

# Learning Set Functions Under the Optimal Subset Oracle via Equivariant Variational Inference

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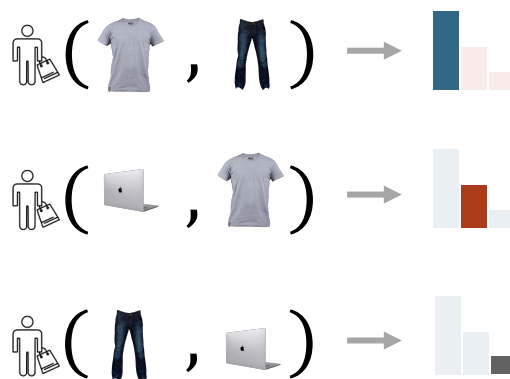
2022-03-11

 Database



Ground set  $V$

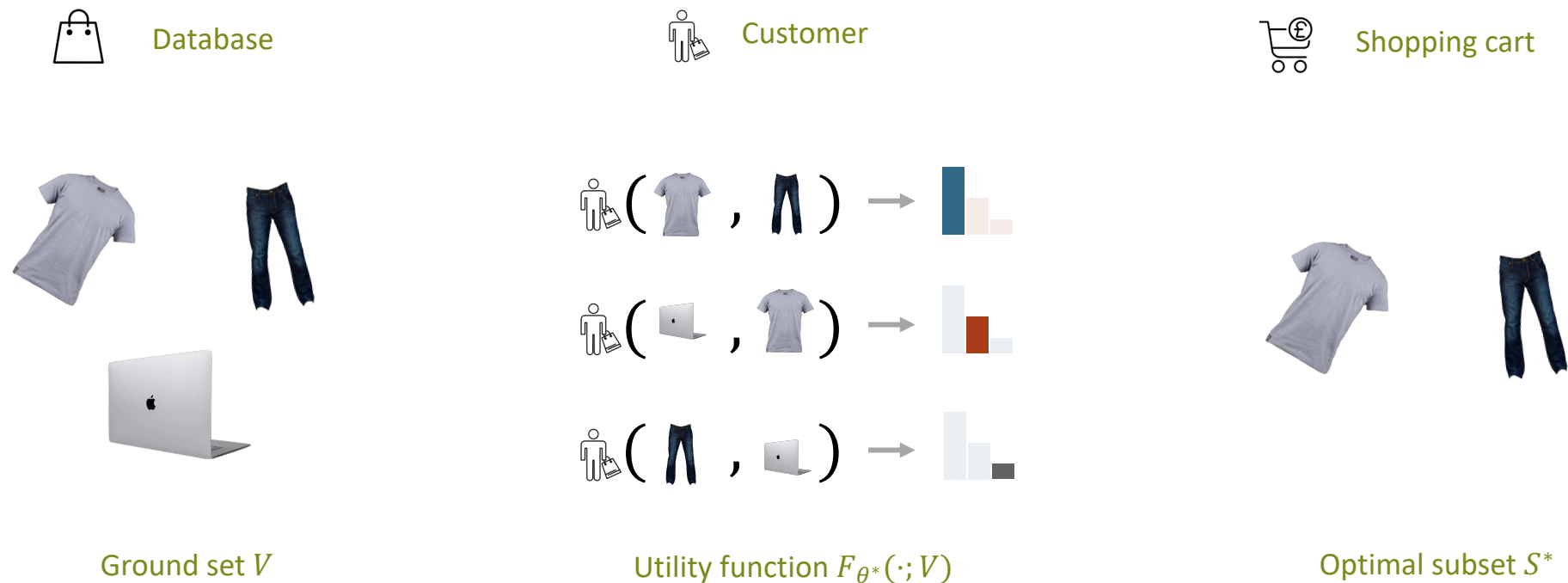
 Customer



 Shopping cart



Optimal subset  $S^*$



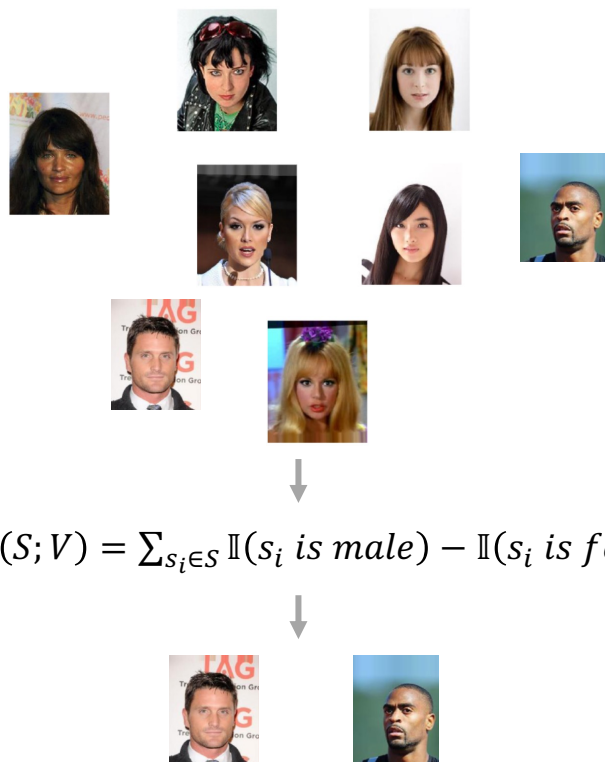
## Data Generation Process:

$$S^* = \operatorname{argmax}_{S \in 2^V} F_{\theta^*}(S; V)$$

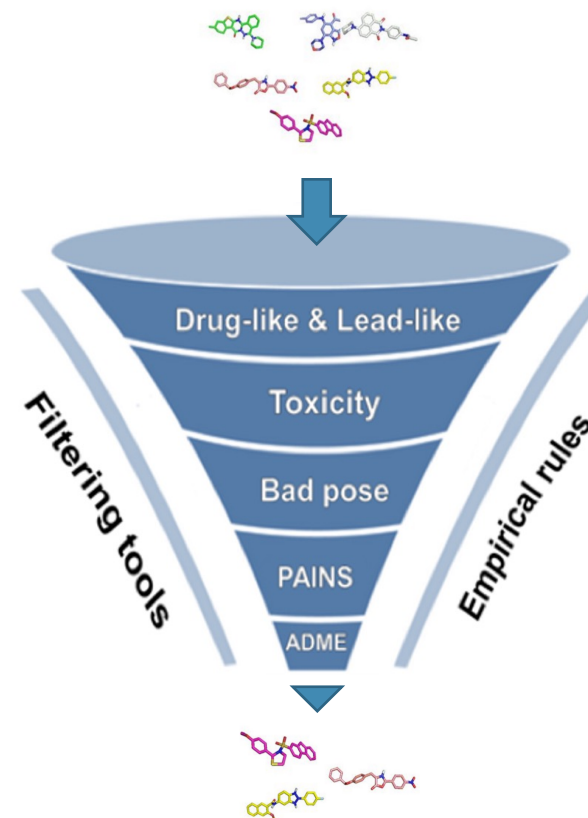
$$\sim \mathbb{p}(S, V) =: \delta_{S=S^*|V}$$

$$S^* = \operatorname{argmax}_{S \in 2^V} F_{\theta^*}(S; V)$$

## Set anomaly detection



## Compound selection



$$S^* = \operatorname{argmax}_{S \in 2^V} F_{\theta^*}(S; V)$$

**Goal:** Learn a surrogate  $F_{\theta}$  to approximate the oracle utility function  $F_{\theta^*}$ .

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**Setting 1, FV oracle:**



}}

Matching via empirical risk minimization



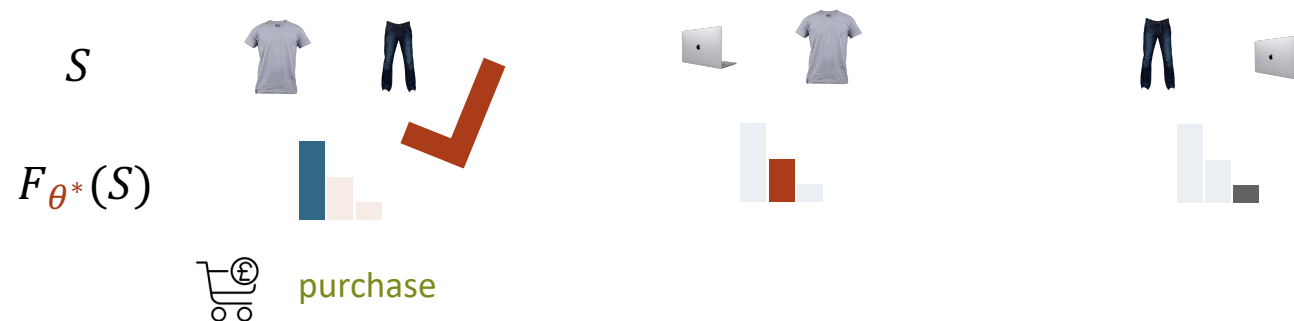
$$\theta^* = \min_{\theta} \sum_i L(F_{\theta}(S_i; V); F_{\theta^*}(S_i; V))$$

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**Setting 1, FV oracle:**

$$\theta^* = \min_{\theta} \sum_i L(F_{\theta}(S_i; V); F_{\theta^*}(S_i; V))$$

Curse of amounts of supervision signals 🤔

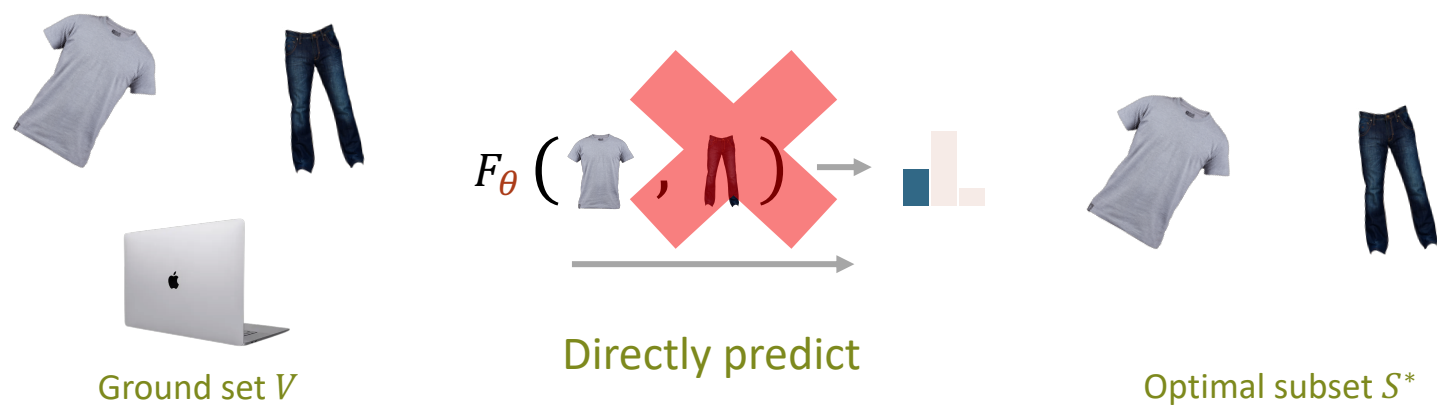
⇒ Training data in form of  $\{(S_i, F_{\theta^*}(S_i; V))\}$  for each  $V$



$$S^* = \operatorname{argmax}_{S \in 2^V} F_{\theta^*}(S; V)$$

**Goal:** Learn a surrogate  $F_{\theta}$  to approximate the oracle utility function  $F_{\theta^*}$ .

**Setting 2, OS oracle:**



⇒ Training data in form of  $\{(S^*, V)\}$



$$\begin{aligned} & \text{Empirical distribution} \\ & \downarrow \\ & \operatorname{argmax}_{\theta} \mathbb{E}_{\mathbb{P}(S^*, V)} [\log p_{\theta}(S^* | V)] \\ & s. t. p_{\theta}(S | V) \propto F_{\theta}(S; V), \forall S \in 2^V \\ & \uparrow \\ & \text{Monotonically grows with the utility function} \end{aligned}$$

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**How to construct a proper set mass function  $p_{\theta}(S|V)$ ?**

$$\begin{aligned} & \operatorname{argmax}_{\theta} \mathbb{E}_{\mathbb{P}(S^*, V)} [\log p_{\theta}(S^* | V)] \\ & \text{s.t. } p_{\theta}(S | V) \propto F_{\theta}(S; V), \forall S \in 2^V \end{aligned}$$

**Desiderata:**

*Permutation invariance*

$$F_{\theta}(\text{👕}, \text{👖}) = F_{\theta}(\text{👖}, \text{👕})$$

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*Permutation invariance*

$$F_{\theta}(\text{👕}, \text{👖}) = F_{\theta}(\text{👖}, \text{👕})$$

*Varying ground set*

$$F_{\theta}(\text{👕}; \text{👕} \text{👖} \text{💻}) \rightarrow \blacksquare \quad F_{\theta}(\text{👕}; \text{👕} \text{👖} \text{💻} \text{📱}) \rightarrow \blacksquare$$

$$\operatorname{argmax}_{\theta} \mathbb{E}_{\mathbb{P}(S^*, V)} [\log p_{\theta}(S^* | V)]$$
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*Differentiability; Minimum prior & Scalability*

$$\begin{aligned} & \operatorname{argmax}_{\theta} \mathbb{E}_{\mathbb{P}(S^*, V)} [\log p_{\theta}(S^* | V)] \\ & s.t. p_{\theta}(S | V) \propto F_{\theta}(S; V), \forall S \in 2^V \end{aligned}$$

## Probabilistic greedy model (PGM):

$$\begin{aligned} & \text{Permutation } \pi = \{s_1, s_2, \dots, s_{|S|}\} \\ & \downarrow \\ & p_{\theta}(S | V) = \sum_{\pi \in \Pi^S} p_{\theta}(\pi | V) \\ & \text{The first } j \text{ chosen elements: } S_j = \{s_1, s_2, \dots, s_j\} \\ & \downarrow \\ & p_{\theta}(\pi | V) = \prod_{j=0}^{|S|-1} \frac{\exp(F_{\theta}(s_{j+1} \cup S_j) / \gamma)}{\sum_{s \in V \setminus S_j} \exp(F_{\theta}(s \cup S_j) / \gamma)} \end{aligned}$$

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## Training:

$$\theta^* = \operatorname{argmax}_{\theta} \sum_{(V, S^*)} \log \sum_{\pi \in \Pi^{S^*}} p_{\theta}(\pi | V)$$

Log-likelihood: approximate with sampling



**DSF:**

$$F_{\theta}(S; V) = \sum_i \overset{\text{Concave function } \rho_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+}{\rho_i} \left( \sum_{s \in S} \overset{\text{Transform } \kappa_i: V \rightarrow \mathbb{R}_+}{\kappa_i}(s) \right)$$

**PGM:**

$$p_{\theta}(S|V) = \sum_{\pi \in \Pi^S} p_{\theta}(\pi|V)$$

$$p_{\theta}(\pi|V) = \prod_{j=0}^{|\mathcal{S}|-1} \frac{\exp(F_{\theta}(s_{j+1} \cup S_j)/\gamma)}{\sum_{s \in V \setminus S_j} \exp(F_{\theta}(s \cup S_j)/\gamma)}$$

	Permutation Invariance	Varying Ground Set	Differentiability	Minimum Prior	Scalability	FV/OS Oracle
DSF	✓	✗	✓	-	-	FV
PGM	✓	✗	✓	✗	✗	OS
EquiVSet (Ours)	✓	✓	✓	✓	✓	OS

$$p_{\theta}(S|V) = \frac{\exp(F_{\theta}(S; V))}{Z}$$

$Z \leftarrow$  Partition function  $Z := \sum_{S \subseteq V} \exp(F_{\theta}(S; V))$

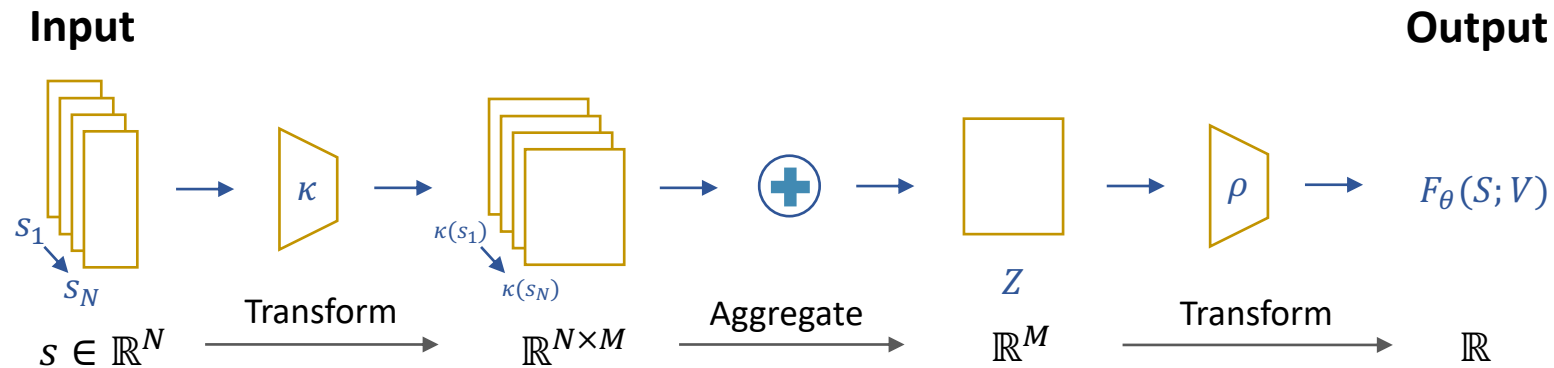
$$p_{\theta}(S|V) = \frac{\exp(F_{\theta}(S; V))}{Z} \leftarrow \text{Partition function } Z := \sum_{S \subseteq V} \exp(F_{\theta}(S; V))$$

**EBMs for Minimum Prior:**

Energy-based modeling has **maximum entropy**

$$p_{\theta}(S|V) = \frac{\exp(F_{\theta}(S; V))}{Z} \leftarrow \text{Partition function } Z := \sum_{S \subseteq V} \exp(F_{\theta}(S; V))$$

## DeepSet for Permutation Invariance:



$$p_{\theta}(S|V) = \frac{\exp(F_{\theta}(S; V))}{Z} \leftarrow \text{Partition function } Z := \sum_{S \subseteq V} \exp(F_{\theta}(S; V))$$

## Training Discrete EBMs:

Contrastive Divergence  $\Rightarrow$  Hard to converge

Score Matching  $\Rightarrow$  NonDifferentiable

Ratio Matching  $\Rightarrow$  Unstable

**Separate training and inference procedure**



$$p_{\theta}(S|V) = \frac{\exp(F_{\theta}(S; V))}{Z} \leftarrow \text{Partition function } Z := \sum_{S \subseteq V} \exp(F_{\theta}(S; V))$$

## Marginal-based Loss:

$\psi \in [0,1]^{|V|}$ : odds that  $s \in V$  shall be selected in the OS  $S^*$

$$\psi^* = \underset{\psi}{\operatorname{argmax}} D(q(S; \psi) \| p_{\theta}(S))$$

$$L(\theta; \psi^*) = \sum_{i=1}^N [-\sum_{j \in S_i^*} \log \psi_j^* - \sum_{j \in V_i \setminus S_i^*} \log(1 - \psi_j^*)]$$

**Cohesive training and inference procedure** 😊

$$p_{\theta}(S|V) = \frac{\exp(F_{\theta}(S; V))}{Z} \leftarrow \text{Partition function } Z := \sum_{S \subseteq V} \exp(F_{\theta}(S; V))$$

## Marginal-based Loss:

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↑

Require  $\psi^*$  is differentiable w.r.t.  $\theta$



Variational distribution  $q(S; \psi) = \prod_{i \in S} \psi_i \prod_{j \notin S} (1 - \psi_j), \psi \in [0, 1]^{|V|}$



$$\min_{\psi} \text{KL}(q(S; \psi) \| p_{\theta}(S))$$

$$\Leftrightarrow \max_{\psi} f_{mt}^{F_{\theta}}(\psi) + \mathbb{H}(q(S; \psi)) =: \text{ELBO}$$



multilinear extension  $f_{mt}^{F_{\theta}}(\psi) := \sum_{S \subseteq V} F_{\theta}(S) \prod_{i \in S} \psi_i \prod_{j \notin S} (1 - \psi_j)$



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multilinear extension  $f_{mt}^{F_{\theta}}(\psi) := \sum_{S \subseteq V} F_{\theta}(S) \prod_{i \in S} \psi_i \prod_{j \notin S} (1 - \psi_j)$

## RNN-like fixed-point iteration:

$$\psi^{(0)} \leftarrow \text{Initialize in } [0,1]^{|V|}$$

$$\psi_{\theta}^{(k)} \leftarrow \left( 1 + \exp \left( -\nabla_{\psi^{(k-1)}} f_{mt}^{F_{\theta}}(\psi^{(k-1)}) \right) \right)^{-1}$$

$$\psi_{\theta}^* \leftarrow \psi_{\theta}^{(K)}$$

} MFVI( $\psi^{(0)}, V, K$ )

$\psi_{\theta}^* = \text{MFVI}(\psi^{(0)}, V, K)$  is **differentiable** w.r.t.  $\theta$



$$L(\theta; \psi^*) = \sum_{i=1}^N [-\sum_{j \in S_i^*} \log \psi_j^* - \sum_{j \in V_i \setminus S_i^*} \log(1 - \psi_j^*)]$$

**Algorithm** DiffMF( $V, S^*$ ):

Initialize variational parameter  $\psi$

$$\psi^{(0)} \leftarrow 0.5 * \mathbf{1}$$

Compute the variational marginals

$$\psi^* \leftarrow \text{MFVI}(\psi^{(0)}, V, K)$$

Update parameter  $\theta$

$$\theta \leftarrow \theta - \eta \nabla_{\theta} L(\theta; \psi^*)$$

$$L(\theta; \psi^*) = \sum_{i=1}^N [-\sum_{j \in S_i^*} \log \psi_j^* - \sum_{j \in V_i \setminus S_i^*} \log(1 - \psi_j^*)]$$

## Training:

Initialize variational parameter  $\psi$

$$\psi^{(0)} \leftarrow 0.5 * \mathbf{1}$$

Compute the variational marginals

$$\psi^* \leftarrow \text{MFVI}(\psi^{(0)}, V, K)$$

Update parameter  $\theta$

$$\theta \leftarrow \theta - \eta \nabla_{\theta} L(\theta; \psi^*)$$

## Inference:

$$S = \text{topN}(\psi^*)$$

$$\psi^* = \text{MFVI}(\psi^{(0)}, V, K)$$

- Expensive computation complexity

$$\psi_{\theta}^{(k)} \leftarrow \left( 1 + \exp \left( -\nabla_{\psi^{(k-1)}} f_{mt}^{F_{\theta}}(\psi^{(k-1)}) \right) \right)^{-1}$$

↑  
expensive sampling loop per data point

- Discard interaction pattern

$$q(S; \psi) = \prod_{i \in S} \psi_i \prod_{j \notin S} (1 - \psi_j), \psi \in [0, 1]^{|V|}$$

↑  
Independent assumption

$$\psi_{\theta}^{(k)} \leftarrow \left( 1 + \exp \left( -\nabla_{\psi^{(k-1)}} f_{mt}^{F_{\theta}}(\psi^{(k-1)}) \right) \right)^{-1}$$

↑  
expensive sampling loop per data point

Approximate with neural networks:

neural network  $q_{\phi}: \mathbb{R}^{|\mathcal{V}|} \rightarrow \mathbb{R}^{|\mathcal{V}|}$

$$L = \text{KL}(q_{\phi}(S; \psi) \| p_{\theta}(S))$$

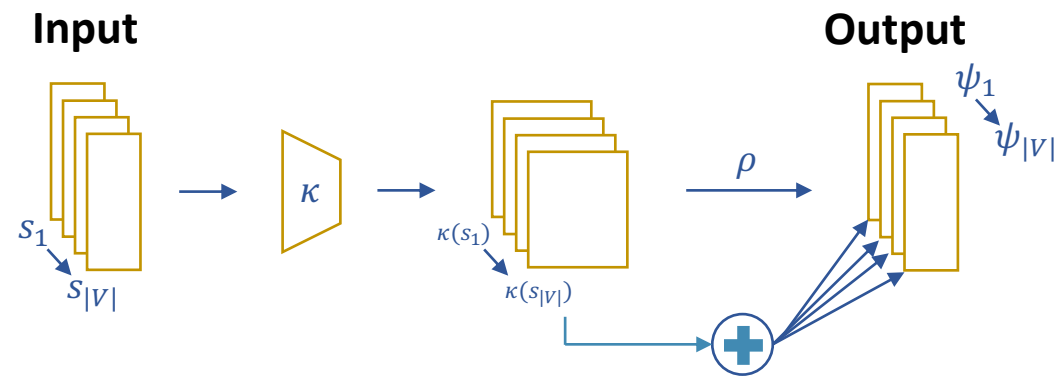
$$= f_{mt}^{F_{\theta}}(\psi) + \mathbb{H}(q_{\phi}(S; \psi)) + \text{const}$$

$q_{\phi}(S; \psi)$  should satisfy **equivariant**

$$\psi_{\theta}^{(k)} \leftarrow \left( 1 + \exp \left( -\nabla_{\psi^{(k-1)}} f_{mt}^{F_{\theta}}(\psi^{(k-1)}) \right) \right)^{-1}$$

↑  
expensive sampling loop per data point

## EquiNet( $V; \phi$ ) with permutation equivariance



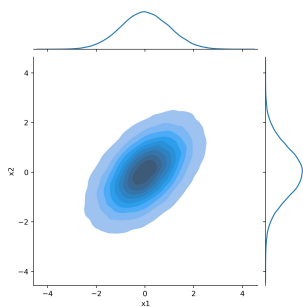
$$q(S; \psi) = \prod_{i \in S} \psi_i \prod_{j \notin S} (1 - \psi_j), \psi \in [0, 1]^{|V|}$$

↑  
Independent assumption

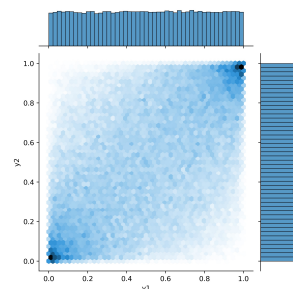
Correlation-aware inference:

$$q(s_1, s_2, \dots, s_{|V|}) = c \left( Q_1(s_1), Q_2(s_2), \dots, Q_{|V|}(s_{|V|}) \right) \prod_{i=1}^{|V|} q_i(s_i)$$

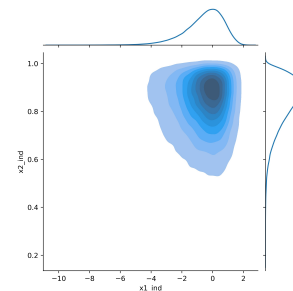
↓ Copula density      ↓ Marginal distribution  
↑ Cumulative distribution function



$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \right)$$

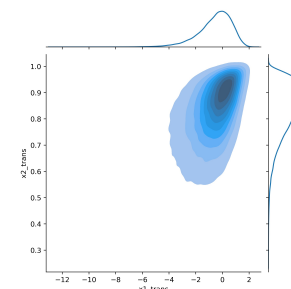


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \sim c(Q_1(x_1), Q_2(x_2))$$



$$\begin{aligned} x_{1trans} &\sim \text{Gumbel} \\ x_{2trans} &\sim \text{Beta} \end{aligned}$$

→ Copula



$$q(s_1, s_2, \dots, s_{|V|}) = \underset{\uparrow}{c} \left( Q_1(s_1), Q_2(s_2), \dots, Q_{|V|}(s_{|V|}) \right) \prod_{i=1}^{|V|} q_i(s_i)$$

Apply Gaussian copula here

Induce Gaussian copula:

Sample an auxiliary noise

$$g \sim N(0, \Sigma) \rightarrow \text{Covariance matrix, parameterized by neural network}$$

Apply element-wise Gaussian CDF

$$u = \Phi_{diag(\Sigma)}(g)$$

Obtain binary sample

$$s = \mathbb{I}(\psi \leq u) \rightarrow \text{Original logits of Bernoulli}$$



$$\phi^* = \operatorname{argmax}_{\phi} f_{mt}^{F\theta}(\psi) + \mathbb{H}(q_{\phi}(S; \psi)) := \text{ELBO}$$

$$\theta^* = \operatorname{argmin}_{\theta} \sum_{i=1}^N [-\sum_{j \in S_i^*} \log \psi_j^* - \sum_{j \in V_i \setminus S_i^*} \log(1 - \psi_j^*)]$$

$\uparrow$   
 $\psi^* = \text{MFVI}(\psi^{(0)}, V, K)$

**Algorithm** EquiVSet( $V, S^*$ ):

Update parameter  $\phi$

$$\phi \leftarrow \phi + \eta \nabla_{\phi} \text{ELBO}(\phi) \quad \rightarrow \text{Optimize } \phi$$

Initialize variational parameter

$$\psi^{(0)} \leftarrow \text{EquiNet}(V; \phi)$$

One step fixed point iteration

$$\psi^* \leftarrow \text{MFVI}(\psi^{(0)}, V, K = 1)$$

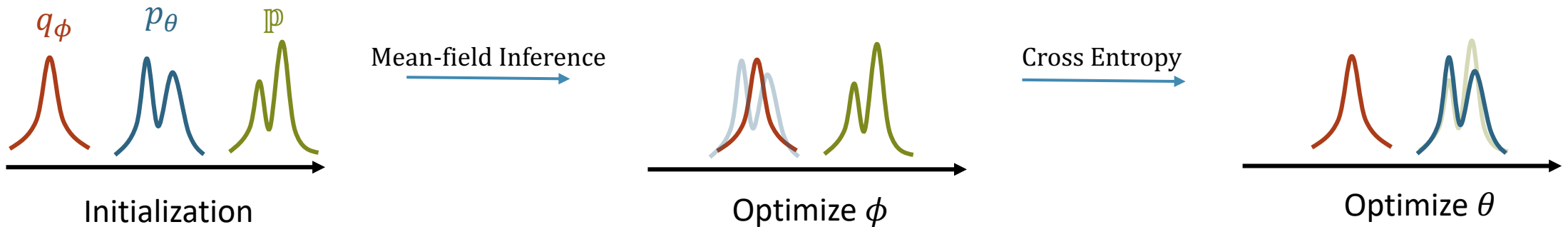
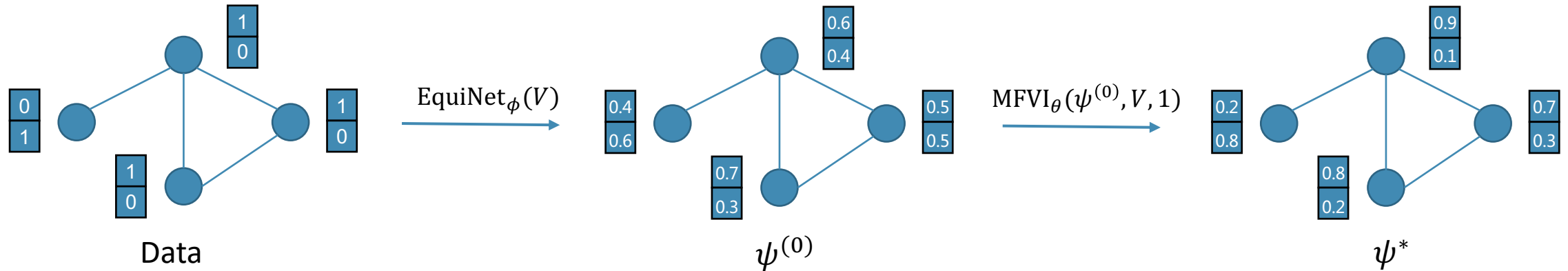
} Mean-field  
Iteration

Update parameter  $\theta$

$$\theta \leftarrow \theta - \eta \nabla_{\theta} L(\theta; \psi^*) \quad \rightarrow \text{Optimize } \theta$$

EquiVSet is a cooperative training framework

$$\theta^* = \operatorname{argmin}_{\theta} \sum_{i=1}^N [-\sum_{j \in S_i^*} \log \psi_j^* - \sum_{j \in V_i \setminus S_i^*} \log(1 - \psi_j^*)]$$



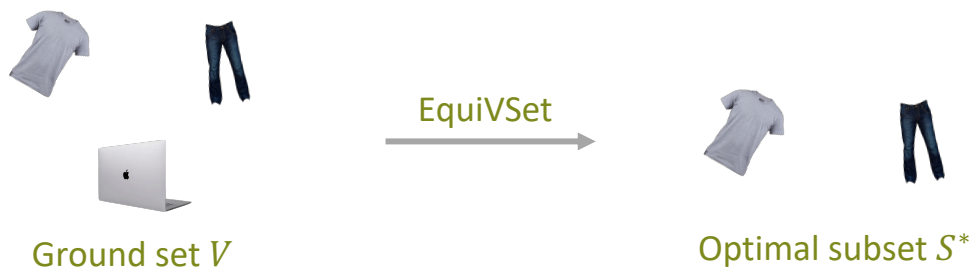


Table 2: Product recommendation results in the MJC metric on the Amazon dataset.

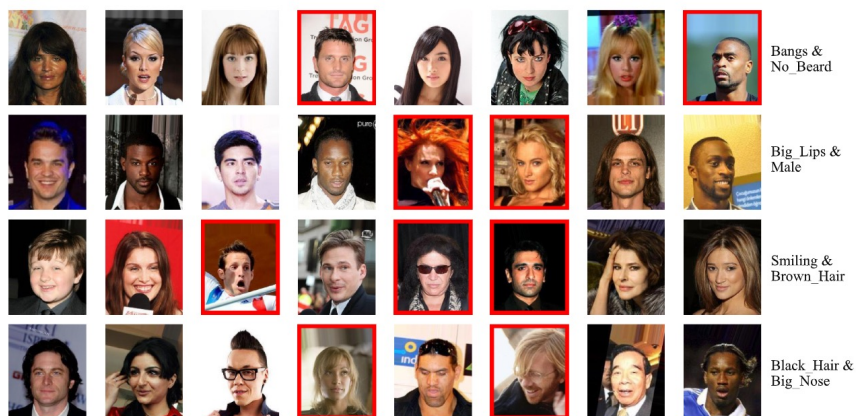
Categories	Random	PGM	DeepSet (NoSetFn)	DiffMF (ours)	EquiVSet <sub>ind</sub> (ours)	EquiVSet <sub>copula</sub> (ours)
Toys	0.0832	0.4414 ± 0.0036	0.4287 ± 0.0047	0.6147 ± 0.0102	0.6491 ± 0.0152	<b>0.6762 ± 0.0221</b>
Furniture	0.0651	0.1746 ± 0.0069	0.1758 ± 0.0072	0.1744 ± 0.0121	<b>0.1775 ± 0.0108</b>	0.1724 ± 0.0091
Gear	0.0771	0.4712 ± 0.0037	0.3806 ± 0.0019	0.5622 ± 0.0171	0.6103 ± 0.0193	<b>0.6973 ± 0.0119</b>
Carseats	0.0659	<b>0.2330 ± 0.0115</b>	0.2121 ± 0.0096	0.2229 ± 0.0104	0.2141 ± 0.0073	0.2149 ± 0.0123
Bath	0.0763	0.5638 ± 0.0077	0.4241 ± 0.0058	0.6901 ± 0.0061	0.6457 ± 0.0200	<b>0.7567 ± 0.0095</b>
Health	0.0758	0.4493 ± 0.0024	0.4481 ± 0.0041	0.5650 ± 0.0092	0.6315 ± 0.0153	<b>0.7003 ± 0.0159</b>
Diaper	0.0839	0.5802 ± 0.0092	0.4572 ± 0.0050	0.7011 ± 0.0112	0.7344 ± 0.0199	<b>0.8275 ± 0.0136</b>
Bedding	0.0791	0.4799 ± 0.0061	0.4824 ± 0.0081	0.6408 ± 0.0093	0.6287 ± 0.0195	<b>0.7688 ± 0.0121</b>
Safety	0.0648	0.2495 ± 0.0060	0.2211 ± 0.0044	0.2007 ± 0.0527	0.2250 ± 0.0287	<b>0.2524 ± 0.0285</b>
Feeding	0.0925	0.5596 ± 0.0081	0.4295 ± 0.0021	0.7496 ± 0.0114	0.6955 ± 0.0063	<b>0.8101 ± 0.0074</b>
Apparel	0.0918	0.5333 ± 0.0050	0.5074 ± 0.0036	0.6708 ± 0.0225	0.6465 ± 0.0150	<b>0.7521 ± 0.0114</b>
Media	0.0944	0.4406 ± 0.0092	0.4241 ± 0.0105	0.5145 ± 0.0105	0.5506 ± 0.0072	<b>0.5694 ± 0.0105</b>

$$\text{Metric: MJC} := \frac{1}{|\mathcal{D}_t|} \sum_{(V, S^*) \in \mathcal{D}_t} \frac{|S^* \cap S|}{|S^* \cup S|}$$

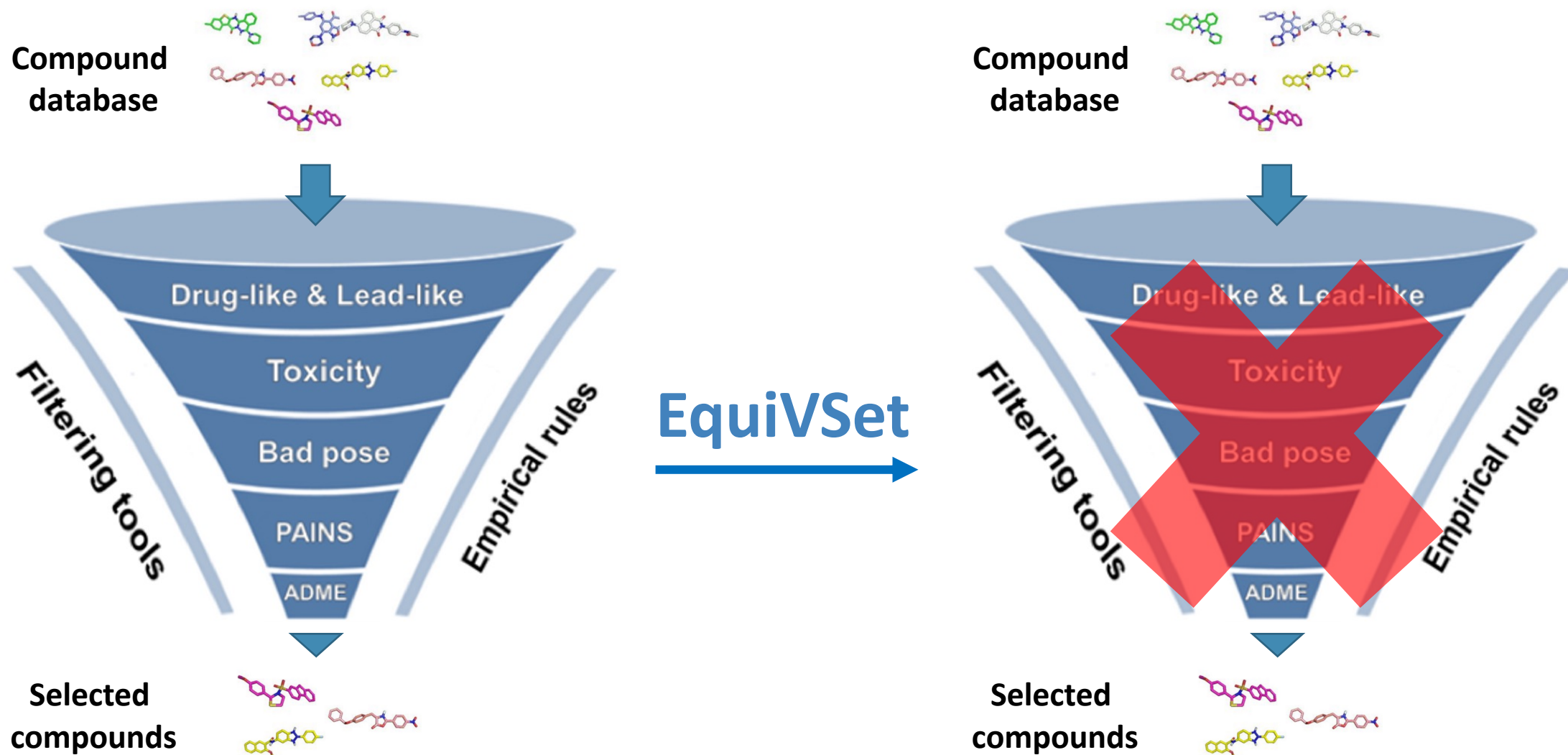


Table 3: Set anomaly detection results in the MJC metric.

Method	Double MNIST	CelebA
Random	0.0816	0.2187
PGM	0.3031 $\pm$ 0.0118	0.4812 $\pm$ 0.0064
DeepSet (NoSetFn)	0.1108 $\pm$ 0.0031	0.3915 $\pm$ 0.0133
DiffMF (ours)	<b>0.6064 <math>\pm</math> 0.0133</b>	0.5455 $\pm$ 0.0079
EquiVSet <sub>ind</sub> (ours)	0.4054 $\pm$ 0.0122	0.5310 $\pm$ 0.0123
EquiVSet <sub>copula</sub> (ours)	0.5878 $\pm$ 0.0068	<b>0.5549 <math>\pm</math> 0.0053</b>



# Application: Compound Selection





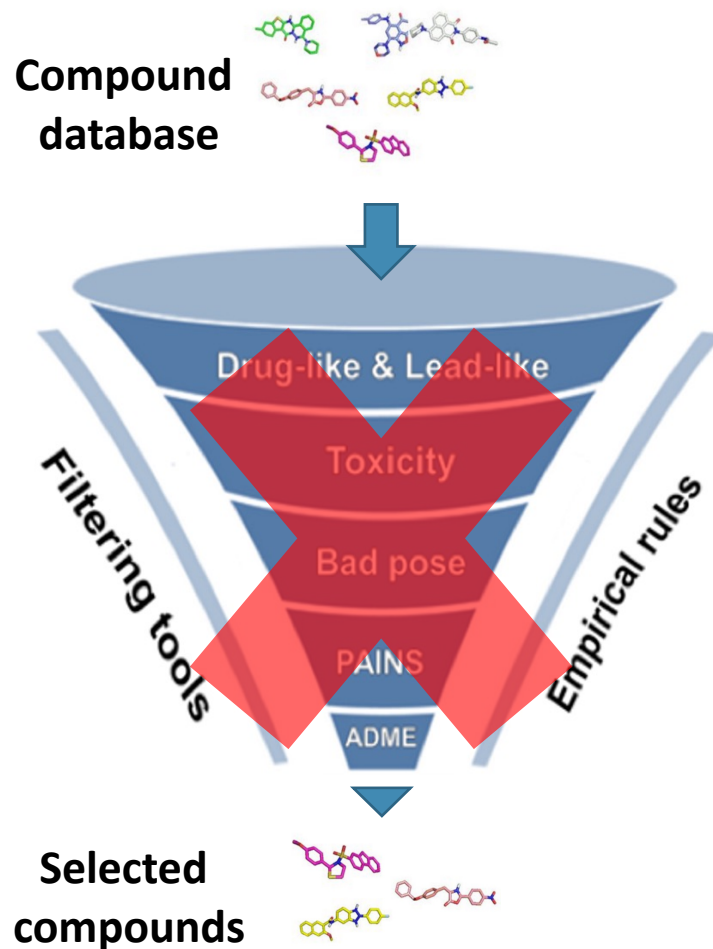


Table 4: Compound selection results in the MJC metric.

Method	PDBBind	BindingDB
Random	0.0725	0.0267
PGM	$0.3499 \pm 0.0087$	$0.1760 \pm 0.0055$
DeepSet (NoSetFn)	$0.3189 \pm 0.0034$	$0.1615 \pm 0.0074$
DiffMF (ours)	$0.3534 \pm 0.0143$	$0.1894 \pm 0.0021$
EquiVSet <sub>ind</sub> (ours)	<b><math>0.3553 \pm 0.0049</math></b>	<b><math>0.1904 \pm 0.0034</math></b>
EquiVSet <sub>copula</sub> (ours)	$0.3536 \pm 0.0083$	$0.1875 \pm 0.0032$



Thank you!