Light Scattering by Leaf Layers with Application to Canopy Reflectance Modeling: The SAIL Model

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The scattering and extinction coefficients of the SAIL canopy reflectance model are derived for the case of a fixed arbitrary leaf inclination angle and a random leaf azimuth distribution. The SAIL model includes the uniform model of G. H. Suits as a special case and its main characteristics are that canopy variables such as leaf area index and the leaf inclination distribution function are used as input parameters and that it provides more realistic angular profiles of the directional reflectance as a function of the view angle or the solar zenith angle.

I. Introduction

For a powerful and accurate processing and interpretation of multispectral remote sensing data from vegetated areas it is of vital importance to gain fundamental insight into the interaction between incident light and leaf canopies. This may improve the possibilities of extracting useful information from remotely sensed data, for instance by separating the influence of the measurement conditions, such as the solar zenith angle and the angle of view, on the intensity of the reflected radiation from the influence of the object itself. Also it is important to establish relations between detected signals and object variables, since this is the key to a quantitative interpretation of the data. Canopy reflectance modeling is an inexpensive tool that can provide such relations quickly and under controlled conditions.

Suits (1972) has developed an analytical canopy reflectance model that calculates the directional reflectance in the observer's direction as a function of canopy parameters as well as parameters describing the measurement conditions. The Suits model is an extension of the so-called AGR model of Allen, Gayle, and Richardson (1970), which, in turn, is an extension of the Kubelka–Munk (1931) theory of light scattering and extinction in diffusing media in general. The KM theory is considered a two-flux theory, since only two types of radiant flux are involved, namely a diffuse downward flux E_- and a diffuse upward flux E_+. The relations between these fluxes are expressed by two simultaneous linear differential equations with two coefficients. In the AGR model also a direct solar flux E_s is included, making it a three-flux theory with three differential equations (called Duntley equations) and five coefficients. Similarly, the Suits model in essence is a four-flux theory, with four differential equations and nine coefficients. The flux type added by Suits is associated with the radiance in the direction of observation, L_o. It is defined by $E_o = \pi L_o$ and it can be interpreted as the irradiance from a Lambertian surface if its radiance were equal to L_o. Using Bunnik's (1978) notation for the Suits coeffi-

0034-4257/84/$3.00

cients, the system of four differential equations is given by

$$dE_s/dx = kE_s, \tag{1a}$$

$$dE_-/dx = -sE_s + aE_- - \sigma E_+, \tag{1b}$$

$$dE_+/dx = s'E_s + \sigma E_- - aE_+, \tag{1c}$$

$$dE_o/dx = wE_s + vE_- + uE_+ - KE_o. \tag{1d}$$

Equation (1d) is introduced here because of its close connection with other radiative transfer theories. In this respect the term $J = wE_s + vE_- + uE_+$ can be identified as the source function describing the generation of internal radiance, and K is the extinction coefficient. Suits (1972) used a differential equation for the probability of direct line of sight from outside the canopy, in addition to one for the internal radiance contributions to the radiance at the top of a layer. It can be shown that both ways of description are equivalent, but the form of system (1) is more compact, more general, and more comprehensible, while it still fully expresses the essence of the Suits model for a layer.

Methods of solution of system (1) are not discussed here, as these are given elsewhere (Suits, 1972; Bunnik, 1978; Chance and Cantu, 1975; and Slater, 1980). Instead, this paper focuses on the estimation of the Suits coefficients for a canopy layer. The reason for this is that Suits's approach of taking horizontal and vertical leaf area projections to calculate the scattering and extinction coefficients is too drastic. This conclusion is based on experience with canopy reflectance calculations with Suits's model as a function of the view angle. It appears that simulations of reflectance variations with the view angle variation of a line scanner result in "V"-shaped profiles, which is highly unrealistic. This type of angular response is caused by the function $\tan\theta_o$, where θ_o is the view angle, which appears in the coefficients w, v, u, and K as a multiplier of the vertical leaf area projection.

In an attempt to improve the angular responses of the Suits model, a detailed analysis of extinction and scattering of radiant flux by leaf layers has been performed. The result of this is the SAIL model (from Scattering by Arbitrarily Inclined Leaves), which calculates the nine Suits coefficients for a given total leaf area index and leaf inclination distribution function of the layer. The SAIL model includes Suits's uniform model as a special case, since the simplified morphology of a canopy layer according to Suits can be expressed by a degenerate leaf inclination distribution of only horizontal and vertical leaves.

2. Canopy Layer Morphology

The idealized morphology of a canopy layer assumed for the SAIL model is given by the following characteristics:

the layer is horizontal and infinitely extended;

the only canopy components are small and flat leaves;

the layer is homogenous.

Further it is assumed that the leaf area index (= total one-sided leaf area per unit layer area) equals L and that the distribution of leaf orientations can be described by a leaf area orientation density function $g(\theta_l, \varphi_l)$, where θ_l and φ_l are the polar zenith angle and the azimuth angle of the leaf's upward normal, $\mathbf{l}$, respectively. The fraction of the leaf area index oriented such that the leaf's normal is within a cone of solid angle $d\Omega_l$ is given by

$$d^2L(\theta_l, \varphi_l) = Lg(\theta_l, \varphi_l)d\Omega_l = Lg(\theta_l, \varphi_l)\sin\theta_l d\theta_l d\varphi_l. \tag{2}$$

If it is also assumed that the leaf's azimuth is distributed at random, it is more convenient to use the leaf inclination density function $f(\theta_l)$, which can be derived from Eq. (2) by integration with respect to φ_l. This yields:

$$f(\theta_l) = 2\pi g(\theta_l, \varphi_l)\sin\theta_l. \tag{3}$$

From this it follows, by the way, that for a random distribution of leaf orientation, which gives $g(\theta_l, \varphi_l) = 1/2\pi$, the leaf inclination density function is of type spherical and given by $f(\theta_l) = \sin\theta_l$.

The fraction of the leaf area index oriented such that the leaf inclination is within the interval θ_l to $\theta_l + d\theta_l$ and the leaf's azimuth is within the interval φ_l to $\varphi_l + d\varphi_l$, can be expressed as a function of $f(\theta_l)$ by combining Eq. (2) and (3). This yields

$$d^2L(\theta_l, \varphi_l) = L\frac{d\varphi_l}{2\pi}f(\theta_l)\,d\theta_l, \tag{4}$$

which is only valid for a random distribution of leaf azimuth.

Although for calculation of optical characteristics such as the canopy reflectance the layer thickness appears to be a redundant parameter, it is included here for compatibility with earlier publications. If the thickness of a layer equals h, a parameter called differential leaf area index or leaf area density, L', can be defined by

$$L' = L/h. \tag{5}$$

The vertical dimension is represented by x, where $x = -h$ for the bottom of the layer and $x = 0$ for the top of the layer, and the fraction of the leaf area index between the levels x and $x + dx$ is given by

$$dL(x) = (L/h)\,dx = L'\,dx. \tag{6}$$

For the development of the SAIL model it was assumed that the leaf azimuth angle exhibits a random distribution. This assumption is reasonable since only a few plant species have been reported to show a definite heliotropic behavior. Summarized, this means that for the SAIL model the only parameters describing the morphology of a canopy layer are the leaf area index L, the leaf inclination density function $f(\theta_l)$, and the layer thickness h.

3. Radiometric Considerations

The radiant flux densities E_s, E_-, E_+, and E_o mentioned in system (1) are defined as radiant fluxes per unit horizontal layer area and per unit wavelength interval. The spectral character is implicitly assumed. For the calculation of intercepted and scattered fluxes, a spherical coordinate system, in which the leaf orientation, the position of the sun and the direction of observation can be indicated, is used. This system is illustrated in Figure 1.

Directions are specified by a zenith angle θ and an azimuth angle φ. For leaf orientation, the sun, and the observer the angles are (θ_l, φ_l), (θ_s, φ_s), and (θ_o, φ_o),

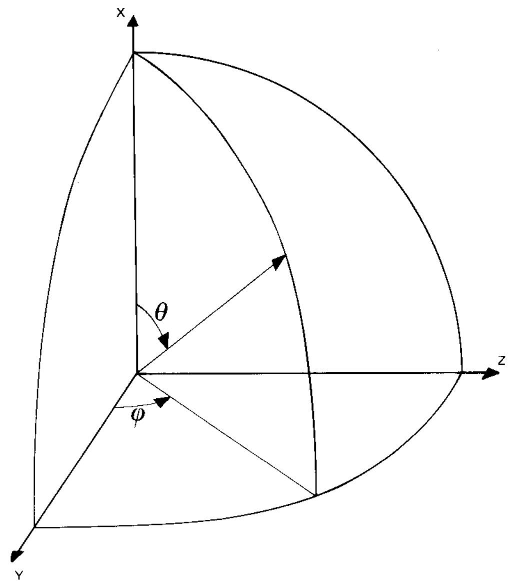

FIGURE 1. Definition of zenith angle θ and azimuth angle φ.

respectively. The azimuthal difference between sun and observer is called ψ. Without loss of generality it can be assumed that $\varphi_s = 0$, and $\varphi_o = \psi$. The use of unit vectors facilitates the calculation of projections of leaf area and layer area in the directions of the sun and of observation. Vectors **l** and **n** represent the normals to the leaf and the layer, and **s** and **o** indicate the position of the sun and the observer, respectively. These are illustrated in Figure 2, relative to a leaf area element dA, and can be expressed in the zenith and azimuth angles as follows:

$$\mathbf{l} = (\cos\theta_l; \sin\theta_l \cos\varphi_l; \sin\theta_l \sin\varphi_l)$$
$$\mathbf{n} = (1;0;0)$$
$$\mathbf{s} = (\cos\theta_s; \sin\theta_s; 0)$$
$$\mathbf{o} = (\cos\theta_o; \sin\theta_o \cos\psi; \sin\theta_o \sin\psi)$$

Leaf area projections and layer area projections are necessary to determine radiometric quantities of the layer from those of individual leaves and vice versa. If, for

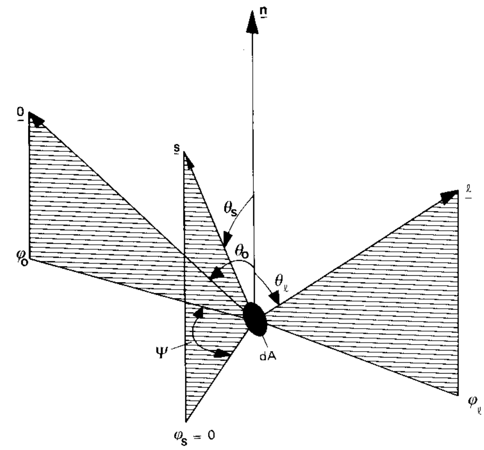

FIGURE 2. Orientations of unit vectors **l**, **n**, **s**, and **o** relative to a leaf area element dA.

instance, the solar irradiance from direction **s** incident on the layer is given by E_s, then the associate irradiance on a leaf with orientation **l** equals $f_s E_s$, where f_s is a conversion factor given by

$$f_s = (\mathbf{s}\cdot\mathbf{l})/(\mathbf{s}\cdot\mathbf{n})$$
$$= \cos\theta_l[1 + \tan\theta_s \tan\theta_l \cos\varphi_l]. \quad (7)$$

Similarly, if the radiance in a direction **o** of an individual leaf with orientation **l** is given by $\mathscr{L}_o$, then the associate radiance of the layer at that location in direction **o** equals $f_o \mathscr{L}_o$, where f_o is a conversion factor given by

$$f_o = (\mathbf{o}\cdot\mathbf{l})/(\mathbf{o}\cdot\mathbf{n})$$
$$= \cos\theta_l[1 + \tan\theta_o \tan\theta_l \cos(\varphi_l - \psi)]. \quad (8)$$

If $\theta_s + \theta_l > \pi/2$, then the product $\tan\theta_s \tan\theta_l$ is greater than 1, which implies that f_s becomes negative if the leaf azimuth angle φ_l is greater than a transition angle β_s given by

$$\beta_s = \arccos(-1/\tan\theta_s \tan\theta_l). \quad (9)$$

Since φ_l is also defined for $\varphi_l > \pi$, it is concluded that f_s is negative for $\beta_s < \varphi_l < 2\pi - \beta_s$ if $\theta_s + \theta_l > \pi/2$. In this case it can be stated that the bottom side of the leaf is illuminated. For $\theta_s + \theta_l < \pi/2$ the factor f_s is positive for any φ_l, which means that in that case always the top side of the leaf is illuminated.

Similar considerations for the factor f_o lead to the conclusion that for $\theta_o + \theta_l > \pi/2$ the bottom side of the leaf is observed for the leaf azimuth interval $\beta_o + \psi < \varphi_l < 2\pi - \beta_o + \psi$, where the transition angle β_o is given by

$$\beta_o = \arccos(-1/\tan\theta_o \tan\theta_l). \quad (10)$$

Also, for $\theta_o + \theta_l < \pi/2$, f_o is positive for any φ_l, which means that always the top side of the leaf is observed.

For conversion of the diffuse irradiances E_- and E_+ on individual leaves into those for the layer and vice versa a factor f_d could be introduced. However, a detailed analysis has shown that this factor is independent of the leaf orientation and equal to 1, so the diffuse irradiances on leaf and layer are equal. Of greater significance is the distinction between a fraction of diffuse flux incident at or scattered from one side of the leaf and a complementary fraction associated with the other side of the leaf. These fractions are called f_1 and f_2, where f_1 refers to the greater of the two, and is given by

$$f_1 = (1 + \cos\theta_l)/2, \quad (11)$$

and where f_2 is given by

$$f_2 = (1 - \cos\theta_l)/2. \quad (12)$$

For the downward diffuse irradiance E_- the fractions f_1 and f_2 are illustrated relative to a leaf area element dA in Figure 3. This shows the division of the upper hemisphere in the two parts associated with both fractions.

In the next sections the factors f_s, f_o, f_1 and f_2 are employed to define extinction efficiencies Q_{ex} and scattering efficiencies Q_{sc} for individual leaves, from

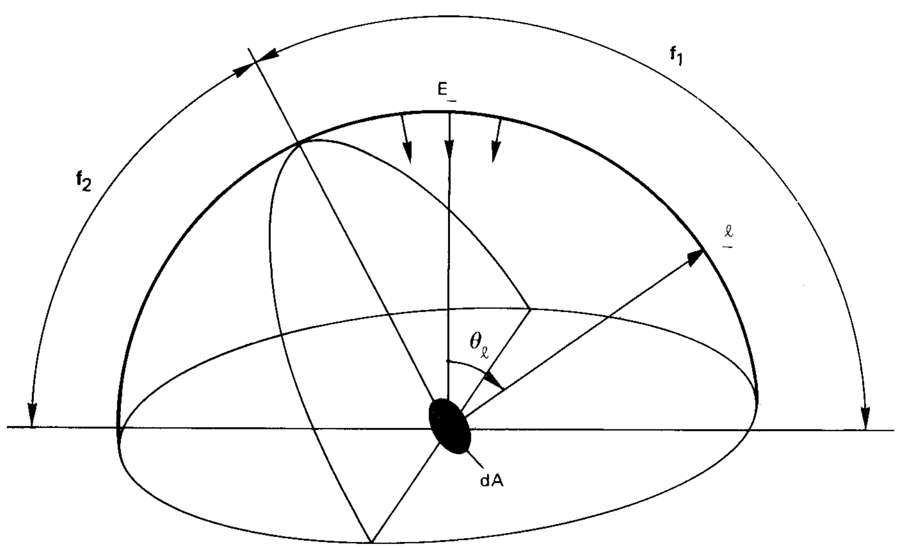

FIGURE 3. Illustration of the fractions f_1 and f_2 of a downward diffuse irradiance E_- incident at the two sides of a leaf area element dA.

which the Suits coefficients of the layer can be found by integration with respect to leaf azimuth and inclination.

4. Extinction

The extinction efficiencies of single leaves describe their capability to intercept radiant flux. They are called $Q_{ex}(E_s)$, $Q_{ex}(E_-)$, $Q_{ex}(E_+)$ and $Q_{ex}(E_o)$.

In general, the extinction coefficient is found as follows: The fraction of leaf area relative to layer area of an infinitesimal layer of thickness dx equals $L'dx$. Of this, a fraction having leaf inclinations between θ_l and $\theta_l + d\theta_l$, and having a leaf azimuth angle between φ_l and $\varphi_l + d\varphi_l$ equals $f(\theta_l)d\theta_l d\varphi_l/2\pi$. The product of both fractions gives

$$d^3L(x,\theta_l,\varphi_l) = L'dx\frac{d\varphi_l}{2\pi}f(\theta_l)\,d\theta_l. \tag{13}$$

If the extinction efficiency for some type of flux E_i is given by $Q_{ex}(E_i)$, then the flux intercepted by the fraction $d^3L(x,\theta_l,\varphi_l)$ follows from

$$d^3E_i = E_i Q_{ex}(E_i) d^3L(x,\theta_l,\varphi_l).$$

Integrating this with respect to φ_l and θ_l gives the total flux intercepted, dE_i, by

$$dE_i = E_i L' dx \int_0^{\pi/2}\int_0^{2\pi} Q_{ex}(E_i)\frac{d\varphi_l}{2\pi} \cdot f(\theta_l)\,d\theta_l. \tag{15}$$

The extinction coefficient c is defined by $c = (dE_i/dx)/E_i$, and applying this definition, one finds

$$c = \frac{L'}{2\pi}\int_0^{\pi/2}\int_0^{2\pi} Q_{ex}(E_i)\,d\varphi_l f(\theta_l)\,d\theta_l. \tag{16}$$

For practical purposes, the integration with respect to θ_l is approximated by a summation of n finite intervals $\Delta\theta_l$, for which the leaf inclination frequencies are given by $F(\theta_l)$. In this case

$$c = \frac{L'}{2\pi}\sum_1^n F(\theta_l)\int_0^{2\pi} Q_{ex}(E_i)\,d\varphi_l = \sum_1^n F(\theta_l)c(\theta_l), \tag{17}$$

in which $c(\theta_l)$ represents the extinction coefficient for fixed leaf inclination θ_l and random leaf azimuth. The extinction coefficient so defined is given by

$$c(\theta_l) = \frac{L'}{2\pi}\int_0^{2\pi} Q_{ex}(E_i)\,d\varphi_l. \tag{18}$$

The single leaf extinction efficiencies $Q_{ex}(E_i)$ for the flux types E_s, E_-, E_+, and E_o are simply equal to the conversion factors $|f_s|$, f_d, f_d, and $|f_o|$, respectively. For f_s and f_o the absolute value is taken because these factors may become negative. As f_d is constant and equal to 1, the extinction coefficient for the diffuse fluxes E_- and E_+ is obtained directly by

$$\kappa(\theta_l) = L'. \tag{19}$$

The extinction coefficient for E_s is called $k(\theta_l)$ and found by integrating f_s in two parts:

Let $f_{st} = 2\int_0^{\beta_s} f_s\, d\varphi_l$, and $f_{sb} = 2\int_{\beta_s}^{\pi} - f_s\, d\varphi_l$, which yields

$$f_{st} = \cos\theta_l [2\beta_s + 2\sin\beta_s \tan\theta_s \tan\theta_l], \quad (20)$$

and

$$f_{sb} = \cos\theta_l [2\beta_s - 2\pi + 2\sin\beta_s \tan\theta_s \tan\theta_l]. \quad (21)$$

Then $k(\theta_l) = (L'/2\pi)(f_{st} + f_{sb})$, or

$$k(\theta_l) = \frac{2}{\pi} L' [(\beta_s - \pi/2)\cos\theta_l + \sin\beta_s \tan\theta_s \sin\theta_l]. \quad (22)$$

In case $\theta_s + \theta_l < \pi/2$, no real solution of β_s exists. Since f_{st} for that case equals $2\pi\cos\theta_l$, and f_{sb} should be zero, and as both results are found by setting $\beta_s = \pi$, it is concluded that the formula for $k\ (\theta_l)$ is valid also for the case $\theta_s + \theta_l < \pi/2$ if β_s is assumed to be equal to π. Similar to $k(\theta_l)$, the extinction coefficient for E_o, $K(\theta_l)$, is found by integration of f_o in two parts, which yields

$$K(\theta_l) = \frac{2}{\pi} L' [(\beta_o - \pi/2)\cos\theta_l + \sin\beta_o \tan\theta_o \sin\theta_l], \quad (23)$$

and in which β_o is set equal to π if $\theta_o + \theta_l < \pi/2$.

The extinction coefficient for diffuse upward or downward flux, κ, has not been defined previously in connection with the Suits model. In the Suits model the attenuation coefficient for diffuse flux, a, is used, which is found by subtracting a diffuse forward scattering coefficient, σ', from the extinction coefficient κ, or

$$a = \kappa - \sigma'. \quad (24)$$

This can be understood as a correction applied to the intercepted flux, because a fraction continues its way in the same direction via forward scattering, and thus does not contribute to attenuation.

5. Scattering

The scattering efficiency factors are given in the form of $Q_{sc}(E_1, E_2)$, where E_1 refers to the type of incident flux and E_2 to the type of scattered flux. For their derivation it was assumed that individual leaves act as perfect Lambertian diffusors, with a hemispherical reflectance ρ and a hemispherical transmittance τ, for both sides of the leaves. For single leaves with orientation (θ_l, φ_l) the scattering efficiencies are presented in Table 1.

Scattering coefficients for a fixed leaf inclination θ_l and random leaf azimuth are found by a procedure similar to the one outlined in the previous section for extinction. In general, the scattering coefficient is defined by

$$b = (dE_2/dx)/E_1,$$

and for fixed θ_l and random φ_l the scattering coefficient $b(\theta_l)$ can be found from

$$b(\theta_l) = \frac{L'}{2\pi} \int_0^{2\pi} Q_{sc}(E_1, E_2)\, d\varphi_l. \quad (25)$$

The names of the different scattering coefficients so obtained for the possible combinations of E_1 and E_2 are presented in Table 2.

TABLE 1 Scattering Efficiency Factors $Q_{sc}(E_1, E_2)$ for Single Leaves

E_2 \ E_1		E_s: $f_s > 0$	E_s: $f_s < 0$	E_-	E_+
E_o	$f_o > 0$	$f_s \rho f_o$	$-f_s \tau f_o$	$(\rho f_1 + \tau f_2) f_o$	$(\tau f_1 + \rho f_2) f_o$
	$f_o < 0$	$-f_s \tau f_o$	$f_s \rho f_o$	$-(\tau f_1 + \rho f_2) f_o$	$-(\rho f_1 + \tau f_2) f_o$
E_-		$f_s(\tau f_1 + \rho f_2)$	$-f_s(\rho f_1 + \tau f_2)$	$f_1(\tau f_1 + \rho f_2) + f_2(\rho f_1 + \tau f_2)$	$f_1(\rho f_1 + \tau f_2) + f_2(\tau f_1 + \rho f_2)$
E_+		$f_s(\rho f_1 + \tau f_2)$	$-f_s(\tau f_1 + \rho f_2)$	$f_1(\rho f_1 + \tau f_2) + f_2(\tau f_1 + \rho f_2)$	$f_1(\tau f_1 + \rho f_2) + f_2(\rho f_1 + \tau f_2)$

TABLE 2 Names of Scattering Coefficients for the Possible Combinations of Incident and Scattered Flux, E_1 and E_2

E_2 \ E_1	E_s	E_-	E_+
E_o	$w(\theta_l)$	$v(\theta_l)$	$u(\theta_l)$
E_-	$s(\theta_l)$	$\sigma'(\theta_l)$	$\sigma(\theta_l)$
E_+	$s'(\theta_l)$	$\sigma(\theta_l)$	$\sigma'(\theta_l)$

The coefficients $\sigma(\theta_l)$ and $\sigma'(\theta_l)$, describing backscatter and forward scattering of the diffuse fluxes E_- and E_+, are equal to the associate scattering efficiencies Q_{sc} of Table 1, multiplied by L' since the factors f_1 and f_2 are independent of φ_l. Taking sum and difference,

$$\sigma(\theta_l) + \sigma'(\theta_l) = L'\left[f_1^2 + 2f_1 f_2 + f_2^2\right](\rho + \tau) = L'(\rho + \tau),$$

and

$$\sigma(\theta_l) - \sigma'(\theta_l) = L'\left[f_1^2 - 2f_1 f_2 + f_2^2\right](\rho - \tau) = L'(\rho - \tau)\cos^2\theta_l.$$

From this it follows that

$$\sigma(\theta_l) = L'\left(\frac{\rho + \tau}{2} + \frac{\rho - \tau}{2}\cos^2\theta_l\right), \quad (26)$$

$$\sigma'(\theta_l) = L'\left(\frac{\rho + \tau}{2} - \frac{\rho - \tau}{2}\cos^2\theta_l\right). \quad (27)$$

The attenuation coefficient $a(\theta_l) = \kappa(\theta_l) - \sigma'(\theta_l)$ is given by

$$a(\theta_l) = L'\left(1 - \frac{\rho + \tau}{2} + \frac{\rho - \tau}{2}\cos^2\theta_l\right). \quad (28)$$

The scattering coefficients for the direct solar flux E_s, $s'(\theta_l)$ and $s(\theta_l)$, are obtained by employing the factors f_{st} and f_{sb} that were used in the derivation of $k(\theta_l)$. This yields

$$s'(\theta_l) = \frac{L'}{2\pi}\left[f_{st}(\rho f_1 + \tau f_2) + f_{sb}(\tau f_1 + \rho f_2)\right],$$

and

$$s(\theta_l) = \frac{L'}{2\pi}\left[f_{st}(\tau f_1 + \rho f_2) + f_{sb}(\rho f_1 + \tau f_2)\right].$$

Taking sum and difference, it is found that

$$s'(\theta_l) + s(\theta_l) = \frac{L'}{2\pi}(\rho + \tau)(f_{st} + f_{sb}),$$

and

$$s'(\theta_l) - s(\theta_l) = \frac{L'}{2\pi}(\rho - \tau)(f_{st} - f_{sb})(f_1 - f_2).$$

Since the extinction coefficient for direct solar flux $k(\theta_l)$ was given by

$$k(\theta_l) = \frac{L'}{2\pi}(f_{st} + f_{sb})$$

it follows that

$$s'(\theta_l) + s(\theta_l) = (\rho + \tau)k(\theta_l).$$

The difference $f_{st} - f_{sb}$ equals $2\pi \cos\theta_l$ and $f_1 - f_2$ equals $\cos\theta_l$, so

$$s'(\theta_l) - s(\theta_l) = L'(\rho - \tau)\cos^2\theta_l.$$

From this $s'(\theta_l)$ and $s(\theta_l)$ are found to be equal to

$$s'(\theta_l) = \frac{\rho + \tau}{2}k(\theta_l) + \frac{\rho - \tau}{2}L'\cos^2\theta_l, \tag{29}$$

$$s(\theta_l) = \frac{\rho + \tau}{2}k(\theta_l) - \frac{\rho - \tau}{2}L'\cos^2\theta_l. \tag{30}$$

Similarly, it can be shown that the scattering coefficients $v(\theta_l)$ and $u(\theta_l)$ are given by

$$v(\theta_l) = \frac{\rho + \tau}{2}K(\theta_l) + \frac{\rho - \tau}{2}L'\cos^2\theta_l, \tag{31}$$

$$u(\theta_l) = \frac{\rho + \tau}{2}K(\theta_l) - \frac{\rho - \tau}{2}L'\cos^2\theta_l. \tag{32}$$

The bidirectional scattering coefficient $w(\theta_l)$ can be found by integration of the function $Q_{sc}(E_s, E_o)$ with respect to φ. This task is quite laborious since several different cases can be distinguished for which various solutions are obtained. However, it turns out to be possible to express the result in one formula which includes all cases. This formula reads:

$$w(\theta_l) = \frac{L'}{2\pi}\left\{[\pi\rho - \beta_2(\rho + \tau)](2\cos^2\theta_l + \sin^2\theta_l \tan\theta_s \tan\theta_o \cos\psi) + (\rho + \tau)\sin\beta_2\left[\frac{2\cos^2\theta_l}{\cos\beta_s \cos\beta_o} + \cos\beta_1 \cos\beta_3 \sin^2\theta_l \tan\theta_s \tan\theta_o\right]\right\} \tag{33}$$

in which the auxiliary azimuth angles β_1, β_2, and β_3 are determined from a decision table as follows:

If:	β_1	β_2	β_3
$\psi \leqslant \lvert\beta_s - \beta_o\rvert$	ψ	$\lvert\beta_s - \beta_o\rvert$	$2\pi - \beta_s - \beta_o$
$\lvert\beta_s - \beta_o\rvert < \psi < 2\pi - \beta_s - \beta_o$	$\lvert\beta_s - \beta_o\rvert$	ψ	$2\pi - \beta_s - \beta_o$
$\psi \geqslant 2\pi - \beta_s - \beta_o$	$\lvert\beta_s - \beta_o\rvert$	$2\pi - \beta_s - \beta_o$	ψ

6. SAIL Coefficients for a Suits Canopy Layer

A Suits canopy layer can be defined as a leaf layer that consists of horizontal and vertical leaves exclusively. Let the leaf area density be given by L' and the fractions of horizontal and vertical leaf area by $F(0)$ and $F(\pi/2)$, respectively. The horizontal and vertical leaf area densities H' and V' are now defined by

$$H' = L'F(0), \qquad V' = L'F(\pi/2). \quad (34)$$

In Suits's nomenclature H' and V' are equal to $\sigma_h n_h$ and $\sigma_v n_v$, where σ_h and σ_v are the average horizontal and vertical leaf area, and n_h and n_v are the associate numbers of leaves per unit volume. If any of the SAIL coefficients is symbolized by $z(\theta_l)$, then the corresponding coefficient for a Suits canopy is given by

$$z = [H'z(0) + V'z(\pi/2)]/L'. \quad (35)$$

Substitution of $\theta_l = 0$ and $\theta_l = \pi/2$ in the SAIL coefficients leads to the following results:

$$a(0) = L'\left[1 - \frac{\rho+\tau}{2} + \frac{\rho-\tau}{2}\right] = L'(1-\tau);$$

$$a(\pi/2) = L'\left[1 - \frac{\rho+\tau}{2}\right];$$

$$\sigma(0) = L'\left[\frac{\rho+\tau}{2} + \frac{\rho-\tau}{2}\right] = L'\rho;$$

$$\sigma(\pi/2) = L'(\rho+\tau)/2;$$

$$k(0) = \left(\frac{2}{\pi}\right)L'(\pi - \pi/2) = L';$$

$$k(\pi/2) = \frac{2}{\pi}L'\tan\theta_s;$$

$$s'(0) = \frac{\rho+\tau}{2}L' + \frac{\rho-\tau}{2}L' = L'\rho;$$

$$s'(\pi/2) = \frac{2}{\pi}L'\frac{\rho+\tau}{2}\tan\theta_s;$$

$$s(0) = \frac{\rho+\tau}{2}L' - \frac{\rho-\tau}{2}L' = L'\tau;$$

$$s(\pi/2) = \frac{2}{\pi}L'\frac{\rho+\tau}{2}\tan\theta_s;$$

$$K(0) = \left(\frac{2}{\pi}\right)L'(\pi - \pi/2) = L';$$

$$K(\pi/2) = \frac{2}{\pi}L'\tan\theta_o;$$

$$v(0) = \frac{\rho+\tau}{2}L' + \frac{\rho-\tau}{2}L' = L'\rho;$$

$$v(\pi/2) = \frac{2}{\pi}L'\frac{\rho+\tau}{2}\tan\theta_o;$$

$$u(0) = \frac{\rho+\tau}{2}L' - \frac{\rho-\tau}{2}L' = L'\tau;$$

$$u(\pi/2) = \frac{2}{\pi}L'\frac{\rho+\tau}{2}\tan\theta_o;$$

$$w(0) = \frac{L'}{2\pi}\cdot\pi\rho\cdot 2 = L'\rho.$$

Regarding the substitution of $\theta_l = \pi/2$ to determine $w(\pi/2)$ according to (33), it is noted that the term $\cos^2\theta_l/(\cos\beta_s\cos\beta_o)$ in (33) becomes indefinite for $\theta_l = \pi/2$. However, this term equals $\sin^2\theta_l\tan\theta_s\tan\theta_o$ if both β_s and β_o are less than π, which is true since both are equal to $\pi/2$. The auxiliary angles β_1, β_2, and β_3 in this case are equal to 0, ψ, and π, respectively. Substitution of $\theta_l = \pi/2$ in (33) then gives

$$\begin{aligned} w(\pi/2) &= \frac{L'}{2\pi}[[\pi\rho - \psi(\rho+\tau)]\tan\theta_s \\ &\quad \times \tan\theta_o\cos\psi + (\rho+\tau)\sin\psi \\ &\quad \times \{2 + 1\cdot(-1)\}\tan\theta_s\tan\theta_o] \\ &= \frac{L'}{2\pi}[[\sin\psi + (\pi-\psi)\cos\psi]\rho \\ &\quad + [\sin\psi - \psi\cos\psi]\tau] \\ &\quad \times \tan\theta_s\tan\theta_o. \end{aligned}$$

If the SAIL coefficients thus found are substituted according to (35), it can be verified that the resulting coefficients are identical to those given for the Suits model in Verhoef and Bunnik (1975) and Suits (1983).

7. View Angle Responses of SAIL Coefficients

The dependence of the directional canopy reflectance on the angle of view will be determined largely by the dependence of some of the extinction and scattering coefficients on the view angle. In this respect the extinction coefficient in the direction of view, $K(\theta_l)$, and the bidirectional scattering coefficient, $w(\theta_l)$, can be selected as the ones responsible for most of the angular variation of the reflectance with the view angle θ_o. In order to investigate the view angle dependence of $K(\theta_l)$ and $w(\theta_l)$, these were plotted as a function of θ_o for four different values of θ_l, namely 0° (horizontal), 30°, 50° and 90° (vertical), for L' equal to one. Figure 4 illustrates the dependence of $K(\theta_l)$ on the view angle. It appears that for horizontal leaves $K(\theta_l)$ is constant and equal to 1. For $\theta_l = 30°$, $K(\theta_l)$ is constant up to $\theta_o = 60°$ and then starts to increase, whereas for $\theta_l = 50°$, it is constant up to $\theta_o = 40°$ and then also increases. For vertical leaves $K(\theta_l)$ increases immediately according to the $\tan\theta_o$ function. It can be concluded that $K(\theta_l)$ is constant and equal to $L'\cos\theta_l$ for $\theta_o < 90° - \theta_l$. At the transition angle $\bar{\theta}_o = 90° - \theta_l$, $K(\theta_l)$ is equal to $L'\cos(90° - \bar{\theta}_o) = L'\sin\bar{\theta}_o$. In Figure 4 these transition angles are indicated by a dot. The dependence of the bidirectional scattering coefficient $w(\theta_l)$ on the view angle is illustrated in Figure 5. Here the view angle variation of a line scanner is simu-

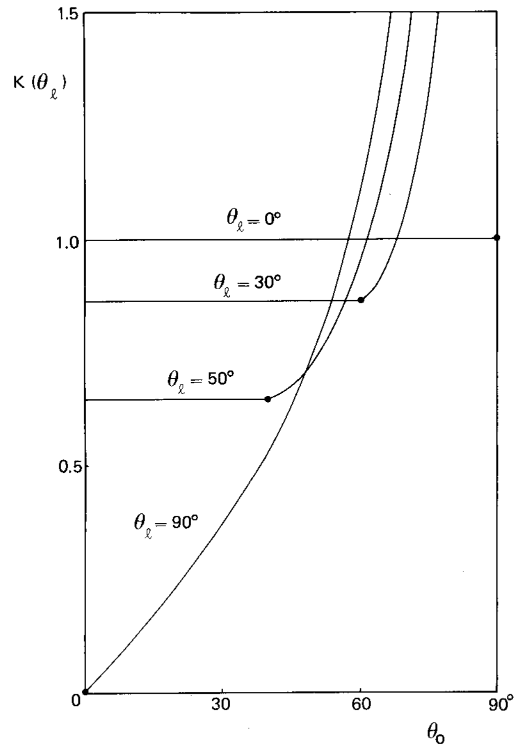

FIGURE 4. Extinction coefficient $K(\theta_l)$ for $L' = 1\ \mathrm{m}^{-1}$ as a function of the view angle for different leaf inclination angles.

lated for the case when the azimuth of the view angle coincides with the azimuth of the sun. The nadir point ($\theta_o = 0°$) is placed in the middle. The left corresponds to the down-sun situation, or $\psi = 0°$, whereas the right side refers to the situation of the sun opposite the view direction, or $\psi = 180°$. The solar zenith angle $\theta_s = 35°$, leaf reflectance and transmittance are $\rho = 13.5$ percent and $\tau = 5.5$ percent, which is representative for a green wheat leaf at a wavelength of 550 nm (green). The transition angle $\bar{\theta}_o$ is indicated in the curves of $w(\theta_l)$ again. From Figure 5 it appears that for horizontal leaves $w(\theta_l)$ equals the leaf reflectance and remains constant. For $\theta_l = 30°$ the curve of $w(\theta_l)$ is characterized by a monotonous decrease by a function of the

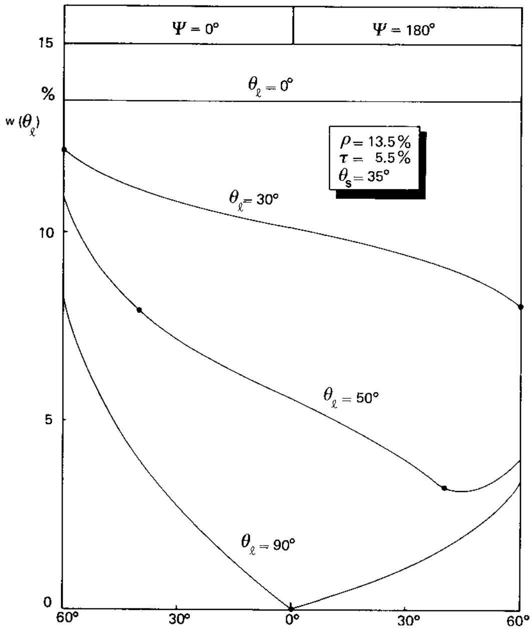

FIGURE 5. Bidirectional scattering coefficient $w(\theta_l)$ for $L' = 1 \text{ m}^{-1}$ as a function of the view angle for different leaf inclination angles.

type $p \tan\theta_o \cos\psi + q$, where p and q are constants that depend on θ_s, and θ_l and ρ, which follows from applying equation (33). For $\theta_l = 50°$ the same type of function is obtained for view angles $\theta_o <$ 40°, but for view angles greater than 40° the behavior gradually changes to one of a tendency to increase with increasing θ_o. This is caused by the fact that beyond the transition view angle $\bar{\theta}_o = 90° - \theta_l$ a part of the interaction is determined by scattering via transmission, since for a fraction of the leaves the bottom side is observed. For vertical leaves the transition view angle is given by $\bar{\theta}_o = 0°$ (nadir view), so the range of θ_o for which the function $p \tan\theta_o \cos\psi + q$ would apply has reduced to zero. In this case the curve of $w(\theta_l)$ is given by a function of the type $p \tan\theta_o$, where p is proportional to the leaf reflectance ρ for the left side of Figure 5, and to the leaf transmittance τ for the right side. This "V"-shape type of function is responsible for a similar type of behavior of the directional canopy reflectance as calculated according to Suits's model.

Considering that the "V"-shape behavior only occurs for exactly vertical leaves and that in real canopies the probability of a vertical leaf is as small as of any exact inclination angle, it can be concluded that this type of response will be absent for real canopies.

A second important observation from Figures 4 and 5 is that the angular responses of the extinction and scattering coefficients typical for leaves of arbitrary inclination angle can never be reproduced by taking a weighted average of those for the horizontal and vertical leaf area projections, so this approach of estimating the extinction and scattering coefficients has to be rejected.

8. Computer Implementation of the SAIL Model

For the present computer implementation of the SAIL model the leaf inclination distribution function (LIDF) of a canopy layer is discretized to an array of inclination frequencies for 13 distinct inclination angles θ_l located at the centers of the intervals 0°–10°, 10°–20°,...,70°–80° and 80°–82°, 82°–84°,...,88°–90°. So the LIDF is approximated by a set of frequencies $F(\theta_l)$ of which the sum equals one and where the associate values of θ_l are equal to 5°, 15°,...,75° and 81°, 83°,..., 89°. The refinement of the interval 80°–90° is applied because the extinction coefficient K and the scattering coefficients u, v, and w are very sensitive to variations of the

LIDF in this region θ_l if the view angle θ_o is close to nadir, which is the situation for most satellite remote sensing missions. The extinction and scattering coefficients are calculated by evaluating the associate SAIL functions at the 13 values of θ_l, applying a weight factor $F(\theta_l)$ and summing the results. If $z(\theta_l)$ represents one of these functions, the associate coefficient for the ensemble thus follows from

$$z = \sum_{i=1}^{13} F(\theta_{li}) z(\theta_{li}), \qquad (36)$$

where θ_{li} is the center inclination angle of interval i.

Since for the Suits model a similar procedure is followed for the leaf inclination angles θ_{li} equal to 0° and 90°, it is concluded that this is a special case of the SAIL model.

9. Examples of Canopy Bidirectional Reflectance Profiles Generated by the SAIL Model

In order to give an impression of the performance of the SAIL model and to compare its results with those of the uniform Suits model, calculations of the bidirectional reflectance of a single layer leaf canopy on a Lambertian soil were carried out for a wide range of view angles and solar zenith angles. The bidirectional reflectance r_{so}, defined here as the ratio of the flux densities E_o and E_s at the top of the canopy if the diffuse downward irradiance from the sky is 0, is found by solving system (1) for the boundary conditions of a purely direct solar irradiance incident at the top of the canopy and a Lambertian soil reflectance at the bottom. All calculations were made for a moderate canopy leaf area index L of 2 and a spherical leaf inclination distribution. For the emulation of Suit's model the horizontal and vertical leaf area indices H and V were obtained by calculating the associate leaf area projections from the continuous version of the spherical leaf angle distribution. This yields $H = 0.5L$ and $V = (\pi/4)L$. Further for both models the ensemble was assumed to consist of green wheat leaves and a sandy loam soil. The results are presented as a function of the observation angle θ_o for different solar zenith angles θ_s and for the azimuth configuration of a line scanner with $\psi = 0°$ and 180°.

For a wavelength in the green at 550 nm the profiles for the SAIL model and for Suits's model are shown in Figures 6 and 7, respectively. Comparing these, it can be concluded that both predict quite spectacular variations of the bidirectional reflectance with the view angle and the solar zenith angle, which are of about the same magnitude for both models. However, there are also significant differences, especially if θ_o or θ_s is small. The curves for the Suits model all show the characteristic break at the nadir point ($\theta_o = 0°$), whereas the SAIL model shows a smooth transition from the case $\psi = 0°$ (downsun) to the case $\psi = 180°$ (sun opposite). Finally it can be noted that for nadir view angle the variation due to a changing solar zenith angle for the SAIL model is only moderate in comparison with the variation predicted by Suits's model.

In Figures 8 and 9 the profiles for the near infrared are presented. Here both models predict a much more symmetric behavior with respect to the nadir point, which is caused by the smaller relative difference of single leaf reflectance and transmittance in the near infrared. The differences between the results for both

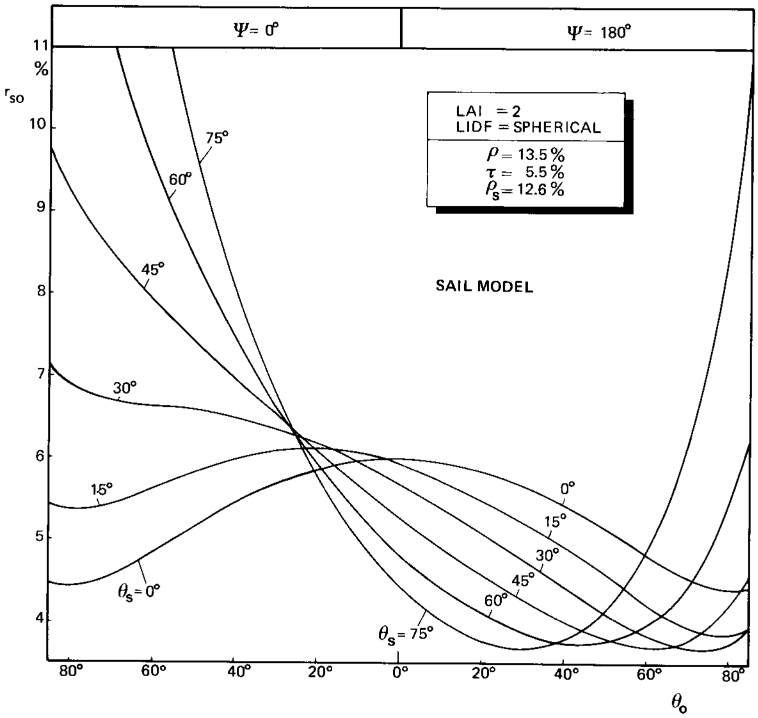

FIGURE 6. Bidirectional reflectance profiles in the green for SAIL model.

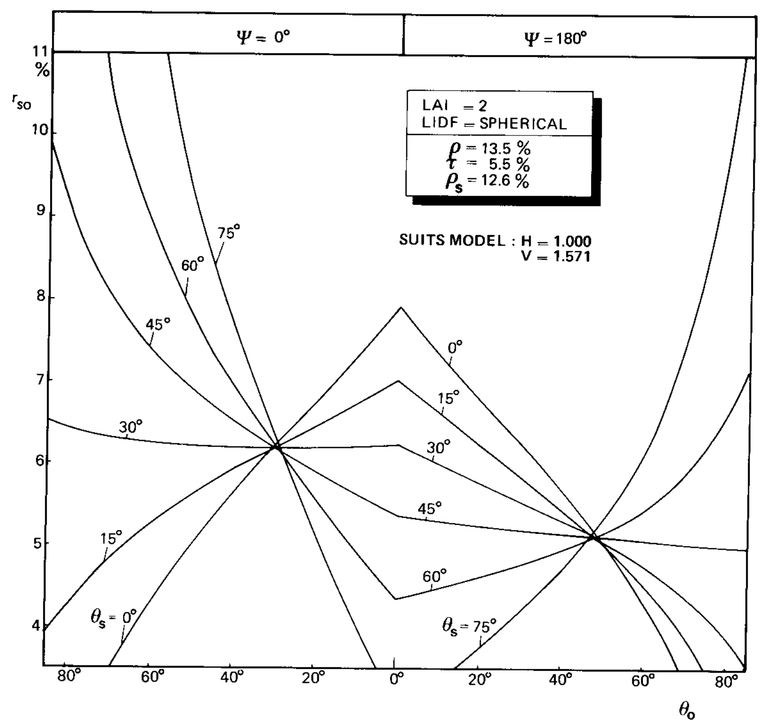

FIGURE 7. Bidirectional reflectance profiles in the green for Suits's model.

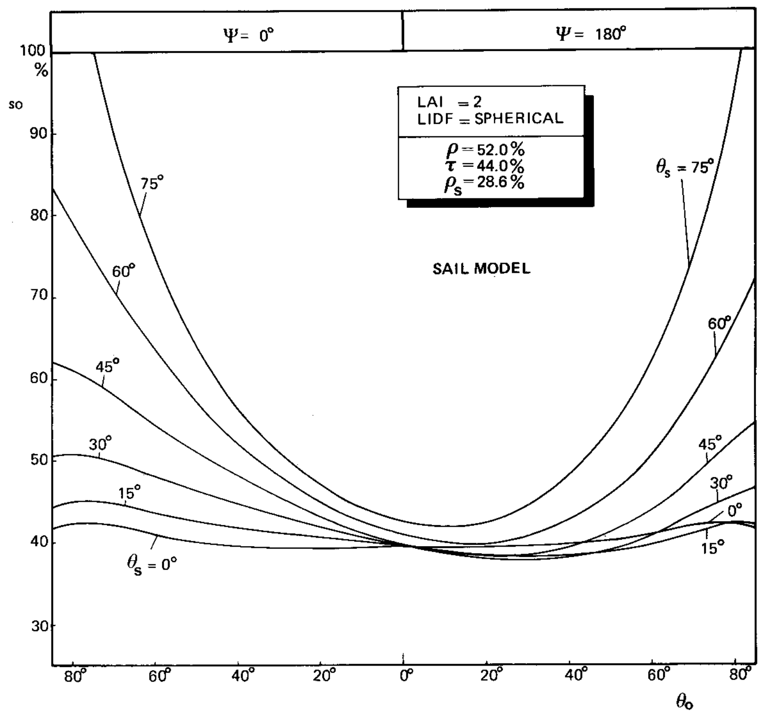

FIGURE 8. Bidirectional reflectance profiles in the near infrared for SAIL model.

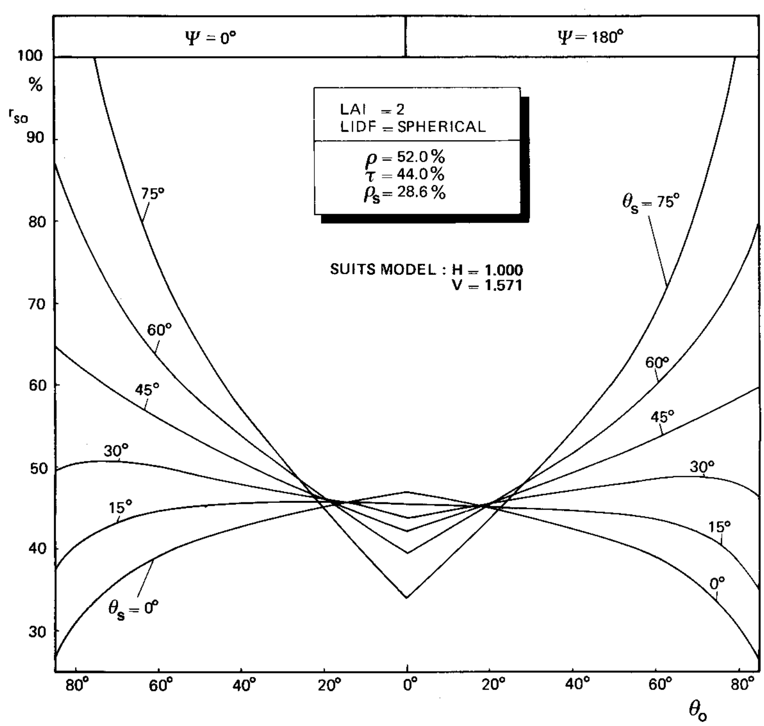

FIGURE 9. Bidirectional reflectance profiles in the near infrared for Suits's model.

models are of the same nature as in the green region of the spectrum with one significant exception: for nadir view the Suits reflectance decreases with increasing solar zenith angle, whereas the SAIL model gives an initially constant reflectance which even increases a little for solar zenith angles greater than 40°. This is consistent with a similar type of difference between the results of a four-layer Suits model for Penjamo wheat and experimental data, as reported by Chance and LeMaster (1977), so these data appear to support the results of the SAIL model already.

10. Conclusions

A detailed analysis of light scattering and extinction by Lambertian leaves of arbitrary inclination has been performed. As a result of this it is concluded that Suits's approach of simplifying the canopy morphology to horizontal and vertical leaf area projections, in order to estimate the scattering and extinction coefficients of a canopy layer, has to be re-examined. This is caused by the fact that each leaf inclination angle generates its own characteristic spatial pattern of intercepting and scattering radiation, which cannot be reproduced by taking a weighted average of the patterns associated with horizontal and vertical leaves.

The SAIL model is an improved version of Suits's canopy reflectance model in that the extinction and scattering coefficients of a layer are calculated on the basis of a given leaf area index and a leaf inclination distribution, in addition to the usual parameters describing the optical properties of single leaves and those associated with measurement conditions. Since the calculation of canopy reflectance is the same for both models, the uniform Suits model is included as a special case.

Comparison calculations show that in bidirectional reflectance profiles as a function of the view angle the break at the nadir point, characteristic for Suits's model, disappears if the SAIL model is used. The greatest deviations of the Suits reflectances from SAIL reflectances are found if the solar zenith angle or the view angle from nadir is smaller than 45°, which means that for simulations with nadir view the use of Suits's model needs reexamination.

A nadir view simulation with varying solar zenith angle for the near infrared reflectance, obtained by the SAIL model, yields a trend consistent with experimental data on Penjamo wheat, as reported by Chance and LeMaster (1977).

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Received March 21, 1984; revised May 25, 1984