the law of diminishing returns. To test this effect quadratic terms were added for the attributes price and user-friendly website, which each contain 4 levels. For the attributes good experience and shipping cost no quadratic terms were added as these attributes concern binary variables and the law of diminishing returns only applies to variables with more than two levels. The systematic utility of the quadratic model (q) is described by the following function:

$$V_i^q((p_i, ge_i, sc_i, uw_i); \beta) = ASC_{ON} + \beta_p \cdot p_i + \beta_{pq} \cdot p_i^2 + \beta_{ge} \cdot ge_i + \beta_{sc} \cdot sc_i + \beta_{uw} \cdot uw_i + \beta_{uwq} \cdot uw_i^2 \quad (42)$$

It should be noted that both the linear term and the quadratic terms are included for the two attributes instead of replacing the linear terms. The reason for this, is that both the linear and non-linear effect can be captured this way. When only including the quadratic term, it is assumed that the linear effect is zero. This is often not a reasonable assumption.

Logarithmic multinomial logit model

Another approach that is often used to account for the law of diminishing terms is through the use of logarithmic terms for these attributes. Molin et al. (2017) found in their research that the model with a logarithmic term has an equally good fit as a model with the linear term and quadratic term and a better fit than the model with only linear terms. In the third model (l), logarithmic terms are used for the attributes price and user-friendly website, which each contain 4 levels. For the attributes good experience and shipping cost no quadratic terms were added as these are binary variables. This results in the following formula for the systematic utility of this model:

$$V_i^l((p_i, ge_i, sc_i, uw_i); \beta) = ASC_{ON} + \beta_{pln} \cdot ln(p_i) + \beta_{ge} \cdot ge_i + \beta_{sc} \cdot sc_i + \beta_{uwln} \cdot ln(uw_i) \quad (43)$$

It should be noted that for this model only the logarithmic terms are included for the two attributes and the linear terms are left out. This reduces the number of parameters and the complexity of the model.

The Biogeme codes that were used to estimate the three different models can be found in Appendix A.5.

6.5 Results multinomial logit models

The estimation reports from PandasBiogeme of the three different models can be found in Appendix A.6. The parameters were considered to be statistically significant when the p-value was below 0.05. Each significant p-value has been indicated with an asterisk in the table below.

	Linear MNL		**Quadratic MNL**		**Logarithmic MNL**	
Parameter	value	p-value	value	p-value	value	p-value
ASC_{ON}	2.99	0*	6.46	0.000287*	13.5	0*
β_{GE}	2.540	0*	2.550	0*	2.550	0*
β_{P}	-0.173	0*	-0.414	0.000738*		
β_{SC}	-0.466	1.06E-05*	-0.535	3.77E-06*	-0.482	5.47E-06*
β_{UW}	0.242	2.53E-07*	-0.011	0.972		
β_{PQ}			0.004	0.0489*		
β_{UWQ}			0.047	0.447		
β_{PLN}					-4.550	0*
β_{UWLN}					0.472	1.97E-06*

Table 9: Parameter Values

It should be noted that parameters B_{UW} and B_{UWQ} are not significant in the quadratic multinomial logit model. Also the parameter B_{PQ} is just below the significance value of 0.05. In contrast, all other linear parameters in the quadratic model are significant. Additionally, for the linear and the logit multinomial logit model all parameters are significant. The insignificant parameters might be an indication that the quadratic term does not describe the behavior of the attributes 'user-friendly website' and 'price' as well as the linear terms do. This will be investigated in the next section where the goodness of fit of the models will be compared.

All the parameters have the expected signs. The attributes price and shipping cost have a negative sign and the attributes good experience and user-friendly website have a positive sign. This indicates that when the price of the product increases or when shipping cost are included, this will negatively effect the utility. On the contrary, when the attributes good experience and user-friendly website increase, this will have a positive effect on the utility. The value of β_{GE} is relatively big compared to the other parameters in all three models. A possible explanation for this is that the predictor variable good experience might have strong effect on the utility. Another explanation could be that the predictor variable has a large range. However, this explanation seems unlikely as the attribute concerns a binary variable. Additionally, the relatively high parameter value could be caused by correlation in the model. If the variable good experience is highly correlated with another attribute, the parameter estimate might be inflated. This effect is called multicollinearity (Dabo-Niang et al., 2008). The correlation matrix has been checked for large values during the composition of the alternatives for the stated preference survey. This correlation matrix indicated the correlation between alternatives. However, this does not exclude the possibility of correlation between attributes. Typically, correlation between attributes are captured in nested logit models. However, as there was no foldover design added to the choice sets of the stated preference survey, it is not possible to determine the interaction effects in this survey.

The alternative-specific constant (ASC) indicates the initial preference of a respondent for an alternative. It describes the utility of an alternative when the contribution of all four attributes is equal to 0, capturing the other elements that may affect the preference of respondents. In the current case there were two alternative specific constants; one representing online shopping (ASC_{ON}) and one representing the opt-out option, which is shopping in a physical store (ASC_{OUT}). In all of the three models ASC_{OUT} was fixed at 0. If ASC_{ON} was a positive value, this indicated that the respondent would choose online shopping over shopping in a physical store. If ASC_{ON}

was a negative value, this indicated that respondents have a strong initial preference to shop in physical stores.
In all three estimated models, the ASC for online shopping was positive and significant, indicating the base preference of the respondent for online shopping. Consumers thus tend to prefer online shopping over a physical store when only considering elements other than the four attributes considered in this study. The respondent will only select the opt-out alternative when the attribute levels have such values that a higher utility is generated for the opt-out option than the value of the ASC for online shopping.
It should be noted that the ASC makes up only one part of the utility function and that the attributes effect the overall utility of each alternative. Therefore, no conclusions can be drawn about the initial preference of the respondent based solely on the ASC values. Further analysis is required.

6.5.1 Comparison of model fit

In order to select the best model, the model fit of the estimated models was compared. The statistical tests that were performed in order to do this, are described in Appendix A.7. It was found that the linear model is the best fitting model of the three considered models. The linear model was selected to assess the relative importance of the attributes.

6.6 Interpretation Results

In order to determine the contribution of the different attributes to the utilities, the utility range of each parameter will be evaluated. This utility range describes the possible values that the predictor variable can contribute to the utility given the parameter value and the range of the variable. To illustrate, if there is a binary predictor variable, then the range of the associated utility values is limited to two values. In the table below, the utility range has been composed for each attribute.

Attribute	Parameter	Min attribute value	Max attribute value	Min utility contribution	Max utility contribution	Utility range
Good experience	2.54	1	4	2.54	10.16	7.62
Price	-0.173	0	1	0	-0.173	0.173
Shipping cost	-0.466	0	1	0	-0.466	0.466
User-friendly website	0.242	1	4	0.242	0.968	0.726

Table 10: utility range

The utility range can be used to evaluate the relative importance of each attribute. Ranking the utility ranges, will give an estimate of the importance of each attribute. Ordering the utility ranges from high to low, gives the following order of attributes: good experience, user-friendly website, shipping cost and price. The attributes good experience and user-friendly website play an important role in the decision of the respondent. These attributes have a positive effect, so an increase in one of these variables will lead to a relatively big increase in the utility function. The attributes shipping cost and price have a relatively small contribution to the utility. These attributes were determined to have negative effect, so an increase in price or shipping cost will lead to a relatively small decrease in the utility function. All parameters have the expected sign.

6.7 Conclusion

The multinomial logit model with a linear distribution for the attributes in the utility functions was found to be the model that best fit the output data of the stated preference survey. The alternative specific constant and parameters that were estimated for this model, were all significant. It was found that the ASC regarding online shopping had a positive value, indicating the initial preference of respondents to shop online instead of in physical store. Furthermore, it was found that the attribute of good experience had the biggest contribution to the utilities of online retailer 1 and 2. This effect was positive. Also, the attribute user-friendly website had a positive contribution to the utility functions, but this effect was much smaller. The attributes shipping cost and price had a relatively small, negative contribution to the utility functions.

7 The online retailer game

The online retailer game will be used to model the Dutch online market of electronic products and the price setting behavior of the online retailers in this market. The first step, prior to the the composition of the game, was a literature review to identify the type of game to be selected. The performed literature review can be found in Appendix A.8. In this chapter the online retailer game will be composed. The game will be non-cooperative. This research aims to find a Nash equilibrium for this game. The online retailer game contains the following characteristics:

- **Player set:** the player set is described by $N = \{1, ..., n\}$, with $i \in N$ being an online retailer.
- **Strategy set:** for every player $i \in N$ the strategy set is given by $P_i \in [0, \overline{p_i}]$ with $\overline{p_i} \in \mathbb{R}_+$. 0 represents the minimal price as no negative prices can be set by the players of the game. It should be noted that the strategy set is continuous, such that each online retailer i can set any price $p_i \in P_i$ within the defined range. A vector of strategies is denoted by $p = (p_i)_{i \in N}$, with $p_{-i} = (p_j)_{j \in N \setminus \{i\}}$. The set of all strategies is denoted by $P = X_{i \in N} P_i$.
- c_i**:** for every player i the cost of the product is denoted by $c_i \in \mathbb{R}_+$.
- β_p**:** the price parameter indicates the price sensitivity of the consumers, with $\beta_p \in \mathbb{R}_+$.
- β_0^i**:** this parameter indicates the initial attractiveness of online retailer i, with $\beta_0^i \in \mathbb{R}_+$.
- **Payoff function:** For every player $i \in N$, the payoff function $f_i : P \to \mathbb{R}_+$ is defined by the following formula[3]:

$$f_i(p) = \frac{e^{\beta_0^i - \beta_p \cdot p_i}}{\sum_{j \in N} e^{\beta_0^j - \beta_p \cdot p_j} + 1} \cdot (p_i - c_i), \ \forall p \in P \tag{44}$$

  The payoff function describes the expected payoff for online retailer i given its strategy and the strategies of the other players, which is calculated through multiplying the probability for player i to be selected by the consumers with the difference between the price and the cost of the product for player i. In this formula the value 1 is obtained through e^{β_0}, which represents the opt-out alternative. β^0, is fixed at 0 as discussed in the discrete choice model chapter.

- **Information:** Each player knows the possible prices they can set and the cost that they face. Also, each player knows the payoff function and how this function maps the prices chosen by the other players to their payoff. The prices chosen by the other players are not known. There is incomplete information.
- **Preferences:** It is assumed that each player i prefers p_i^* over p_i when it holds that $f_i(p_i^*, p_{-i}) > f_i(p_i, p_{-i})$ for a given p_i^*.
- **Timing:** the price choices in the game are made simultaneously, meaning the players set their prices at the same time.

Defining the online retailer game as a sextuple, gives the following output:

$$(N; (P_i)_{i \in N}; (f_i)_{i \in N}; \beta_p; (\beta_0^i)_{i \in N}; (c_i)_{i \in N}) \tag{45}$$

7.1 Existence Nash equilibrium

In order to evaluate the existence of a Nash equilibrium, two versions of the online retailer game were studied: one with a continuous strategy set and one with a discrete strategy set. The results will be discussed in the next section.

[3] It should be noted that the payoff function could also be multiplied by the number of online retailers N. However, this was excluded from this research as it is a constant and will therefore not affect the relative ratios of the revenues.

7.1.1 Online retailer game with a continuous strategy set

In this section the online retailer game with a continuous strategy set will be considered. This game has been composed in the previous section. The Nash equilibrium is a solution concept you can determine for a game. One possible approach to investigate the existence of a Nash equilibrium is to analyze the payoff functions of the players using a theorem. Arrow & Debreu (1954) published a theorem to prove the existence of a Nash equilibrium.

Theorem Arrow & Debreu (1954), Pure Nash Equilibrium Existence: *Consider the strategic game $(N; (S_i)_{i\in N}; (g_i)_{i\in N})$ with N the finite players set, S_i the strategy set for player i and g_i the payoff function for player i. When for each $i \in N$ the following applies, then there exists a Nash equilibrium:*

1. *$g_i(s_i, s_{-i})$ is quasi-concave in s_i*
2. *$g_i(s_i, s_{-i})$ is continuous in s_i*
3. *Strategy set S_i is convex, non-empty and compact*

If the online retailer game meets these three criteria it can be proven that a Nash equilibrium exists. To evaluate whether this is the case, three Lemmas shall be introduced that each tackle one of the criteria.

Lemma 1: In the online retailer game $(N; (P_i)_{i\in N}; (f_i)_{i\in N}; \beta_p; (\beta_0^i)_{i\in N}; (c_i)_{i\in N})$, the payoff function f_i is quasi-concave in p_i, $\forall i \in N$.

Proof: A function f is quasi-concave if it is non-decreasing up till a certain point x_0 and then non-increasing afterwards (Arrow & Enthoven, 1954). A function is non-decreasing on the interval $[a, b]$, if it holds that $g(x) \leq g(x')\ \forall x, x' \in [a, b]$, where $x' \geq x$. A function is non-increasing on the interval $[a, b]$, if it holds that $g(x) \geq g(x')\ \forall x, x' \in [a, b]$, where $x' \geq x$. The payoff function f_i can be proven to be quasi-concave if f_i is non-decreasing for $x \leq x_0$ and f_i is non-increasing for $x > x_0$, for some $x_0 \in \mathbb{R}_+$.

Let $i \in N$. Then, the payoff function for player i is given by:

$$f_i(p) = \frac{e^{\beta_0^i - \beta_p \cdot p_i}}{e^{\beta_0^i - \beta_p \cdot p_i} + (\sum_{j\in N\setminus\{i\}} e^{\beta_0^j - \beta_p \cdot p_j}) + 1} \cdot (p_i - c_i),\ \forall p \in P \tag{46}$$

Firstly the derivative over p_i of f_i will be studied. In order to evaluate it, the function shall first be simplified. Let $C = (\sum_{j\in N\setminus\{i\}} e^{\beta_0^j - \beta_p \cdot p_j}) + 1$, then the function can be formulated as:

$$f_i(p) = \frac{e^{\beta_0^i - \beta_p \cdot p_i}}{e^{\beta_0^i - \beta_p \cdot p_i} + C} \cdot (p_i - c_i),\ \forall p \in P \tag{47}$$

The derivative over p_i of this simplified function is represented by the function below. The derivations to arrive at this formula can be found in appendix A.9:

$$\frac{d}{dp_i}(f_i(p)) = \frac{e^{\beta_0^i}(e^{\beta_0^i} + e^{\beta_p p_i}(\beta_p \cdot C \cdot (c_i - p_i) + C))}{(e^{\beta_0^i} + Ce^{\beta_p \cdot p_i})^2} \tag{48}$$

Firstly, it should be noted that the constant C in equation (48) will always be positive as it is a sum of positive terms, namely the sum of exponential functions and the positive constant $+1$. Additionally, the denominator of function (48) will always result in positive values as it consists of a sum of exponential functions, which is squared.

In order to meet the condition of quasi-concavity, it should be proven that the function (48) is positive up till a certain point x_0 and negative after, such that the function (46) is non-decreasing and then non-increasing. To evaluate this, it suffices to focus on the numerator, as the denominator has been identified as a strictly positive term. Considering the first term of the numerator (1) $e^{\beta_0^i}$, it is observed that this term will always result in positive values as it is an exponential function. Therefore, it suffices to focus on the second term of the numerator (2) $e^{\beta_0^i} + e^{\beta_p p_i}(\beta_p \cdot C \cdot (c_i - p_i) + C))$. Firstly, term 2 shall be rewritten, which gives the following result:

$$e^{\beta_0^i} + e^{\beta_p p_i}\left(\beta_p \cdot C \cdot \left(c_i - p_i + \frac{1}{\beta_p}\right)\right) \tag{49}$$

Substituting by $A = e^{\beta_0^i}$, $D = \beta_p \cdot C$ and $c_i' = c_i + \frac{1}{\beta_p}$, gives:

$$A + e^{\beta_p p_i} \cdot (D \cdot (c_i' - p_i)) \tag{50}$$

Note that $A \geq 0$, because the exponential function results in positive values. Additionally, $D \geq 0$ as it was defined for the online retailer game that $\beta_p \in \mathbb{R}_+$ and it was previously stated that $C \geq 0$.

Consider the situation when $p_i = 0$ for the above function (50). For the online retailer game it was defined that $c_i \geq 0$ and $\beta_p \in \mathbb{R}_+$. Therefore, the term c_i' in the above function (50) will always result in positive values. Subsequently, when $p_i = 0$, the term $(c_i' - p_i)$ will result in positive values. Lastly, the multiplication by the positive terms D and $e^{\beta_p p_i}$ and the addition of the positive term A will cause the output of function (50) to be positive when $p_i = 0$.

The next step will be to investigate the behavior of function (50) for different values of p_i. This will be done through taking the derivative over p_i of function (50):

$$-e^{\beta_p \cdot p_i} \cdot D \cdot (\beta_p(p_i - c_i') + 1) \tag{51}$$

Given that $D = \beta_p \cdot C$, the above function can be rewritten as:

$$-e^{\beta_p \cdot p_i} \cdot \beta_p \cdot C \cdot (\beta_p(p_i - c_i') + 1) \tag{52}$$

From function (52) it can be the case that the function is either (a) first positive and then becomes negative and stays negative or (b) it starts of negative and remains negative. To determine whether function (52) meets situation (a) or (b), the behavior of the term $(p_i - c_i')$ in function (52) should be considered. When $p_i < c_i'$ the term $(p_i - c_i')$ in function (52) will result in a negative value. Subsequently, this negative value is multiplied with the price parameter β_p, where $\beta_p \in \mathbb{R}_+$. The resulting value of the term $\beta_p(p_i - c_i')$ will therefore be negative when $p_i < c_i'$. Subsequently, a value of 1 is added to the term $\beta_p(p_i - c_i')$. If the negative value of $\beta_p(p_i - c_i')$ is smaller than -1, the addition of the positive term $+1$ will cause a negative value. If the negative value of $\beta_p(p_i - c_i')$ is bigger than -1, the addition of the positive term $+1$ will result in a positive value. Consider the situation where the term $(\beta_p(p_i - c_i') + 1)$ results in a negative value. The multiplication by the positive terms β_p and C will result in a negative value. However, the multiplication by the negative term $-e^{\beta_p \cdot p_i}$ will result in a positive value for function (52). Additionally, Consider the situation where $(\beta_p(p_i - c_i') + 1)$ results in a positive value. The multiplication by the positive terms β_p and C will then result in a positive value. In contrast, the multiplication by the negative term $-e^{\beta_p \cdot p_i}$ will result in a negative value for function (52). To summarize, it is observed that when p_i is much smaller than c_i, function (52) results in a positive value and when p_i is slightly smaller than c_i', function (52) results in a negative value. Furthermore, the behavior of function (52) will be analyzed when $p_i > c_i'$. The term $(p_i - c_i')$ will then result in a positive value. Subsequently, the term $(\beta_p(p_i - c_i') + 1)$ will result in a positive value. The multiplication by the positive terms β_p and C will result in a positive value. However, the multiplication by the negative term $-e^{\beta_p \cdot p_i}$ will result in a negative value for function (52). Considering the behavior of function (52), it can be concluded that there is a point up till which the function results in positive values. After this point the function results in negative values. The function (52) goes to $-\infty$ as p_i goes to ∞, because the term $e^{\beta_p \cdot p_i}$ and the term $(p_i - c_i')$ grow as p_i grows. It can be concluded that situation (a) applies for function (52) as the function is first positive and then negative.

Now that the behavior of function (52) has been analyzed, the behavior of function (50) will be explored. It was determined that function (52) is positive and then negative till infinity. Furthermore, it is proven that the payoff function is continuous (Lemma 2). Therefore, it can be concluded that there is a point p_i^*, where the function (52) is equal to $-A$. When this is the case, function (50) and thus the nominator of function (48) will be equal to zero, such that the derivative of the payoff function (48) results in a positive value. After this point the nominator of the payoff function (48) will result in a negative value as it was shown that function (52) goes to $-\infty$ as p_i goes to ∞. It was stated that the denominator always results in a positive value, therefore the payoff function (48) will be negative when the nominator is negative. It can be concluded that the payoff function (46) is non-decreasing up till the point p_i^* and non-increasing after, such that the function is quasi-concave over its entire domain. The same applies for the sub-domain $[0, \overline{p_i}]$ of the function, which is considered for the online retailer game.

Lemma 2: In the online retailer game $(N; (P_i)_{i \in N}; (f_i)_{i \in N}; \beta_p; (\beta_0^i)_{i \in N}; (c_i)_{i \in N})$, the payoff function f_i is continuous in p_i, $\forall i \in N$.

Proof: The payoff function of player i is a composition of continuous functions:

$$f_i(p) = \frac{e^{\beta_0^i - \beta_p \cdot p_i} \cdot (p_i - c_i)}{e^{\beta_0^i - \beta_p \cdot (p_i - c_i)} + (\sum_{j \in N \setminus \{i\}} e^{\beta_0^j - \beta_p \cdot p_j}) + 1}, \; \forall p \in P \tag{53}$$

The exponential functions $e^{\beta_0^i - \beta_p \cdot p_i}$, $e^{\beta_0^{-i} - \beta_p \cdot p_{-i}}$ are continuous, since $f(x) = a \cdot e^{b \cdot x}$ is continuous for any values of x and any given $a, b \in \mathbb{R}$ (Máté, 2015). Additionally, the term 1 is continuous.

Next, the denominator of the payoff function shall be evaluated on continuity. The denominator is the sum of continuous functions $f(x) = a \cdot e^{b \cdot x}$. In general, if there is a finite sum of continuous functions, its sum will also be continuous (Russell, 2020).

Subsequently, the nominator shall be investigated. The term $p_i - c_i$ is continuous in p_i as subtracting a constant from a continuous function results in a continuous function. The nominator therefore consists of two continuous terms, namely $e^{\beta_0^i - \beta_p \cdot p_i}$ and $p_i - c_i$. According to the product rule of continuous function, the product of two continuous function is a function that is continuous everywhere (Sleziak, 2003). It can be concluded that the nominator results in a continuous function.

Lastly, according to the quotient rule for continuous functions $h(x) = f(x)/g(x)$ is continuous for all x, when $f(x)$ and $g(x)$ are both continuous functions and $g(x)$ is always bigger than zero (Russell, 2020). The denominator $g(x)$ is a summation of terms in the form $a \cdot e^{b \cdot x}$, which always produces a positive value as output, is non-zero. It can be concluded that the payoff function is continuous everywhere in p_i.

Lemma 3: In the online retailer game $(N; (P_i)_{i \in N}; (f_i)_{i \in N}; \beta_p; (\beta_0^i)_{i \in N}; (c_i)_{i \in N})$, the strategy set P_i is non-empty, compact and convex.

Proof: The strategy set $P_i \in [0, \overline{p_i}]$ is non-empty, because $0 \in [0, \overline{p_i}]$. The strategy set contains always the element $\overline{p_i}$, so it can be concluded that the strategy space is nonempty.

The Heine-Borel theorem, states that a subset S of Euclidean space $\mathbb{R}^n$ is compact if and only if it is closed an bounded (Moore, 2007). The strategy set $P_i \in [0, \overline{p_i}]$ is said to be closed if it contains all its boundary points. In the current case 0 and $\overline{p_i}$ are the boundary points and are included, thus the strategy set $P_i \in [0, \overline{p_i}]$ is closed. Additionally, a strategy set is said to be bounded if there is an upper and lower limit, such that the strategy set lies within these limits. When the lower limit is set equal to -1 and the upper limit is set to $\overline{p_i} + 1$, the whole set $P_i \in [0, \overline{p_i}]$ lies within the bounded interval. Therefore, the strategy set $P_i \in [0, \overline{p_i}]$ is bounded. According to the Heine-Borel theorem it can be concluded that the strategy set $P_i \in [0, \overline{p_i}]$ is is compact as it has been proven that the strategy set is closed and bounded.

To determine whether the strategy set $P_i \in [0, \overline{p_i}]$ is a convex set, it needs to be determined that for any two points in P_i, the line segment that connects these two points on the real line, lies entirely within P_i. Let $p_1, p_2 \in [0, \overline{p_i}]$, then the set of points that connect these two points p_1 and p_2 on the real line, is given by:

$$\{\lambda \cdot p_1 + (1 - \lambda) \cdot p_2 \mid \lambda \in [0, 1]\} \tag{54}$$

For every element of the set of the above equation, it needs to be shown that it belongs to P_i. In doing so, it is observed that for any $\lambda \in [0, 1]$ the following holds:

$$\lambda \cdot p_1 + (1 - \lambda) \cdot p_2 \geq \lambda \cdot p_1 \geq 0 \tag{55}$$

This first part of the above equation holds as $p_2 \geq 0$ and $(1-\lambda) \geq 0$. The last part of the above equation holds as $\lambda \geq 0$ and $p_1 \geq 0$. Furthermore, it holds that:

$$\lambda \cdot p_1 + (1-\lambda) \cdot p_2 \leq p_2 \leq \overline{p_i} \tag{56}$$

The first part of the equation holds as $p_1 < p_2$ and $\lambda \leq 1$. The last part of the equation holds as $p_2 \leq \overline{p_i}$.

Hence, for every $\lambda \in [0, 1]$, it is known that $\lambda \cdot p_1 + (1-\lambda) \cdot p_2 \in [0, \overline{p_i}]$. Therefore, $P_i \in [0, \overline{p_i}]$ is a convex set.

Theorem: *The online retailer game* $(N; (P_i)_{i\in N}; (f_i)_{i\in N}; \beta_p; (\beta_0^i)_{i\in N}; (c_i)_{i\in N})$ *has at least one pure strategy Nash equilibrium.*

Proof: All three conditions of the theorem from Arrow & Debreu (1954) are met with Lemma 1, Lemma 2 and Lemma 3. Therefore, it can be concluded that at least one pure strategy Nash equilibrium exists for the online retailer game.

It should be noted that the theorem can only be used to prove the existence of a Nash equilibrium, nothing is known about the uniqueness of the Nash equilibrium.

7.1.2 Online retailer game with a discrete strategy set

A Nash equilibrium has been proven to exist for a game with a continuous strategy set. However, when evaluating the uniqueness of the Nash equilibrium it was determined that proving or disproving the uniqueness of the Nash equilibrium was not possible due to time constraints. Therefore, a numerical approach is considered in this section. This approach considers the online retailer game with a discrete strategy set, such that $P_i \in \{0, 1, ..., 40\}, \forall i \in N$. A lower bound of 0 is used for that strategy set as the online retailers cannot set a negative price. Additionally, an upper bound of 40 is set for the strategy set as this captures the total range of prices in the current market. The rest of the elements of this game are equal to the elements of the general game, such that the game can be formulated by the following tuple:

$$(N; (P_i)_{i\in N}; (f_i)_{i\in N}; \beta_p; (\beta_0^i)_{i\in N}; (c_i)_{i\in N}) \tag{57}$$

In order to identify the Nash equilibria of the discrete game a Python code was developed, which is provided in Appendix A.10. This code starts with an initial set of strategies and checks for every player whether it is possible to improve from its initial strategy, while keeping the strategy of the other players the same. The payoff of the player for each price from 0 to 40 is compared to the payoff for the player with its initial strategy. If each player cannot improve from the initial strategy vector, this indicates a Nash equilibrium. Every possible combination of strategies has been tested as an initial vector, such that the code was able to identify all possible Nash equilibria.

To evaluate the uniqueness of the Nash equilibrium of the discrete online retailer game, an initial game will be composed with the estimated parameters from the discrete choice model chapter. Subsequently, the effect of parameter changes on the uniqueness of the Nash equilibrium shall be tested. The following parameters were estimated with the discrete choice model:

- $ASC_{ON} = 2.99$
- $ASC_{OUT} = \beta^0 = 0$
- $\beta_{ge} = 2.540$
- $\beta_p = -0.173$
- $\beta_{sc} = -0.466$
- $\beta_{uw} = 0.242$

In order to compose the initial preference parameter β_0^i of consumers for each online retailer i, the following formula was used:

$$\beta_0^i = ASC_{ON} + 2.540 \cdot ge_i - 0.466 \cdot sc_i + 0.242 \cdot uw_i \tag{58}$$

For the discrete game a variety of 2, 3 and 4 online retailers was considered. These online retailers consisted out of online retailer Circulus and its three biggest competitors regarding electronic products: competitor A, B and C. The attribute values for the attributes good experience, shipping cost and user-friendly website are provided for each player in the table below.

Online retailer	**Player**	**GE**	**SC**	**UW**
Competitor A	1	0	1	2
Online retailer Circulus	2	1	1	4
Competitor B	3	1	0	4
Competitor C	4	1	1	3

Table 11: attribute values 4 players

These values were composed using insights from the last section of the survey. In this section respondents were asked to indicate their preferred online retailer and why. This gave insight in the probable values of the three attributes for each online retailer. Additionally, the websites of the online retailers were investigated to see whether there were shipping cost or not. Lastly, the composed values were evaluated together with the pricing experts of online retailer Circulus. These experts observe the online market daily and could therefore use insights from experience to determine whether the composed values were realistic. Using the values from the above table, the initial preference parameter could be estimated for the four online retailers under consideration.

The cost parameter was determined based on ratio estimates for the initial discrete game. It was assumed that the bigger the company, the lower the purchasing cost due to more suppliers and contract deals. The cost of online retailer Circulus, which were known, were used as a reference point for this. The relative size of the competitor compared to online retailer Circulus was estimated and this ratio was used to determine the purchasing cost for each competitor. To illustrate, when online retailer Circulus would have a cost of 5 euros per product and competitor A is twice the size of online retailer Circulus regarding market share, competitor A will face cost of 2.50 euros per product. This resulted in the following estimated costs for the four online retailers:

Online retailer	**Cost**
Competitor A	14
Online retailer Circulus	16
Competitor B	16
Competitor C	17

Table 12: estimated cost per online retailer

The complete set of estimated parameters, for the initial discrete online retailer game with 2, 3 and 4 players, has been listed in an overview in the table below:

Parameter	**2 players**	**3 players**	**4 players**
β_0^i	$\beta_0^1 = 3.008$ $\beta_0^2 = 6.032$	$\beta_0^1 = 3.008$ $\beta_0^2 = 6.032$ $\beta_0^3 = 6.498$	$\beta_0^1 = 3.008$ $\beta_0^2 = 6.032$ $\beta_0^3 = 6.498$ $\beta_0^4 = 5.79$
β_p	0.173	0.173	0.173
c_i	$c_1 = 14$ $c_2 = 16$	$c_1 = 14$ $c_2 = 16$ $c_3 = 16$	$c_1 = 14$ $c_2 = 16$ $c_3 = 16$ $c_4 = 17$

Table 13: estimated parameter values initial discrete online retailer game

In reality, the parameters may vary from these estimates. Therefore, the robustness of the game to parameter changes should be tested. A test framework was composed in order to observe the effect of parameter changes on the uniqueness of the Nash equilibrium of the discrete online retailer game. For each test the initial game was used as a basis and a single parameter was varied. The tests were performed for 2, 3 and 4 player games. The following parameter ranges were tested, which lead to a total of 480 testing instances:

Parameter	**Testing range**	**Effect**
Cost - c_i	(14, 15, 16, 17)	Varied across players
Initial preference - β_0^i	(1, 5, 9)	Varied across players
Price - β_p	(0.1, 0.2, ..., 0.9)	Same for each player

Table 14: Test framework parameters

The resulting Nash equilibria for each test can be found in Appendix A.11. For the test framework with varying cost, unique Nash equilibria were found for the game with 2 online retailers. This suggested that the two-player game is robust to cost parameter changes, meaning that the outcome will always converge to an unique Nash equilibrium. However, for the three player game several instances occurred where two Nash equilibria were found. This means that there were two sets of strategies where no player could improve their payoff by changing their strategy, given the strategies of the other players. The same effect occurred for the four player game. For the three player game two Nash equilibria occurred in 12.5 percent of the tested instances. For the four player game two Nash equilibria were identified for 9.4 percent of the tested instances. It can be concluded that the robustness of the game to cost parameter changes depends on the number of players.

For the test framework with varying values for the initial preference parameter β_0^i, a unique Nash equilibrium was found for each combination for the game with 2 players. In contrast, for the game with 3 or 4 players instances with two Nash equilibria occurred. For both the 3 and 4 player games two Nash equilibria occurred in 14.8 percent of the tested instances. The robustness of the game to changes in parameter β_0^i depends on the number of players.

Lastly, the test framework with varying values for the price parameter β_p resulted in solely unique Nash equilibria for the game with 2 or 3 players. For the game with 4 players and a varying price parameter, two Nash equilibria were found when the price parameter took a value of 0.1. Two Nash equilibria were identified for 11 percent of the testing instances with the four player game. The robustness of the discrete online retailer game to changes in parameter β_p depends on the number of players.

Answering sub-research question 4.b., whether the Nash equilibrium is unique, depends on the situation. Under certain parameter combinations and with a certain number of players a unique Nash equilibrium is identified, while with other instances two Nash equilibria occur. It could be the case that the two Nash equilibria are an effect of the discrete approach of this game. It is expected that when the step size is decreased an unique Nash equilibrium can be identified for each testing instance. This hypothesis was tested through reducing the size of the discrete steps of the strategy set to 0.5. Consider the instance from the test framework, where two Nash equilibria were identified at (20, 25, 25) and (20, 26, 26), when $c_1 = 14, c_2 = 16, c_3 = 16, \beta_0^1 = 1, \beta_0^2 = 5, \beta_0^5 = \beta_0^1 = 5, \beta_p = 0.173$ and the considered price range is $[0, 1, ..., 40]$. Now, assume that the price range is given by $[0, 0.5, ..., 40]$. With this smaller step size a unique Nash equilibrium is identified at (20.0, 25.5, 25.5). After checking all other instances with two Nash equilibria and studying the setting with a price range with smaller steps, it was concluded that a discrete game with a smaller step size allows for the identification of a unique Nash equilibrium for every tested instance. By reducing the step size, the discrete game will converge to the continuous game. Based on the numerical experiments of this study, it is therefore hypothesized that the continuous game contains a unique Nash equilibrium.

Sub research question 4.c focuses on which Nash equilibrium should be selected when multiple Nash equilibria are identified. For the current testing framework, the instances with two Nash equilibria were evaluated. The payoff of each player from one Nash equilibrium was compared to the payoff of each player from the other Nash equilibrium. This was done for each testing instance with two Nash equilibria. It was found that, in each instance, one of the equilibria resulted in a higher payoff for all players than the other Nash equilibrium. Typically, this was the Nash equilibrium with higher prices for each player. These Nash equilibria are said to be dominant in terms of payoff. The online retailers are expected to select this dominant Nash equilibrium as they all have the goal to maximize revenue. It should be noted that the approach of selecting the dominant Nash equilibrium is sub-optimal as it was found that reducing the step size of the discrete strategy set allows for the identification of an unique Nash equilibrium for each instance. This approach is preferable when time resources allow it.

7.2 The online retailer game for Circulus

In this section the discrete online retailer game will be used to model the situation for online retailer Circulus and its three biggest competitors regarding electronic products: competitor A, competitor B and competitor C. The Nash equilibrium will be identified for the composed game and will be used to determine a beneficial pricing strategy for online retailer Circulus.

For the composition of the discrete online retailer game the parameters from the previous section in Table 13 were used. Furthermore, a discrete strategy set of $P_i \in \{0, 1, ..., 40\}$ was applied for each of the four players was used. To identify the Nash equilibria, the python code in Appendix A.10 was applied. Two Nash equilibria were identified: $(20, 24, 25, 24)$ and $(20, 24, 26, 25)$. In the first Nash equilibrium competitor A sets a price of 20 euros for the electric toothbrush, online retailer Circulus and competitor C set a price of 24 euros and competitor B sets a price of 25 euros for the electric toothbrush. In the second Nash equilibrium competitor A sets a price of 20 euros for the electric toothbrush, online retailer Circulus sets a price of 24 euros, competitor B sets a price of 25 euros and competitor C sets a price of 26 euros. It should be noted, that in both Nash equilibria competitor A has the lowest price. A probable explanation for this, is the relatively low value for the initial preference parameter for competitor A. This forces competitor A to set a lower price to remain attractive to customers. The prices of online retailer Circulus, competitor B and competitor C have smaller differences, compared to the Nash equilibrium price of online retailer Circulus, due to their parameter values for initial preference and cost.

In order to determine which Nash equilibrium should be selected, the payoffs of the Nash equilibria were compared. The first Nash equilibrium at $(20, 24, 25, 24)$ results in a payoff of 0.17 for competitor A, 2.37 for online retailer Circulus, 3.57 for competitor B and 1.63 for competitor C. The second Nash equilibrium at $(20, 24, 26, 25)$ results in a payoff of 0.19 for competitor A, 2.63 for online retailer Circulus, 3.71 for competitor B and 1.74 for competitor C. The second Nash equilibrium at $(20, 24, 26, 25)$ is dominant regarding the payoff for each player. Therefore, it is reasonable to believe that this Nash equilibrium will be played by the online retailers. However, it was found in the previous section that the unique Nash equilibrium could be identified when reducing the step size of the discrete strategy set. This was also investigated for the considered game, such that the step size of the price range was reduced to 0.5. A unique Nash equilibrium was identified at: (20.0, 24.5, 25.5, 24.5). In this Nash equilibrium competitor A sets a price of 20.0 euros for the electric toothbrush, online retailer Circulus and competitor C set a price of 24.5 euros and competitor B sets a price of 25.5 euros for the electric toothbrush. Subsequently, the step size was reduced even further to steps of 0.25, which resulted in a Nash equilibrium at (20.0, 24.25, 25.5, 24.5). This new approach reveals the slight price difference between online retailer Circulus (24.25 euros) and competitor C (24.5 euros). It is observed that the smaller the step size, the more decimals the unique Nash equilibrium contains. The increase in decimals allows for the observation of small price differences in the Nash equilibrium prices of the online retailers. With the reduced step size the continuous online retailer game is approached. Due to constrained time resources, no smaller step size than 0.25 could be applied. It was decided to select the Nash equilibrium of (20.0, 24.25, 25.5, 24.5) to determine a beneficial pricing strategy for online retailer Circulus, as this estimation contained the most decimals due to the reduced step size of the price range.

7.2.1 Conclusion

It was proven that there exists at least one pure strategy Nash equilibrium for the online retailer game with a continuous strategy set according to the theorem of Arrow & Debreu (1954). Subsequently, the online retailer game with a discrete strategy set was composed for the four players competitor A, online retailer Circulus, competitor B and competitor C. Two Nash equilibria were found for competitor A, online retailer Circulus, competitor B and competitor C, respectively: $(20, 24, 25, 24)$ and $(20, 24, 26, 25)$. The Nash equilibrium at $(20, 24, 26, 25)$ turned out to be dominant regarding the payoff for each player of the game. Additionally, it was found that a unique Nash equilibrium could be identified for the game at (20.0, 24.25, 25.5, 24.5), through reducing the step size of the strategy set to 0.25.

8 Effects of up-pricing for online retailer Circulus

In this chapter the last sub-research question, regarding the effects of up-pricing for online retailer Circulus, will be answered. In order to determine this effect, firstly the robustness of the values of the Nash equilibrium to parameter changes will be considered. Additionally, several risks of up-pricing will be assessed. Lastly, the effect of different goals of the players on up-pricing will be considered.

In the previous chapter it was found that as the parameters vary, the uniqueness of the Nash equilibrium is affected, depending on the number of players in the game. Additionally, the strategies in the Nash equilibrium may be affected by the parameter changes. In the next section the robustness of the Nash equilibrium prices to parameter changes will be evaluated. This will be done through conducting several experiments with the composed test framework for the varying parameters c_i, β_0^i and β_p. For the following section, it should be noted that the parameter values are as defined in Table 13, unless mentioned otherwise. To illustrate, when the price parameter c_i is varied for the experiment, the other parameters are set equal to the values in Table 13.

8.1 Robustness test Nash equilibrium values, cost parameter c_i

For the test framework with the varying cost parameter c_i, it was observed that the strategies in the Nash equilibrium increased as the cost increased. To determine how much the Nash equilibrium prices increased as the costs increased, two experiments were performed on the test framework of the four player game. For both tests the instance where four players each have a cost of 14 euros was defined as the initial situation, the corresponding Nash equilibrium was identified at (20, 22, 23, 22). In the first experiment it was observed for each player how much their Nash equilibrium price increased when their cost increased by 1 euro, while the costs of the other players remained 14 euros. The costs were increased for each player individually, while the costs of the other players remained at 14 euros. For player 1, 2 and 3 it was observed that the Nash equilibrium price increased by 1 euro as the cost increased by 1 euro. However, for player 4 it was observed that the Nash equilibrium remained the same as the cost increased by 1 euro. This experiment was repeated through comparing how much the Nash equilibrium increased when the cost of one player increased by 2 or 3 euros, while the costs of the other players remained 14 euros. Again it was observed that the increase was lower for player 4 than for player 1, 2 and 3 when only one of the players faced increased costs.
For the second experiment, the Nash equilibrium price when all players have costs of 14 euros was compared to the Nash equilibrium price when all players have a cost of 15 euros. The costs were increased with the same value for all players at the same time. It was found that for each player The Nash equilibrium price increased by 1 euro as the cost for each player increased by 1 euro. This same effect was observed when comparing the Nash equilibrium price when all players have a cost of 14 euros to the Nash equilibrium prices when all players have a cost of 16 or 17 euros. The increase was then the same for each player.
It is concluded that as the costs increase the Nash equilibrium prices increases with a certain rate, where the rate depends on how the costs differ per player per instance. Therefore, cost variations are an important factor to consider for online retailer Circulus when applying up-pricing. Up-pricing will be beneficial when the cost remain stable or decrease, allowing for increased revenue. Additionally, up-pricing may still be beneficial when the costs increase slightly, a long as the Nash equilibrium prices are higher than the current price of the online retailer with the increased cost.

8.2 Robustness test Nash equilibrium values, initial preference parameter β_0^i

For the test framework with the varying initial preference parameter β_0^i, it was observed that the distance between the Nash equilibrium prices of the players increased as the parameter varied among players. Two experiments were performed with the four player game, where the situation in which all players have a value of 1 for β_0^i was defined as the initial situation. The corresponding Nash equilibrium was identified at (20, 22, 22, 23). In the first experiment, the value for β_0^i was increased to 5 for one player at a time, while the value for β_0^i remained the same for all other players. For player 1 an increase of 7 euros was observed for the Nash equilibrium price, for player

2 and 3 an increase of 6 euros and for player 4 an increase of 5 euros. The increase in the Nash equilibrium price differs among players as their value for β_0^i increases to 5, while the parameter value of β_0^i remains at 1 for the other players. This same effect was observed when the value of β_0^i was increased to 9, while the other players maintained a value of 1 for β_0^i.
In the second experiment, the situation where all players have a value of 1 for β_0^i, was compared to the situation where all players have a value of 5 for β_0^i and the situation where all players have a value of 9. The Nash equilibrium prices increased at different rates for the players when the value of β_0^i increased from 1 to 5 for all players together. For player 1 an increase of 2 euros was observed, for player 2 and 3 an increase of 1 euro and for player 4 no increase was observed. Additionally, when the value of β_0^i increased from 1 to 9 for all players together, the Nash equilibrium price of player 1 increased by 2 euros, the price of player 2 and 3 increased by 2 euros and the price of player 1 increased by 1 euro. It is observed that how much the Nash equilibrium price increases for each player, as the parameter β_0^i increases, depends on the combination of parameter values for β_0^i for all players in the game.
It is concluded that as the parameter β_0^i increases for a player, the Nash equilibrium price increases for that player, where the rate of increase depends on the combination of values of β_0^i from all players in the game. Therefore, when applying up-pricing the initial preference for online retailer Circulus and its competitors should be considered. It is more likely that up-pricing is beneficial in a setting where online retailer Circulus has a higher value for the initial preference parameter than the other online retailers.

8.3 Robustness test Nash equilibrium values, price parameter β_p

For the test framework with the varying price parameter β_p, it was observed that the strategies of the Nash equilibrium resulted in lower prices as the value of the price parameter increased. One experiment was applied for the four player game to evaluate the decrease in Nash equilibrium prices as the price parameter increased by steps of 0.1. It was observed that although the price parameter increased gradually for all four players, the Nash equilibrium prices of the player decreased at different rates. To illustrate, when the price parameter increased from 0.2 to 0.3 for all players, the Nash equilibrium shifted from (19, 23, 24, 23) to (17, 20, 21, 21). For players 1 and 4 the prices decrease by 2 euros, while the prices decrease by 3 euros for player 2 and 3. These different rates of decrease may be due to the influence of the differing values of the players for the other parameters c_i and β_0^i. The same effect was observed for the other test instances.
It is concluded that the Nash equilibrium prices of the players decrease as the price parameter β_p increases, where the rate of decrease differs per player. This indicates that in order for up-pricing to be a successful pricing strategy, the price sensitivity of the consumer should be considered. In case the demand of the product concerns a high level of elasticity, meaning that it is responsive to price changes, even a small increase in price could lead to a much bigger decrease in sales. Up-pricing will be beneficial for relatively inelastic products. Customers may then accept a higher price without reducing their amount of purchases significantly.

8.4 Risks up-pricing

For the considered situation with online retailer Circulus and its three biggest competitors, it can be concluded that up-pricing is a beneficial strategy for online retailer Circulus under the condition that each competitor sets its Nash equilibrium price and that the parameter values hold. However, in reality there may be alternate initial price scenarios that would not benefit from up-pricing. Consider the initial situation where each player currently has a price of 35 euros for the electric toothbrush. The identified Nash equilibrium at (20.0, 24.25, 25.5, 24.5) from the online retailer game would then advise the online retailers to lower their price instead of applying up-pricing. Additionally, the opt-out option for consumers constrains the online retailers to keep the prices below a certain level. When all online retailers offer a high unattractive price, the consumer may decide to buy the product at a physical store instead. This decreases the revenue of the entire online market. Due to this effect, up-pricing will only be beneficial until this level is reached. Furthermore, there is the risk of competitors deviating from their Nash equilibrium price. Competitors may adjust their price as online retailer Circulus increases its price to remain competitive. If the competitors do not set their Nash equilibrium price, this may lead to a price war between the players or losing market shares when consumers perceive the prices as too high.

To summarize, as the parameter values shift, the Nash equilibrium shifts. This can cause up-pricing to be potentially more or less beneficial. Therefore, the parameter values for each player and their impact on the online retailer game should be considered when analyzing the benefits and drawbacks of up-pricing. Also other risks such as becoming unattractive to customers due to too high prices or competitors deviating from their Nash equilibrium price, should be considered when analyzing the effects of up-pricing.
For the considered online retailer game, it is concluded that up-pricing is a beneficial strategy under the condition that the parameter values result in a Nash equilibrium with higher prices than the current prices of the online retailers and under the condition that each competitor selects its Nash equilibrium price.

8.5 Different pricing strategies

In the proposed online retailer game, each player has the same preference: revenue maximization. However, in reality it is observed that online retailers may differ in their pricing strategy. Typically, competitors may aspire to have the lowest price in the market to ensure attractiveness to customers.

Consider the situation where the fourth player has a different objective than the other three players. In particular, it is now assumed that player 1, 2 and 3 maintain the goal of revenue maximization, while player 4 will always set the price equal to the lowest price of the other three players, such that $p_4 = min(p_1, p_2, p_3)$. For this game it should be investigated whether a Nash equilibrium can be identified for player 1, 2 and 3 considering that player 4 will set the lowest price in the market. The discrete online retailer game was altered such that, player 4 (competitor C) has the strategy to set the lowest price in the market. The other players maintain the strategy of revenue maximization. A python code, which can be found in Appendix A.10, was composed to investigate the existence of a Nash equilibrium for this new setting.

In the new scenario an unique Nash equilibrium was identified at (20, 24, 25, 20). Additionally, with a reduced step size of 0.25 for the price range, a Nash equilibrium was identified at (20.0, 23.75, 24.75, 20.0). This Nash equilibrium was compared to the Nash equilibrium at (20.0, 24.25, 25.5, 24.5) of the online retailer game where all four players have the goal to maximize revenue. It was observed that player 1 maintained an optimal strategy of 20 euros as this still maximized its revenue when considering the prices of the other online retailers. In contrast, the Nash equilibrium prices of player 2 and 3 were slightly smaller as a response to the new strategy of player 4 to set the lowest price. Player 4 followed its new strategy and set a price of 20 euros in the new Nash equilibrium.

It can be concluded that for the considered scenario, up-pricing would still be a beneficial strategy for online retailer Circulus when setting a price of 23.75 euros. However, competitors with a different goal, such as setting the lowest price in the market, will not follow.

Other possible pricing strategies that the online retailers could apply concern setting a relatively high price to facilitate high quality customer service. Through pricing the products at a premium price, additional resources such as faster shipping or personalized shopping assistance can be allocated to enhance the shopping experience for the customer. This can contribute to customer loyalty. Another pricing strategy that may be applied is setting the average price of the price range observed among competitors. This pricing strategy aims to balance maintaining reasonable profits and attracting price-sensitive customers. Although this approach may be beneficial for the competitive position against other online retailers, a disadvantage could be the vulnerability to price fluctuations in the market. Lastly, a pricing strategy that may be applied by the online retailers is setting the price equal to the highest price in the market, such that $p_4 = max(p_1, p_2, p_3)$. This can be considered as a form of up-pricing. However, it differs from the up-pricing strategy considered in this research as setting the price equal to the highest price in the market does not consider the behavior of other competitors in the market. When the competitive dynamics of the market are not considered, the increase in price could lead to a loss of market share when price-sensitive customers switch to competitors with a lower price. Additionally, customer loyalty might be endangered when increasing the price without adding value.
It is expected that these different pricing strategies will affect the identified Nash equilibrium of

the online retailer game. However, due to limited time resources not all possible pricing strategies and their effect on the Nash equilibrium could be examined. For the product under consideration, the composed online retailer game and the selected pricing strategies were expected to sufficiently reflect the market dynamics in order to draw meaningful conclusions.

9 Discussion

In this chapter, the conclusions of the results will be composed and the research questions will be answered. Subsequently, the recommendation to online retailer Circulus is discussed. Also, the scientific contribution of this research will be evaluated. Lastly, the limitations of this research and implications for future research shall be assessed.

9.1 Conclusion

The goal of this research was to investigate the conditions under which up-pricing is a beneficial pricing strategy. This research was performed for the Dutch online retailer Circulus. The main research question was formulated in the following manner:

Main research question: *Under which conditions is up-pricing a beneficial strategy for online retailer Circulus?*

In order to answer the main research question, several sub-research questions were composed. In this section each sub-research question will be answered, in order to answer the main research question.

The first sub-research question investigated the factors that could influence the choice of a consumer to select an online retailer. Through an interview with a data specialist at online retailer Circulus and a rapid literature review it was found that there were 12 influencing factors that play an important role in how consumers choose to buy from a specific online retailer. The factors were listed in Table 4.

The next sub-research questions focused on which discrete choice models exist in literature to model consumer behavior. A literature review with a snowballing approach was used, which identified the multinomial logit model and the nested logit model. The multinomial logit model assigns a utility to each alternative, such that the probability to be selected can be determined for each alternative. The multinomial logit model assumes that the alternatives are independent. The nested logit model is an extension of the multinomial logit model and relaxes the assumption of independent alternatives, such that dependence among alternatives can be captured.

Sub-research question 3.a investigated which input data can be used to estimate the discrete choice model. It was found in literature that survey data was an appropriate approach to do this. Two main approaches were identified: revealed and stated preference surveys. A revealed preference survey gathers preferences of respondents based on observed choices in the real world, while stated preference surveys ask the respondents to indicate their choice for a predefined scenario. For the current research a stated preference survey was performed as no suitable data was available to perform a revealed preference survey. The stated preference survey was focused on four influencing factors: good experience, price, shipping cost and user-friendly website. Furthermore, the survey focused on one specific product: the electric toothbrush.

Subsequently, sub-research question 3.b assessed which discrete choice model represented consumer behavior the best. In order to answer this question, the resulting data from the stated preference survey was used to compose three different discrete choice models. These choice models differed in their distribution of terms for the utility functions. The base model assumed that all terms were linearly distributed for the utility function. The quadratic model extended the base model, through adding two quadratic parameters for the four-level attributes. Lastly, the logarithmic model assumed linear distributions for the binary attributes and a logarithmic distribution for the four-level attributes. Using statistical tests, it was found that the base model, with a linear distribution for all attributes in the utility function, fitted the survey data the best.

Sub-research question 4.a and 4.b investigated the existence and uniqueness of a Nash equilibrium of the online retailer game. The online retailer game was composed to model the Dutch online market of electronic products for online retailer Circulus and its three biggest competitors. Firstly, a game with a continuous strategy set was composed, such that each player i could choose any price $p_i \in P_i$, where $P_i \in [0, \overline{p_i}]$. For this game it was proven, according to the theorem of Arrow & Debreu (1954), that there exists a Nash equilibrium. However, due to time constraints the uniqueness of the Nash equilibrium could not be proven or disproven. For that reason, a discrete version of the online retailer game was considered, such that $P_i \in [0, 1, ..., 40]$. A python code was used to find the Nash equilibria for a test framework, where the number of online retailers and the parameters regarding cost, price and initial preference were alternated. This resulted in a unique Nash equilibrium for the games with two players. In contrast, for the games with three and four players, instances occurred where two Nash equilibria were found. Therefore, in order to answer sub-research question 4.c, regarding which Nash equilibrium to select, the instances with two Nash equilibria were evaluated. It was found that, in all cases, the Nash equilibrium with higher prices resulted in a higher payoff for each player of the game. Therefore, it is expected that the online retailers will select the strategy from this dominant Nash equilibrium. Additionally, it was found that with a smaller step size of 0.5 for the price range, a unique Nash equilibrium could be identified for each instance of the discrete online retailer game. Through reducing the continuous online retailer game could be approached. The smallest step size that could be applied considering time constraints was 0.25. With this step size a unique Nash equilibrium was identified at (20.0, 24.25, 25.5, 24.5), where competitor A charges a price of 20 euros, online retailer Circulus sets a price of 24.25 euros, competitor B a price of 25.5 euros and competitor C a price of 24.5 euros.

The last research question focused on the effects of up-pricing for online retailer Circulus. It should be noted that this effect could only be determined for the electric toothbrush under consideration. Currently, competitor A and online retailer Circulus have a price of 20 euros for this product, competitor B has a price of 26 euros and competitor C has a price of 23 euros. According to the unique Nash equilibrium of the discrete online retailer game competitor A should set a price of 20 euros, online retailer Circulus a price of 24.25 euros, competitor B a price of 25.5 euros and competitor C a price of 24.5 euros. This would result in an increased payoff for each player. Therefore, according to the findings of the game, it is expected that when online retailer Circulus applies up-pricing and increases its price to 24.25 euros, competitor C will follow this action and set a higher price. competitor B will decrease its price from 26 to 25.5 euros, but maintain a higher price than online retailer Circulus. In contrast, competitor A is expected not to follow and will maintain a price of 20 euros. It can be concluded that up-pricing is a beneficial strategy for online retailer Circulus in the modelled situation when the price of the resulting Nash equilibrium is selected by online retailer Circulus. However, the benefits of up-pricing were found to be strongly dependent on the parameter values. It was observed that as the cost parameter increased the Nash equilibrium prices increased for the online retailers to compensate, such that up-pricing may no longer be beneficial. Additionally, the Nash equilibrium prices decreased as the price parameter increased, which reduced the possibilities for up-pricing. In contrast, an increase in the initial preference parameter caused an increase in the Nash equilibrium prices, allowing for increased revenue when applying up-pricing.

It was shown that up-pricing is only a beneficial strategy when the current prices of the online retailers are lower than the prices of the identified Nash equilibrium. Subsequently, it was found that up-pricing is beneficial until a certain price level, which constrains the online retailers from setting a too high unattractive price. Another identified risk to the beneficial effects of up-pricing, considered the possibility of competitors deviating from their Nash equilibrium price to remain competitive.

Lastly, a unique Nash equilibrium was found for the game where the online retailers varied in their goal. In this altered game three players had the goal to maximize revenue, while the fourth player had the goal to set its price equal to the lowest price in the market. Up-pricing could then still be applied through setting a price of 23.75 euros for online retailer Circulus. However, competitors with a different goal, such as setting the lowest price in the market, will not follow.

To conclude, it depends on the parameter values whether up-pricing causes an increase in revenue. It was observed that parameter changes cause alterations in the prices of the Nash equilibrium. For the studied situation with online retailer Circulus the estimated parameter values caused the Nash equilibrium prices to be higher than the current prices of the online retailers, such that up-pricing will be a beneficial strategy. On the other hand, if the parameters for cost, initial preference and price would have been varied such that the Nash equilibrium prices were lower than the current prices of the competitors, down-pricing would have been the most beneficial strategy. To summarize, the success of up-pricing depends on the parameter values of the considered online retailer game. It is concluded that up-pricing is a beneficial strategy under the condition that the parameter values result in a Nash equilibrium with higher prices than the current prices of the online retailers and under the condition that each competitor selects its Nash equilibrium price. This answers the main research question of this study.

9.2 Recommendation online retailer Circulus

The model of this research suggests that up-pricing may be a beneficial strategy for online retailer Circulus as it increases revenue. However, the model also indicates that up-pricing is a sensitive matter. When the amount of customers decreases as all competitors increase their price, up-pricing may no longer be beneficial. Therefore, the situation should be evaluated thoroughly before increasing the price. The composed model suggests a price increase for Circulus for the studied toothbrush to 24.25. According to the composed Nash equilibrium, each online retailer will face an increase in revenue when all online retailer set their Nash equilibrium price. It is believed that the predictions of the composed model are accurate as long as the estimation of the costs are realistic. If this is the case, it could be wise for online retailer Circulus to set the Nash equilibrium price of 24.25 for the electric toothbrush to allow for an increase in revenue.

Additionally, online retailer Circulus is advised to use the composed framework to estimate the possibilities for up-pricing for other electronic products. It is expected that the parameters from the discrete choice model can be generalized to the electronic product groups of online retailer Circulus. These parameters and the game can be used by online retailer Circulus to determine the Nash equilibrium of prices for other electronic products as well.

If the proposed method turns out to be successful in practice, online retailer Circulus could decide to estimate the parameters for different product groups. This can be done through the use of a stated preference survey for the electric toothbrush, as proposed in this study. Additionally, the cost parameter could be estimated through taking their own average cost as a reference point, as was done in this research. It should be noted that this approach is time consuming. Therefore, it would be best to compose the model for several different products of different product groups, such that the results of a product can be generalized to the product group.

Furthermore, online retailer Circulus is advised to evaluate their performance on the different influencing factors that were tackled in this study. As it turns out, price is not the only important factor influencing consumer behavior. A good previous experience and an user-friendly website tend to play an important role in the decision of a consumer to buy from a specific online retailer. Online retailer Circulus could increase their attractiveness to consumers by scoring well on these attributes.

Lastly, there could be other influencing factors that play a role in the decision of a consumer when considering other products of online retailer Circulus. It was observed during the interview with the survey data expert of online retailer Circulus that much information is available on possible influencing factors and how online retailer Circulus scores on these factors in comparison to their competitors. Therefore, it might be useful to improve the communication between the pricing department and the survey data department. The pricing department would then have insight in the performance regarding the influencing factors and could alter the prices of products according to this performance and its expected effect on the choice of the consumer.

9.3 Scientific contribution

The scientific contribution of this research is the development of a model to analyze the behavior of consumers when selecting an online retailer for electronic products, considering the influencing factors price, good experience, shipping cost and user-friendly website. It was proven that the studied influencing factors had a significant effect on the behavior of a consumer when selecting an online retailer. Additionally, a model was created to analyze how competitors react to price changes, considering the influencing factors. It was proven that a Nash equilibrium could be found for this model.

The research has several practical implications, as it could help online retailers to understand the factors that influence consumer behavior and to make strategic decisions accordingly. Additionally, this research contributes to the existing literature on the integration of discrete choice modelling and game theory. A framework is provided that can be used to analyze various market segments. Overall, this research has made a contribution in the field of economics, decision sciences and game theory.

9.4 Limitations and implications future research

In this section the limitations that were encountered in this research and implications for future research will be discussed. Firstly, the limitations and implications for future research regarding the discrete choice model will be covered. Subsequently, the limitations and implications for future research regarding the online retailer game will be evaluated.

9.4.1 Discrete choice model

The first limitation that was encountered during the composition of the discrete choice model concerned the orthogonal fractional factorial design for the stated preference survey. The composed orthogonal fractional factorial design only captured the main effects between variables and not the interaction effects. This was done due to time constraints. To measure interaction effects a foldover design had to be added, which would mean that two different versions of the survey would have to be tested and twice the number of required respondents would be needed. It would be interesting for future research to investigate the interaction effects and how this influences the attractiveness of online retailers. This would allow for the estimation of a nested logit model.

A limitation of the stated preference survey was that it was focused on one product. This limited the generalizability of results as no conclusions can be drawn for other product groups of online retailer Circulus. It is expected that the parameter values will differ as the products differ. To illustrate, when considering a product from the product group toys the price parameter might be more important as consumers who buy toys often tend to be more price sensitive than consumers who buy an electric toothbrush. Furthermore, different parameters might be of importance when considering a different product. When considering toys the parameter regarding product safety might be important for instance. Future research could regenerate the results for different product groups to see how these differ from the results for electronic products.

The third limitation of the discrete choice model concerns the alternative-specific constant (ASC) in the multinomial logit model. It was found that the respondent had an initial preference for online shopping. This could be explained by the benefits that online shopping has over physical shopping, such as the small amount of time and effort required for online shopping compared to going to a physical store. Aside from an initial preference, it may be possible that the preference for online shopping is observed due to the fact that respondents were provided with little to no information about the physical store alternative in the stated preference survey. In real life more factors play a role in this decision. More research should be performed on whether people tend to prefer online shopping over going to a physical store and what factors play a role in this.

Considering the estimated parameters of the discrete choice model, price had a smaller parameter value than expected. There could be several possible explanations for this. It might be the case that respondents felt that 35 euros was still a reasonable price for the product, causing the other attributes to play a more important role in the decision of the respondents. However, it was observed that when both alternatives had a price of 35 euros, the opt-out option was often selected. Another explanation might be that the other factors were simply perceived as more important by the respondents, causing the attribute of price to have a relatively low value. Future research could investigate whether this same effect occurs for other products to get an insight in the cause of this effect.

9.4.2 Online retailer game

During the composition of the online retailer game, there were several limitations encountered that could be interesting topics for future research. Firstly, a limitation of the online retailer game with a continuous choice set was that the uniqueness of the Nash equilibrium could not be proven due to time constraints. The payoff functions of the online retailer game with a continuous choice set were proven to be quasi-concave, implying the existence of a global maximum. At this global maximum the payoff of the online retailer is maximized given the prices of the other online retailers. If it can be proven mathematically that the payoff functions intersect at their global maximum, this proves the uniqueness of the Nash equilibrium. The global maximum of each payoff function can be found through setting the derivative equal over p_i equal to zero and solving this function for p_i. Additionally, it should be proven whether the global maxima coincide to prove the uniqueness of the Nash equilibrium. Solving these functions and finding the intersections of the payoff functions for multiple online retailers turned out to be too time consuming during this research and therefore a numerical approach was used instead. It would be interesting for future research to use the proposed approach to see whether the Nash equilibrium of the online retailer game with a continuous strategy can be proven to be unique.

Furthermore, a limitation of the online retailer game with a discrete strategy set was that only the four biggest competitors were included. It would be interesting to see in future research what the effect of including smaller competitors would be. Including smaller competitors would allow for a more realistic market representation. Smaller competitors may exert competitive pressure on the larger competitors, which may disrupt the Nash equilibrium of the current online retailer game.

Additionally, in the discrete online retailer game, competitors that contain both a social platform and a physical store were excluded. As the online retailer game focused on online platforms, it was expected that including such a competitor would not be fitting as the initial preference for the hybrid store could not be represented in the game. It might be interesting in future research to see whether the outcome changes if also competitors with a physical store as well as an online platform would be included.

Moreover, a limitation in this research concerned the testing framework that was applied for the discrete online retailer game. For this testing framework the initial game was used as a basis and a single parameter was varied. Ideally, the parameters would have been varied at the same time and all combinations would have been tested. However, this was not possible in this research due to limited available resources. It was expected that the chosen selection of combinations would capture the effects of the varying parameters sufficiently. It would be interesting for future research to apply an extended testing framework with all possible combinations to see whether an unique Nash equilibrium can be identified and to see how the values of the Nash equilibrium change as the parameter values change.

Furthermore, a limitation of the discrete online retailer game was the step size of the discrete strategy set. Steps of 1 were used as the Python code would otherwise take too long. This turned out to complicate the identification of an unique Nash equilibrium. For future research it would be good to use smaller step sizes for the discrete strategy set, such that the unique Nash equilibrium can be identified for every instance of the testing framework.

Another limitation of the online retailer game concerned the selected goals for the online retailers. The initial discrete game considered the goal of revenue maximization for all four players. Additionally, an altered discrete game considered the situation where three players maintained the goal of revenue maximization, while the fourth player had the goal to set its price equal the lowest price in the market. For this altered game a unique Nash equilibrium could still be identified, but the prices were lower than the Nash equilibrium prices of the online retailer game where all online retailers had the goal to maximize revenue. The benefits of up-pricing also depend on the goals of the online retailers. In reality there are many more different goals that online retailers may apply. This may disrupt or alter the identified Nash equilibrium. Therefore, it would be interesting for future research to investigate how different goals for the online retailers affect the Nash equilibrium of the online retailer game and whether up-pricing can then still be a beneficial strategy.

Additionally, a limitation concerned the estimation of the cost parameter for the online retailer game. This was done through using the relative sizes of the companies and using the average cost of online retailer Circulus as a reference point. To test the robustness of the model to parameter changes, the test framework was applied and it was observed that the values of the Nash equilibrium shifted as the parameters changed. For the cost it was observed that as the costs increased, the Nash equilibrium increased. Therefore, it is important for future research to achieve a very accurate estimation of the cost parameter. This could be done through interviews with the different competitors. However, an issue that may arise concerns the data privacy of the online retailers. Also for the other parameters regarding initial preference and price, an accurate estimation is important, since it was observed that alterations in these parameters caused the Nash equilibrium to shift. It was found that the uniqueness of the Nash equilibrium was not affected by the parameter changes, but the values of the Nash equilibrium were. Future research should focus on obtaining manners to get as accurate parameter estimates as possible.

Lastly, the online retailer game supposes that the online retailers will set their Nash equilibrium prices. However, in reality there may be a wide range of factors that can cause retailers to deviate from the Nash equilibrium. Examples of such factors are that competitors may lack information about the action of competitors, consumer trends, strategic behavior or external factors such as economic conditions or government regulations. The Nash equilibrium provides a theoretical framework to gain insight in strategic actions, but for its real-life application the limitations caused by these factors should be considered. Future research could focus on identifying the factors and determining their effects on the applicability of a Nash equilibrium in the real world.

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A Appendix

A.1 Logit probabilities derivation

$$P(i) = \int_{\epsilon_i=-\infty}^{\infty} \left(\prod_{j \in C \setminus \{i\}} e^{-e^{-(\epsilon_i + V_i - V_j)}} \right) e^{-\epsilon_i} e^{-e^{-\epsilon_i}} d\epsilon_i \tag{59}$$

When i is included, this gives $V_i - V_i = 0$ (Train, 2009). Then the following holds:

$$e^{-e^{-(\epsilon_i + V_i - V_i)}} = e^{-e^{-(\epsilon_i + 0)}} = e^{-e^{-\epsilon_i}} \tag{60}$$

The formula can therefore be rewritten as:

$$P(i) = \int_{\epsilon_i=-\infty}^{\infty} \left(\prod_{j \in C} e^{-e^{-(\epsilon_i + V_i - V_j)}} \right) e^{-\epsilon_i} d\epsilon_i \tag{61}$$

Using the rule $\prod exp(x) = exp(\sum x)$, the formula is rewritten as:

$$P(i) = \int_{\epsilon_i=-\infty}^{\infty} exp \left(-\sum_{j \in C} e^{-(\epsilon_i + V_i - V_j)} \right) e^{-\epsilon_i} d\epsilon_i \tag{62}$$

$$P(i) = \int_{\epsilon_i=-\infty}^{\infty} exp \left(-e^{-\epsilon_i} \sum_{j \in C} e^{-(V_i - V_j)} \right) e^{-\epsilon_i} d\epsilon_i \tag{63}$$

Next defining $t = e^{-\epsilon_i}$ such that $dt = -e^{-\epsilon_i} d\epsilon_i$

$$P(i) = \int_{\epsilon_i=-\infty}^{\infty} exp \left(-t \sum_{j \in C} e^{-(V_i - V_j)} \right) - dt \tag{64}$$

It should be noted that when ϵ_i goes to ∞, t goes to 0 and as ϵ_i goes to $-\infty$, t goes to ∞:

$$P(i) = \int_{t=\infty}^{t=0} exp \left(-t \sum_{j \in C} e^{-(V_i - V_j)} \right) - dt \tag{65}$$

$$P(i) = \int_{t=0}^{t=\infty} exp \left(-t \sum_{j \in C} e^{-(V_i - V_j)} \right) dt \tag{66}$$

$$P(i) = \left. \frac{exp \left(-t \sum_{j \in C} e^{-(V_i - V_j)} \right)}{-\sum_{j \in C} e^{-(V_i - V_j)}} \right|_{t=0}^{t=\infty} \tag{67}$$

$$P(i) = \frac{1}{\sum_{j \in C} e^{-(V_i - V_j)}} = \frac{e^{V_i}}{\sum_{j \in C} e^{V_j}} \tag{68}$$

A.2 Orthogonal fractional factorial design

A common approach to compose the choice sets for a stated preference survey is a fractional factorial design. In this context, orthogonal indicates that the levels of the attribute are independent such that a change in the level of one attribute does not affect the levels of the other attributes (Gunst & Mason, 2009). To illustrate, when given the attributes price, shipping cost, good experience and user-friendly website, an orthogonal design ensures that a change in the level of price does not affect the levels of the other attributes shipping cost, good experience and user-friendly website. Orthogonal designs are an appropriate approach for choice experiments as they allow for an efficient estimation of the marginal effects of each attribute on the choice of the respondent. Additionally, fractional factorial designs allow for a reduced number of attribute level combinations to be tested, while still ensuring that the main effects of each attribute can be estimated. This is done through a systematic selection of a subset of possible attribute level combinations. The reduced number of choice sets, reduces the burden placed on the respondent.

When using the Ngene software package to compose an orthogonal fractional factorial design, the software uses the following steps to ensure that the design is orthogonal (ChoiceMetrics, 2018):

1. The user is asked to specify the attributes and each attribute level.
2. Ngene generates a candidate set of designs that includes all possible combinations of the attributes and attribute levels.
3. Ngene evaluates the different candidate designs to determine whether the criteria for orthogonality are met. These criteria include a balanced distribution of levels for each attribute and minimal correlations between the attributes. Lastly, the optimal design, which meets the orthogonality criteria, is selected from the candidate set by Ngene. This optimal design is the smallest fractional factorial design, which allows for the estimation of the main effect of each attribute.
4. After composing the orthogonal fractional factorial design, Ngene allows the user to edit the design. For instance, the user can modify the design through adding an opt-out option.

To summarize, Ngene generates candidate designs, evaluates them for orthogonality and selects the optimal design, which is the smallest fractional factorial design and meets the criteria of orthogonality. This approach ensures that the orthogonal fractional factorial design is balanced, efficient and allows for the estimation of the main effects of each attribute.

A.3 Choice sets

Ngene Syntax

```
? My design
Design
; alts = alt1, alt2
; orth = seq
; rows = 16
; model:

U(alt1) = beta_price * price[20, 25, 30, 35] + beta_goodexperience * goodexperience[0,1]
+ beta_shippingcost * shippingcost[0,1] + beta_sitequality * sitequality[1, 2, 3, 4]/

U(alt2) = beta_price * price + beta_goodexperience * goodexperience
+ beta_shippingcost * shippingcost + beta_sitequality * sitequality $
```

Figure 8: Syntax Ngene

Ngene output

Design								
Choice situation	alt1.price	alt1.goodexperience	alt1.shippingcost	alt1.sitequality	alt2.price	alt2.goodexperience	alt2.shippingcost	alt2.sitequality
1	20	0	0	1	25	0	0	4
2	30	1	1	1	30	0	1	3
3	20	1	0	3	35	1	0	3
4	30	0	1	3	35	1	1	4
5	35	0	0	1	20	1	1	4
6	25	1	1	1	30	0	0	4
7	35	1	0	3	35	0	1	2
8	25	0	1	3	25	1	1	1
9	20	1	1	4	20	0	0	1
10	30	0	0	4	20	1	0	3
11	20	0	1	2	30	1	0	2
12	30	1	0	2	25	1	0	2
13	35	1	1	4	35	0	0	1
14	25	0	0	4	20	0	1	2
15	35	0	1	2	30	1	1	1
16	25	1	0	2	25	0	1	3
Correlations (Pearson Product Moment)								
Attribute	alt1.price	alt1.goodexperience	alt1.shippingcost	alt1.sitequality	alt2.price	alt2.goodexperience	alt2.shippingcost	alt2.sitequality
alt1.price	1	0	0	0	0.2	0.111803	0.447214	-0.1
alt1.goodexperience	0	1	0	0	0.33541	-0.5	-0.25	-0.111803
alt1.shippingcost	0	0	1	0	0.33541	0	0	-0.33541
alt1.sitequality	0	0	0	1	-0.05	0	-0.111803	-0.55
alt2.price	0.2	0.33541	0.33541	-0.05	1	0	0	0
alt2.goodexperience	0.111803	-0.5	0	0	0	1	0	0
alt2.shippingcost	0.447214	-0.25	0	-0.111803	0	0	1	0
alt2.sitequality	-0.1	-0.111803	-0.33541	-0.55	0	0	0	1

Figure 9: Output Ngene

A.4 Stated preference survey

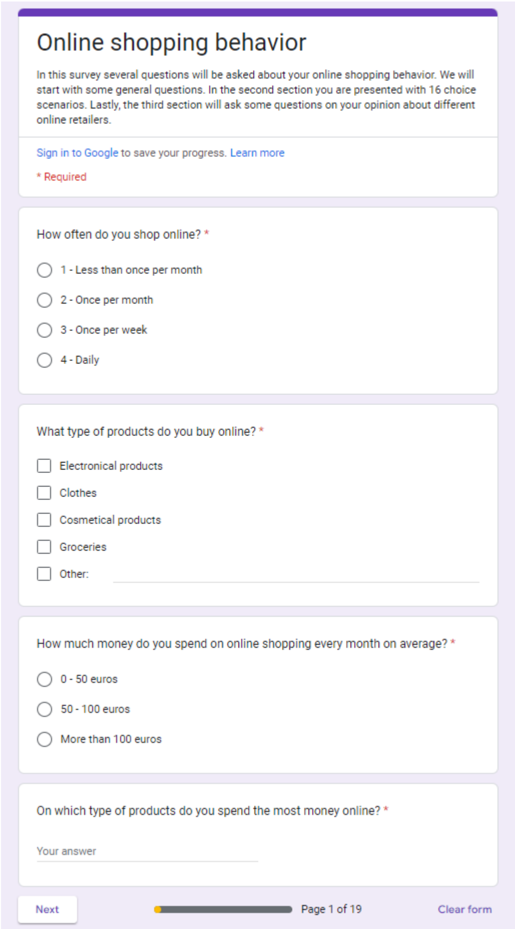

Figure 10: stated preference survey - part 1

Stated preference

For this part of the survey please imagine that you are going to buy an electric toothbrush. Your current toothbrush is broken, so you quickly need a new one. You know which electric toothbrush you want to buy, but you are doubting which online retailer you should select. The online retailers differ in the following aspects:

- **Price**: although the product is the same, the retailers ask different prices. The prices that the online retailers ask for the electric toothbrush will range from 20 to 35 euros. *(20, 25, 30, 35)*
- **Good experience**: a trustworthy online retailer is known for its good reputation regarding past purchasing experiences. There has either been a good previous experience or no good previous experience. *(good, bad)*
- **Shipping cost**: whether the online retailer requires shipping cost or not. It can be assumed that every retailer has the same shipping cost, which is around 3 euros. *(yes, no)*
- **User-friendly website**: this will be rated in stars ranging from 1 to 4 stars. 4 stars indicate that the site is easy to use, such that you can easily find the product you are looking for. *(1, 2, 3, 4 stars)*.

In the next section you will be presented with 16 questions. Each question asks you to select an online retailer based on the above factors. You can also decide not to buy the electric toothbrush when you feel the offers from both of the two online retailers are unfavorable. You will then not have a new electric toothbrush from an online retailer, but will need to go to a physical store to buy the product. The price of the toothbrush in the physical store is not known beforehand.

This is the electric toothbrush you want to buy, the product is the same for every online retailer.

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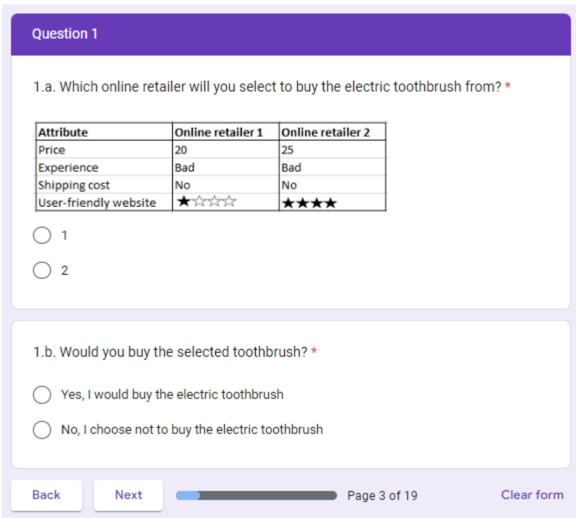
Figure 11: stated preference survey - part 2



Question 1

1.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	20	25
Experience	Bad	Bad
Shipping cost	No	No
User-friendly website	★☆☆☆	★★★★

○ 1

○ 2

1.b. Would you buy the selected toothbrush? *

○ Yes, I would buy the electric toothbrush

○ No, I choose not to buy the electric toothbrush

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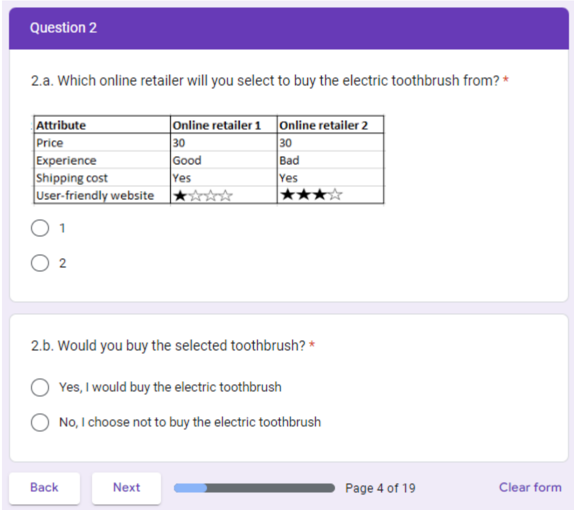
Figure 12: stated preference survey - part 3



Question 2

2.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	30	30
Experience	Good	Bad
Shipping cost	Yes	Yes
User-friendly website	★☆☆☆	★★★☆

◯ 1

◯ 2

2.b. Would you buy the selected toothbrush? *

◯ Yes, I would buy the electric toothbrush

◯ No, I choose not to buy the electric toothbrush

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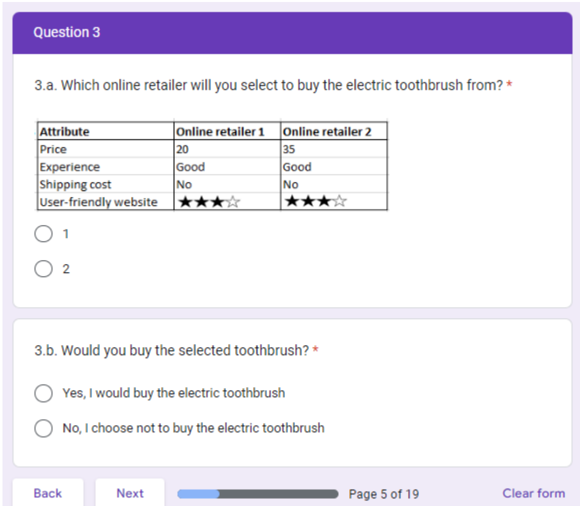
Figure 13: stated preference survey - part 4



Question 3

3.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	20	35
Experience	Good	Good
Shipping cost	No	No
User-friendly website	★★★☆	★★★☆

◯ 1

◯ 2

3.b. Would you buy the selected toothbrush? *

◯ Yes, I would buy the electric toothbrush

◯ No, I choose not to buy the electric toothbrush

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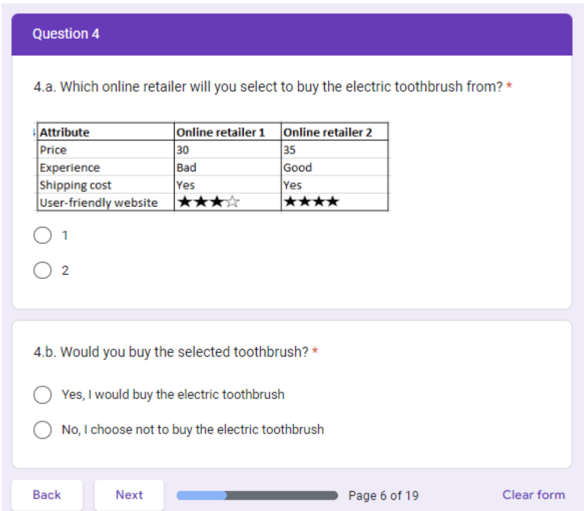
Figure 14: stated preference survey - part 5



Question 4

4.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	30	35
Experience	Bad	Good
Shipping cost	Yes	Yes
User-friendly website	★★★☆	★★★★

◯ 1

◯ 2

4.b. Would you buy the selected toothbrush? *

◯ Yes, I would buy the electric toothbrush

◯ No, I choose not to buy the electric toothbrush

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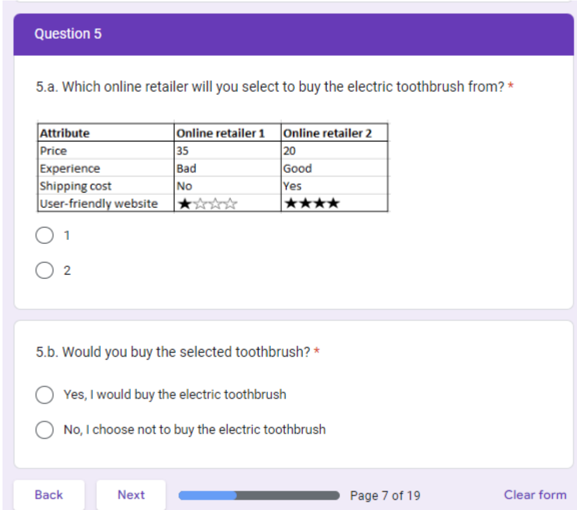
Figure 15: stated preference survey - part 6



Question 5

5.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	35	20
Experience	Bad	Good
Shipping cost	No	Yes
User-friendly website	★☆☆☆	★★★★

○ 1

○ 2

5.b. Would you buy the selected toothbrush? *

○ Yes, I would buy the electric toothbrush

○ No, I choose not to buy the electric toothbrush

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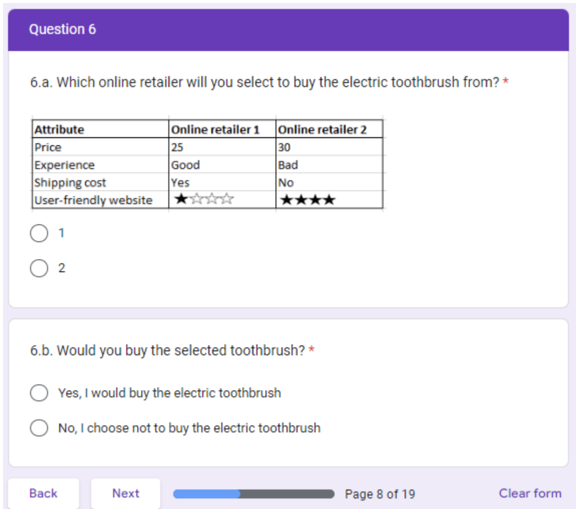
Figure 16: stated preference survey - part 7



Question 6

6.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	25	30
Experience	Good	Bad
Shipping cost	Yes	No
User-friendly website	★☆☆☆	★★★★

○ 1

○ 2

6.b. Would you buy the selected toothbrush? *

○ Yes, I would buy the electric toothbrush

○ No, I choose not to buy the electric toothbrush

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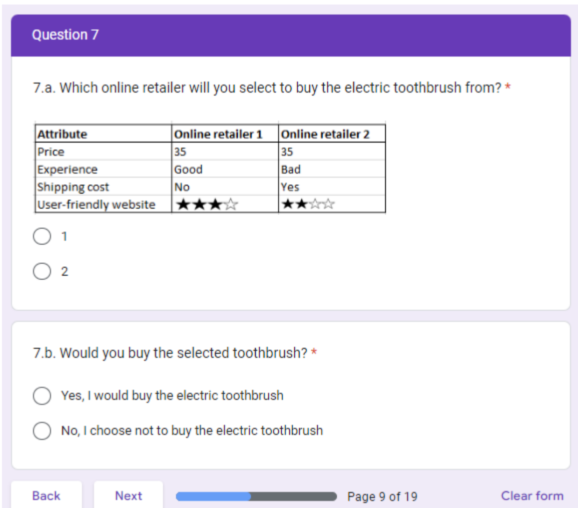
Figure 17: stated preference survey - part 8



Question 7

7.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	35	35
Experience	Good	Bad
Shipping cost	No	Yes
User-friendly website	★★★☆	★★☆☆

○ 1

○ 2

7.b. Would you buy the selected toothbrush? *

○ Yes, I would buy the electric toothbrush

○ No, I choose not to buy the electric toothbrush

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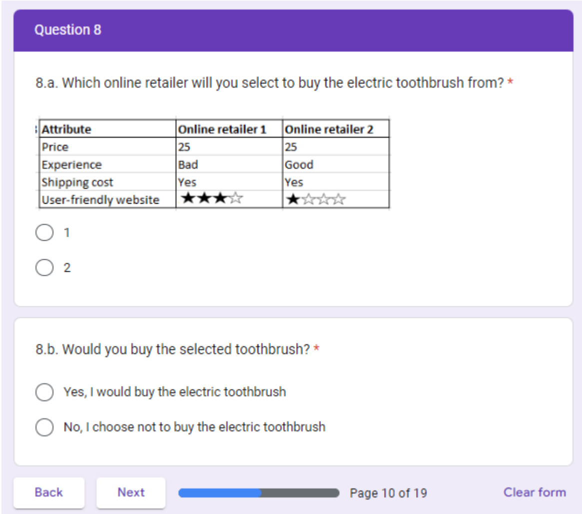
Figure 18: stated preference survey - part 9


Question 8

8.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	25	25
Experience	Bad	Good
Shipping cost	Yes	Yes
User-friendly website	★★★☆	★☆☆☆

1

2

8.b. Would you buy the selected toothbrush? *

Yes, I would buy the electric toothbrush

No, I choose not to buy the electric toothbrush

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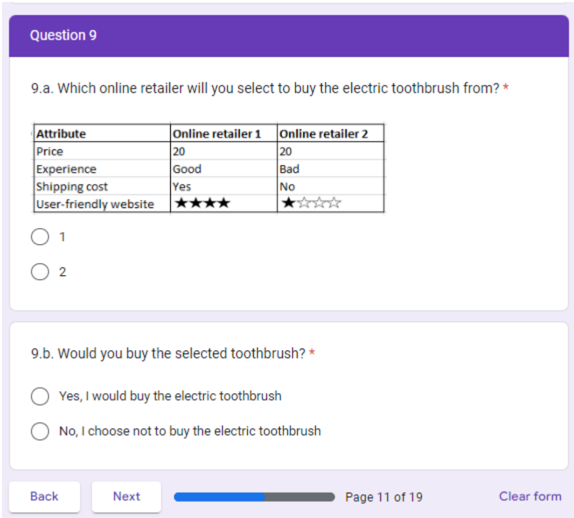
Figure 19: stated preference survey - part 10


Question 9

9.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	20	20
Experience	Good	Bad
Shipping cost	Yes	No
User-friendly website	★★★★	★☆☆☆

1

2

9.b. Would you buy the selected toothbrush? *

Yes, I would buy the electric toothbrush

No, I choose not to buy the electric toothbrush

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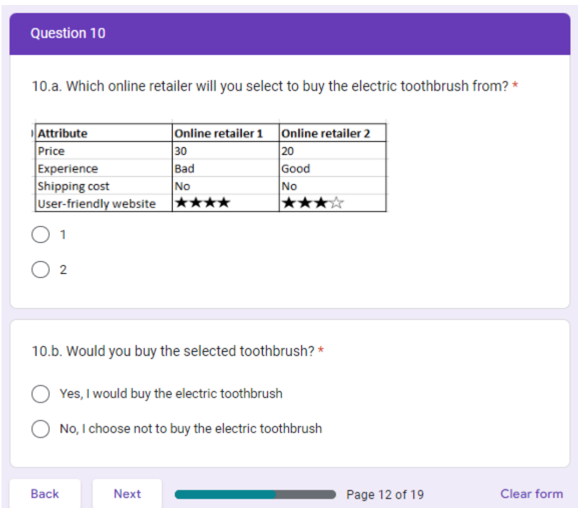
Figure 20: stated preference survey - part 11


Question 10

10.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	30	20
Experience	Bad	Good
Shipping cost	No	No
User-friendly website	★★★★	★★★☆

1

2

10.b. Would you buy the selected toothbrush? *

Yes, I would buy the electric toothbrush

No, I choose not to buy the electric toothbrush

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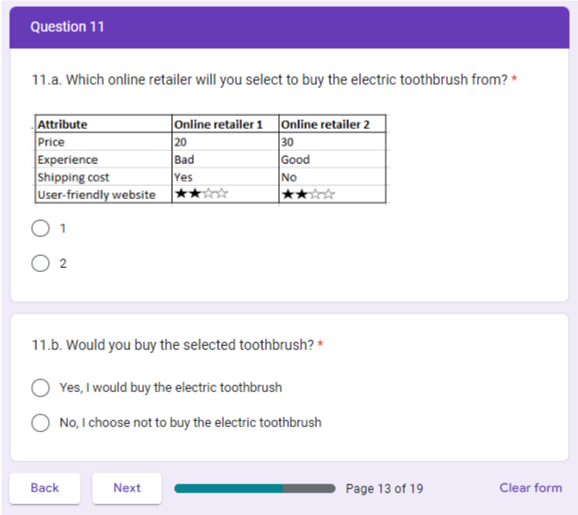
Figure 21: stated preference survey - part 12



Question 11

11.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	20	30
Experience	Bad	Good
Shipping cost	Yes	No
User-friendly website	★★☆☆	★★☆☆

◯ 1

◯ 2

11.b. Would you buy the selected toothbrush? *

◯ Yes, I would buy the electric toothbrush

◯ No, I choose not to buy the electric toothbrush

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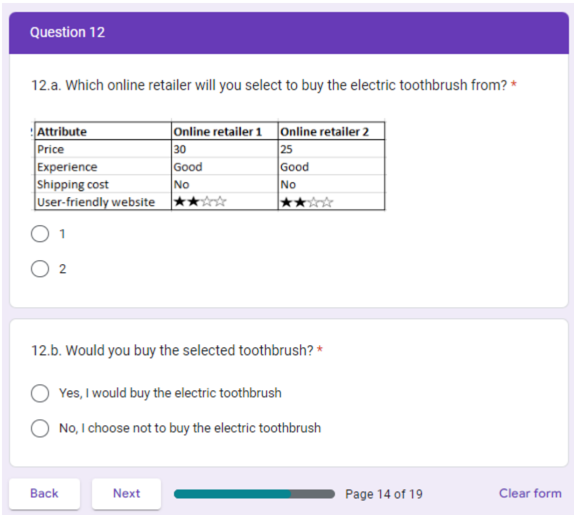
Figure 22: stated preference survey - part 13



Question 12

12.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	30	25
Experience	Good	Good
Shipping cost	No	No
User-friendly website	★★☆☆	★★☆☆

◯ 1

◯ 2

12.b. Would you buy the selected toothbrush? *

◯ Yes, I would buy the electric toothbrush

◯ No, I choose not to buy the electric toothbrush

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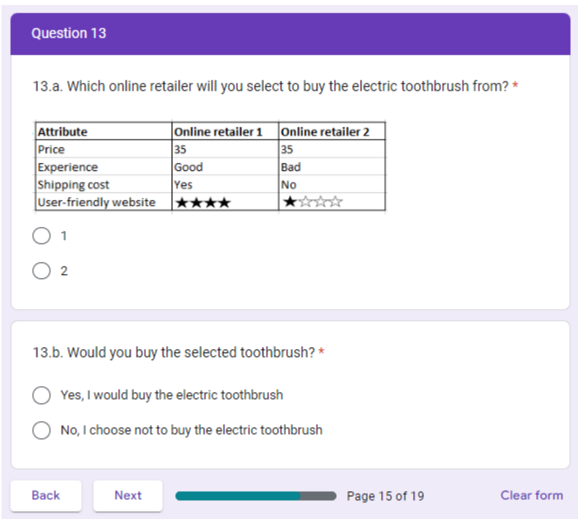
Figure 23: stated preference survey - part 14



Question 13

13.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	35	35
Experience	Good	Bad
Shipping cost	Yes	No
User-friendly website	★★★★	★☆☆☆

◯ 1

◯ 2

13.b. Would you buy the selected toothbrush? *

◯ Yes, I would buy the electric toothbrush

◯ No, I choose not to buy the electric toothbrush

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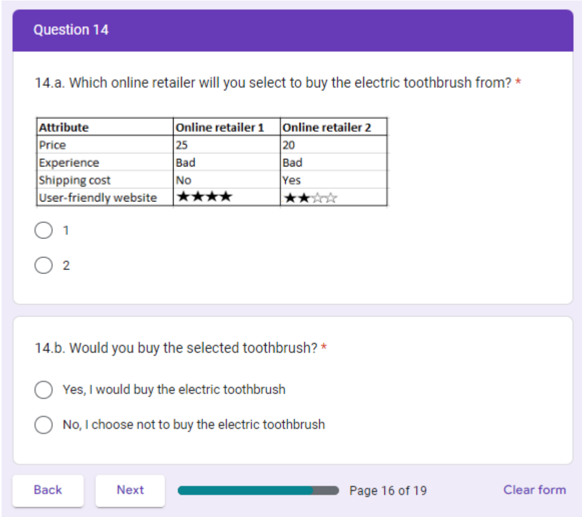
Figure 24: stated preference survey - part 15


Question 14

14.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	25	20
Experience	Bad	Bad
Shipping cost	No	Yes
User-friendly website	★★★★	★★☆☆

1

2

14.b. Would you buy the selected toothbrush? *

Yes, I would buy the electric toothbrush

No, I choose not to buy the electric toothbrush

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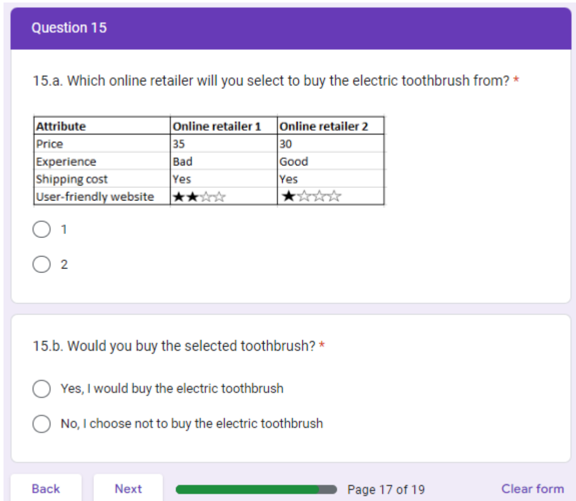
Figure 25: stated preference survey - part 16


Question 15

15.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	35	30
Experience	Bad	Good
Shipping cost	Yes	Yes
User-friendly website	★★☆☆	★☆☆☆

1

2

15.b. Would you buy the selected toothbrush? *

Yes, I would buy the electric toothbrush

No, I choose not to buy the electric toothbrush

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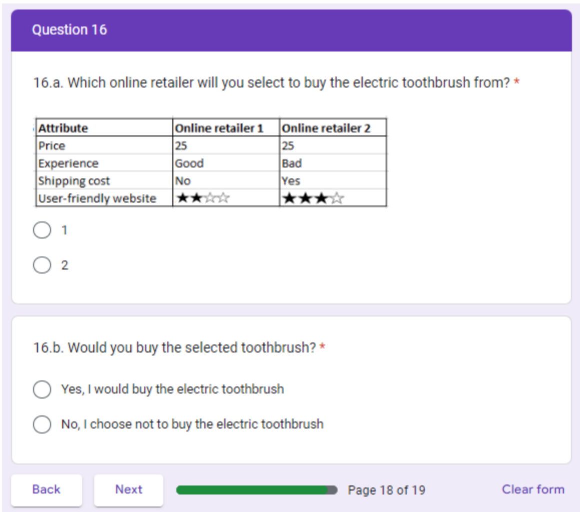
Figure 26: stated preference survey - part 17


Question 16

16.a. Which online retailer will you select to buy the electric toothbrush from? *

Attribute	Online retailer 1	Online retailer 2
Price	25	25
Experience	Good	Bad
Shipping cost	No	Yes
User-friendly website	★★☆☆	★★★☆

1

2

16.b. Would you buy the selected toothbrush? *

Yes, I would buy the electric toothbrush

No, I choose not to buy the electric toothbrush

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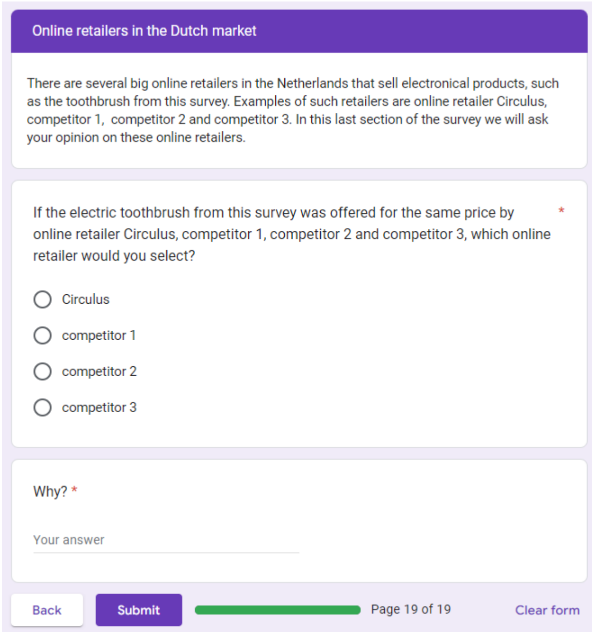
Figure 27: stated preference survey - part 18



Online retailers in the Dutch market

There are several big online retailers in the Netherlands that sell electronical products, such as the toothbrush from this survey. Examples of such retailers are online retailer Circulus, competitor 1, competitor 2 and competitor 3. In this last section of the survey we will ask your opinion on these online retailers.

If the electric toothbrush from this survey was offered for the same price by online retailer Circulus, competitor 1, competitor 2 and competitor 3, which online retailer would you select? *

- Circulus
- competitor 1
- competitor 2
- competitor 3

Why? *

Your answer

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Figure 28: stated preference survey - part 19

A.5 Biogeme code estimation multinomial logit models

```
import pandas as pd
import biogeme.database as db
import biogeme.biogeme as bio
import biogeme.models as models
from biogeme.expressions import Beta

data = pd.read_excel("outputsurvey.xlsx")

print(data.describe())

database = db.Database("Base_model", data)

globals().update(database.variables)

print(database.getSampleSize())


#list of parameters to be estimated
ASC_ONLINE = Beta('ASC_ONLINE',0,None,None,0)
ASC_OPTOUT  = Beta('ASC_OPTOUT',0,None,None,1)
B_PRICE = Beta('B_PRICE',0,None,None,0)
B_GE = Beta('B_GE',0,None,None,0)
B_SC = Beta('B_SC',0,None,None,0)
B_UW = Beta('B_UW',0,None,None,0)

#specify utility functions
V1 = ASC_ONLINE + \
    B_PRICE * Price1 + \
    B_GE * GE1 + \
    B_SC * SC1 + \
    B_UW * UW1
V2 = ASC_ONLINE + \
 B_PRICE * Price2 + \
 B_GE * GE2 + \
 B_SC * SC2 + \
 B_UW * UW2
V3 = ASC_OPTOUT

#associate utility functions with number of alternatives
V = {1 : V1,
     2 : V2,
     0 : V3}

av = {1: 1,
      2: 1,
      0: 1}

logprob = models.loglogit(V, av, CHOICE)

biogeme = bio.BIOGEME(database, logprob)
biogeme.modelName = 'Basis_model'

# Calculate the null log likelihood
biogeme.calculateNullLoglikelihood(av)

results = biogeme.estimate()

pandasResults = results.getEstimatedParameters()
print(pandasResults)
```

Figure 29: Base Model - python code biogeme

```python
import pandas as pd
import biogeme.database as db
import biogeme.biogeme as bio
import biogeme.models as models
from biogeme.expressions import Beta

data = pd.read_excel("outputsurvey.xlsx")

database = db.Database("Quadratic_model", data)

globals().update(database.variables)

#list of parameters to be estimated
ASC_ONLINE = Beta('ASC_ONLINE',0,None,None,0)
ASC_OPTOUT  = Beta('ASC_OPTOUT',0,None,None,1)
B_PRICE = Beta('B_PRICE',0,None,None,0)
B_PRICEQUA = Beta('B_PRICEQUA',0,None,None,0)
B_GE = Beta('B_GE',0,None,None,0)
B_SC = Beta('B_SC',0,None,None,0)
B_UW = Beta('B_UW',0,None,None,0)
B_UWQUA = Beta('B_UWQUA',0,None,None,0)

#specify utility functions
V1 = ASC_ONLINE + \
    B_PRICE * (Price1) + \
    B_PRICEQUA * (Price1**2) + \
    B_GE * GE1 + \
    B_SC * SC1 + \
    B_UW * (UW1) +\
    B_UWQUA * (UW1**2)
V2 = ASC_ONLINE + \
    B_PRICE * (Price2) + \
    B_PRICEQUA * (Price2**2) + \
    B_GE * GE2 + \
    B_SC * SC2 + \
    B_UW * (UW2) +\
    B_UWQUA * (UW2**2)
V3 = ASC_OPTOUT

#associate utility functions with number of alternatives
V = {1 : V1,
     2 : V2,
     0 : V3}

av = {1: 1,
      2: 1,
      0: 1}

logprob = models.loglogit(V, av, CHOICE)

biogeme = bio.BIOGEME(database, logprob)
biogeme.modelName = 'Quadratic_model'

# Calculate the null log likelihood
biogeme.calculateNullLoglikelihood(av)

results = biogeme.estimate()

pandasResults = results.getEstimatedParameters()
print(pandasResults)
```

Figure 30: Quadratic Model - python code biogeme

```python
import pandas as pd
import biogeme.database as db
import biogeme.biogeme as bio
import biogeme.models as models
from biogeme.expressions import Beta
import math

data = pd.read_excel("outputsurveylog.xlsx")

database = db.Database("Logit_model", data)

globals().update(database.variables)

#list of parameters to be estimated
ASC_ONLINE = Beta('ASC_ONLINE',0,None,None,0)
ASC_OPTOUT  = Beta('ASC_OPTOUT',0,None,None,1)
B_PRICELN = Beta('B_PRICELN',0,None,None,0)
B_GE = Beta('B_GE',0,None,None,0)
B_SC = Beta('B_SC',0,None,None,0)
B_UWLN = Beta('B_UWLN',0,None,None,0)

#specify utility functions
V1 = ASC_ONLINE + \
    B_PRICELN * Price1ln + \
    B_GE * GE1 + \
    B_SC * SC1 + \
    B_UWLN * UW1ln
V2 = ASC_ONLINE + \
 B_PRICELN * Price2ln + \
 B_GE * GE2 + \
 B_SC * SC2 + \
 B_UWLN * UW2ln
V3 = ASC_OPTOUT

#associate utility functions with number of alternatives
V = {1 : V1,
     2 : V2,
     0 : V3}

av = {1: 1,
      2: 1,
      0: 1}

logprob = models.loglogit(V, av, CHOICE)

biogeme = bio.BIOGEME(database, logprob)
biogeme.modelName = 'Logit_model'

# Calculate the null log likelihood
biogeme.calculateNullLoglikelihood(av)

results = biogeme.estimate()

pandasResults = results.getEstimatedParameters()
print(pandasResults)
```

Figure 31: Logarithmic Model - python code biogeme

A.6 Biogeme code output

Report file:	Basis_model.html
Database name:	Base_model

Estimation report

Number of estimated parameters:	5
Sample size:	1552
Excluded observations:	0
Null log likelihood:	-1705.046
Init log likelihood:	-1705.046
Final log likelihood:	-1152.44
Likelihood ratio test for the null model:	1105.212
Rho-square for the null model:	0.324
Rho-square-bar for the null model:	0.321
Likelihood ratio test for the init. model:	1105.212
Rho-square for the init. model:	0.324
Rho-square-bar for the init. model:	0.321
Akaike Information Criterion:	2314.881
Bayesian Information Criterion:	2341.617
Final gradient norm:	1.6434E-03
Nbr of threads:	8
Algorithm:	Newton with trust region for simple bound constraints
Proportion analytical hessian:	100.0%
Relative projected gradient:	8.029673e-07
Relative change:	0.001644858119311502
Number of iterations:	5
Number of function evaluations:	16
Number of gradient evaluations:	6
Number of hessian evaluations:	6
Cause of termination:	Relative gradient = 8e-07 <= 6.1e-06
Optimization time:	0:00:00.093950

Estimated parameters

Name	Value	Rob. Std err	Rob. t-test	Rob. p-value
ASC_ONLINE	2.99	0.279	10.7	0
B_GE	2.54	0.131	19.4	0
B_PRICE	-0.173	0.0112	-15.5	0
B_SC	-0.466	0.106	-4.41	1.06e-05
B_UW	0.242	0.0469	5.16	2.53e-07

Figure 32: Base Model - biogeme output

Report file:	Quadratic_model.html
Database name:	Quadratic_model

Estimation report

Number of estimated parameters:	7
Sample size:	1552
Excluded observations:	0
Null log likelihood:	-1705.046
Init log likelihood:	-1705.046
Final log likelihood:	-1150.321
Likelihood ratio test for the null model:	1109.451
Rho-square for the null model:	0.325
Rho-square-bar for the null model:	0.321
Likelihood ratio test for the init. model:	1109.451
Rho-square for the init. model:	0.325
Rho-square-bar for the init. model:	0.321
Akaike Information Criterion:	2314.642
Bayesian Information Criterion:	2352.073
Final gradient norm:	2.0648E-03
Nbr of threads:	8
Algorithm:	Newton with trust region for simple bound constraints
Proportion analytical hessian:	100.0%
Relative projected gradient:	8.377117e-07
Relative change:	5.080738963228471e-10
Number of iterations:	6
Number of function evaluations:	19
Number of gradient evaluations:	7
Number of hessian evaluations:	7
Cause of termination:	Relative change = 5.08e-10 <= 1e-05
Optimization time:	0:00:00.316673

Estimated parameters

Name	Value	Rob. Std err	Rob. t-test	Rob. p-value
ASC_ONLINE	6.46	1.78	3.63	0.000287
B_GE	2.55	0.133	19.1	0
B_PRICE	-0.414	0.123	-3.38	0.000738
B_PRICEQUA	0.00441	0.00224	1.97	0.0489
B_SC	-0.535	0.116	-4.62	3.77e-06
B_UW	-0.0111	0.318	-0.035	0.972
B_UWQUA	0.047	0.0619	0.76	0.447

Figure 33: Quadratic Model - biogeme output

Report file: Logit_model.html
Database name: Logit_model

Estimation report

Number of estimated parameters: 5
Sample size: 1552
Excluded observations: 0
Null log likelihood: -1705.046
Init log likelihood: -1705.046
Final log likelihood: -1152.644
Likelihood ratio test for the null model: 1104.804
Rho-square for the null model: 0.324
Rho-square-bar for the null model: 0.321
Likelihood ratio test for the init. model: 1104.804
Rho-square for the init. model: 0.324
Rho-square-bar for the init. model: 0.321
Akaike Information Criterion: 2315.289
Bayesian Information Criterion: 2342.025
Final gradient norm: 2.4224E-03
Nbr of threads: 8
Algorithm: Newton with trust region for simple bound constraints
Proportion analytical hessian: 100.0%
Relative projected gradient: 5.137824e-06
Relative change: 7.271037008593773e-07
Number of iterations: 7
Number of function evaluations: 22
Number of gradient evaluations: 8
Number of hessian evaluations: 8
Cause of termination: Relative change = 7.27e-07 <= 1e-05
Optimization time: 0:00:00.211376

Estimated parameters

Name	Value	Rob. Std err	Rob. t-test	Rob. p-value
ASC_ONLINE	13.5	0.894	15.1	0
B_GE	2.55	0.13	19.6	0
B_PRICELN	-4.55	0.29	-15.7	0
B_SC	-0.482	0.106	-4.55	5.47e-06
B_UWLN	0.472	0.0992	4.76	1.97e-06

Figure 34: Logarithmic Model - biogeme output

A.7 Comparison of fit multinomial logit models

There are several different approaches possible to determine the goodness of fit of the models to the data. One approach is to compare McFadden's Rho-square for the different models. This goodness-of-fit measure compares the likelihood of the final model to the likelihood of the initial model. In the initial model all parameter values are set to 0. Rho-square can take any value between 0 and 1, where 0 indicates that the model does not explain the uncertainty better than the initial model and 1 indicates that the final model is a perfect fit to the data (Abdulhafedh et al., 2017). There is not a fixed value for the the Rho-square that indicates a good fit, since the context and goals of the analysis play an important role. A rule-of-thumb that is often applied is that Rho-square values above 0.2 indicate a reasonable fit and values above 0.4 indicate a good fit (Hauber et al., 2016). The number of parameters in the model also affect Rho-square, such that Rho-square increases as the number of parameters increases. The adjusted Rho-square accounts for this effect such that the models can be compared. The adjusted Rho-square can be calculated with the following formula:

$$\rho^2_{adjusted} = 1 - \frac{LL_{final} - k}{LL_{initial}} \tag{69}$$

In this formula LL_{final} represents the log-likelihood of the estimated model and $LL_{initial}$ represents the log-likelihood of the initial model with all parameter values set to 0. k indicates the number of parameters to be estimated in the model. The following results were obtained for the three models under consideration:

	Linear	**Quadratic**	**Logarithmic**
Parameters	5	7	5
Initial log likelihood	-1705.046	-1705.046	-1705.046
Final log likelihood	-1152.440	-1150.321	-1152.644
Rho-square adjusted	0.321167875	0.321237667	0.32104823

Table 15: Rho-square adjusted

It can be observed that all three models have a similar value of 0.32 for Rho-square adjusted, which lies with in the range of 0.2 and 0.4 and thus indicates a reasonable model fit. It should be noted that the adjusted Rho-square cannot be interpreted as a direct measure of the predictive accuracy of the models as it only indicates the relative improvement of the fitted model compared to the initial model. In order to know which model fits the data better, the next step will be to use the likelihood ratio statistics (LRS) in order to compare the log-likelihood of the different models. This test compares the goodness of fit of two models, where one model is an extended version of the other model as it contains additional parameters (Hauber et al., 2016). The simpler model contains less parameters than the extended model. The LRS can be calculated with the following formula:

$$LRS = -2 \cdot (LL_{model1} - LL_{model2}) \tag{70}$$

When the value of LRS exceeds the corresponding value in the χ^2-table, it can be concluded that the extended model fits the data better (Van Berkum & Di Bucchianico, 2016). This value is found by the degrees of freedom, which is equal to the difference in parameters of the models and the significance level. The significance level is set at 5 percent. The LRS can be applied to compare the linear model and the quadratic model as the linear model contains 5 parameters and the quadratic model 7. The quadratic model is a more complex version of the linear model as it contains the 5 linear parameters from the linear model and 2 additional parameters of a quadratic nature.

$$LRS = -2 \cdot (-1152.440 + 1150.321) = 4.238 \tag{71}$$

There are 2 degrees of freedom as this is equal to the difference in number of parameters of the different models. The χ^2-value corresponding to 2 degrees of freedom and a 5 percent significance level is 5.991. The LRS does not exceed this value and therefore it can be concluded that the linear model fits the data better than the quadratic model.

In order to compare the linear model and the logarithmic model, the LRS method cannot be applied as both models contain 5 parameters. Also the explanatory variables of the models differ, as the linear model contains linear terms for the attributes 'price' and 'user-friendly website' and the logarithmic model contains logarithmic terms for these attributes. In such case, the AIC and BIC measures can be used (Hauber et al., 2016). Models with lower AIC and BIC values indicate a better fit to the data. The AIC and BIC values can be obtained with the following formulas:

$$AIC = -2 \cdot LL_{final} + k \cdot 2 \tag{72}$$

$$BIC = -2 \cdot LL_{final} + k \cdot ln(n) \tag{73}$$

Where k represents the number of parameters in the model and n represents the sample size, which is equal to 97 in this case. Filling out these formulas results in an AIC and BIC of 2314.88 and 2327.75, respectively for the linear model. For the logarithmic model AIC and BIC are 2315.29 and 2328.16, respectively. The AIC and BIC of the linear model are smaller and it can therefore be concluded that the linear model fits the data better than the logarithmic model.

To conclude, the linear model is the best fitting model of the three considered models.

A.8 Literature review game theory

Game theory is a mathematical discipline that has often been applied to economical contexts (Peters, 2015). This discipline focuses on decision making when various players are involved. In non-cooperative games, also called strategic or competitive games, players take decisions for themselves to maximize their individual payoff (Elkind & Rothe, 2016). The aim of non-cooperative games is to investigate what happens in society when players make independent rational strategic decisions (Fujiwara-Greve, 2015). The price competition in e-commerce markets can be described as a non-cooperative game. Therefore, non-cooperative games will be the focus of this research. To model a non-cooperative game for the situation at online retailer Circulus, the following elements are needed:

- **Player set**: number of strategic rational decision-makers. A rational player chooses the best action based on his preferences, given all actions available to him (Osborne et al., 2004). The set of players is represented by $N = \{1, ..., n\}$, where $n \geq 1$ (Peters, 2015).
- **Strategy set**: a plan of strategy for a player given the circumstances that can occur in the game. Each player i, where $i \in N$ has an own strategy set, which is denoted by S_i (Peters, 2015). A strategy is denoted by $s_i \in S_i$. The strategy set may be of a discrete or continuous nature. When the game contains a discrete strategy set, the player can select one strategy from a discrete set of available strategies (Webb, 2007). In contrast, when the game contains a continuous strategy set, the player can select one strategy from an infinite set of strategies (Felix, 2017). A set of strategies is denoted by $s = (s_i, s_{-i})_{i \in N}$, with $s_{-i} = (s_j)_{j \in N|\{i\}}$. This set of of strategies s lies in a domain S, which is the Cartesian product of all possible strategy sets of the players $S_1 \times S_2 \times ... \times S_n \to \mathbb{R}$.
- **Payoff function / preferences**: the payoff function represents the preferences of the decision-maker. In economic theory this function is often defined as the utility function. The payoff function for player i is defined as $f_i(s_i, s_{-i}) \to \mathbb{R}$ and represents the payoff for every player $i \in N$, which is obtained by the strategy from player i (s_i) and the strategies from the other players (s_{-i}).
- **Information**: the information available to the players in every stage of the game. This may concern information about the payoff function, about the actions of other players (sequential games) or about the strategy set for instance (Osborne et al., 2004).
- **Preferences**: the preference of players indicates how the players value their outcomes (Osborne et al., 2004). For instance, a player that values revenue maximization will choose a strategy that maximizes its expected payoff.
- **Timing**: whether choices in the game are made simultaneously or sequentially by players. In simultaneous games players decide at the same time, while sequential decisions are made in a consecutive order (Peters, 2015). In sequential games a player can directly anticipate on the decision of the other player.

A solution concept that is often used to analyze a non-cooperative game is the Nash equilibrium. Under a given Nash equilibrium, it is assumed that each rational player will choose the best available action through considering the actions of the other players (Osborne et al., 2004). In doing so, the player must form a so-called belief about the actions of the other players. Given that each player chooses its action based on rational choice and their beliefs about the actions of the other players and given that the beliefs of every player about the actions of the other players are correct, a Nash equilibrium may exist. In a Nash equilibrium, no player has an incentive to deviate from this strategy, given that the other players do not deviate from their strategies as well and this applies to every player of the game. A game may contain a unique Nash equilibrium, multiple Nash equilibria or no Nash equilibrium (Basar et al., 2010). A strategy combination $s^* = (s_i^*)_{i \in N}$ is a Nash equilibrium if for every player $i \in N$ and all $s_i \in S_i$, it holds that (Peters, 2015):

$$f_i(s_i^*, s_{-i}^*) \geq f_i(s_i, s_{-i}^*), \tag{74}$$

Applications of game theory in an e-commerce context

A game that has often been applied in e-commerce literature is the Bertrand Game. This game concerns the strategic choice of pricing (Fujiwara-Greve, 2015). In the Bertrand's game the different players determine their price simultaneously and demand is represented by a logit model (Li & Huh, 2011). It is a non-cooperative, simultaneous game. For this game a distinction is made between the situation where products are differentiated and products are substitutes. When products are differentiated, the products from different firms are not the same to consumers, they may prefer a firm that does not have the lowest price in the market. When products are substitutes, the consumer has no preference and will buy the product with the lowest price. In economics such products are called perfect substitutes (Fujiwara-Greve, 2015). The Bertrand's model states that each player gains 50 percent of the market share when both players have the same price. When the prices are not the same, the company with the lowest price gains all market share. An equilibrium is found at the marginal cost of the product. Moving above that price only gives the opponent the opportunity to seize the entire market (Chatterjee & Samuelson, 2001).

Non-cooperative games have not only been used in e-commerce literature when players compete in price, but can also be used when players compete in quantity. A famous game theoretic approach to tackle this issue is the Cournot model. In this model the price of the products depends on the total quantity offered. The strategy of each player is the quantity offered and the payoff for each player is the profit function, which is equal to the revenue minus the cost. Players determine their quantities simultaneously. A Nash Cournot equilibrium can be found through solving the profit maximization problem (Peters, 2015).

Deceree et al. (2021) used non-cooperative game theory in an e-commerce environment to evaluate customer satisfaction. They composed a game to determine the optimal strategy for each player, considering the action of the other player. An optimal strategy indicated which elements to improve to increase customer satisfaction for each of the online retailers.

Furthermore, non-cooperative games have been applied in literature for assortment planning for online retailers. The game applied in the article of Saberi et al. (2019) allows online retailers to determine an optimal strategy for assortment planning, considering the assortment planning of competitors in the online industry. A non-cooperative Stackelberg game is applied. The Stackelberg game is similar to the Cournot game as both consider competition in quantity. However, in the Stackelberg game there is a leader who chooses first and the follower players react through choosing sequentially. The leader therefore receives a significant advantage. Besbes & Sauré (2016) extended the approach of Saberi et al. (2019) in their research. They determined an optimal strategy for assortment planning and the corresponding optimal pricing strategy for each product.

To summarize, non-cooperative games have been used in literature to determine an optimal strategy for increasing payoff for online retailers. This strategy may be based on setting the optimal price or may be focused on the optimal quantity to offer. Other research focused on an optimal strategy considering the improvement of customer satisfaction or an optimal strategy for assortment planning. All these optimal strategies were determined through the composition of a non-cooperative game in an e-commerce context.

A.9 Derivative payoff function online retailer game

For simplicity the following substitution is used in the derivations:

- A $= \beta_0^i$
- B $= \beta_p$
- x $= p_i$

This results in the following formula:

$$f_i(x) = \frac{e^{A-B\cdot x} \cdot (x-c)}{e^{A-B\cdot x} + C} \tag{75}$$

To find the derivative of the given function, we will use the quotient rule of differentiation:

$$(\frac{f(x)}{g(x)})' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2} \tag{76}$$

where

$$f(x) = e^{A-B\cdot x} \cdot (x-c) \tag{77}$$

and

$$g(x) = e^{A-B\cdot x} + C \tag{78}$$

Applying the quotient rule, where the derivative over x is taken for $f(x)$ and $g(x)$, gives the following:

$$f_i' = \frac{(e^{A-B\cdot x}(B\cdot(c-x)+1)(e^{A-B\cdot x}+C) - (e^{A-B\cdot x}(x-c)(-Be^{A-B\cdot x})}{(e^{A-B\cdot x}+C)^2} \tag{79}$$

Simplifying the above equation, gives:

$$f_i' = \frac{e^A(e^A + e^{B\cdot x}(B\cdot C\cdot(c-x)+C))}{(e^A + Ce^{B\cdot x})^2} \tag{80}$$

A.10 Python code non-cooperative game

```python
import numpy as np

# Define the range of prices
price_range = np.arange(0, 40, 1)
c_A = 14
c_B = 16
b0_A = 3.008
b0_B = 6.032
bp = 0.173

# Define the set of initial situations
initial_situations = [(i, j) for i in price_range for j in price_range]

# Initialize lists to store initial situations where A and B cannot improve
cannot_improve_A = []
cannot_improve_B = []

# Loop over each initial situation
for init in initial_situations:

    # Unpack the initial situation
    init_A, init_B = init

    # Set the initial payoffs
    payoff_init_A = (np.exp(b0_A - bp * init_A) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(0))) * (init_A - c_A)
    payoff_init_B = (np.exp(b0_B - bp * init_B) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(0))) * (init_B - c_B)

    # Initialize the improvement variables
    improvement_A = False
    improvement_B = False

    # Loop over prices for player A
    for price_A in price_range:

        # Calculate the payoff for player A at the current price
        total_utility = np.exp(b0_A - bp * price_A) + np.exp(b0_B - bp * init_B) + np.exp(0)
        payoff_A = (np.exp(b0_A - bp * price_A) / total_utility) * (price_A - c_A)

        # Check if the payoff is higher than the initial payoff
        if payoff_A > payoff_init_A:
            #print(f"Player A can improve from ({init_A},{init_B}) by setting price {price_A}")
            improvement_A = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_A == max(price_range) and not improvement_A:
            print(f"Player A cannot improve from ({init_A},{init_B})")

    # Loop over prices for player B
    for price_B in price_range:

        # Calculate the payoff for player B at the current price
        total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * price_B) + np.exp(0)
        payoff_B = (np.exp(b0_B - bp * price_B) / total_utility) * (price_B - c_B)

        # Check if the payoff is higher than the initial payoff
        if payoff_B > payoff_init_B:
            #print(f"Player B can improve from ({init_A},{init_B}) by setting price {price_B}")
            improvement_B = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_B == max(price_range) and not improvement_B:
            print(f"Player B cannot improve from ({init_A},{init_B})")
```

Figure 35: 2 player non-cooperative game, part 1

```python
        # Check if the price has reached the maximum and no improvement was found
        elif price_B == max(price_range) and not improvement_B:
            print(f"Player B cannot improve from ({init_A},{init_B})")

    # Check if both players cannot improve from the initial situation
    if not improvement_A and not improvement_B:
        cannot_improve_A.append(init_A)
        cannot_improve_B.append(init_B)

# Print the initial situations where both players cannot improve
print("Initial situations where both players cannot improve:")
for init_A, init_B in zip(cannot_improve_A, cannot_improve_B):
    print(f"({init_A},{init_B})")
```

Figure 36: 2 player non-cooperative game, part 2

```python
import numpy as np

# Define the range of prices
price_range = np.arange(0, 40, 1)
c_A = 14
c_B = 16
c_C = 16
b0_A = 3.008
b0_B = 6.032
b0_C = 6.498
bp = 0.173

# Define the set of initial situations
initial_situations = [(i, j, k) for i in price_range for j in price_range for k in price_range]

# Initialize lists to store initial situations where A, B, and C cannot improve
cannot_improve_A = []
cannot_improve_B = []
cannot_improve_C = []

# Loop over each initial situation
for init in initial_situations:

    # Unpack the initial situation
    init_A, init_B, init_C = init

    # Set the initial payoffs
    payoff_init_A = (np.exp(b0_A - bp * init_A) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * init_C) + np.exp(0))) * (init_A - c_A)
    payoff_init_B = (np.exp(b0_B - bp * init_B) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * init_C) + np.exp(0))) * (init_B - c_B)
    payoff_init_C = (np.exp(b0_C - bp * init_C) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * init_C) + np.exp(0))) * (init_C - c_C)

    # Initialize the improvement variables
    improvement_A = False
    improvement_B = False
    improvement_C = False

    # Loop over prices for player A
    for price_A in price_range:

        # Calculate the payoff for player A at the current price
        total_utility = np.exp(b0_A - bp * price_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * init_C) + np.exp(0)
        payoff_A = (np.exp(b0_A - bp * price_A) / total_utility) * (price_A - c_A)

        # Check if the payoff is higher than the initial payoff
        if payoff_A > payoff_init_A:
            #print(f"Player A can improve from ({init_A},{init_B},{init_C}) by setting price {price_A}")
            improvement_A = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_A == max(price_range) and not improvement_A:
            print(f"Player A cannot improve from ({init_A},{init_B},{init_C})")

    # Loop over prices for player B
    for price_B in price_range:

        # Calculate the payoff for player B at the current price
        total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * price_B) + np.exp(b0_C - bp * init_C) + np.exp(0)
        payoff_B = (np.exp(b0_B - bp * price_B) / total_utility) * (price_B - c_B)
```

Figure 37: 3 player non-cooperative game, part 1

```python
    # Loop over prices for player B
    for price_B in price_range:

        # Calculate the payoff for player B at the current price
        total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * price_B) + np.exp(b0_C - bp * init_C) + np.exp(0)
        payoff_B = (np.exp(b0_B - bp * price_B) / total_utility) * (price_B - c_B)

        # Check if the payoff is higher than the initial payoff
        if payoff_B > payoff_init_B:
            #print(f"Player B can improve from ({init_A},{init_B},{init_C}) by setting price {price_B}")
            improvement_B = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_B == max(price_range) and not improvement_B:
            print(f"Player B cannot improve from ({init_A},{init_B},{init_C})")

    # Loop over prices for player C
    for price_C in price_range:

        # Calculate the payoff for player B at the current price
        total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * price_C) + np.exp(0)
        payoff_C = (np.exp(b0_C - bp * price_C) / total_utility) * (price_C - c_C)

        # Check if the payoff is higher than the initial payoff
        if payoff_C > payoff_init_C:
            improvement_C = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_C == max(price_range) and not improvement_C:
            print(f"Player C cannot improve from ({init_A},{init_B},{init_C})")

    # Check if both players cannot improve from the initial situation
    if not improvement_A and not improvement_B and not improvement_C:
        cannot_improve_A.append(init_A)
        cannot_improve_B.append(init_B)
        cannot_improve_C.append(init_C)

# Print the initial situations where both players cannot improve
print("Initial situations where all players cannot improve:")
for init_A, init_B, init_C in zip(cannot_improve_A, cannot_improve_B, cannot_improve_C):
    print(f"({init_A},{init_B},{init_C})")
```

Figure 38: 3 player non-cooperative game, part 2

```python
import numpy as np

# Define the range of prices
price_range = np.arange(0, 40, 1)
c_A = 14
c_B = 14
c_C = 14
c_D = 14
b0_A = 3.008
b0_B = 6.032
b0_C = 6.498
b0_D = 5.79
bp = 0.173

# Define the set of initial situations
initial_situations = [(i, j, k, l) for i in price_range for j in price_range for k in price_range for l in price_range]

# Initialize lists to store initial situations where A, B, and C cannot improve
cannot_improve_A = []
cannot_improve_B = []
cannot_improve_C = []
cannot_improve_D = []

# Loop over each initial situation
for init in initial_situations:

    # Unpack the initial situation
    init_A, init_B, init_C, init_D = init

    # Set the initial payoffs
    payoff_init_A = (np.exp(b0_A - bp * init_A) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B)
                                                   + np.exp(b0_C - bp * init_C) + np.exp(b0_D - bp * init_D) + np.exp(0))) * (init_A - c_A)
    payoff_init_B = (np.exp(b0_B - bp * init_B) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B)
                                                   + np.exp(b0_C - bp * init_C) + np.exp(b0_D - bp * init_D) + np.exp(0))) * (init_B - c_B)
    payoff_init_C = (np.exp(b0_C - bp * init_C) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B)
                                                   + np.exp(b0_C - bp * init_C) + np.exp(b0_D - bp * init_D) + np.exp(0))) * (init_C - c_C)
    payoff_init_D = (np.exp(b0_D - bp * init_D) / (np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B)
                                                   + np.exp(b0_C - bp * init_C) + np.exp(b0_D - bp * init_D) + np.exp(0))) * (init_D - c_D)

    # Initialize the improvement variables
    improvement_A = False
    improvement_B = False
    improvement_C = False
    improvement_D = False

    # Loop over prices for player A
    for price_A in price_range:
```

Figure 39: 4 player non-cooperative game, part 1

```
# Loop over prices for player A
for price_A in price_range:

    # Calculate the payoff for player A at the current price
    total_utility = np.exp(b0_A - bp * price_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * init_C)
    + np.exp(b0_D - bp * init_D) + np.exp(0)
    payoff_A = (np.exp(b0_A - bp * price_A) / total_utility) * (price_A - c_A)

    # Check if the payoff is higher than the initial payoff
    if payoff_A > payoff_init_A:
        improvement_A = True

    # Check if the price has reached the maximum and no improvement was found
    elif price_A == max(price_range) and not improvement_A:
        print(f"Player A cannot improve from ({init_A},{init_B},{init_C},{init_D})")

# Loop over prices for player B
for price_B in price_range:

    # Calculate the payoff for player A at the current price
    total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * price_B) + np.exp(b0_C - bp * init_C)
    + np.exp(b0_D - bp * init_D) + np.exp(0)
    payoff_B = (np.exp(b0_B - bp * price_B) / total_utility) * (price_B - c_B)

    # Check if the payoff is higher than the initial payoff
    if payoff_B > payoff_init_B:
        improvement_B = True

    # Check if the price has reached the maximum and no improvement was found
    elif price_B == max(price_range) and not improvement_B:
        print(f"Player B cannot improve from ({init_A},{init_B},{init_C},{init_D})")

# Loop over prices for player C
for price_C in price_range:

    # Calculate the payoff for player A at the current price
    total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * price_C)
    + np.exp(b0_D - bp * init_D) + np.exp(0)
    payoff_C = (np.exp(b0_C - bp * price_C) / total_utility) * (price_C - c_C)

    # Check if the payoff is higher than the initial payoff
    if payoff_C > payoff_init_C:
        improvement_C = True

    # Check if the price has reached the maximum and no improvement was found
    elif price_C == max(price_range) and not improvement_C:
        print(f"Player C cannot improve from ({init_A},{init_B},{init_C},{init_D})")

# Loop over prices for player D
for price_D in price_range:
```

Figure 40: 4 player non-cooperative game, part 2

```python
    # Loop over prices for player D
    for price_D in price_range:

        # Calculate the payoff for player A at the current price
        total_utility = np.exp(b0_A - bp * init_A) + np.exp(b0_B - bp * init_B) + np.exp(b0_C - bp * init_C)
        + np.exp(b0_D - bp * price_D) + np.exp(0)
        payoff_D = (np.exp(b0_D - bp * price_D) / total_utility) * (price_D - c_D)

        # Check if the payoff is higher than the initial payoff
        if payoff_D > payoff_init_D:
            improvement_D = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_D == max(price_range) and not improvement_D:
            print(f"Player D cannot improve from ({init_A},{init_B},{init_C},{init_D})")

    # Check if both players cannot improve from the initial situation
    if not improvement_A and not improvement_B and not improvement_C and not improvement_D:
        cannot_improve_A.append(init_A)
        cannot_improve_B.append(init_B)
        cannot_improve_C.append(init_C)
        cannot_improve_D.append(init_D)

# Print the initial situations where both players cannot improve
print("Initial situations where all players cannot improve:")
for init_A, init_B, init_C, init_D in zip(cannot_improve_A, cannot_improve_B, cannot_improve_C, cannot_improve_D):
    print(f"({init_A},{init_B},{init_C},{init_D})")
```

Figure 41: 4 player non-cooperative game, part 3

The discrete online retailer game with different goals

```python
# Define the range of prices
price_range = np.arange(0, 40, 1)
c_A = 14
c_B = 16
c_C = 16
c_D = 17
b0_A = 3.008
b0_B = 6.032
b0_C = 6.498
b0_D = 5.79
bp1 = 0.173
bp2 = 0.173
bp3 = 0.173
bp4 = 0.173

initial_situations = []
for i in price_range:
    for j in price_range:
        for k in price_range:
            for l in price_range:
                if (l == i and i < j and l < k) or (l == j and j < i and j < k) or (l == k and k < i and k < j):
                    initial_situations.append((i, j, k, l))

# Initialize lists to store initial situations where A, B, and C cannot improve
cannot_improve_A = []
cannot_improve_B = []
cannot_improve_C = []
cannot_improve_D = []

# Loop over each initial situation
for init in initial_situations:
```

Figure 42: 4 player discrete game, different strategies, part 1

```python
# Loop over each initial situation
for init in initial_situations:

    # Unpack the initial situation
    init_A, init_B, init_C, init_D = init

    # Set the initial payoffs
    payoff_init_A = (np.exp(b0_A - bp1 * init_A) / (np.exp(b0_A - bp1 * init_A) + np.exp(b0_B - bp2 * init_B)
                                                    + np.exp(b0_C - bp3 * init_C) + np.exp(b0_D - bp4 * init_D)
                                                    + np.exp(0))) * (init_A - c_A)
    payoff_init_B = (np.exp(b0_B - bp2 * init_B) / (np.exp(b0_A - bp1 * init_A) + np.exp(b0_B - bp2 * init_B)
                                                    + np.exp(b0_C - bp3 * init_C) + np.exp(b0_D - bp4 * init_D)
                                                    + np.exp(0))) * (init_B - c_B)
    payoff_init_C = (np.exp(b0_C - bp3 * init_C) / (np.exp(b0_A - bp1 * init_A) + np.exp(b0_B - bp2 * init_B)
                                                    + np.exp(b0_C - bp3 * init_C) + np.exp(b0_D - bp4 * init_D)
                                                    + np.exp(0))) * (init_C - c_C)
    payoff_init_D = (np.exp(b0_D - bp4 * init_D) / (np.exp(b0_A - bp1 * init_A) + np.exp(b0_B - bp2 * init_B)
                                                    + np.exp(b0_C - bp3 * init_C) + np.exp(b0_D - bp4 * init_D)
                                                    + np.exp(0))) * (init_D - c_D)

    # Initialize the improvement variables
    improvement_A = False
    improvement_B = False
    improvement_C = False

    # Loop over prices for player A
    for price_A in price_range:

        # Calculate the payoff for player A at the current price
        total_utility = np.exp(b0_A - bp1 * price_A) + np.exp(b0_B - bp2 * init_B) + np.exp(b0_C - bp3 * init_C)
        + np.exp(b0_D - bp4 * init_D) + np.exp(0)
        payoff_A = (np.exp(b0_A - bp1 * price_A) / total_utility) * (price_A - c_A)

        # Check if the payoff is higher than the initial payoff
        if payoff_A > payoff_init_A:
            improvement_A = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_A == max(price_range) and not improvement_A:
            improvement_A = False
```

Figure 43: 4 player discrete game, different strategies, part 2

```python
    # Loop over prices for player B
    for price_B in price_range:

        # Calculate the payoff for player A at the current price
        total_utility = np.exp(b0_A - bp1 * init_A) + np.exp(b0_B - bp2 * price_B) + np.exp(b0_C - bp3 * init_C)
        + np.exp(b0_D - bp4 * init_D) + np.exp(0)
        payoff_B = (np.exp(b0_B - bp2 * price_B) / total_utility) * (price_B - c_B)

        # Check if the payoff is higher than the initial payoff
        if payoff_B > payoff_init_B:
            improvement_B = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_B == max(price_range) and not improvement_B:
            improvement_B = False

    # Loop over prices for player C
    for price_C in price_range:

        # Calculate the payoff for player A at the current price
        total_utility = np.exp(b0_A - bp1 * init_A) + np.exp(b0_B - bp2 * init_B) + np.exp(b0_C - bp3 * price_C)
        + np.exp(b0_D - bp4 * init_D) + np.exp(0)
        payoff_C = (np.exp(b0_C - bp3 * price_C) / total_utility) * (price_C - c_C)

        # Check if the payoff is higher than the initial payoff
        if payoff_C > payoff_init_C:
            improvement_C = True

        # Check if the price has reached the maximum and no improvement was found
        elif price_C == max(price_range) and not improvement_C:
            improvement_C = False

    # Check if players cannot improve from the initial situation
    if not improvement_A and not improvement_B and not improvement_C:
        cannot_improve_A.append(init_A)
        cannot_improve_B.append(init_B)
        cannot_improve_C.append(init_C)
        cannot_improve_D.append(init_D)

Print the initial situations where both players cannot improve
int("Initial situations where all players cannot improve:")
r init_A, init_B, init_C, init_D in zip(cannot_improve_A, cannot_improve_B, cannot_improve_C, cannot_improve_D):
```

Figure 44: 4 player discrete game, different strategies, part 3

```python
    # Check if players cannot improve from the initial situation
    if not improvement_A and not improvement_B and not improvement_C:
        cannot_improve_A.append(init_A)
        cannot_improve_B.append(init_B)
        cannot_improve_C.append(init_C)
        cannot_improve_D.append(init_D)

# Print the initial situations where both players cannot improve
print("Initial situations where all players cannot improve:")
for init_A, init_B, init_C, init_D in zip(cannot_improve_A, cannot_improve_B,
                                          cannot_improve_C, cannot_improve_D):
    print(f"({init_A},{init_B},{init_C},{init_D})")
```

Figure 45: 4 player discrete game, different strategies, part 4

A.11 Testing framework discrete non-cooperative game

Cost player 1	Cost player 2	NE1
14	14	21, 29
14	15	21, 30
14	16	21, 30
14	17	21, 30
15	14	22, 30
15	15	22, 30
15	16	22, 30
15	17	22, 31
16	14	23, 30
16	15	23, 30
16	16	23, 31
16	17	23, 31
17	14	23, 30
17	15	23, 30
17	16	23, 31
17	17	23, 31

Table 16: Test framework cost, 2 players

Cost player 1	Cost player 2	Cost player 3	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
14	14	14	20, 24, 25				
14	14	15	20, 24, 26				
14	14	16	20, 24, 27				
14	14	17	20, 24, 27				
14	15	14	20, 24, 25	20, 25, 26	0.22, 3.47, 5.69	0.26, 3.79, 6.10	20, 25, 26
14	15	15	20, 25, 26				
14	15	16	20, 25, 27				
14	15	17	20, 25, 27				
14	16	14	20, 25, 26				
14	16	15	20, 25, 26	20, 26, 27	0.26, 3.41, 5.59	0.31, 3.71, 5.97	20, 26, 27
14	16	16	20, 26, 27				
14	16	17	20, 26, 28				
14	17	14	20, 26, 26				
14	17	15	20, 26, 27				
14	17	16	20, 26, 27				
14	17	17	20, 26, 28				
15	14	14	21, 24, 26				
15	14	15	21, 24, 26				
15	14	16	21, 24, 27				
15	14	17	21, 24, 27				
15	15	14	21, 25, 26				
15	15	15	21, 25, 26				
15	15	16	21, 25, 27				
15	15	17	21, 25, 27				
15	16	14	21, 25, 26				
15	16	15	21, 25, 26	21, 26, 27	0.22, 3.43, 5.63	0.25, 3.74, 6.02	21, 25, 26
15	16	16	21, 26, 27				
15	16	17	21, 26, 28				
15	17	14	21, 26, 26				
15	17	15	21, 26, 27				
15	17	16	21, 26, 27				
15	17	17	21, 26, 28				
16	14	14	22, 24, 26				
16	14	15	22, 24, 26				
16	14	16	22, 24, 27				
16	14	17	22, 24, 27	22, 25, 28	0.18, 4.61, 4.37	0.22, 4.97, 4.72	22, 25, 28
16	15	14	22, 25, 26				
16	15	15	22, 25, 26				
16	15	16	22, 25, 27				
16	15	17	22, 25, 28				
16	16	14	22, 25, 26				
16	16	15	22, 25, 26	22, 26, 27	0.18, 3.46, 5.66	0.21, 3.77, 6.06	22, 26, 27
16	16	16	22, 26, 27				
16	16	17	22, 26, 28				
16	17	14	22, 22, 26				
16	17	15	22, 26, 27				
16	17	16	22, 26, 27				
16	17	17	22, 26, 28				
17	14	14	23, 24, 26				
17	14	15	23, 24, 26				
17	14	16	23, 24, 27				
17	14	17	23, 24, 27	23, 25, 28	0.16, 4.63, 4.39	0.18, 5.00, 4.75	23, 25, 28
17	15	14	23, 25, 26				
17	15	15	23, 25, 26				
17	15	16	23, 25, 27				
17	15	17	23, 25, 28				
17	16	14	23, 25, 26				
17	16	15	23, 25, 26	23, 26, 27	0.16, 3.47, 5.69	0.19, 3.79, 6.10	23, 26, 27
17	16	16	23, 26, 27				
17	16	17	23, 26, 28				
17	17	14	23, 26, 26				
17	17	15	23, 26, 27				
17	17	16	23, 26, 27	23, 27, 28	0.19, 3.41, 5.59	0.21, 3.71, 5.97	23, 27, 28
17	17	17	23, 27, 28				

Table 17: Test framework cost, 3 players

Cost player 1	Cost player 2	Cost player 3	Cost player 4	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
14	14	14	14	20, 22, 23, 22				
14	14	14	15	20, 22, 23, 22	20, 22, 24, 23	0.12, 2.42, 3.65, 1.66	0.13, 2.70, 3.80, 1.78	20, 22, 24, 23
14	14	14	16	20, 22, 24, 23				
14	14	14	17	20, 23, 24, 24				
14	14	15	14	20, 22, 24, 22				
14	14	15	15	20, 22, 24, 23				
14	14	15	16	20, 22, 24, 23	20, 23, 25, 24	0.14, 2.70, 3.42, 1.56	0.16, 3.00, 3.76, 1.76	20, 23, 25, 24
14	14	15	17	20, 23, 25, 24				
14	14	16	14	20, 23, 25, 22				
14	14	16	15	20, 23, 25, 23				
14	14	16	16	20, 23, 25, 24				
14	14	16	17	20, 23, 25, 24				
14	14	17	14	20, 23, 26, 22				
14	14	17	15	20, 23, 26, 23				
14	14	17	16	20, 23, 26, 24				
14	14	17	17	20, 23, 26, 24				
14	15	14	14	20, 23, 24, 22				
14	15	14	15	20, 23, 24, 23				
14	15	14	16	20, 23, 24, 23				
14	15	14	17	20, 23, 24, 24				
14	15	15	14	20, 23, 24, 22				
14	15	15	15	20, 23, 24, 23				
14	15	15	16	20, 23, 24, 23	20, 23, 25, 24	0.15, 2.40, 3.62, 1.65	0.16, 2.67, 3.76, 1.76	20, 23, 25, 24
14	15	15	17	20, 23, 25, 24				
14	15	16	14	20, 23, 25, 22				
14	15	16	15	20, 23, 25, 23				
14	15	16	16	20, 23, 25, 24				
14	15	16	17	20, 23, 25, 24	20, 24, 26, 25	0.16, 2.67, 3.38, 1.54	0.19, 2.96, 3.71, 1.74	20, 24, 26, 25
14	15	17	14	20, 23, 26, 22				
14	15	17	15	20, 24, 26, 23				
14	15	17	16	20, 24, 26, 24				
14	15	17	17	20, 24, 26, 25				
14	16	14	14	20, 24, 24, 22				
14	16	14	15	20, 24, 24, 23				
14	16	14	16	20, 24, 24, 24				
14	16	14	17	20, 24, 24, 24				
14	16	15	14	20, 24, 24, 22				
14	16	15	15	20, 24, 25, 23				
14	16	15	16	20, 24, 25, 24				
14	16	15	17	20, 24, 25, 24				
14	16	16	14	20, 24, 25, 22				
14	16	16	15	20, 24, 25, 23				
14	16	16	16	20, 24, 25, 24				
14	16	16	17	20, 24, 25, 24	20, 24, 26, 25	0.17, 2.37, 3.57, 1.63	0.19, 2.63, 3.71, 1.74	20, 24, 26, 25
14	16	17	14	20, 24, 26, 22				
14	16	17	15	20, 24, 26, 23				
14	16	17	16	20, 24, 26, 24				
14	16	17	17	20, 24, 26, 25				
14	17	14	14	20, 25, 24, 22				
14	17	14	15	20, 25, 24, 23				
14	17	14	16	20, 25, 24, 24				
14	17	14	17	20, 25, 24, 24	20, 25, 25, 25	0.17, 1.94, 4.59, 1.58	0.19, 2.18, 4.77, 1.71	20, 25, 25, 25
14	17	15	14	20, 25, 25, 22				
14	17	15	15	20, 25, 25, 23				
14	17	15	16	20, 25, 25, 24				
14	17	15	17	20, 25, 25, 25				
14	17	16	14	20, 25, 25, 22				
14	17	16	15	20, 25, 25, 23				
14	17	16	16	20, 25, 26, 24				
14	17	16	17	20, 25, 26, 25				
14	17	17	14	20, 25, 26, 22				
14	17	17	15	20, 25, 26, 23				
14	17	17	16	20, 25, 26, 24				
14	17	17	17	20, 25, 26, 25				

Table 18: Test framework cost, 4 players - part 1

Cost player 1	Cost player 2	Cost player 3	Cost player 4	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
15	14	14	14	21, 22, 23, 22				
15	14	14	15	21, 22, 23, 22	21, 23, 24, 23	0.11, 2.43, 3.67, 1.67	0.12, 2.71, 4.03, 1.89	21, 23, 24, 23
15	14	14	16	21, 23, 24, 23				
15	14	14	17	21, 23, 24, 23				
15	14	15	14	21, 22, 24, 22				
15	14	15	15	21, 23, 24, 23				
15	14	15	16	21, 23, 24, 23	21, 23, 25, 24	0.12, 2.71, 3.63, 1.65	0.14, 3.02, 3.78, 1.77	21, 23, 25, 24
15	14	15	17	21, 23, 25, 24				
15	14	16	14	21, 23, 25, 22				
15	14	16	15	21, 23, 25, 23				
15	14	16	16	21, 23, 25, 24				
15	14	16	17	21, 23, 25, 24				
15	14	17	14	21, 23, 26, 22				
15	14	17	15	21, 23, 26, 23				
15	14	17	16	21, 23, 26, 24				
15	14	17	17	21, 23, 26, 24				
15	15	14	14	21, 23, 24, 22				
15	15	14	15	21, 23, 24, 23				
15	15	14	16	21, 23, 24, 23				
15	15	14	17	21, 23, 24, 24				
15	15	15	14	21, 23, 24, 22				
15	15	15	15	21, 23, 24, 23				
15	15	15	16	21, 23, 24, 23	21, 23, 25, 24	0.12, 2.41, 3.63, 1.65	0.14, 2.68, 3.78, 1.77	21, 23, 25, 24
15	15	15	17	21, 23, 25, 24				
15	15	16	14	21, 23, 25, 22				
15	15	16	15	21, 23, 25, 23				
15	15	16	16	21, 23, 25, 24				
15	15	16	17	21, 23, 25, 24	21, 24, 26, 25	0.14, 2.68, 3.40, 1.55	0.16, 2.98, 3.73, 1.75	21, 24, 26, 25
15	15	17	14	21, 23, 26, 22				
15	15	17	15	21, 24, 26, 23				
15	15	17	16	21, 24, 26, 24				
15	15	17	17	21, 24, 26, 25				
15	16	14	14	21, 24, 24, 22				
15	16	14	15	21, 24, 24, 23				
15	16	14	16	21, 24, 24, 24				
15	16	14	17	21, 24, 24, 24				
15	16	15	14	21, 24, 24, 22				
15	16	15	15	21, 24, 25, 23				
15	16	15	16	21, 24, 25, 24				
15	16	15	17	21, 24, 25, 24				
15	16	16	14	21, 24, 25, 22				
15	16	16	15	21, 24, 25, 23				
15	16	16	16	21, 24, 25, 24				
15	16	16	17	21, 24, 25, 24	21, 24, 26, 25	0.15, 2.38, 3.59, 1.64	0.16, 2.65, 3.73, 1.75	21, 24, 26, 25
15	16	17	14	21, 24, 26, 22				
15	16	17	15	21, 24, 26, 23				
15	16	17	16	21, 24, 26, 24				
15	16	17	17	21, 24, 26, 25				
15	17	14	14	21, 25, 24, 22				
15	17	14	15	21, 25, 24, 23				
15	17	14	16	21, 25, 24, 24				
15	17	14	17	21, 25, 24, 24	21, 25, 25, 25	0.14, 1.95, 4.61, 1.59	0.16, 2.19, 4.79, 1.72	21, 25, 25, 25
15	17	15	14	21, 25, 25, 22				
15	17	15	15	21, 25, 25, 23				
15	17	15	16	21, 25, 25, 24				
15	17	15	17	21, 25, 25, 25				
15	17	16	14	21, 25, 25, 22				
15	17	16	15	21, 25, 25, 23				
15	17	16	16	21, 25, 26, 24				
15	17	16	17	21, 25, 26, 25				
15	17	17	14	21, 25, 26, 22				
15	17	17	15	21, 25, 26, 23				
15	17	17	16	21, 25, 26, 24				
15	17	17	17	21, 25, 26, 25				

Table 19: Test framework cost, 4 players - part 2

Cost player 1	Cost player 2	Cost player 3	Cost player 4	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
16	14	14	14	22, 22, 23, 22				
16	14	14	15	22, 22, 23, 22	22, 23, 24, 23	0.09, 2.33, 3.68, 1.67	0.10, 2.72, 4.05, 1.90	22, 23, 24, 23
16	14	14	16	22, 23, 24, 23				
16	14	14	17	22, 23, 24, 24				
16	14	15	14	22, 22, 24, 22				
16	14	15	15	22, 23, 24, 23				
16	14	15	16	22, 23, 24, 23	22, 23, 25, 24	0.10, 2.72, 3.64, 1.66	0.12, 3.03, 3.79, 1.78	22, 23, 25, 24
16	14	15	17	22, 23, 25, 24				
16	14	16	14	22, 23, 25, 22				
16	14	16	15	22, 23, 25, 23				
16	14	16	16	22, 23, 25, 24				
16	14	16	17	22, 23, 25, 24				
16	14	17	14	22, 23, 26, 22				
16	14	17	15	22, 23, 26, 23				
16	14	17	16	22, 23, 26, 24				
16	14	17	17	22, 23, 26, 24				
16	15	14	14	22, 23, 24, 22				
16	15	14	15	22, 23, 24, 23				
16	15	14	16	22, 23, 24, 23				
16	15	14	17	22, 23, 24, 24				
16	15	15	14	22, 23, 24, 22				
16	15	15	15	22, 23, 24, 23				
16	15	15	16	22, 23, 24, 23	22, 23, 25, 24	0.10, 2.42, 3.64, 1.66	0.12, 2.69, 3.79, 1.78	22, 23, 25, 24
16	15	15	17	22, 23, 25, 24				
16	15	16	14	22, 23, 25, 22				
16	15	16	15	22, 23, 25, 23				
16	15	16	16	22, 23, 25, 24				
16	15	16	17	22, 23, 25, 24	22, 24, 26, 25	0.12, 2.69, 3.41, 1.55	0.13, 2.99, 3.75, 1.76	22, 24, 26, 25
16	15	17	14	22, 23, 26, 22				
16	15	17	15	22, 24, 26, 23				
16	15	17	16	22, 24, 26, 24				
16	15	17	17	22, 24, 26, 25				
16	16	14	14	22, 24, 24, 22				
16	16	14	15	22, 24, 24, 23				
16	16	14	16	22, 24, 24, 24				
16	16	14	17	22, 24, 24, 24				
16	16	15	14	22, 24, 24, 22				
16	16	15	15	22, 24, 25, 23				
16	16	15	16	22, 24, 25, 24				
16	16	15	17	22, 24, 25, 24				
16	16	16	14	22, 24, 25, 22				
16	16	16	15	22, 24, 25, 23				
16	16	16	16	22, 24, 25, 24				
16	16	16	17	22, 24, 25, 24	22, 24, 26, 25	0.12, 2.39, 3.60, 1.64	0.14, 2.66, 3.75, 1.76	22, 24, 26, 25
16	16	17	14	22, 24, 26, 22				
16	16	17	15	22, 24, 26, 23				
16	16	17	16	22, 24, 26, 24				
16	16	17	17	22, 24, 26, 25				
16	17	14	14	22, 25, 24, 22				
16	17	14	15	22, 25, 24, 23				
16	17	14	16	22, 25, 24, 24				
16	17	14	17	22, 25, 24, 24	22, 25, 25, 25	0.12, 1.96, 4.63, 1.60	0.13, 2.20, 4.81, 1.72	22, 25, 25, 25
16	17	15	14	22, 25, 25, 22				
16	17	15	15	22, 25, 25, 23				
16	17	15	16	22, 25, 25, 24				
16	17	15	17	22, 25, 25, 25				
16	17	16	14	22, 25, 25, 22				
16	17	16	15	22, 25, 25, 23				
16	17	16	16	22, 25, 26, 24				
16	17	16	17	22, 25, 26, 25				
16	17	17	14	22, 25, 26, 22				
16	17	17	15	22, 25, 26, 23				
16	17	17	16	22, 25, 26, 24				
16	17	17	17	22, 25, 26, 25				

Table 20: Test framework cost, 4 players - part 3

Cost player 1	Cost player 2	Cost player 3	Cost player 4	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
17	14	14	14	23, 22, 23, 22				
17	14	14	15	23, 22, 23, 22	23, 23, 24, 23	0.07, 2.44, 3.68, 1.68	0.09, 2.73, 4.06, 1.90	23, 23, 24, 23
17	14	14	16	23, 23, 24, 23				
17	14	14	17	23, 23, 24, 24				
17	14	15	14	23, 22, 24, 22				
17	14	15	15	23, 23, 24, 23				
17	14	15	16	23, 23, 24, 23	23, 23, 25, 24	0.09, 2.73, 3.65, 1.66	0.10, 3.04, 3.80, 1.78	23, 23, 25, 24
17	14	15	17	23, 23, 25, 24				
17	14	16	14	23, 23, 25, 22				
17	14	16	15	23, 23, 25, 23				
17	14	16	16	23, 23, 25, 24				
17	14	16	17	23, 23, 25, 24				
17	14	17	14	23, 23, 26, 22				
17	14	17	15	23, 23, 26, 23				
17	14	17	16	23, 23, 26, 24				
17	14	17	17	23, 23, 26, 24				
17	15	14	14	23, 23, 24, 22				
17	15	14	15	23, 23, 24, 23				
17	15	14	16	23, 23, 24, 23				
17	15	14	17	23, 23, 24, 24				
17	15	15	14	23, 23, 24, 22				
17	15	15	15	23, 23, 24, 23				
17	15	15	16	23, 23, 24, 23	23, 23, 25, 24	0.09, 2.42, 3.65, 1.66	0.10, 2.70, 3.80, 1.78	23, 23, 25, 24
17	15	15	17	23, 23, 25, 24				
17	15	16	14	23, 23, 25, 22				
17	15	16	15	23, 23, 25, 23				
17	15	16	16	23, 23, 25, 24				
17	15	16	17	23, 23, 25, 24	23, 24, 26, 25	0.10, 2.70, 3.42, 1.56	0.12, 3.00, 3.76, 1.76	23, 24, 26, 25
17	15	17	14	23, 23, 26, 22				
17	15	17	15	23, 24, 26, 23				
17	15	17	16	23, 24, 26, 24				
17	15	17	17	23, 24, 26, 25				
17	16	14	14	23, 24, 24, 22				
17	16	14	15	23, 24, 24, 23				
17	16	14	16	23, 24, 24, 24				
17	16	14	17	23, 24, 24, 24				
17	16	15	14	23, 24, 24, 22				
17	16	15	15	23, 24, 25, 23				
17	16	15	16	23, 24, 25, 24				
17	16	15	17	23, 24, 25, 24				
17	16	16	14	23, 24, 25, 22				
17	16	16	15	23, 24, 25, 23				
17	16	16	16	23, 24, 25, 24				
17	16	16	17	23, 24, 25, 24	23, 24, 26, 25	0.10, 2.40, 3.62, 1.65	0.12, 2.67, 3.76, 1.76	23, 24, 26, 25
17	16	17	14	23, 24, 26, 22				
17	16	17	15	23, 24, 26, 23				
17	16	17	16	23, 24, 26, 24				
17	16	17	17	23, 24, 26, 25				
17	17	14	14	23, 25, 24, 22				
17	17	14	15	23, 25, 24, 23				
17	17	14	16	23, 25, 24, 24				
17	17	14	17	23, 25, 24, 24	23, 25, 25, 25	0.10, 1.96, 4.65, 1.60	0.11, 2.20, 4.83, 1.73	23, 25, 25, 25
17	17	15	14	23, 25, 25, 22				
17	17	15	15	23, 25, 25, 23				
17	17	15	16	23, 25, 25, 24				
17	17	15	17	23, 25, 25, 25				
17	17	16	14	23, 25, 25, 22				
17	17	16	15	23, 25, 25, 23				
17	17	16	16	23, 25, 26, 24				
17	17	16	17	23, 25, 26, 25				
17	17	17	14	23, 25, 26, 22				
17	17	17	15	23, 25, 26, 23				
17	17	17	16	23, 25, 26, 24				
17	17	17	17	23, 25, 26, 25				

Table 21: Test framework cost, 4 players - part 4

B0 player 1	B0 player 2	NE1
1	1	20, 22
1	5	20, 28
1	9	20, 39
5	1	27, 22
5	5	24, 25
5	9	21, 37
9	1	39, 22
9	5	38, 22
9	9	26, 27

Table 22: Test framework β_0^i, 2 players

B0 player 1	B0 player 2	B0 player 3	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
1	1	1	20, 22, 22				
1	1	5	20, 22, 28				
1	1	9	20, 22, 39				
1	5	1	20, 28, 22				
1	5	5	20, 25, 25	20, 26, 26	0.10, 3.53, 3.53	0.12, 3.76, 3.76	20, 26, 26
1	5	9	20, 23, 38				
1	9	1	20, 39, 22				
1	9	5	20, 38, 23				
1	9	9	20, 27, 27	20, 28, 28	0.003, 5.46, 5.46	0.004, 5.95, 5.95	20, 28, 28
5	1	1	27, 22, 22				
5	1	5	24, 22, 25				
5	1	9	21, 22, 37				
5	5	1	24, 25, 22				
5	5	5	23, 24, 24				
5	5	9	21, 23, 35				
5	9	1	21, 37, 22				
5	9	5	21, 35, 23				
5	9	9	20, 27, 27	20, 28, 28	0.18, 5.30, 5.30	0.21, 5.75, 5.75	20, 28, 28
9	1	1	39, 22, 22				
9	1	5	38, 22, 23				
9	1	9	26, 22, 27				
9	5	1	38, 23, 22				
9	5	5	35, 23, 23				
9	5	9	26, 22, 27				
9	9	1	26, 27, 22				
9	9	5	26, 27, 22				
9	9	9	23, 24, 24	24, 25, 25	3.35, 2.50, 2.50	3.72, 2.81, 2.81	24, 25, 25

Table 23: Test framework β_0^i, 3 players

B0 player 1	B0 player 2	B0 player 3	B0 player 4	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
1	1	1	1	20, 22, 22, 23				
1	1	1	5	20, 22, 22, 28				
1	1	1	9	20, 22, 22, 39				
1	1	5	1	20, 22, 28, 23				
1	1	5	5	20, 22, 26, 26				
1	1	5	9	20, 22, 23, 38				
1	1	9	1	20, 22, 39, 23				
1	1	9	5	20, 22, 39, 24				
1	1	9	9	20, 22, 28, 28				
1	5	1	1	20, 28, 22, 23				
1	5	1	5	20, 26, 22, 26				
1	5	1	9	20, 23, 22, 38				
1	5	5	1	20, 25, 25, 23	20, 26, 26, 23	0.10, 3.50, 3.50, 0.06	0.12, 3.72, 3.72, 0.07	20, 26, 26, 23
1	5	5	5	20, 24, 24, 25				
1	5	5	9	20, 23, 23, 36				
1	5	9	1	20, 23, 38, 23				
1	5	9	5	20, 23, 36, 24				
1	5	9	9	20, 22, 28, 28				
1	9	1	1	20, 39, 22, 23				
1	9	1	5	20, 39, 22, 24				
1	9	1	9	20, 28, 22, 28				
1	9	5	1	20, 38, 23, 23				
1	9	5	5	20, 36, 23, 24				
1	9	5	9	20, 28, 22, 28				
1	9	9	1	20, 27, 27, 23	20, 28, 28, 23	0.003, 5.46, 5.46, 0.002	0.004, 5.95, 5.95, 0.002	20, 28, 28, 23
1	9	9	5	20, 27, 27, 23	20, 28, 28, 23	0.003, 5.36, 5.36, 0.11	0.004, 5.82, 5.82, 0.13	20, 28, 28, 23
1	9	9	9	20, 25, 25, 25				
5	1	1	1	27, 22, 22, 23				
5	1	1	5	24, 22, 22, 26				
5	1	1	9	21, 22, 22, 37				
5	1	5	1	24, 22, 25, 23				
5	1	5	5	23, 22, 24, 25				
5	1	5	9	21, 22, 23, 36				
5	1	9	1	21, 22, 37, 23				
5	1	9	5	21, 22, 35, 23				
5	1	9	9	20, 22, 28, 28				
5	5	1	1	24, 25, 22, 23				
5	5	1	5	23, 24, 22, 25				
5	5	1	9	21, 23, 22, 36				
5	5	5	1	23, 24, 24, 23				
5	5	5	5	22, 23, 23, 24				
5	5	5	9	21, 22, 22, 34				
5	5	9	1	21, 23, 35, 23				
5	5	9	5	21, 22, 34, 23				
5	5	9	9	20, 22, 27, 28				
5	9	1	1	21, 37, 22, 23				
5	9	1	5	21, 35, 22, 23				
5	9	1	9	20, 28, 22, 28				
5	9	5	1	21, 35, 23, 23				
5	9	5	5	21, 34, 22, 23				
5	9	5	9	20, 27, 22, 28				
5	9	9	1	20, 27, 27, 23	20, 28, 28, 23	0.18, 5.30, 5.30, 0.002	0.21, 5.74, 5.7, 0.002	20, 28, 28, 23
5	9	9	5	20, 27, 27, 23				
5	9	9	9	20, 25, 25, 25				
9	1	1	1	39, 22, 22, 23				
9	1	1	5	38, 22, 22, 24				
9	1	1	9	26, 22, 22, 27	27, 22, 22, 28	6.47, 0.002, 0.002, 4.54	7.00, 0.003, 0.003, 4.99	27, 22, 22, 28
9	1	5	1	38, 22, 23, 23				
9	1	5	5	35, 22, 23, 23	36, 22, 23, 24	15.58, 0.01, 0.76, 0.65	15.87, 0.02, 0.88, 0.74	36, 22, 23, 24
9	1	5	9	26, 22, 22, 27	27, 22, 22, 28	6.35, 0.002, 0.12, 4.45	6.85, 0.003, 0.14, 4.87	27, 22, 22, 28
9	1	9	1	26, 22, 27, 23				
9	1	9	5	26, 22, 27, 23				
9	1	9	9	23, 22, 24, 25	24, 22, 25, 25	3.52, 0.001, 2.63, 2.21	3.72, 0.001, 2.81, 2.50	24, 22, 25, 25
9	5	1	1	38, 23, 22, 23				
9	5	1	5	35, 23, 22, 23	36, 23, 22, 24	15.58, 0.76, 0.01, 0.65	15.87, 0.88, 0.02, 0.74	36, 23, 22, 24
9	5	1	9	26, 22, 22, 27	27, 22, 22, 28	6.35, 0.12, 0.002, 4.45	6.85, 0.14, 0.003, 4.87	27, 22, 22, 28
9	5	5	1	35, 23, 23, 23				
9	5	5	5	34, 22, 22, 23				
9	5	5	9	26, 22, 22, 27				
9	5	9	1	26, 22, 27, 23				
9	5	9	5	26, 22, 27, 23				
9	5	9	9	23, 22, 24, 25				
9	9	1	1	26, 27, 22, 23				
9	9	1	5	26, 27, 22, 23				
9	9	1	9	23, 24, 22, 25	24, 25, 22, 25	3.52, 2.63, 0.001, 2.21	3.72, 2.81, 0.001, 2.50	24, 25, 22, 25
9	9	5	1	26, 27, 22, 23				
9	9	5	5	26, 27, 22, 23				
9	9	5	9	23, 24, 22, 25				
9	9	9	1	23, 24, 24, 23	24, 25, 25, 23	3.35, 2.50, 2.50, 0.001	3.72, 2.81, 2.81, 0.001	24, 25, 25, 23
9	9	9	5	23, 24, 24, 23				
9	9	9	9	22, 24, 24, 24				

Table 24: Test framework β_0^i, 4 players

Bp	NE1
0.1	26,39
0.2	20,27
0.3	18,21
0.4	17,19
0.5	16,18
0.6	16,18
0.7	15,17
0.8	15,17
0.9	15,17

Table 25: Test framework β_p, 2 players

Bp	NE1
0.1	25, 33, 36
0.2	19, 24, 25
0.3	17, 21, 21
0.4	17, 19, 19
0.5	16, 18, 18
0.6	16, 18, 18
0.7	15, 17, 17
0.8	15, 17, 17
0.9	15, 17, 17

Table 26: Test framework β_p, 3 players

Bp	NE1	NE2	Payoff NE1	Payoff NE2	Dominant NE
0.1	24, 30, 32, 30	24, 31, 33, 30	0.27, 4.34, 6.47, 3.16	0.29, 4.51, 6.67, 3.39	24, 31, 33, 30
0.2	19, 23, 24, 23				
0.3	17, 20, 21, 21				
0.4	17, 19, 19, 20				
0.5	16, 18, 18, 19				
0.6	16, 18, 18, 19				
0.7	15, 17, 17, 18				
0.8	15, 17, 17, 18				
0.9	15, 17, 17, 18				

Table 27: Test framework β_p, 4 players