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Combining multiple prioritized knowledge bases by negotiation☆

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Abstract

Recently, several belief negotiation models have been introduced to deal with the problem of belief merging. A negotiation model usually consists of two functions: a negotiation function and a weakening function. A negotiation function is defined to choose the weakest sources and these sources will weaken their point of view using a weakening function. However, the currently available belief negotiation models are based on classical logic, which makes them difficult to define weakening functions. In this paper, we define a prioritized belief negotiation model in the framework of possibilistic logic. The priority between formulae provides us with important information to decide which beliefs should be discarded. The problem of merging uncertain information from different sources is then solved by two steps. First, beliefs in the original knowledge bases will be weakened to resolve inconsistencies among them. This step is based on a prioritized belief negotiation model. Second, the knowledge bases obtained by the first step are combined using a *conjunctive* operator which may have a reinforcement effect in possibilistic logic.

Keywords: Possibility theory; Belief merging; Negotiation

1. Introduction

Belief merging deals with problems of obtaining a coherent belief base from several inconsistent belief bases representing sources of information [1,3,12,18,20–23]. In [20–23], the merging operators were defined by some postulates. Recently, a class of general operators called DA^2 (DA^2 means a distance between interpretations and two aggregation functions) merging operators was proposed which encodes many previous merging operators as specific cases [19]. Although these merging operators satisfy some good properties, they are too *ideal*. Namely, the agents cannot communicate or negotiate.

In recent years, some belief merging methods based on belief negotiation models were proposed to make the merging process more *active* [12,13,18]. Belief negotiation models based methods deal with the merging problem by several rounds of negotiation or competition. In each round, some sources are chosen by a negotiation function, then these sources have to weaken their point of view using a weakening function. However, both Konieczny's belief negotiation model and Booth's belief negotiation model are defined in purely propositional logic systems. Therefore it is difficult to define a weakening function.

☆ This paper is an extended version of a conference paper [24].

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The importance of priorities in inconsistency handling has been addressed by many researchers in recent years, e.g. [5,17,23]. Priority between formulae provides us with important information to decide which beliefs should be discarded. So it is helpful to consider priority when we define a belief negotiation model. Possibilistic logic [16] provides a good framework to express priorities and reason with uncertain information. In possibilistic logic, each classical first order formula is attached to a number or a weight, denoting the *necessity degree* of the formula. The necessity degrees can be interpreted as the levels of priority of formulae.

Many merging operators in possibilistic logic have been proposed [3,4]. When sources of information are *strongly in conflict*, two classes of operators are often applied. One is called normalized conjunctive operators and the other is called disjunctive operators. However, both classes of operators have their disadvantages. One of the disadvantages of the normalized conjunctive operators is that they may be very sensitive to rather small variations of *necessity degrees* around 0 [4]. The problem with the disjunctive operators is that the result of merging may be very imprecise and too much original information is lost. Furthermore, the existing merging operators are too *static*, that is, agents cannot interact with each other to reach agreement.

In this paper, we propose a prioritized belief negotiation model, where priorities between formulae are handled in the framework of possibilistic logic. Each source of beliefs is represented as a *possibilistic belief base* (PBB). The procedure of merging different sources of beliefs is carried out in two steps. The first step is called a negotiation step, beliefs in some of the original knowledge bases will be weakened to make it possible for them to be added together consistently (this step is called *social contraction* in [13]). Some negotiation functions and weakening functions will be defined by considering the priority in this step. The second step is called a combination step, the knowledge bases obtained by the first step are combined using a *conjunctive* operator which may have a reinforcement effect in possibilistic logic [3,7].

This paper is organized as follows. Section 2 gives some preliminaries. We introduce Konieczny's belief game model in Section 3. In Section 4, we give a brief review of possibilistic logic. Semantic and syntactical combination rules in possibilistic logic are introduced in Section 5. In Section 6, our prioritized belief negotiation model is presented. In Section 7, we give some particular negotiation functions and weakening functions. In Section 8, we instantiate the prioritized belief negotiation model and provide an example to illustrate the new merging methods. A comparison of our merging methods in this paper with some previous merging methods is given in Section 9. Finally, we conclude the paper in Section 10.

2. Preliminaries

In this paper, we consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$. Ω denotes the set of possible worlds, where each possible world is a function from $\mathcal{P}$ to $\{\top, \bot\}$ ($\top$ denotes truth value *true* and $\bot$ denotes the truth value *false*). A model of a formula ϕ is a possible world ω which makes the formula *true*. We use $mod(\phi)$ to denote the set of models of formula ϕ, i.e., $mod(\phi) = \{w \in \Omega | w \models \phi\}$. Deduction in classical propositional logic is denoted by symbol $\vdash$ as usual. A literal is an atom p or its negation $\neg p$. We will denote literals by $l, l_1, \ldots$, atoms in $\mathcal{P}$ by $p, q, r, \ldots$, and classical formulae by $\phi, \psi, \gamma, \ldots$. Given two formulae ϕ and ψ, ϕ and ψ are equivalent, denoted $\phi \equiv \psi$, if and only if $\phi \vdash \psi$ and $\psi \vdash \phi$. A formula ϕ is consistent if and only if $mod(\phi) \neq \emptyset$.

A belief base φ is a consistent propositional formula (or, equivalently, a finite set of propositional formulae $\{\phi_1, \ldots, \phi_n\}$ such that $\phi_1 \wedge \cdots \wedge \phi_n$ is consistent). Let $\varphi_1, \ldots, \varphi_n$ be n belief bases (not necessarily different). A *belief profile* is a multi-set Ψ consisting of those n belief bases: $\Psi = (\varphi_1, \ldots, \varphi_n)$. The *conjunction* of the belief bases of Ψ is denoted as $\bigwedge \Psi$, i.e., $\bigwedge \Psi = \varphi_1 \wedge \cdots \wedge \varphi_n$. $\bigsqcup$ and $\sqsubseteq$ are used to denote the *union* and *inclusion* of belief profiles, respectively. A belief profile Ψ is consistent if and only if $\bigwedge \Psi$ is consistent. Two belief profiles Ψ_1 and Ψ_2 are said to be equivalent ($\Psi_1 \equiv \Psi_2$) if and only if there is a bijection f between Ψ_1 and Ψ_2 such that $\forall \varphi \in \Psi_1$, $\varphi \equiv f(\varphi)$, where $f(\varphi)$ is the image of φ in Ψ_2. $\mathcal{E}$ denotes the set of all finite non-empty belief profiles.

3. Belief game model

A belief game model (BGM) [18] is developed from Booth's belief negotiation model [13] which provides a framework for merging sources of beliefs incrementally. It consists of two functions. One is called a negotiation function, which selects from every belief profile in $\mathcal{E}$ a subset of belief bases. The other is called a weakening function, which aims to weaken the beliefs of a selected source.

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Definition 1. A *negotiation function* is a function $g\colon \mathcal{E}\to\mathcal{E}$ such that:

(n1) $g(\Psi) \sqsubseteq \Psi$,
(n2) If $\bigwedge \Psi \not\equiv \top$, then $\exists\varphi\in g(\Psi)$ s.t. $\varphi\not\equiv\top$,
(n3) If $\Psi\equiv\Psi'$, then $g(\Psi)\equiv g(\Psi')$.

The first two conditions guarantee a non-empty subset is chosen from a belief profile to be weakened. The third condition is about irrelevance of syntax.

Definition 2. A *weakening function* is a function $\blacktriangledown\colon \mathcal{L}\to\mathcal{L}$ such that:

(w1) $\varphi\vdash\blacktriangledown(\varphi)$,
(w2) If $\varphi \equiv \blacktriangledown(\varphi)$, then $\varphi \equiv \top$,
(w3) If $\varphi \equiv \varphi'$, then $\blacktriangledown(\varphi) \equiv \blacktriangledown(\varphi')$.

The first two conditions ensure that a base will be replaced by a strictly weaker one unless the base is already a tautological one. The last condition is an irrelevance of syntax requirement, i.e., the result of weakening depends only on the information conveyed by a base, not on its syntactical form.

A weakening function can be extended as follows. Let Ψ' be a subset of Ψ, $\blacktriangledown_{\Psi'}(\Psi) = \{\blacktriangledown(\varphi) \mid \varphi \in \Psi'\}\sqcup\{\varphi \mid \varphi \in \Psi \setminus \Psi'\}$.

Definition 3. A BGM is a pair $\mathcal{N} = \langle g, \blacktriangledown\rangle$ where g is a negotiation function and $\blacktriangledown$ is a weakening function. The solution to a belief profile Ψ for a BGM $\mathcal{N} = \langle g, \blacktriangledown\rangle$, noted as $\mathcal{N}(\Psi)$, is the belief profile $\Psi_{\mathcal{N}}$, defined as

- $\Psi_0 = \Psi$,
- $\Psi_{i+1} = \blacktriangledown_{g(\Psi_i)}(\Psi_i)$,
- $\Psi_{\mathcal{N}}$ is the first Ψ_i that is consistent.

4. Possibilistic logic

Possibilistic logic [16] is a weighted logic where each classical logic formula is associated with a level of priority.

The semantics of possibilistic logic is based on the notion of a *possibility distribution* which is a mapping π from Ω to the unit interval [0,1]. The unit interval can be replaced by any totally ordered scale. $\pi(\omega)$ represents the degree of compatibility of the interpretation ω with the available beliefs about the real world. $\pi(\omega) = 0$ means that the interpretation ω is impossible to be the real world, and $\pi(\omega) = 1$ means that nothing prevents ω from being the real world, while $0 < \pi(\omega) < 1$ means that ω is only somewhat possible to be the real world. When $\pi(\omega) > \pi(\omega')$, ω is preferred to ω' for being the real world. A possibility distribution is said to be normal if and only if there exist a $\omega\in\Omega$, such that $\pi(\omega) = 1$. Given two possibility distributions π and π', π is said to be less specific (or less informative) than π' if for all $\omega\in\Omega$, $\pi(\omega)\geqslant\pi'(\omega)$ and $\exists\omega\in\Omega$, $\pi(\omega) > \pi'(\omega)$.

From a possibility distribution π, two measures defined on a set of propositional formulae can be determined. One is the possibility degree of formula ϕ, and is defined as $\Pi_\pi(\phi) = \max\{\pi(\omega)\colon \omega\models\phi\}$. The other is the necessity degree of formula ϕ, and is defined as $N_\pi(\phi) = 1 - \Pi_\pi(\neg\phi)$. The possibility degree of ϕ evaluates to what extent ϕ is consistent with knowledge expressed by π and the necessity degree of ϕ evaluates to what extent ϕ is entailed by the available knowledge. $N_\pi(\phi) = 1$ means that ϕ is a totally certain piece of knowledge, while $N_\pi(\phi) = 0$ expresses the complete lack of knowledge of priority about ϕ, but does not mean that ϕ is or should be false. We have $N_\pi(true) = 1$ and $N_\pi(\phi\wedge\psi) = \min(N_\pi(\phi), N_\pi(\psi))$ for all ϕ and ψ.

A PBB is a set of possibilistic formulae of the form $B = \{(\phi_i, a_i)\colon i = 1, \ldots, n\}$, where $a_i \in (0, 1]$ and they are meant to be the necessity degrees of ϕ_i, for $i = 1, \ldots, n$. The classical base associated with B is denoted as B^*, namely $B^* = \{\phi_i | (\phi_i, a_i) \in B\}$. A PBB B is consistent if and only if its classical base B^* is consistent. A possibilistic belief profile $\mathcal{KP}$ is a multi-set of PBBs which are not necessarily different. We use $\cup(\mathcal{KP})$ to denote the union of knowledge bases in $\mathcal{KP}$. $\mathcal{KP} = (B_1, \ldots, B_n)$ is consistent if and only if $B_1^*\cup\cdots\cup B_n^*$ is consistent. We use $\mathcal{PE}$ to denote the set of all finite non-empty possibilistic belief profiles and $\mathcal{K}$ to denote the set of all the PBBs.

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Definition 4. Let B be a PBB, and $a \in (0,1]$. The a-cut (resp. strict a-cut) of B is $B_{\geqslant a} = \{\phi \in B^* | (\phi, b) \in B$ and $b \geqslant a\}$ (resp. $B_{>a} = \{\phi \in B^* | (\phi, b) \in B$ and $b > a\}$).

The *inconsistency degree* of B, which defines its level of inconsistency, is defined as $Inc(B) = \max\{a_i | B_{\geqslant a_i}$ is inconsistent$\}$, where $Inc(B) = 0$ if B is consistent.

Let B and B' be two PBBs. B and B' are said to be equivalent, denoted $B \equiv_s B'$, if and only if $\forall a \in (0,1]$, $B_{\geqslant a} \equiv B'_{\geqslant a}$. Two possibilistic belief profiles $\mathcal{KP}_1$ and $\mathcal{KP}_2$ are said to be equivalent, denoted $\mathcal{KP}_1 \equiv_s \mathcal{KP}_2$, if and only if there is a bijection between them such that each PBB of $\mathcal{KP}_1$ is equivalent to its image in $\mathcal{KP}_2$.

Definition 5. Let B be a *PBB*. Let (ϕ, a) be a piece of information with $a > Inc(B)$. (ϕ, a) is said to be a consequence of B, denoted $B \vdash_\pi (\phi, a)$, iff $B_{\geqslant a} \vdash \phi$.

It is required that weights of possibilistic formulae which are consequences of B be greater than the inconsistency degree of B. This is because for any possibilistic formula (ϕ, a), if $a \leqslant Inc(B)$, then $B_{\geqslant a} \vdash \phi$. That is, (ϕ, a) can be inferred from B trivially. $B \vdash_\pi B'$ denotes $B \vdash_\pi (\phi, a)$ for all $(\phi, a) \in B'$. $B \equiv_s B'$ if and only if $B \vdash_\pi B'$ and $B' \vdash_\pi B$.

Although possibilistic inference is inconsistency tolerant, it suffers from the *drowning problem* [2]. That is, given an inconsistent possibilistic knowledge base B, formulae whose certainty degrees are not larger than $Inc(B)$ are completely useless for non-trivial deductions. For instance, let $B = \{(\phi, 0.9), (\neg\phi, 0.8), (\gamma, 0.6), (\psi, 0.7)\}$, it is clear that B is equivalent to $B' = \{(\phi, 0.9), (\neg\phi, 0.8)\}$ because $Inc(B) = 0.8$. So $(\psi, 0.7)$ and $(\gamma, 0.6)$ are not used in the possibilistic inference.

Given a PBB B, a unique *possibility distribution*, denoted by π_B, can be obtained by the principle of minimum specificity [16]. For all $\omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall (\phi_i, a_i) \in B, \quad \omega \models \phi_i, \\ 1 - \max\{a_i | \omega \not\models \phi_i, (\phi_i, a_i) \in B\} & \text{otherwise.} \end{cases} \tag{1}$$

5. Semantic and syntactical combination rules in possibilistic logic

Many combination rules in possibilistic logic have been proposed [3,7]. Let B_1 and B_2 be two PBBs, π_1 and π_2 be their associated possibility distributions. Semantically, a two place function $\oplus$ from $[0,1] \times [0,1]$ to [0,1] is applied to aggregate π_1 and π_2 into a new possibility distribution $\pi_\oplus$, i.e., $\pi_\oplus(\omega) = \pi_1(\omega) \oplus \pi_2(\omega)$. Generally, the operator $\oplus$ is very weakly constrained, i.e., the only requirements for it are the following properties [3]:

(1) $1 \oplus 1 = 1$, and
(2) if $a \geqslant c$, $b \geqslant d$ then $a \oplus b \geqslant c \oplus d$, where $a, b, c, d \in [0,1]$ (monotonicity).

The first property states that if two sources agree that an interpretation ω is fully possible, then the result of merging should confirm it. The second property is the monotonicity condition, that is, a degree resulting from a combination cannot decrease if the degrees to be combined increase.

We now consider some specific operators.

Definition 6 (*Benferhat et al. [3]*). A disjunctive operator is a two place function $\oplus$: $[0,1] \times [0,1] \rightarrow [0,1]$ such that for all $a \in [0,1]$, $a \oplus 1 = 1 \oplus a = 1$.

Examples of disjunctive operators are the maximum operator and the *probabilistic sum* operator defined by $a \oplus b = a + b - ab$.

Definition 7 (*Benferhat et al. [3]*). A conjunctive operator is a two place function $\oplus$: $[0,1] \times [0,1] \rightarrow [0,1]$ such that for all $a \in [0,1]$, $a \oplus 1 = 1 \oplus a = a$.

Examples of conjunctive operators are the minimum operator and the product operator.

Definition 8 (*Benferhat et al. [3]*). A reinforcement operator is a two place function $\oplus$: $[0,1] \times [0,1] \rightarrow [0,1]$ such that for all $a, b \neq 1$ and $a, b \neq 0$, $a \oplus b < \min(a, b)$.

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Examples of reinforcement operator are the product operator and the *Lukasiewicz* t-norm $\max(0, a + b - 1)$. It is clear that a conjunctive operator may be a reinforcement operator.

In the case of n sources $B_1, \ldots, B_n$, the semantic combination of their possibility distributions $\pi_1, \ldots, \pi_n$ can be performed easily when $\oplus$ is associative. That is, we have $\pi_\oplus(\omega) = (\cdots((\pi_1(\omega) \oplus \pi_2(\omega)) \oplus \pi_3(\omega)) \oplus \cdots) \oplus \pi_n(\omega)$. When the operator is not associative, it needs to be generalized as a unary operator defined on vector $(a_1, \ldots, a_n)$ of real numbers from [0,1] such that:

(1) $\oplus(1, \ldots, 1) = 1$, and
(2) if $\forall i = 1, \ldots, n,\ a_i \geqslant b_i$ then $\oplus(a_1, \ldots, a_n) \geqslant \oplus(b_1, \ldots, b_n)$, where $a_i, b_i \in [0, 1]$.

The syntactical counterpart of the fusion of π_1 and π_2 is to obtain a PBB whose possibility distribution is $\pi_\oplus$. In [6], it has been shown that this knowledge base has the following form:

$$B_\oplus = \{(\phi_i, 1 - (1 - a_i)\oplus 1) : (\phi_i, a_i) \in B_1\} \cup \{(\psi_j, 1 - 1\oplus(1 - b_j)) : (\psi_j, b_j) \in B_2\} \cup \{(\phi_i \vee \psi_j, 1 - (1 - a_i)\oplus(1 - b_j)) : (\phi_i, a_i) \in B_1 \text{ and } (\psi_j, b_j) \in B_2\}. \quad (2)$$

That is, we have $\pi_{B_\oplus}(\omega) = \pi_\oplus(\omega) = \pi_1(\omega)\oplus\pi_2(\omega)$, for all $\omega\in\Omega$, where $\pi_{B_\oplus}$ is the possibility distribution associated to $B_\oplus$. It is clear that when $\oplus = \min$, $B_1 \oplus B_2 \equiv_s B_1 \cup B_2$. Whilst when $\oplus = \max$, $B_1 \oplus B_2 \equiv_s \{(\phi\vee\psi, \min(a, b)) : (\phi, a) \in B_1, (\psi, b) \in B_2\}$. It is often assumed that an operator used to combine possibility distributions should be both commutative and associative, i.e., $a\oplus b = b\oplus a$ and $a\oplus(b\oplus c) = (a\oplus b)\oplus c$. When $\oplus$ is associative, the syntactic computation of the resulting base is easily generalized to n sources. The syntactic generalization for a non-associative operator can be carried out as follows.

Proposition 9 (*Benferhat et al. [3]*). *Let $B_1, \ldots, B_n$ be a set of n PBB and $(\pi_1, \ldots, \pi_n)$ be their associated possibility distributions. Let $\pi_{B_\oplus}$ be the result of combining $(\pi_1, \ldots, \pi_n)$ with $\oplus$. The possibilistic knowledge base associated to $\pi_{B_\oplus}$ is*

$$B_\oplus = \{(D_j, 1 - \oplus(x_1, \ldots, x_n)) : j = 1, \ldots, n\}, \quad (3)$$

where D_j $(j = 1, \ldots, n)$ are disjunctions of size j between formulae ϕ_i taken from different B_i's $(i = 1, \ldots, n)$ and x_i is either equal to $1 - a_i$ or 1 depending, respectively, on whether ϕ_i belongs to D_j or not.

When the original PBBs B_1 and B_2 are consistent, i.e., $B_1 \cup B_2$ is consistent, conjunctive operators exploit the symbolic complementarities between sources.

Proposition 10 (*Benferhat et al. [3]*). *Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic profile such that the classical base $B_1^* \cup \cdots \cup B_n^*$ is consistent. Let $\oplus$ be a conjunctive operator. Then, $B_\oplus^* \equiv B_1^* \cup \cdots \cup B_n^*$.*

When a reinforcement operator is chosen, then all the common information is recovered with a higher degree. That is, if a formula is inferred from each possibilistic knowledge base with a positive degree, then this formula should be inferred from the fused base with a higher degree.

Proposition 11 (*Benferhat et al. [3]*). *Let B_1 and B_2 be such that $B_1^* \cup B_2^*$ is consistent. Let $\oplus$ be a reinforcement operator. Let ϕ be such that $B_1 \vdash_\pi (\phi, a)$ and $B_2 \vdash_\pi (\phi, b)$, where a and b are strictly positive. Then*

$$B_\oplus \vdash_\pi (\phi, c)$$

with $c > \max(a, b)$ if $a, b \in (0, 1)$, and $c = 1$ if $a = 1$ or $b = 1$.

By Propositions 10 and 11, when the union of original PBBs is consistent, it is advisable to use a conjunctive operator or an operator which is both conjunctive and has reinforcement effect because all the formulae in these PBBs are kept in the resulting PBB and their necessity degrees may be reinforced.

By Proposition 10, suppose $\oplus$ is a conjunctive operator, $B_\oplus$ is inconsistent when $B_1 \cup \cdots \cup B_n$ is inconsistent. We have two ways to handle the inconsistency. The first way is to restore a consistent PBB by deleting some conflicting

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formulae from $B_{\oplus}$ [4]. The merging operators obtained in this way are called normalized conjunctive operators. For example, one of the normalized conjunctive operators deletes those formulae in the resulting base whose weights are not larger than the inconsistency degree. So normalized conjunctive operators also have the *drowning problem*. The other way is to ignore the inconsistency and apply possibilistic consequence relation to infer conclusions [9] (note that possibilistic consequence relation is inconsistency tolerant).

6. A prioritized belief negotiation model

In this section, we propose a prioritized belief negotiation model to generalize the BGM [18], where priorities between formulae are handled in the framework of possibilistic logic. Each source of beliefs is represented as a PBB. We assume that the original PBBs are self-consistent.

Definition 12. A *negotiation function* is a function $g: \mathcal{PE} \to \mathcal{PE}$ such that:

(N1) $g(\mathcal{KP}) \sqsubseteq \mathcal{KP}$,
(N2) If $\mathcal{KP}$ is inconsistent and $\exists B \in \mathcal{KP}$ s.t. $B^* \not\equiv \top$, then $\forall B' \in g(\mathcal{KP})$, $(B')^* \not\equiv \top$.

Condition N1 is directly generalized from condition n1 in BGM. Condition N2 states that the negotiation function will not select the PBB whose classical base is equivalent to the *tautology* if there is a PBB whose classical base is not equivalent to the *tautology*. That is, we do not choose the *tautology* to weaken if possible. This condition is to ensure that our prioritized belief negotiation model always terminates. Our negotiation function relies on the syntactical form of the PBBs, because every formula is attached a weight in a PBB, and we need to consider the syntax of the PBB in some cases. A negotiation function g is called *syntax-independent* if it satisfies the following condition.

(N3) If $\mathcal{KP} \equiv_s \mathcal{KP}'$, then $g(\mathcal{KP}) \equiv_s g(\mathcal{KP}')$.

Next we will give the definition of a weakening function.

Definition 13. A *weakening function* is a function $\blacktriangledown: \mathcal{K} \times \mathcal{PE} \times \mathcal{PE} \to \mathcal{K}$ such that: for each triple consisting of a PBB B and two possibilistic profiles $\mathcal{KP}$ and $\mathcal{KP}'$, if $\mathcal{KP}' \sqsubseteq \mathcal{KP}$ and $B \in \mathcal{KP}'$, then $\blacktriangledown_{\mathcal{KP},\mathcal{KP}'}(B)$ should satisfy the conditions (W1) and (W2) below, otherwise $\blacktriangledown_{\mathcal{KP},\mathcal{KP}'}(B) = B$.

(W1) $\blacktriangledown_{\mathcal{KP},\mathcal{KP}'}(B) \subseteq B$,
(W2) If $B = \blacktriangledown_{\mathcal{KP},\mathcal{KP}'}(B)$, then $B^* \equiv \top$.

Condition W1 says that the weakened base contains no more information than the original one. Condition W2 states that a PBB which is selected by a negotiation function must have its belief weakened unless it does not contain any meaningful information. Unlike the weakening function in BGM, our weakening function only weakens the PBBs in a subset of possibilistic belief profile and keeps other PBBs unchanged. When weakening a PBB, our weakening function may take into account other PBBs in the possibilistic profile. So it is context-dependent. Furthermore, the priority between formulae in a PBB makes the construction of weakening function easy. For example, for a PBB B, we can define a weakening function which deletes *conflicting* formulae of B with the lowest priority.

We can extend a weakening function on possibilistic belief profiles as follows: let $\mathcal{KP}'$ be a subset of $\mathcal{KP}$, $\blacktriangledown_{\mathcal{KP},\mathcal{KP}'}(\mathcal{KP}) = \{\blacktriangledown_{\mathcal{KP},\mathcal{KP}'}(B) : B \in \mathcal{KP}\}$.

Definition 14. A prioritized belief negotiation model is a pair $\mathcal{N} = \langle g, \blacktriangledown \rangle$ where g is a negotiation function and $\blacktriangledown$ is a weakening function. The solution to a possibilistic belief profile $\mathcal{KP}$ for a belief negotiation model $\mathcal{N} = \langle g, \blacktriangledown \rangle$, denoted $\mathcal{N}(\mathcal{KP})$, is the belief profile $\mathcal{KP}_{\mathcal{N}}$ defined as

- $\mathcal{KP}_0 = \mathcal{KP}$,
- $\mathcal{KP}_{i+1} = \blacktriangledown_{\mathcal{KP}_i, g(\mathcal{KP}_i)}(\mathcal{KP}_i)$,
- $\mathcal{KP}_{\mathcal{N}}$ is the first $\mathcal{KP}_i$ that is consistent.

Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. The combination of PBBs in $\mathcal{KP}$ is divided into two steps.

Step 1: PBBs in $\mathcal{KP}$ are weakened using a prioritized belief negotiation model to obtain a consistent belief profile $\mathcal{KP}_{\mathcal{N}}$.

Step 2: PBBs in $\mathcal{KP}_{\mathcal{N}}$ are combined using a conjunctive operator which may have a reinforcement effect (usually we choose a commutative and associative operator such as the product operator).

The idea is that the information of the original belief bases is weakened to make them consistent and then their common beliefs or goals will be reinforced.

7. Negotiation and weakening functions

7.1. Negotiation function

7.1.1. Distance between two PBBs

The first category of negotiation functions is based on a distance between two PBBs. In this subsection, we define a distance function based on the quantity of conflict defined in [25].

The following is the definition of a distance between two PBBs, which is a simple extension of the distance between two classical belief bases in [18].

Definition 15. A (pseudo) distance between two PBBs is a function $d: \mathcal{K} \times \mathcal{K} \rightarrow [0, +\infty)$ such that:

- $d(B, B') = 0$ iff $B^* \cup B'^* \nvdash \perp$,
- $d(B, B') = d(B', B)$.

Clearly, a very simple distance can be defined as follows: $d_D(B, B') = 0$ if $B^* \cup B'^* \nvdash \perp$ and $d_D(B, B') = 1$ otherwise.

Now we introduce a new distance between two PBBs based on the *weighted prime implicants* (WPIs) [25]. This can be used to define a distance between two PBBs.

An *implicant* of a belief base φ is a conjunction of literals D such that $D \vdash \varphi$ and D does not contain two complementary literals.

Definition 16. A *prime implicant* of a belief base φ is an implicant D of φ such that for every other implicant D' of φ, $D \nvdash D'$.

Prime implicants are often used in knowledge compilation to make the deduction tractable. Suppose $D_1, \ldots, D_k$ are all the prime implicants of φ, we have $\varphi \vdash \phi$ iff for every prime implicant D_i, $D_i \vdash \phi$, for any ϕ.

Now we define the WPI of a PBB. Let us first define the WPI for PBB $B = \{(\phi_1, a_1), \ldots, (\phi_n, a_n)\}$ where ϕ_i are clauses, and a clause is a disjunction of literals. For a more general PBB, we can decompose it as an equivalent PBB whose formulae are clauses by the min-decomposability of necessity measures, i.e., $N(\wedge_{i=1,k}\phi_i) \geqslant m \Leftrightarrow \forall i, N(\phi_i) \geqslant m$ [15]. That is, a possibilistic formula $(\phi_1 \wedge \cdots \wedge \phi_k, a)$ can be equivalently decomposed as a set of possibilistic formulae $(\phi_1, a), \ldots, (\phi_k, a)$.

Let $B = \{(\phi_1, a_1), \ldots, (\phi_n, a_n)\}$ be a PBB where ϕ_i are clauses. A weighted implicant of B is $D = \{(\psi_1, b_1), \ldots, (\psi_k, b_k)\}$, a PBB, such that $D \vdash_\pi B$, where ψ_i are literals such that no two complementary literals exist. Let D and D' be two weighted implicants of B, D is said to be *subsumed* by D' iff $D \neq D'$, $D'^* \subseteq D^*$ and $\forall(\psi_i, a_i) \in D$, $\exists(\psi_i, b_i) \in D'$ with $b_i \leqslant a_i$ (b_i is 0 if $\psi_i \in D^*$ but $\psi_i \notin D'^*$).

Definition 17. Let $B = \{(\phi_1, a_1), \ldots, (\phi_n, a_n)\}$ be a PBB where ϕ_i are clauses. A WPI of B is D such that:

(1) D is a weighted implicant of B,
(2) $\nexists\, D'$ of B such that D is subsumed by D'.

Let us look at an example to illustrate how to construct WPIs.

Example 18. Let $B = \{(p, 0.8), (q \vee r, 0.5), (q \vee \neg s, 0.6)\}$ be a PBB. The WPIs of B are $D_1 = \{(p, 0.8), (q, 0.6)\}$, $D_2 = \{(p, 0.8), (r, 0.5), (\neg s, 0.6)\}$, and $D_3 = \{(p, 0.8), (q, 0.5), (\neg s, 0.6)\}$.

The WPI generalizes the prime implicant.

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Lemma 19. *Let* $B = \{(\phi_1, 1), \ldots, (\phi_n, 1)\}$ *be a PBB where all the formulae have weight* 1, *i.e., B is a classical knowledge base.*[1] *D is a weighted implicant of B iff D is an implicant of B.*

Proof. A PBB $D = \{(\psi_1, 1), \ldots, (\psi_k, 1)\}$, where ψ_j $(j = 1, k)$ are literals, is a weighted implicant of B iff $D \vdash_\pi (\phi_i, 1)$ for all $(\phi_i, 1) \in B$ and there are no two complementary literals. According to [16], $D \vdash_\pi (\phi_i, 1)$ iff $D \vdash \phi_i$ for all i. So D is a weighted implicant of B if and only if D is an implicant of B. □

Lemma 20. *Let* $B = \{(\phi_1, 1), \ldots, (\phi_n, 1)\}$ *be a PBB where all the formulae have weight* 1. *Let* D *and* D' *be two weighted implicants of* B *and* $D \neq D'$. *Then* D *is subsumed by* D' *and only if* $D \vdash D'$.

Proof. Since D and D' are two weighted implicants of B, by Lemma 19, D and D' are implicants of B. So $D \vdash D'$ iff $D' \subset D$ iff D is subsumed by D'. □

Proposition 21. *Let* $B = \{(\phi_1, 1), \ldots, (\phi_n, 1)\}$ *be a PBB where all the formulae have weight* 1. *Then* D *is a WPI of* B *iff* D *is a prime implicant of* B.

Proof. The proof is clear by Lemmas 19 and 20 and Definitions 16 and 17. □

However, given PBB B, if D is a WPI of B, then D^* is not necessary to be a prime implicant of B^*. A counterexample can be found in Example 18, where D_3 is a WPI, but $D_3^* = \{p, q, \neg s\}$ is not a prime implicant of B^*.

Definition 22. Let p be a propositional symbol, $\sim$ is the complementation operation such that $\sim p$ is $\neg p$ and $\sim(\neg p)$ is p. This operation is not in the object language but will be used to make definitions clearer.

We define the quantity of conflict between two WPIs.

Definition 23. Let B_1 and B_2 be two PBBs. Suppose C and D are WPIs of B_1 and B_2, respectively, then the quantity of conflict between C and D is defined as

$$q_{\mathrm{Con}}(C, D) = \sum_{(l,a)\in C \text{ and } (\sim l,b)\in D} \min(a, b). \tag{4}$$

When the weights associated with all the formulae are 1, $q_{\mathrm{Con}}(C, D)$ is the cardinality of the set of atoms which are in conflict in $C \cup D$.

Definition 24. Let B_1 and B_2 be two PBBs. Suppose $\mathcal{C}$ and $\mathcal{D}$ are the sets of WPIs of B_1 and B_2, respectively, then the quantity of conflict between B_1 and B_2 is defined as

$$Q_{\mathrm{Con}}(B_1, B_2) = \min\{q_{\mathrm{Con}}(C, D) | C \in \mathcal{C}, D \in \mathcal{D}\}. \tag{5}$$

The quantity of conflict between B_1 and B_2 measures information that is in conflict between B_1 and B_2. It has been proved in [25] that the quantity of conflict between two classical belief bases is the Dalal distance between them [14]. So we can define a distance function d_C based on the quantity of conflict such that $d_C(B_1, B_2) = Q_{\mathrm{Con}}(B_1, B_2)$ (it is easy to check that d_C satisfies the requirements of a distance function in Definition 15).

Proposition 25. *Let* $B_1 \equiv_s B_1'$ *and* $B_2 \equiv_s B_2'$. *Then* $d_C(B_1, B_2) = d_C(B_1', B_2')$.

Proof. Since $B_i \equiv_s B_i'$ $(i = 1, 2)$, we have $D \vdash_\pi B_i$ iff $D \vdash_\pi B_i'$, for an arbitrary PBB D. So D is a weighted implicant of B_i iff it is a weighted implicant of B_i'. By Definition 17, D is a WPI of B_i if and only if it is a WPI of B_i'. By Eq. (4) it is clear that $Q_{\mathrm{Con}}(B_1, B_2) = Q_{\mathrm{Con}}(B_1', B_2')$, so $d_C(B_1, B_2) = d_C(B_1', B_2')$. □

Proposition 25 tells us that the distance function d_C is syntax-independent.

[1] B is used to denote both a PBB consisting of possibilistic formulae whose weights are 1 and its classical base.

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7.1.2. Distance-based negotiation function

Before defining our distance-based negotiation function, we need to introduce the aggregation function defined in [18].

Definition 26. An aggregation function is a total function *f* associating a non-negative integer to every finite tuple of non-negative integers and verifying the following conditions:

- If $x \leqslant y$, then $f(x_1, \ldots, x, \ldots, x_n) \leqslant f(x_1, \ldots, y, \ldots, x_n)$ (non-decreasingness).
- $f(x_1, \ldots, x_n) = 0$ iff $x_1 = \cdots = x_n = 0$ (minimality).
- For every non-negative integer x, $f(x, \ldots, x) = x$ (identity).

Two most commonly used aggregation functions are the maximum and the sum Σ.

Now we can define the distance-based negotiation function.

Definition 27. Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. A distance-based ordering between two different PBBs B_i and B_j is defined as follows:

$$B_i \prec_d B_j \quad \text{iff either } B_i^* \equiv \top \text{ and } B_j^* \not\equiv \top \text{or } B_i^* \not\equiv \top \text{ and } B_j^* \not\equiv \top, \text{ but } f(d(B_i, B_1), \ldots, d(B_i, B_n)) < f(d(B_j, B_1), \ldots, d(B_j, B_n)),$$

where *f* is an aggregation function, and d is a distance function between two PBBs.

That is, when $B_i \prec_d B_j$, we say that B_j is "further" from the group than B_i.

Definition 28. Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. A distance-based negotiation function is defined as follows: for all $B_i \in \mathcal{KP}$,

$$B_i \in g^{d,f}(\mathcal{KP}) \quad \text{iff} \quad \nexists B_j \in \mathcal{KP} \text{ such that } B_i \prec_d B_j.$$

In Definition 28, those sources that are "furthest" from the group are weakened. We now check that $g^{d,f}(\mathcal{KP})$ is a negotiation function. First, according to Definition 28, N1 is satisfied. N2 is also satisfied by definition of the distance-based ordering.

When $d = d_D$ or $d = d_C$, we have the following proposition analyzing the computational issues of their corresponding negotiation functions.

Proposition 29. *Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. Let $d = d_D$ or $d = d_C$. f is an aggregation function. $g^{d,f}$ is the distance-based negotiation function. Determining whether $B_i (B_i \in \mathcal{KP})$ belongs to $g^{d,f}(\mathcal{KP})$ or not is in Δ_2^p, where Δ_2^p denotes the set of decision problems decidable by a polynomial-time Turing machine with an NP oracle.*

Proof. By the definition of d_D, we need one call to an NP oracle to compute the distance $d_D(B_i, B_j)$ between two belief bases B_i and B_j. So for each $B_i \in \mathcal{KP}$, we need $n-1$ calls to an NP oracle to compute $f(d_D(B_i, B_1), \ldots, d_D(B_i, B_n))$. To check whether $B_i \in g^{d_D,f}(\mathcal{KP})$, we need to check whether $B_i^* \equiv \top$ and compute $f(d_D(B_j, B_1), \ldots, d_D(B_j, B_n))$ for all $j \in \{1, \ldots, n\}$ in the worst case. So it needs no more than n^2 calls to an NP oracle to check whether $B_i \in g^{d_D,f}(\mathcal{KP})$.

By [26], we need only two calls to an NP oracle to compute the distance $d_C(B_i, B_j)$ between two belief bases B_i and B_j. Similar to the proof for $g^{d_D,f}$, we can prove that it needs no more than $2n^2$ calls to an NP oracle to check whether $B_i \in g^{d_D,f}(\mathcal{KP})$. □

The following corollary follows from Proposition 25.

Corollary 30. $g^{d_C,f}$ *satisfies the condition* N3, *that is, it is syntax-independent.*

7.1.3. Conflict-based negotiation function

Priority provides an easy way for us to deal with inconsistency. In belief revision and belief merging, an implicit or explicit priority is often assumed. The inconsistency of a PBB can be resolved by dropping those conflicting formulae with the lowest priority in a minimally inconsistent subbase [8,17]. A natural negotiation function can be defined by selecting those PBBs which contain conflicting formulae in the lowest level of the union of all the original PBBs.

We first introduce some definitions in [5].

Definition 31 (*Benferhat et al. [5]*). A subbase C of PBB B is said to be minimally inconsistent if and only if it satisfies the following two requirements:

- $C^* \vDash \bot$,
- $\forall \phi \in C^*,\ C^* - \{\phi\} \nvDash \bot$.

Definition 32 (*Benferhat et al. [5]*). A possibilistic formula $(\phi,\ a)$ is said to be in *conflict* in B iff it belongs to some minimally inconsistent subbase of B.

Next, we define the weakest conflicting formula in a PBB.

Definition 33. Let B be an inconsistent PBB. A possibilistic formula (ϕ, a) is said to be a weakest conflicting formula in B iff it satisfies

- (ϕ, a) is in conflict in B,
- $\forall (\psi, b) \in B$, if $b < a$, then (ψ, b) is not in conflict in B.

In the following, we define a negotiation function which selects those PBBs which contain a weakest conflicting formula in the union of all the original PBBs.

Definition 34. Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. A weakest-conflict-based negotiation function is defined as follows:

$$g^{\mathrm{wc}}(\mathcal{KP}) = \{B_i \in \mathcal{KP} | \exists \phi \in B_i,\ \phi \text{ is a weakest conflicting formula in } \cup (\mathcal{KP})\}.$$

The weakest-conflict-based negotiation function is recommended to be used with the weakest-conflict-based weakening function that will be defined in the next subsection. We check that g^{wc} is a negotiation function. N1 is clearly satisfied. We consider N2. Suppose $\mathcal{KP}$ is inconsistent and there exists $B_j \in \mathcal{KP}$ such that $B_j^* \not\equiv \top$, since g^{wc} only selects those PBBs containing a weakest conflicting formula, for all $B' \in g^{\mathrm{wc}}(\mathcal{KP})$, we have $(B')^* \not\equiv \top$.

The negotiation function g^{wc} does not satisfy N3. Let us look at an example.

Example 35. Let $\mathcal{KP} = \{B_1, B_2\}$ and $\mathcal{KP}' = \{B_1', B_2'\}$, where $B_1 = \{(\phi, 0.7),\ (\psi, 0.5)\}$, $B_2 = \{(\neg\phi, 0.6)\}$, $B_1' = \{(\phi, 0.7), (\psi, 0.5), (\phi, 0.3)\}$ and $B_2' = \{(\neg\phi, 0.6)\}$. It is easy to check that $\mathcal{KP} \equiv_s \mathcal{KP}'$. However, $g^{\mathrm{wc}}(\mathcal{KP}) = \{B_2\}$ (since $(\neg\phi, 0.6)$ is a weakest conflicting formula) and $g^{\mathrm{wc}}(\mathcal{KP}') = \{B_1'\}$ (since $(\phi, 0.3)$ is a weakest conflicting formula). So $g^{\mathrm{wc}}(\mathcal{KP}) \not\equiv_s g^{\mathrm{wc}}(\mathcal{KP}')$.

7.2. Weakening function

The priority derived from the necessity degrees of possibilistic formulae allows us to define some interesting weakening functions. The first weakening function deletes the weakest conflicting formulae in a belief base.

Definition 36. Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. A possibilistic formula (ϕ, a) is said to be the weakest conflicting formula of $B_i \in \mathcal{KP}$ iff

- (ϕ, a) is in conflict in $\cup(\mathcal{KP})$,
- $\forall (\psi, b) \in B_i$, if $b < a$, then (ψ, b) is not in conflict in $\cup(\mathcal{KP})$.

Definition 37. Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile and $\mathcal{KP}'$ be a subset of $\mathcal{KP}$. Let $B \in \mathcal{KP}'$ and $C = \{(\phi, a) \in B | (\phi, a)$ is a weakest conflicting formula of B in $\cup(\mathcal{KP})\}$. Let $\alpha = \min\{a \in (0, 1]\colon \exists \phi, (\phi, a) \in B\}$.

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The weakest-conflict-based (WC for short) weakening function is defined as for each $B \in \mathcal{KP}$,

$$\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}(B) = \begin{cases} B \backslash C & \text{if } C \neq \emptyset; \\ \{(\phi, a) \in B : a \neq \alpha\}, & \text{otherwise.} \end{cases}$$

That is, for each B which is selected by a negotiation function, if it contains a conflicting formula, then the WC-weakening function deletes those weakest conflicting formulae in it. Otherwise, it simply deletes those formulae in B that have the least priority. We check that $\blacktriangledown^{\mathrm{wc}}$ is a weakening function. First, it is clear that W1 is satisfied. Second, W2 is satisfied because the result of the weakening function $\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}$ drops some formulae from the original knowledge base.

The weakening function defined above needs to compute the conflicting formulae, which is computationally too hard. In the following, we define a weakening function which does not need to compute conflicting formulae.

Definition 38. Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile and $\mathcal{KP}'$ be an arbitrary subset of $\mathcal{KP}$. $B \in \mathcal{KP}'$. Let $\alpha = \min\{a \in (0, 1] : \exists \phi, (\phi, a) \in B\}$. The blind-optimized weakening function is defined as $\blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}(B) = \{(\phi, a) \in B : a \neq \alpha\}$.

The blind-optimized weakening function deletes formulae with the least priority. The weakening function applies when the agent does not know which formula is in conflict in the PBB, so it simply deletes those formulae that have the least priority. Similar to the WC-weakening function, it is easy to show that the blind-optimized weakening function is a weakening function.

8. Instantiating the framework and examples

8.1. Instantiation

Different combinations of the negotiation functions and the weakening functions will result in different prioritized belief negotiation models and then different belief merging methods. In the examples given below, we assume that after some PBBs are weakened, the combination operator is the minimum, i.e., the PBBs are conjoined.

- $\langle g^{\mathrm{wc}}, \blacktriangledown^{\mathrm{wc}} \rangle$ [2] : This merging method deletes the conflicting formulae from the lower levels, i.e., weights of formulae are lower. That is, the agents always choose the weakest information to discard. This idea can be found in [8].
- $\langle g^{d_D, f^{\mathrm{Max}}}, \blacktriangledown^{\mathrm{wc}} \rangle$: In this case, every PBB which is in conflict with any other PBBs deletes their weakest conflicting formulae in each round. This merging method usually deletes more formulae than the merging method based on $\langle g^{\mathrm{wc}}, \blacktriangledown^{\mathrm{wc}} \rangle$.
- $\langle g^{d_D, f^{\Sigma}}, \blacktriangledown^{\mathrm{wc}} \rangle$: In this case, in each round of negotiation, those PBBs which have the greatest number of PBBs in conflict will be selected and have their weakest conflicting formulae deleted.
- $\langle g^{d_C, f^{\Sigma}}, \blacktriangledown^{\mathrm{wc}} \rangle$: In this case, in each round of negotiation, those PBBs which have more quantities of information in conflict with other PBBs will be selected and have their weakest conflicting formulae deleted.
- $\langle g^{d_C, f^{\Sigma}}, \blacktriangledown^{\mathrm{bo}} \rangle$: In this case, in each round of negotiation, those PBBs which have more quantities of information in conflict with other PBBs will be selected and have all the formulae in their lowest layers deleted. This merging method deletes more formulae than the merging method based on $\langle g^{d_C, f^{\Sigma}}, \blacktriangledown^{\mathrm{wc}} \rangle$. However, it is computationally simpler.

In the examples above, we require that the combination rule used in the second step of merging be the minimum. If we relax this restriction, we can get more interesting merging methods. For example, in the case of $\langle g^{\mathrm{wc}}, \blacktriangledown^{\mathrm{wc}} \rangle$, if we further assume that the combination operator is the product operator, then we can get a merging method which has a reinforcement effect.

[2] For simplicity, we will ignore the subscript of the weakening functions.

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To choose between different negotiation functions and weakening functions, we need to consider the following criteria:

- *Computational complexity*:

(1) Negotiation function: We proposed two different kinds of negotiation functions. One is the distance-based negotiation function (DBF) and the other is the conflict-based negotiation function (CBF). Usually, the computational complexity of CBF is harder than that of DBF under the usual assumptions of complexity theory (i.e., $\Delta_2^p \subset \Sigma_2^p$). This is because determining if a given formula ϕ is in conflict in a belief base is a hard task (that is, Σ_2^p-complete as shown in [11]). In contrast, we have shown that computational complexities of negotiation functions based on both d_D and d_C are in Δ_2^p.
(2) Weakening function: We proposed two weakening functions. One is the WC weakening function and the other is the blind-optimized weakening function. The WC weakening function needs to determine if a given formula is in conflict in a belief base, which is a hard task. Whilst, the blind-optimized weakening function can be computed in polynomial time. Therefore, the computational complexity of the WC weakening function is much harder than that of the blind-optimized weakening function under the usual assumptions of complexity theory.

- *Information loss*: Weakening functions delete information in belief bases that are chosen by a negotiation function. When choosing between different weakening functions, an important criterion is to consider the amount of information deleted. Intuitively, we may prefer those weakening functions which delete less information from a belief base.

Proposition 39. *Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ be a possibilistic belief profile. Let $\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}$ and $\blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}$ be the WC-weakening function and the blind-optimized weakening function, respectively. Suppose $B_i \in \mathcal{KP}'$. Let $\alpha = \min\{a \in (0,1]: \exists\phi, (\phi, a) \in B_i\}$. If there is a $(\phi,\alpha) \in B_i$ such that (ϕ,α) is in conflict in $\cup(\mathcal{KP})$, then $\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}(B_i) \vdash_\pi \blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}(B_i)$.*

Proof. Since there is a $(\phi,\alpha) \in B_i$ such that (ϕ,α) is in conflict in $\cup(\mathcal{KP})$, by Definitions 37 and 38, we have $\blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}(B_i) \subseteq \blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}(B_i)$. So $\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}(B_i) \vdash_\pi \blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}(B_i)$. □

Proposition 39 shows that the blind-optimized weakening function deletes more original information than the WC weakening function in some case. However, the proposition may not hold in some special cases. Let us look at a counter-example. [3]

Example 40. Let $\mathcal{KP} = \{B_1, B_2\}$ be a possibilistic belief profile, where $B_1 = \{(\neg q, 0.8)\}$ and $B_2 = \{(p, 0.5), (q, 0.7)\}$, where p and q are atoms. It is clear that q is in conflict in $B_1 \cup B_2$. Suppose $\mathcal{KP}' = \{B_2\}$. We then have that $\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}(B_2) = \{(p, 0.5)\}$, whilst $\blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}(B_2) = \{(q, 0.7)\}$. So $\blacktriangledown^{\mathrm{wc}}_{\mathcal{KP},\mathcal{KP}'}(B_2) \nvdash_\pi \blacktriangledown^{\mathrm{bo}}_{\mathcal{KP},\mathcal{KP}'}(B_2)$.

In the following, we use $\Delta_{\mathcal{N},\oplus}$ to denote the possibilistic merging operator obtained by a prioritized belief negotiation model $\mathcal{N}$ and a conjunctive operator $\oplus$ ($\oplus$ may have the reinforcement effect).

8.2. Illustrative example

In this section, we give an example to illustrate two prioritized belief negotiation model based merging methods, i.e., those based on $\langle g^{d_D, f^\Sigma}, \blacktriangledown^{\mathrm{wc}}\rangle$ and $\langle g^{d_C, f^\Sigma}, \blacktriangledown^{\mathrm{wc}}\rangle$.

Example 41. Three people are talking about origins of human beings and planets. Their opinions are summarized as weighted logical sentences in a possibilistic belief profile $\mathcal{KP} = \{A, B, C\}$, where

$A = \{(p, 0.4), (q \to r, 1), (s, 0.8), (\neg s \to \neg r, 0.9)\}$,
$B = \{(q, 0.8), (\neg s, 0.6), (e, 0.8)\}$,
$C = \{(\neg p, 0.8), (\neg q, 0.6), (e \to r, 0.4)\}$,

- p represents "there were human beings in Mars before"
- q represents "scientists have detected some strange signals from outer space"

[3] This example is given by one of the reviewers.

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- r represents “there are aliens in other planets”
- s represents “the ancestors of human are gorillas”
- e represents “the earth was created by chance, not by a creator”.

In this example, C is quite sure that there were no human beings in Mars before and is unsure that if the earth was created by chance, then there are aliens in other planets too.

Now we will see how they can negotiate with each other to make their opinions coherent.

- *Method* 1: $\langle g^{d_D,f^{\Sigma}}, \blacktriangledown^{wc}\rangle$ and $\oplus =$ Lukasiewicz t-norm:
  Since A, B and C are in conflict with each other, $g^{d_D,f^{\Sigma}}(\mathcal{KP}) = \mathcal{KP}$. So A is replaced by $\blacktriangledown^{wc}(A) = \{(q\rightarrow r, 1), (s, 0.8), (\neg s\rightarrow\neg r, 0.9)\}$,[4] B is replaced by $\blacktriangledown^{wc}(B) = \{(q, 0.8), (e, 0.8)\}$ and C is replaced by $\blacktriangledown^{wc}(C) = \{(\neg p, 0.8), (\neg q, 0.6)\}$. Now $\blacktriangledown^{wc}(B)$ and $\blacktriangledown^{wc}(C)$ are still in conflict, and they will have to weaken their beliefs in the second round. So $\blacktriangledown^{wc}(B) = \{(e, 0.8)\}$ and $\blacktriangledown^{wc}(C) = \{(\neg p, 0.8)\}$. In this case, we have reached a consistent possibilistic belief profile. By combining $\blacktriangledown^{wc}(A)$, $\blacktriangledown^{wc}(B)$ and $\blacktriangledown^{wc}(C)$ using Lukasiewicz t-norm, we have the following result of merging (in the following, we delete the redundant formulae from the resulting knowledge base to make it easier to read):

$$\begin{aligned}\Delta_{\mathcal{N},\oplus}(\mathcal{KP}) \equiv_s \{&(q\rightarrow r, 1), (s, 0.8), (\neg s\rightarrow\neg r, 0.9), (e, 0.8), (\neg p, 0.8),\\ &(e\vee\neg p, 1), (\neg q \vee r \vee e, 1), (s\vee e, 1), (s\vee\neg r\vee e, 1),\\ &(\neg p\vee\neg q \vee r, 1), (\neg p\vee s, 1), (\neg p\vee s\vee\neg r, 1), (\neg p\vee\neg q\vee r\vee e, 1),\\ &(\neg p\vee s\vee e, 1), (\neg p\vee s\vee\neg r\vee e, 1)\}.\end{aligned}$$

- *Method* 2: $\langle g^{d_C,f^{\Sigma}}, \blacktriangledown^{wc}\rangle$ and $\oplus =$ Lukasiewicz t-norm:
  Since $\mathcal{KP}$ is not consistent, we need to compute the distance from each PBB to others using $g^{d_C,f^{\Sigma}}$. $d_C(A, B) = 0.6$, $d_C(A, C) = 0.4$, $d_C(B, C) = 0.6$. Since $d_C(A, B)+d_C(A, C) = 1$, $d_C(B, A)+d_C(B, C) = 1.2$, and $d_C(C, A)+d_C(C, B) = 1$, in the first round, we have $g^{d_C,f^{\Sigma}}(\mathcal{KP}) = \{B\}$. So B is replaced by $\blacktriangledown^{wc}(B) = \{(q, 0.8), (e, 0.8)\}$. The obtained belief profile is still inconsistent, we must then go to the second round. Now $d_C(A, B) = 0$, $d_C(A, C) = 0.4$, $d_C(B, C) = 0.6$. Since $d_C(A, B) + d_C(A, C) = 0.4$, $d_C(B, A) + d_C(B, C) = 0.6$, and $d_C(C, A)+d_C(C, B) = 1$, we have $g^{d_C,f^{\Sigma}}(\mathcal{KP}) = \{C\}$. C is then replaced by $\blacktriangledown^{wc}(C) = \{(\neg p, 0.8), (e\rightarrow r, 0.4)\}$. The obtained belief profile is inconsistent again, we must now go to the third round. $d_C(A, B) = 0$, $d_C(A, C) = 0.4$, $d_C(B, C) = 0$. Since $d_C(A, B) + d_C(A, C) = 0.4$, $d_C(B, A) + d_C(B, C) = 0$, and $d_C(C, A) + d_C(C, B) = 0.4$ $g^{d_C,f^{\Sigma}}(\mathcal{KP} = \{A, C\}$. A is then replace by $\blacktriangledown^{wc}(A) = \{(q\rightarrow r, 1), (s, 0.8), (\neg s\rightarrow\neg r, 0.9)\}$ and C is replaced by $\blacktriangledown^{wc}(C) = \{e\rightarrow r, 0.4)\}$. The obtained belief profile is consistent, and the result of merging is

$$\begin{aligned}\Delta_{\mathcal{N},\oplus}(\mathcal{KP}) \equiv_s \{&(q\rightarrow r, 1), (s, 0.8), (\neg s\rightarrow\neg r, 0.9), (q, 0.8), (e, 0.8),\\ &(e\rightarrow r, 0.4), (s\vee\neg e\vee r, 1), (q\vee s, 1), (q\vee\neg e\vee r, 1), (s\vee e, 1)\}.\end{aligned}$$

It is clear that the negotiation process in the second method is more complex than that of the first one. However, in the second merging method, C drops the beliefs with high priority, whilst A and B both drop their weakest beliefs. Therefore, C loses the game.

8.3. Logical properties

We consider the logical properties of the merging operators obtained by the prioritized belief negotiation models.

In [28], in order to evaluate the split–combination operators, we adapted logical properties for merging operators in the propositional setting to possibilistic logic. Let Δ be a possibilistic merging operator, and $\mathcal{KP}$, $\mathcal{KP}_1$ and $\mathcal{KP}_2$ be possibilistic profiles, then we have

(P1) $\Delta(\mathcal{KP})$ is consistent.
(P2) Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$. If $B_1 \cup \cdots \cup B_n$ is consistent, then $(\Delta(\mathcal{KP}))^* \equiv (B_1 \cup \cdots \cup B_n)^*$ and $\forall\phi$, if there exists i such that $B_i\vdash_{\pi}(\phi, a)$ then there exists b such that $b\geqslant a$ and $\Delta(\mathcal{KP})\vdash_{\pi}(\phi, b)$.

[4] To make the notation simpler, we will ignore the subscript of the weakening functions. Moreover, we do not use subscripts to denote the different weakening steps of the bases.

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(P3) If $\mathcal{KP}_1 \equiv_s \mathcal{KP}_2$, then $\Delta(\mathcal{KP}_1) \equiv_s \Delta(\mathcal{KP}_2)$.
(P4) Let $\mathcal{KP} = \{B_1, \ldots, B_n\}$ and $\bigcup \mathcal{KP} = B_1 \cup \cdots \cup B_n$. Let $(\phi, a) \in \cup(\mathcal{KP})$. If (ϕ, a) is free in $\cup\mathcal{KP}$, i.e., (ϕ, a) is not in conflict in $\cup(\mathcal{KP})$, then $\Delta(\mathcal{KP}) \vdash_\pi (\phi, b)$, where $b \geqslant a$.

(P1) says that the resulting PBB should be consistent. (P2) is adapted from a postulate in [9]. It states that when there is no conflict among original knowledge bases, the operator should not only restore all the original information, but also have a reinforcement effect on the weights of formulae. (P3) is the principle of irrelevance of syntax, i.e., if two possibilistic profiles are equivalent, then the PBBs resulting from the two merging will be equivalent. (P4) requires that the resulting PBB contain all the formulae which are not involved in conflict.

First, it is clear that the resulting PBB of merging must be consistent. That is, we have the following proposition.

Proposition 42. *Let $\oplus$ be a conjunctive operator. The operator $\Delta_{\mathcal{N},\oplus}$ satisfies* (P1).

The proof of Proposition 42 is clear by considering Definition 14.

Proposition 43. *Let $\oplus$ be a conjunctive operator which may have reinforcement effect. The operator $\Delta_{\mathcal{N},\oplus}$ satisfies* (P2).

Proof. By Definition 14, if $\mathcal{KP}$ is consistent, then $\mathcal{KP}_\mathcal{N} = \mathcal{KP}$. So $\Delta_{\mathcal{N},\oplus}(\mathcal{KP}) = B_\oplus$, where $B_\oplus$ is defined by Eq. 3. Since $\oplus$ is a conjunctive operator, we have $(\Delta_{\mathcal{N},\oplus}(\mathcal{KP}))^* \equiv (B_1 \cup \cdots \cup B_n)^*$. Furthermore, since $\oplus$ is a conjunctive operator which may have reinforcement effect, we have if $\exists i$ such that $B_i \vdash_\pi (\phi, a)$ then $\exists b$ such that $b \geqslant a$ and $\Delta_{\mathcal{N},\oplus}(\mathcal{KP}) \vdash_\pi (\phi, b)$. □

We also have the following two propositions.

Proposition 44. *Let $\oplus$ be a conjunctive operator. Let $\mathcal{N} = \langle g^{d,f}, \blacktriangledown \rangle$, where $d = d_D$ or d_C, and $\blacktriangledown = \blacktriangledown^{\mathrm{bo}}$. Then the operator $\Delta_{\mathcal{N},\oplus}$ satisfies* (P3). *However, it does not satisfy* (P4) *in general.*

Proof. Let $\mathcal{KP}_1 \equiv_s \mathcal{KP}_2$, where $\mathcal{KP}_1 = \{B_1, \ldots, B_n\}$ and $\mathcal{KP}_2 = \{B'_1, \ldots, B'_n\}$. Without loss of generality, suppose $B_i \equiv_s B'_i$ for all i. By Corollary 30, $g^{d_C,f}$ is independent of the syntax. It is clear that $g^{d_D,f}$ is also independent of the syntax. So $B_i \in g^{d,f}(\mathcal{KP}_1)$ iff $B'_i \in g^{d,f}(\mathcal{KP}_2)$. Since $\blacktriangledown = \blacktriangledown^{\mathrm{bo}}$, we have $\blacktriangledown(B_i) \equiv_s \blacktriangledown(B'_i)$. It is clear that $\oplus$ is a syntax-independent operator. Therefore, the operator $\Delta_{\mathcal{N},\oplus}$ satisfies (P3).

Clearly, $\Delta_{\mathcal{N},\oplus}$ does not satisfy (P4) in general. This is because the weakening function $\blacktriangledown^{\mathrm{bo}}$ deletes all the formulae with least priority, including formulae which is not involved in conflict. □

Proposition 45. *Let $\oplus$ be a conjunctive operator. Let $\mathcal{N} = \langle g^{\mathrm{wc}}, \blacktriangledown^{\mathrm{wc}} \rangle$. Then the operator $\Delta_{\mathcal{N},\oplus}$ satisfies* (P4). *It does not satisfy* (P3) *in general.*

Proof. Since g^{wc} selects those bases which contain the weakest conflicting formulae and then $\blacktriangledown^{\mathrm{wc}}$ deletes only those weakest conflicting formulae, it is clear that $\Delta_{\mathcal{N},\oplus}$ satisfies (P4). However, it does not satisfy (P3) because both g^{wc} and $\blacktriangledown^{\mathrm{wc}}$ are syntax-dependent. □

9. Related work

This paper is closely related to the belief negotiation models proposed by Booth [12,13] and Konieczny [18]. Our prioritized belief negotiation model is different from belief negotiation models in several aspects. First, the original belief bases are prioritized in our model, i.e., a weight is attached to each formula in the belief bases, whilst the negotiation models are based on classical logic. Second, our model is syntax-relevant, i.e., it relies on the syntactical form of the belief bases. This is because every formula is attached to a weight in a PBB, and we need to consider the syntax of the PBB. In contrast, the belief negotiation models are syntax-irrelevant. Third, after belief bases are weakened to be consistent by the prioritized belief negotiation model, we *combine* them using an appropriate operator. In contrast, the merging method based on the belief negotiation model simply takes the conjunction of the belief bases which have been weakened to be coherent.

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Many merging methods have been proposed in possibilistic logic [3–5,7,27]. In [3,4,7], given several PBBs, the semantic combination rules are applied to aggregate the possibility distributions associated with them. The syntactical counterpart of these semantic combination rules are then defined (see Eq. (2)). Next, we compare our merging operators with existing possibilistic merging operators. In generally, our new merging operators differ from existing ones in that existing ones are *static*, while the new operators are more *active*, that is, different agents can compete with each other to reach agreement.

9.1. Disjunctive operator

When belief bases are conflicting, the resulting belief base of the combination rules is inconsistent in general. If we want to get a consistent belief base after merging, then it is more advisable to apply a *disjunctive* operator. In this case, the resulting belief base of merging two belief bases B_1 and B_2 is $B_\oplus = \{(\phi_i \vee \psi_j, 1-(1-a_i)\oplus(1-b_j)) : (\phi_i, a_i)\in B_1, (\psi_j, b_j)\in B_2\}$. A disadvantage of the disjunctive operator based merging method is that too much information is lost after combination.

Let us look at Example 41 again.

Example 46 (*Continue Example 41*). Let $\oplus = \max$. Merging A and B by the maximum we get $B_1 \equiv_s \{(p\vee q, 0.4), (p\vee\neg s, 0.4), (p\vee e, 0.4), (\neg q\vee r\vee\neg s, 0.6), (\neg q\vee r\vee e, 0.8), (q\vee s, 0.8), (s\vee e, 0.8)\}$. We then merge B_1 and C by the maximum, the result is equivalent to

$$
\begin{aligned}
B_{\max} = \{&(\neg p \vee \neg q\vee r\vee\neg s, 0.6), (\neg p \vee \neg q\vee r\vee e, 0.8), (\neg p\vee q\vee s, 0.8),\\
&(\neg p\vee s\vee e, 0.8), (p \vee \neg q\vee\neg s, 0.4), (p\vee\neg q\vee e, 0.4), (\neg q\vee r\vee\neg s, 0.6),\\
&(\neg q\vee r\vee e, 0.6), (\neg q\vee s\vee e, 0.6), (p\vee q\vee\neg e\vee r, 0.4),\\
&(p\vee\neg e\vee\neg s\vee r, 0.4), (q\vee\neg e\vee r\vee s, 0.4)\}.
\end{aligned}
$$

Let us compare $B_{\max}$ with $\Delta_{\mathcal{N},\oplus}(\mathcal{KP})$ obtained by method 1 in Example 41. Most of formulae in $B_{\max}$ can be inferred from $\Delta_{\mathcal{N},\oplus}(\mathcal{KP})$. Only three formulae $(p\vee\neg q\vee\neg s, 0.4)$, $(p\vee q\vee\neg e\vee r, 0.4)$ and $(p\vee\neg e\vee\neg s\vee r, 0.4)$ with low weights are not inferred. Furthermore, $\Delta_{\mathcal{N},\oplus}(\mathcal{KP})$ contains many formulae in the original belief bases, such as $(q\rightarrow r, 1)$, $(s, 0.8)$ and $(\neg p, 0.8)$. In contrast, none of formula in $B_{\max}$ belongs to any original belief bases. So $\Delta_{\mathcal{N},\oplus}(\mathcal{KP})$ contains more important information.

In [28], we have shown that the disjunctive operators satisfy (P1) and (P3). However, it does not satisfy (P2) and (P4) in general. So the new merging operators satisfy more logical properties than the disjunctive operators.

9.2. Normalized conjunctive merging operators

Since the resulting belief base may be inconsistent if we use a conjunctive or reinforcement operator, we need another step to resolve the inconsistency. This can be done by deleting those formulae whose weights are not greater than the inconsistency degree of the resulting belief base. However, if the inconsistency degree is very high (i.e., greater than 0.7), then too much original information will be lost after merging. That is, the normalized conjunctive merging operators suffer from the drowning problem.

Example 47 (*Continue Example 41*). Let $\oplus = \min$. By Equation (2), the resulting belief base of merging A, B and C is $B_{\min} \equiv_s A\cup B\cup C$. It is easy to check that $Inc(B_{\min}) = 0.6$. So after the inconsistency handling step, we get a belief base $B' = \{(\neg q\vee r, 1), (\neg s\rightarrow\neg r, 0.9), (s, 0.8), (q, 0.8), (e, 0.8), (\neg p, 0.8)\}$, which only contains formulae whose necessity degrees are greater than 0.6.

It was shown in [28] that the normalized conjunctive operators satisfy (P1), (P2) and do not satisfy (P3) and (P4) in general. Therefore, our new merging operators satisfy more logical properties than the normalized conjunctive ones.

9.3. Split–combination merging operators

In [27], we introduce a split–combination (S–C) approach for merging individually consistent PBB. The general idea of the S–C approach can be described as follows. Given a set of PBBs B_i, where $i = 1, \ldots, n$, in the first step, we split them into $B_i = \langle C_i, D_i \rangle$ with regard to a splitting method. In the second step, we combine all C_i by the maximum based merging method (the result is a PBB C) and combine all D_i by the minimum based merging method (the result is a PBB D). The final result of the S–C combination method, denoted $B_{S\text{–}C}$, is $C \cup D$. Different S–C methods can be developed by incorporating different ways of splitting the knowledge bases, while retaining the general S–C approach. Two different splitting methods have been given. One is called the *upper-free-degree-based* splitting (U-S) method and the other is called the *free-formula based* splitting (F-S) method. The U-S method splits a PBB with regard to its *upper-free-degree*, which is the minimum weight $a\in[0, 1]$ such that the strict a-cut of the PBB does not contain any conflicting formulae. The F-S method splits a PBB B into two subbases such that one of them contains formulae which are not in conflict in B and the other contains formulae which are in conflict. We call the merging operator based on the U-S method as upper-free-degree based split-combination (U-S–C) operator and the merging operator based on the free-formula based method as free-formula based split-combination (F-S–C) merging operator. We have shown that both merging methods improve the disjunctive operator based merging method. However, they are not advisable to be used to merge belief bases which are strong in conflict, i.e., most formulae of original belief bases are involved in conflict and the inconsistency degree of their union is very high. Let look at Example 41 again.

Example 48 (*Continue Example 41*). Suppose we choose the U-S method to split the belief base. Let $D = A\cup B\cup C$. It is easy to check that the upper-free-degree of D is 1 because every formula in D is involved in conflict. So A, B and C are split into $A = \langle A, \emptyset\rangle$, $B = \langle B, \emptyset\rangle$ and $C = \langle C, \emptyset\rangle$. We then combine A, B and C using the maximum operator, and the resulting belief base is $B_{\max}$ obtained in Example 46. Suppose we choose the F-S method to split the belief bases. Since all formulae in A, B and C are involved in conflict, they are split into $A = \langle A, \emptyset\rangle$, $B = \langle B, \emptyset\rangle$ and $C = \langle C, \emptyset\rangle$. So the result of merging is the same as that of merging by the U-S–C operator, i.e., $B_{\max}$. Therefore, we get a belief base which deletes too much information from the original belief bases.

A difference between the S–C operators and the operator based on prioritized belief negotiation model is that all the conflicting formulae are deleted or weakened by S–C operators whilst only part of them are deleted or weakened in an operator based on the prioritized belief negotiation model. Furthermore, the new operators consider the interaction among different belief bases or agents, i.e., agents play a game and those lose the game will have to give up more beliefs.

According to [28], the U-S–C operator has good logical properties, i.e., it satisfies (P1), (P2) and (P3). It does not satisfies (P4) in general. A problem with the U-S–C operator is that when the inconsistency degree of the union of the original knowledge bases is high, it is close to the disjunctive operator. The F-S–C operator is syntax-dependent, that is, it does not satisfy (P3). However, it satisfies (P1), (P2) and (P4). The operator $\Delta_{\mathcal{N},\oplus}$ where $\mathcal{N} = \langle g^{\mathrm{wc}}, \blacktriangledown^{\mathrm{wc}}\rangle$ satisfies the same set of logical properties as the F-S–C operator. Unlike F-S–C operator which weakens all the conflicting information, $\Delta_{\mathcal{N},\oplus}$ only deletes those weakest conflicting formulae to restore consistency.

According to our analysis above, we can conclude that our new merging operators are good alternatives of possibilistic merging operators.

10. Conclusions

In this paper, we proposed a prioritized belief negotiation model which generalizes Konieczny's belief game model [18]. Our prioritized belief negotiation model may take into account the syntax of the PBBs and we have defined some particular negotiation functions and weakening functions by considering the priorities of formulae in each PBB. We then presented a two-step scenario for merging PBBs based on the prioritized belief negotiation model. In the first step, original PBBs are weakened to make them consistent. Then in the second step, we combine the resulting PBBs using some combination rules in possibilistic logic [3] (the selection of an appropriate combination rule can be based on the selection criteria discussed in [25]). By choosing between different prioritized belief negotiation models, we can get different merging methods. Unlike the previous merging methods in possibilistic logic, our methods consider the interaction among different belief bases or agents, i.e., agents play a game and those lose the game will have to give up more beliefs.

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Our definition of the weakening function simply deletes some formulae from a knowledge base. The investigation of more complicated weakening functions will be left as future work. It is very interesting to use some techniques developed in inconsistency handling (such as the disjunction based methods in [10]) to weaken conflicting formulae.

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