Visual attractiveness in vehicle routing via bi-objective optimization

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Abstract

We consider the problem of designing vehicle routes in a distribution system that are at the same time cost-effective and visually attractive. In this paper we argue that clustering, a popular data mining task, provides a good proxy for visual attractiveness. Our claim is supported by the proposal of a bi-objective capacitated vehicle routing problem in which, in addition to seek for traveling cost minimization, optimizes clustering criteria defined over the customers partitioned in the different routes. The model is solved by a multi-objective evolutionary algorithm to approximate its Pareto frontier. We show, by means of computational experiments, that our model is able to characterize vehicle routing solutions with low routing costs which are, at the same time, attractive according to the visual metrics proposed in the literature.

Keywords: Vehicle routing problem, Visual attractiveness, Clustering

1. Introduction

The vehicle routing problem (VRP) [1] is arguably one of the most classic combinatorial optimization problems arising in the logistics chain. The VRP consists in determining the routes that a certain fleet of vehicles must take

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in order to collect items at known customer locations. Each item typically has a certain size or weight associated. The total amount (in terms of either weight or size) of the quantities collected by a single vehicle cannot exceed its capacity. In the most classical version of the VRP, the data (customer demands, traveling times, time windows, etc.) are assumed to be all known beforehand. A decision maker must then plan ahead the vehicle routes so as to satisfy the demands of the customers at minimum traveling cost. The VRP is, unfortunately, strongly NP-hard even for a single objective as the traveling salesman problem (TSP) [2] can be polynomially reduced to it [3].

In the vehicle routing literature, the problem might be optimized regarding other objectives and constraints such as makespan [4], CO_2 emissions [5], earliness/tardiness of service [6], level of service [7], or fleet size [8]. Sometimes it is also possible or even necessary to integrate several such objectives within multi-objective settings to explicitly account for the often conflicting nature of many of them [9, 10].

Very recently, Rossit et al. [11] wrote an extensive survey on the importance of producing visually attractive solutions for the VRP as they are more likely to be accepted by operators and practitioners, making easier their adoption in practical situations. The attractiveness feature is sometimes considered so important in real applications that their evaluation by practitioners might be done even during the optimization process itself [12, 13, 14]. Visual attractiveness is not a property that can be easily expressed in mathematical terms due to its subjectivity [15]. In an extensive survey presented in [11], the authors state three properties that attractive vehicle routes must have:

i. *compactness*, which means that demand points in one route should be relatively close to each other;

ii. *non-overlapping* or *not-crossing*, which means that the vehicles should keep a certain separation among them while performing their routes so that their routes do not cross each other; and

iii. *low complexity*, which is related to structural characteristics of each route individually (e.g. number of intra-route crossings, number of jagged turns).

Although often conflicting, the cost and visual attractiveness objectives do not always present a negative correlation, e.g. [16, 17] show that the addition of visual constraints also improved the cost of the solutions for some VRP variants.

In Rossit et al. [11], the authors present a series of metrics for (i-iii) which are used to compare the visual attractiveness of VRP solutions. In this article, we argue that partitioning the demand points by means of clustering methods naturally yields the desirable visual properties (i) and (ii). Clustering is a popular data mining technique which, given a set of data points, groups them to produce well-separated and homogeneous subsets, called *clusters* [18]. Homogeneity means that points in the same cluster should be similar whereas separation means that points in different clusters should differ one from the other. Unlike the tradition in the VRP literature of performing clustering and routing sequentially, our framework allows for the simultaneous consideration of both tasks, leading to low-cost, visually attractive routes in a more natural way. With that in mind, we introduce the VRP with integrated minimization of the total routing cost and maximization of the routes' visual attractiveness based on clustering.

The remainder of this article is organized as follows. In Section 2 we present a detailed literature review on clustering methods as a combinatorial optimization problem. Besides, we survey a series of papers in which clustering is used as a sub-routine within optimization methods for the VRP. In Section 3 we provide a brief but precise description of our problem with a formal multi-objective linear-integer formulation, including some illustrative examples. In Section 4 we describe an evolutionary algorithm capable of handling large instances of our problem. In Section 5 we perform a critical and experimental analysis of the VRPs results obtained by our multi-objective evolutionary algorithm on some classical problems from the VRP literature as well as on a real road network. Finally, Section 6 concludes the paper.

2. Related works

The literature on clustering algorithms, criteria and applications is vast. For comprehensive compendiums we refer to Hansen and Jaumard [18], Jain et al. [19], Aggarwal and Reddy [20]. Cluster analysis is the task of grouping data that share similar characteristics, and to separate data that differ. Clustering might be performed in many different ways depending on the chosen clustering criterion, which defines the measure used to tell if a group of objects is either compact or not, and at what extent.

One of the most used types of clustering is that of partitioning, where we look for a partition $P = \{C_1, \ldots, C_K\}$ of a set of data points $O = \{o_1, \ldots, o_n\}$ into K clusters such that: (i) $C_k \neq \emptyset$, for $k = 1, \ldots, K$; (ii) $C_k \cap C_\ell = \emptyset$, for $1 \leqslant k < \ell \leqslant K$; and (iii) $\cup_{k=1}^{K} C_k = O$. The set of all K-partitions of O is denoted $\mathcal{P}(O, K)$. In that setting (see e.g. [21]), clustering can be seen as as a mathematical optimization problem whose objective function $f : \mathcal{P}(O, K) \to \mathbb{R}$, the clustering criterion, defines the optimal solution for the clustering problem given by:

$$\min\{f(P) : P \in \mathcal{P}(O, K)\}. \tag{1}$$

Clustering methods group data points based on the clustering criterion and on the dissimilarity (equiv. similarity) relations between the data points. The dissimilarity d_{ij} between a pair of objects (o_i, o_j) is usually computed as a function of the data attributes, such that d values (usually) satisfy: (i) $d_{ij} = d_{ji} \geqslant 0$, and (ii) $d_{ii} = 0$. Hence, as dissimilarities do not need to obey triangular inequalities, they do not necessarily represent distances.

The clustering criterion f defines how homogeneity is expressed in the clusters to be found [18]. There exists several clustering criteria in the literature. Among them, the *diameter minimization* (DMin) is expressed as

$$\min_{\{C_1, \ldots, C_K\}} \max_{i<j : o_i, o_j \in C_k} \{d_{ij}\}; \tag{2}$$

which declares a cluster as compact if its two data points that differ the most are still alike, or the *minimum sum-of-cliques* (MSC) which aims to minimize the sum of all the dissimilarities between objects in the same cluster, expressed as:

$$\sum_{k=1}^{K} \sum_{i<j : o_i, o_j \in C_k} \{d_{ij}\}. \tag{3}$$

If data points o_i in O correspond to points of a s-dimensional Euclidean Euclidean space, further concepts are useful. Homogeneity of a cluster C_k can then be measured in reference to a cluster center which is not in general a data point belonging to the dataset. A very popular criterion for clustering points in Euclidean space is the minimum sum-of-squares criterion (MSSC) given by:

$$\min \sum_{k=1}^{K} \sum_{i : o_i \in C_k} (\|o_i - y_k\|)^2, \tag{4}$$

where $\| \cdot \|$ is the Euclidean norm and y_k is the centroid of the points o_i in cluster C_k (due to first-order optimality conditions).

The clustering criterion used is determinant to the computational complexity of the associated clustering problem. DMin, MSC and MSSC are NP-hard in general [22, 23, 24]. Consequently, for larger problems, authors usually resort to heuristics, such as the complete-linkage heuristic for diameter minimization [25], or the k-means algorithm for minimum sum-of-squares clustering [26].

Vehicle routing algorithms have, since the very early times, included clustering subroutines to reduce the computational burden associated with the routing of the entire problem. The sweep algorithm introduced by Gillet and Miller [27] is an example of such decomposition. In the sweep algorithm, customers are grouped according to their proximity using polar coordinates. This can be seen as the ordering in which the nodes would be sweeped by an imaginary clock hand. Fisher and Jaikumar [28] proposed a so-called cluster-first-route-second algorithm for vehicle routing problems in which the customers are first grouped according to their proximity solving a generalized assignment problem. For each cluster, a traveling salesman problem (TSP) is then solved. Taillard [29] uses a similar decomposition in which the clustering of the nodes is performed by solving a minimum spanning forest of the nodes, rooted at the depot. A TSP is then solved for each subtree.

Recent heuristics are now less dependent on a pre-clustering of the nodes, mainly because of the additional computational power available that allows the simultaneous routing of several thousands of nodes at once within reasonable time limits. However, some rich vehicle routing problems that are challenging even for medium-size problems still benefit from such decomposition scheme [30, 31, 32]. Concerning the integration of routing and clustering, Mourgaya and Vanderbeck [33] introduces a clustering problem that integrates regionalization and route balancing. A routing decisional layer is only included *a posteriori*. Their analysis suggests that by using the clustering provided by this tactical planning the operator can find well balanced and compact solutions, at the expense of larger routing costs. In [34], the authors penalize vehicle routes that are deemed as non-compact. The penalty, denoted *clustering penalty*, is made proportional to the proximity of the demand points to the median demand point of their routes.

The use of clustering sub-routines within VRP solution methods is also connected to the concept of consistency [35, 36]. From the drivers perspective, routing plans in which customers are well-separated into contiguous, compact and balanced sub-regions are more coherent and consistent to their daily activities. A way of bringing consistency to VRPs solutions is through *districting* the customer locations according to some criteria such as contiguity and balance constraints [37, 38]. Each district is thus responsible for the operations performed inside it. Districts can be understood as clusters with specific strategical objectives. The works of [39, 40] partition service regions into districts using geographical criteria measures that yield compact and balanced sub-regions.

Visual attractiveness plays an important role in the adoption of routing plans, as practitioners may in part drive their logistics decisions based on aesthetical considerations. A few remarkable examples in the literature have successfully incorporated visual attractiveness metrics to enhance the robustness of routing plans. Tang and Miller-Hooks [14] consider a routing problem with shape constraints. These constraints aim at imposing the visual attractiveness of the solutions. The authors consider two such constraints and embed these measures within a heuristic solver. This solver maintains visually attractive routes all along the search, but at the expense of violating other constraints, and stop when the solutions become feasible. Sahoo et al. [12] develop a waste management system that considers —among other criteria— visual attractiveness metrics in the design of the system routes. They consider a simple swapping heuristic that moves stops from one route to another if by doing so the routes become more compact. In Lum et al. [41] the authors consider a minimax k-vehicles windy rural postman problem (MMKWRPP), a problem belonging to the broader class of arc-routing problems. In the MMKWRPP, the objective is to design a set of k vehicle routes to serve a series of arcs in a network, such as to minimize the cost of the most expensive route. The authors propose a cluster-first-route-second heuristic, and consider visual attractiveness at two levels: first to guide the design of the initial clusters, and second within a local search improvement heuristic. In Corberán et al. [42], the authors consider the same problem and now introduce a mathematical model that includes some measures of visual attractiveness explicitly via additional constraints and objectives. The latter gives raise to a multiobjective model that they tackle by means of heuristics.

3. Problem description and mathematical formulation

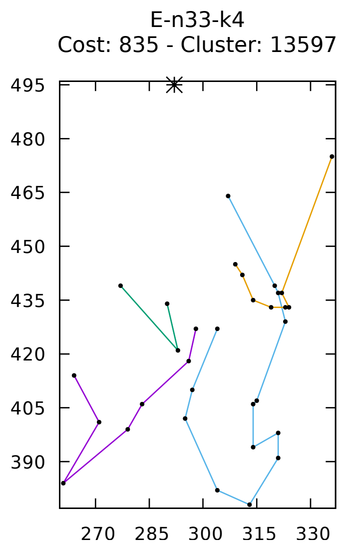
The main contribution of our work is to show that classical clustering methods widely used by the data mining community are able to provide visually attractive VRP solutions. To that purpose, we propose in this section a new bi-objective vehicle routing model that simultaneously optimizes travel distance costs and clustering objectives.

We are given a set of $n+1$ nodes $V = \{0, 1, \ldots, n\}$. The node labeled 0 represents the depot, whereas the remaining nodes represent the customers. The set of customer nodes is denoted V^+. With each customer $i \in V^+$ is associated a demand $a_i > 0$. We are also given a set of K identical vehicles, each of which has a capacity equal to Q. With every pair of nodes $(i, j), i < j$, is associated an edge $\{i, j\}$ with a routing cost c_{ij}. The VRP with simultaneous optimization of the total routing cost and customer clustering is the problem of routing each of the K vehicles, so as to visit every customer node exactly once, while respecting the total demand collected by each vehicle on its route. The objectives are: 1) to minimize the total routing cost; and 2) to minimize (or maximize) a clustering criterion associated to the different vehicle routes. As it may be impossible to find a single solution that optimizes both objectives simultaneously, the real goal of this optimization problem is to find (or at least to approximate) the *Pareto frontier* [43], i.e., the set of all solutions of the problem that are not dominated by any other solution. A solution x is said to be dominated by another solution y if y is at least as good as x for all the objectives, being strictly better for at least one of them.

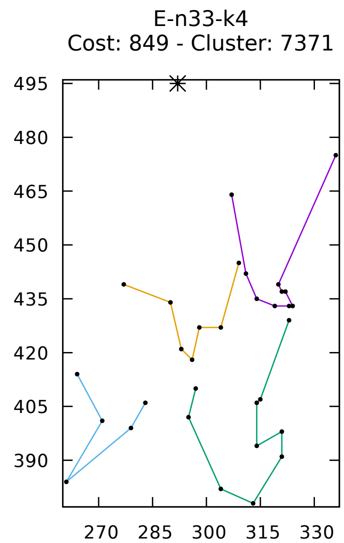
To illustrate, let us consider problem E-n33-k4 from the classical CVRP testbed. The optimal traveling cost solution for this problem has an optimal traveling time of 835, and is shown in Figure 1a. The depot is represented by the $*$ symbol, and the edges used from and to the depot are omitted. A possible clustering measure for this VRP solution could be obtained by MSSC (4), where each customer is located in a position of the Euclidean space under consideration. The MSSC is then computed as the sum of squared Euclidean distances of each customer to the centroid of the customers of the route it belongs to. The MSSC value for the solution in Figure 1a is 13597.

Let us consider another solution to the problem in Figure 1b—namely a Pareto solution as identified by our evolutionary method to be described later — of cost 849 (i.e. fourteen units higher than the optimal routing cost) but with a lower MSSC of 7371. A quick inspection of these two solutions reveals that the routes shown in Figure 1b are more compact (property i.) and more separated from each other (property ii.). One can hence argue that the second solution is more visually attractive than the first one. Finally, a third solution is presented in Figure 1c whose routing cost is 865 and MSSC is 9789. It is not in the Pareto frontier, since it is dominated by the solution of Figure 1b.

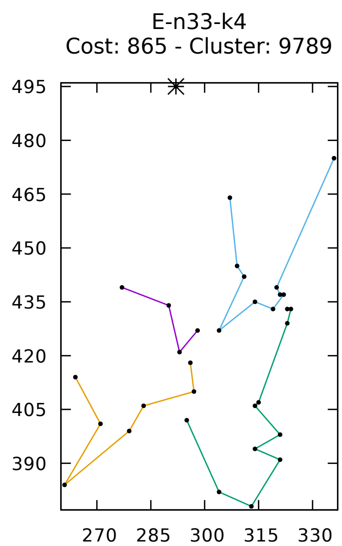
Figure 1: Three solutions for instance E-n33-k4



(a) Solution of minimum traveling cost



(b) A Pareto solution



(c) A dominated solution

The VRP with simultaneous minimization of the total routing cost and optimal clustering can be formulated as a bi-objective mathematical optimization problem, as follows. For each edge $\{i, j\}$, we let x_{ij} be an integer variable representing the number of times that edge $\{i, j\}$ is taken by some vehicle. For depot-to-customer edges $\{0, i\}, i \in V^+$, this variable may take integer values between 0 and 2, whereas for customer-to-customer edges it is a binary variable. We also let y_{ij} be a binary variable taking the value 1 iff nodes i and j are serviced by the same vehicle, for any two nodes $i, j \in V^+, i < j$. Finally, we let $f : \mathbb{B}^{n\times n} \to \mathbb{R}$ be a real-valued function equal to the clustering criterion under optimization. For notational simplicity, for any set $S \subset V$, we denote $x(\delta(S)) = \sum_{i\in S, j\notin S, i<j} x_{ij} + \sum_{i\in S, j\notin S, i>j} x_{ji}$, and if in addition $S \subseteq V^+$, we also let $r(S)$ be a lower bound on the number of vehicles needed to service the customers in S. It is common to define $r(S) = \lceil \sum_{j\in S} a_j/Q \rceil$. The following model —derived from the two-index vehicle-flow formulation of the CVRP introduced by Laporte et al. [44]— is valid for the problem:

$$\min \quad \text{total routing cost} = \sum_{i,j\in V, i<j} c_{ij}x_{ij} \tag{5}$$

$$\max \textit{ or } \min \quad \text{clustering} = f(y) \tag{6}$$

subject to

$$x(\delta(\{i\})) = 2 \qquad i \in V^+ \tag{7}$$

$$x(\delta(\{0\})) = 2K \tag{8}$$

$$x(\delta(S)) \geqslant 2r(S) \qquad S \subseteq V^+, |S| \geqslant 2 \tag{9}$$

$$y_{ij} \geqslant x_{ij} \qquad i, j \in V^+, i < j \tag{10}$$

$$y_{ik} - y_{ij} - y_{jk} + 1 \geqslant 0 \qquad i, j, k \in V^+, i < j < k \tag{11}$$

$$y_{ij} - y_{ik} - y_{jk} + 1 \geqslant 0 \qquad i, j, k \in V^+, i < j < k \tag{12}$$

$$y_{jk} - y_{ij} - y_{ik} + 1 \geqslant 0 \qquad i, j, k \in V^+, i < j < k \tag{13}$$

$$x_{0j} \in \{0, 1, 2\} \qquad j \in V^+ \tag{14}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in V^+, i < j \tag{15}$$

$$y_{ij} \in \{0, 1\} \qquad i, j \in V^+, i < j. \tag{16}$$

In this problem, the two objectives (5)-(6) seek to simultaneously optimize the total routing cost and the chosen clustering criterion, respectively. In particular, objective (5) is defined over variables x whereas objective (6) expresses a clustering criterion function defined over variables y. Yet, both objectives use the Euclidean distances between customers as cost coefficients (i.e., $c_{ij} = d_{ij}$, $\forall i, j \in V$). Constraints (7)-(9) are classical VRP constraints: degree, fleet size and capacity constraints, respectively. Constraints (10)-(13) impose that customers serviced by the same vehicle must belong to the same cluster. More specifically, constraints (10) impose that customers that are visited consecutively in a route are associated to the same cluster, whereas constraints (11)-(13) impose the transitivity of this relationship between customers that are visited in the same route but not in sequence. Finally, constraints (14)-(16) express the integer nature of the variables x and y.

4. Multiobjective evolutionary algorithm

In this section, we present a population-based multi-objective heuristic for our bi-objective optimization problem. We have implemented a NSGA-II algorithm which has been shown to be a very efficient heuristic for solving multi-objective problems in general [45], both in terms of the quality of the solutions found as in terms of their number.

The NSGA-II uses two routines, namely the *ranking* and the *crowding distance*, to sort solutions. The first computes for each solution the number of solutions in the population which are dominated by it. The set of solutions whose rankings are equal defines a Pareto front. Thus, the solutions with ranking equal to zero are in the best Pareto front found so far. Ties are broken by a second criterion, the crowding distance, which defines the distance of a solution to its nearest neighbors in the Pareto front it belongs. The crowding distance contributes to fill possible discontinuities in the Pareto fronts. Let $RC(x)$ and $CL(x)$ stand for the total routing cost and the clustering criterion (here a minimization one) value of a solution x in the population, then the crowding distance of x is computed as:

$$\frac{RC^{suc(x)} - RC^{pred(x)}}{RC^{max} - RC^{min}} + \frac{CL^{suc(x)} - CL^{pred(x)}}{CL^{max} - CL^{min}}, \tag{17}$$

where $suc(x)$ and $pred(x)$ are respectively the solutions that succeeds and preceeds x in its Pareto front in terms of function values. The maximum and minimum routing costs and clustering values in the Pareto front to which x belongs are given by $RC^{max}, RC^{min}, CL^{max}, CL^{min}$, respectively. Solutions corresponding to RC^{min} and CL^{min} are set to have maximum crowding distance.

The pseudo-code of the NSGA-II framework is presented by Algorithm 1.

Algorithm 1 NSGA-II framework

$P_1 \leftarrow$ initial population
$t \leftarrow 1$
repeat
  $Q_t \leftarrow$ genetic operators on P_t + local search
  $R_t \leftarrow P_t \cup Q_t$
  sort R_t solution according to *ranking*
  sort R_t solution according to *crowding distance*
  $t \leftarrow t + 1$
  $P_t \leftarrow selection(R_t)$
until stopping condition satisfied

The NSGA-II algorithm for our bi-objective vehicle routing problem builds an initial population as done by Prins [46], i.e., by combining the solutions obtained by the heuristics of Clarke and Wright [47], Mole and Jameson [48], and Gillett and Miller [49], with solutions randomly generated. The offspring is obtained by the application of the PMX and OX crossover operators largely used in the literature by genetic algorithms for VRP problems (see [50] for a survey). The crossover operators are randomly chosen and applied for two parents randomly selected from P_t. The operators are applied until that Q_t solutions are obtained, and so that $|Q_t| = |P_t|$. The Q_t solutions are in the sequel randomly selected for mutation (with prob. of 30% in our experiments). The mutation operator corresponds to the application of one single random move in one of the following neighborhoods:

- reinsertion: one customer is removed and inserted in another position of the route;
- 2-opt: two non-adjacent arcs are removed and another two are added in such a way that a new route is generated;
- shift(1,0): one customer is transferred from its route to another route;
- swap(1,1): two customers from two different routes are permuted; and

- swap(2,1): two adjacent customers from a route are permuted with a customer from another route,

Each solution is then improved by a Variable Neighborhood Descent (VND, [51]) local search in the same above neighborhoods in that exact order, so that intra-route neighborhoods are used more often due to their lower complexity of exploration. That local search is oriented towards improving routing costs so that neighbouring solutions that deteriorate the clustering criterion under consideration are discarded. Finally, after the solutions are sorted, the next population is obtained by selecting the first $|P_t|$ solutions according to their ranking and crowding distance.

5. Computational experiments

In this section we present and analyze the results of experiments aiming at assessing the visual attractiveness of the VRP solutions produced by the evolutionary algorithm of the previous section on optimizing the proposed bi-objective model. For the experiments, we use a classical dataset from the CVRP literature, namely the instances A-B-E-P available at `http://vrp.atd-lab.inf.puc-rio.br`. In particular for these instances, we assume that that the routing costs c_{ij} are equal to the Euclidean distances between the locations of customers i and j the plane. The NSGA-II algorithm has been implemented in C++ using the GNU g++ compiler v5.4, running under a Linux machine with 4 GB of RAM, with an Intel Core i3-2310M @ 2.1 GHz.

The visual attractiveness of the obtained routes using different clustering criteria are first assessed according to a set of visual metrics. In the sequel, we observe the impact in the routing cost caused by the quest of more visually attractive VRP solutions. Finally, the approximate Pareto frontiers obtained by the NSGA-II heuristics are evaluated in terms of their effectiveness in producing low-cost and visual attractive VRP solutions.

All the obtained VRP solutions can be found at `https://github.com/diegorlima/CVRP-bi-objective`, where they are categorized and illustrated according to the applied clustering criterion f used within our bi-objective model.

5.1. Visual attractiveness metrics

Rossit et al. [11] explore different metrics proposed in the literature for assessing the visual attractiveness of VRP solutions according to properties (i)-(iii) described in section 1. The authors perform an in-depth correlation analysis to reveal any dependence between the metrics and recommend the use of a subset of them. Following the recommendations provided in [11], we evaluate the routes obtained by our VRP model using

- the compactness metric of [13]:

$$comp_r^1 = \frac{avgDist_r}{avgMaxDist_r}, \tag{18}$$

  where $avgDist_r$ is the average distance between two consecutive customers in route r, and $avgMaxDist_r$ is the average distance of the 20% longest distances between two consecutive costumers in route r.

- The compactness metric of [34]:

$$comp_r^2 = \sum_{i \in r} d_{i,m_r}, \tag{19}$$

  where m_r is the customer located in the intermediate position of the route r

- The proximity metric of [52]:

$$prox_r = \frac{|o_r|}{|r|}, \tag{20}$$

  where o_r is the set of customers of route r that are nearer to the median of another route $r' \neq r$ than to its own median. The median of a route r corresponds to the location of the closest customer to the geometric center of r which is calculated from the coordinates of the customers assigned to it.

Our computational results regarding these three metrics are reported concerning average values obtained from the set of K routes.

Another measure computed from the whole set of routes is the inter-route crossing (*cross*) metric [13], which is simply computed as the number of crossings between edges belonging to two distinct routes. This measure does not count edges involving the depot node.

Remark that the above visually attractiveness metrics are not trivially modelled within typical VRP formulations. Consequently, they cannot be straightforwardly incorporated into them.

Finally, we did not select in our study any metric to evaluate the complexity of the individual routes obtained (property iii.), since the clustering objective of our bi-objective VRP model does not yield less complex routes, e.g. with less intra-route crossings, or smaller angles between consecutive customers. That is, the clustering criterion influences how the customers are partitioned among the routes, but plays no role on how to organize the customers to be served by a specific vehicle.

5.2. Clustering criteria

We evaluate our bi-objective VRP for visual attractiveness introduced in section 3 using three distinct clustering criteria $f : \mathbb{R}^{n\times n} \to \mathbb{R}$ commonly used in the data mining literature, namely the diameter minimization (DMin) , the min-sum of cliques (MSC), and the minimum sum-of-squares (MSSC). For modeling DMin minimization, it suffices to replace (6) by the minimization of a variable $D \geqslant 0$ and add constraints

$$D \geqslant d_{ij} y_{ij} \qquad i, j \in V^+, i < j, \tag{21}$$

where $d_{ij} \geqslant 0$ represents hereafter the Euclidean distance between the locations of customers i and j. As such, the resulting bi-objective optimization problem is integer-linear. Analogously, clustering our model with MSC is also integer-linear as (6) is replaced by the minimization of

$$\sum_{i=1}^{n} \sum_{i<j}^{n+1} d_{ij} y_{ij}. \tag{22}$$

Conversely, the MSSC criterion in place of (6) yields a mixed-integer non-linear optimization problem whose objective function is given by

$$\frac{\sum_{i=1}^{n} \sum_{i<j}^{n+1} d_{ij} y_{ij}}{\sum_{i<j} y_{ij}}, \tag{23}$$

due to Huygen's theorem [53]. Note that all the clustering criteria are expressed in terms of variables y only.

5.3. Visual attractiveness

Tables 1 to 4 present the results of NSGA-II on optimizing our bi-objective VRP model with each of the clustering criteria presented in section 5.2. Each NSGA-II run is halted after 400 generations regardless of the criterion used. Our limited computational experiments demonstrated that more generations were not useful in obtaining different Pareto frontiers for the tested instances. The tables report for each visualization metric average improvements yielded by the Pareto frontier solutions over the solutions of minimum traveling cost, which are excluded from the Pareto frontier for average computation. We have verified that our NSGA-II always included the minimum cost solutions in the obtained Pareto frontiers. Therefore, we report "-" whenever the solution of minimum cost is the only one of the frontier. Moreover, if the *cross* metric is already equal to zero in the solution of minimum cost, we report an $*$ which means that no improvement is possible in that case. The tables report average improvements categorized by group instance.

Table 1: Visualization metrics results for instances of group A

	$comp_r^1$			$comp_r^2$			$prox_r$			$cross$		
Instance	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC
A-n32-k5	+7.38%	+3.53%	+4.70%	+12.49%	+20.55%	+18.61%	+16.67%	+52.08%	+45.24%	0.00%	0.00%	0.00%
A-n33-k5	+1.09%	-4.57%	-1.19%	+0.54%	+7.20%	+7.65%	+38.89%	+40.74%	+35.56%	+16.67%	+33.33%	+60.00%
A-n33-k6	-0.52%	-3.19%	-3.46%	-7.03%	-1.40%	+7.20%	-75.00%	-37.50%	+37.50%	*	*	0.00%
A-n34-k5	+4.54%	+7.54%	-0.66%	+8.14%	+14.29%	+13.50%	-11.11%	+50.00%	+28.79%	*	0.00%	*
A-n36-k5	+6.92%	+2.88%	+0.56%	-5.43%	+8.46%	+6.32%	-66.67%	+35.00%	+37.50%	-66.67%	+80.00%	0.00%
A-n37-k5	-5.09%	-6.33%	-5.87%	+13.94%	+22.54%	+16.19%	+54.55%	+75.00%	+65.66%	+100.00%	+100.00%	+66.67%
A-n37-k6	-0.18%	+4.82%	-2.92%	+1.16%	+13.04%	+12.49%	-20.00%	+27.50%	+33.33%	*	+75.00%	*
A-n38-k5	-2.35%	-1.82%	+0.88%	-10.08%	+0.10%	+2.36%	-20.00%	+6.67%	+16.00%	*	*	*
A-n39-k5	-4.38%	-	+24.09%	+6.48%	-	+6.02%	+35.71%	-	0.00%	0.00%	-	0.00%
A-n39-k6	-11.10%	-10.63%	-6.79%	-4.04%	+13.00%	+16.17%	-23.81%	+44.90%	+33.33%	+66.67%	+100.00%	+100.00%
A-n44-k6	+0.97%	-	+0.96%	-43.87%	-	+2.20%	-233.33%	-	0.00%	-700.00%	-	+50.00%
A-n45-k6	-	-	-	-	-	-	-	-	-	-	-	-
A-n45-k7	+22.55%	+16.61%	+20.34%	+20.01%	+24.06%	+20.00%	+33.33%	+46.43%	+33.93%	+66.67%	+83.33%	+78.12%
A-n46-k7	+9.82%	+10.99%	+7.68%	+15.90%	+18.65%	+17.56%	+20.83%	+38.89%	+40.00%	+50.00%	+83.33%	+60.00%
A-n48-k7	-0.95%	+4.80%	+0.50%	+3.22%	+9.03%	+12.27%	+18.00%	+20.00%	+65.00%	*	0.00%	0.00%
A-n53-k7	+13.50%	+15.76%	+15.16%	+4.38%	+5.89%	+5.11%	-10.00%	-10.00%	-17.50%	+50.00%	+25.00%	+25.00%
A-n54-k7	+8.95%	+4.84%	+6.75%	+2.63%	+0.24%	+6.47%	-1.82%	0.00%	-7.58%	0.00%	0.00%	+66.67%
A-n55-k9	+18.77%	+11.00%	+14.78%	-5.82%	+12.36%	+12.41%	-77.78%	+11.11%	+16.05%	-200.00%	+66.67%	+88.89%
A-n60-k9	-2.38%	-	+0.37%	+0.92%	-	+6.03%	+14.29%	-	+42.86%	0.00%	-	-33.33%
A-n61-k9	+4.18%	+2.33%	-3.15%	-1.05%	-1.32%	+0.26%	+18.75%	+9.37%	+6.25%	0.00%	0.00%	0.00%
A-n62-k8	-9.40%	-10.71%	-5.48%	+0.99%	+13.31%	+18.79%	-6.25%	+18.75%	+44.32%	-266.67%	+100.00%	-127.27%
A-n63-k10	-9.53%	-10.70%	+6.38%	-4.72%	+4.37%	+12.64%	-8.93%	+14.29%	+52.38%	-8.33%	+73.33%	+72.22%
A-n63-k9	-	+6.98%	-	-	-0.81%	-	-	+9.09%	-	-	-33.33%	-
A-n64-k9	-2.24%	+9.90%	+11.71%	-0.21%	+11.66%	+15.13%	-17.71%	+28.12%	+31.25%	-33.33%	+45.83%	+66.67%
A-n65-k9	+11.34%	+13.96%	-	+0.52%	+1.02%	-	-8.33%	-6.25%	-	-133.33%	-50.00%	-
A-n69-k9	-	-5.99%	-1.30%	-	+1.77%	+3.41%	-	-5.56%	+13.89%	-	0.00%	*
A-n80-k10	-3.17%	-3.28%	+14.29%	+0.81%	+8.92%	+11.97%	+1.39%	+22.22%	+45.14%	-33.33%	0.00%	+70.83%
AVG	+2,45%	+2,55%	+4,10%	+0,41%	+9,00%	+10,45%	-13,68%	+21,34%	+29,12%	-57,46%	+37,26%	+32,22%

Table 2: Visualization metric results for instances of group B

	$comp_r^1$			$comp_r^2$			$prox_r$			$cross$		
Instance	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC
B-n31-k5	+9.39%	-	+26.47%	-7.62%	-	+6.67%	-66.67%	-	+33.33%	0.00%	-	*
B-n34-k5	-5.43%	-0.87%	+2.55%	-23.33%	+6.83%	+5.81%	-17.86%	+21.43%	+15.87%	*	*	*
B-n35-k5	-2.16%	+0.51%	+1.13%	+22.47%	+38.99%	+43.86%	-33.33%	+50.00%	+40.00%	0.00%	0.00%	0.00%
B-n38-k6	+11.10%	+14.29%	+11.74%	+16.36%	+27.34%	+26.15%	+25.00%	+41.67%	+38.97%	*	0.00%	0.00%
B-n39-k5	+19.48%	+12.24%	+8.97%	+8.24%	+8.63%	+6.52%	-5.56%	+23.33%	+20.83%	-100.00%	0.00%	+25.00%
B-n41-k6	+2.96%	-	+2.44%	-10.29%	-	+3.11%	-33.33%	-	-62.50%	*	-	*
B-n43-k6	-3.63%	-5.96%	-4.00%	+7.48%	+21.02%	+31.68%	+23.47%	+25.71%	+55.36%	-57.14%	-80.00%	+50.00%
B-n44-k7	+18.29%	+21.78%	+11.09%	+27.08%	+31.88%	+33.62%	+20.63%	+33.33%	+44.44%	+46.43%	+85.00%	+83.33%
B-n45-k5	+17.27%	+4.33%	+0.34%	-18.58%	-1.39%	+1.55%	-100.00%	+14.29%	+7.14%	-500.00%	0.00%	-50.00%
B-n45-k6	-	-	-	-	-	-	-	-	-	-	-	-
B-n50-k7	-	+5.02%	+12.08%	-	-3.66%	+10.46%	-	+8.33%	+27.50%	-	-266.67%	+60.00%
B-n50-k8	-7.74%	-15.43%	-10.06%	-2.84%	+5.18%	+19.22%	-5.00%	+16.67%	+38.97%	+5.00%	-40.00%	+66.15%
B-n51-k7	-	-	-	-	-	-	-	-	-	-	-	-
B-n52-k7	+2.60%	+2.97%	+6.81%	+3.77%	+24.53%	+20.99%	-20.00%	+25.00%	+23.33%	+33.33%	+50.00%	-166.67%
B-n56-k7	+6.49%	+20.43%	+25.48%	-9.81%	+19.26%	+13.34%	-47.62%	+3.57%	-28.57%	*	0.00%	*
B-n57-k7	-	-	-	-	-	-	-	-	-	-	-	-
B-n57-k9	+0.39%	+16.56%	+13.74%	-17.43%	+20.71%	+18.79%	-45.00%	+36.67%	+18.75%	-158.33%	+22.22%	+8.33%
B-n63-k10	+10.02%	+7.41%	+14.39%	-0.39%	-4.49%	+7.39%	-44.44%	-8.33%	0.00%	-100.00%	+50.00%	+50.00%
B-n64-k9	-2.28%	-	-3.90%	-7.74%	-	-2.78%	0.00%	-	-13.33%	-50.00%	-	-350.00%
B-n66-k9	+2.59%	-12.43%	-3.61%	-78.58%	+10.58%	-6.73%	-107.14%	+21.43%	+7.14%	-220.00%	+53.33%	+66.67%
B-n67-k10	+1.23%	-0.14%	+1.70%	-7.84%	+4.79%	+3.33%	-8.33%	0.00%	-15.15%	0.00%	+33.33%	+51.52%
B-n68-k9	+1.26%	+1.00%	+19.45%	+10.63%	+19.57%	+20.29%	-11.67%	+13.33%	-5.33%	-25.00%	+20.00%	+16.00%
B-n78-k10	-1.79%	+2.63%	+3.98%	+0.89%	+15.05%	+13.27%	+15.28%	+24.17%	+22.62%	-50.00%	+13.33%	0.00%
AVG	+4,21%	+4,37%	+7,04%	-4,61%	+14,40%	+13,83%	-24,29%	+20,62%	+13,47%	-78,38%	-3,72%	-5,60%

Table 3: Visualization metrics results for instances of group E

	$comp_r^1$			$comp_r^2$			$prox_r$			$cross$		
Instance	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC
E-n22-k4	-1.99%	-4.15%	-4.15%	-2.15%	+6.45%	+6.45%	0.00%	+50.00%	+50.00%	0.00%	0.00%	0.00%
E-n23-k3	+1.43%	+0.64%	-1.68%	+5.85%	+5.40%	+5.09%	+12.50%	+4.17%	-6.25%	*	*	*
E-n30-k3	+2.57%	-	+11.69%	-11.89%	-	-3.48%	-350.00%	-	-150.00%	-100.00%	-	+50.00%
E-n33-k4	+1.43%	-4.41%	-5.02%	+20.09%	+20.37%	+23.02%	+27.78%	+44.44%	+64.81%	0.00%	+16.67%	+83.33%
E-n51-k5	+1.49%	+1.73%	+1.73%	-0.78%	-1.22%	-1.22%	+8.33%	-33.33%	-33.33%	*	0.00%	0.00%
E-n76-k10	-	-	-	-	-	-	-	-	-	-	-	-
E-n76-k14	-	+9.38%	-	-	-0.91%	-	-	0.00%	-	-	0.00%	-
E-n76-k7	-	-5.38%	-6.04%	-	+1.54%	+0.94%	-	+5.45%	-1.52%	-	*	*
E-n76-k8	+14.82%	0.00%	-1.70%	+13.54%	-0.14%	+2.22%	+31.25%	-11.11%	+16.67%	+40.00%	-100.00%	0.00%
E-n101-k14	-	-	-	-	-	-	-	-	-	-	-	-
E-n101-k8	-24.64%	-16.25%	-7.68%	-4.08%	+2.19%	+5.54%	-24.36%	+34.72%	+31.73%	-100.00%	+8.33%	-87.50%
AVG	-0,70%	-2,31%	-1,61%	+2,94%	+4,21%	+4,82%	-42,07%	+11,79%	-3,49%	-32,00%	-12,50%	+7,64%

Table 4: Visualization metrics results for instances of group P

	$comp_r^1$			$comp_r^2$			$prox_r$			$cross$		
Instance	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC
P-n16-k8	+34.70%	+4.48%	-	-9.09%	-6.06%	-	-200.00%	-100.00%	-	*	0.00%	-
P-n19-k2	-	-18.97%	+3.82%	-	+2.70%	+0.23%	-	0.00%	+50.00%	-	+100.00%	0.00%
P-n20-k2	-11.90%	-10.53%	-1.80%	+3.28%	+0.61%	+9.02%	0.00%	+33.33%	+66.67%	0.00%	+50.00%	+100.00%
P-n21-k2	-	-	-	-	-	-	*	*	*	-	-	-
P-n22-k2	-	-	-	-	-	-	-	-	-	-	-	-
P-n22-k8	-10.11%	-10.17%	-	-5.88%	-1.13%	-	-12.50%	-6.25%	-	+25.00%	+12.50%	-
P-n23-k8	-	-	-	-	-	-	-	-	-	-	-	-
P-n40-k5	-9.90%	-5.06%	-1.81%	+3.40%	+2.12%	+2.61%	+25.00%	+25.00%	+25.00%	0.00%	0.00%	0.00%
P-n45-k5	-	+4.48%	-	-	+2.51%	-	-	+41.67%	-	-	0.00%	-
P-n50-k7	+14.52%	+24.35%	+34.33%	+5.47%	+15.51%	+14.38%	-22.50%	+50.00%	+20.00%	*	0.00%	0.00%
P-n50-k10	+1.80%	+4.58%	+11.45%	-31.65%	-4.56%	+2.00%	-58.33%	-16.67%	-0.00%	-66.67%	0.00%	+33.33%
P-n50-k8	-	-	-	-	-	-	-	-	-	-	-	-
P-n51-k10	-	+12.45%	+12.45%	-	+1.43%	+1.43%	-	0.00%	0.00%	-	+50.00%	+50.00%
P-n55-k10	+3.97%	+18.21%	+13.72%	+1.21%	+7.63%	+10.14%	-29.63%	+16.67%	+40.74%	+33.33%	+100.00%	+100.00%
P-n55-k15	-18.26%	-	-	-57.97%	-	-	-70.59%	-	-	-162.50%	-	-
P-n55-k8	+15.53%	+10.39%	+0.61%	+6.86%	+6.86%	+17.25%	-10.00%	+20.00%	+50.00%	*	0.00%	0.00%
P-n55-k7	-4.77%	-15.40%	-15.27%	+1.35%	+8.41%	+7.85%	+18.18%	+43.18%	+54.55%	0.00%	0.00%	0.00%
P-n60-k10	-1.83%	+13.21%	+13.91%	+3.01%	+2.21%	+3.54%	+15.56%	+13.33%	+13.33%	-100.00%	-25.00%	-33.33%
P-n60-k15	-15.20%	-	-	-5.94%	-	-	-31.25%	-	-	*	-	-
P-n65-k10	-8.71%	+1.08%	-2.18%	+0.36%	+0.14%	+3.98%	+23.08%	0.00%	+26.15%	*	*	0.00%
P-n70-k10	-17.60%	-	-	-16.67%	-	-	-77.78%	-	-	-500.00%	-	-
P-n76-k4	+30.09%	-	+15.56%	+0.56%	-	+4.84%	+7.14%	-	+16.07%	0.00%	-	0.00%
P-n76-k5	-1.58%	-	-	+0.44%	-	-	+8.33%	-	-	*	-	-
P-n101-k4	+4.66%	-9.25%	+14.06%	+20.59%	+22.51%	+25.95%	+34.78%	+60.25%	+73.91%	*	*	0.00%
AVG	+0,32%	+1,59%	+7,60%	-4,74%	+4,06%	+7,94%	-22,38%	+12,03%	+33,57%	-77,08%	+22,12%	+19,23%

We remark from Tables 1 to 4 that:

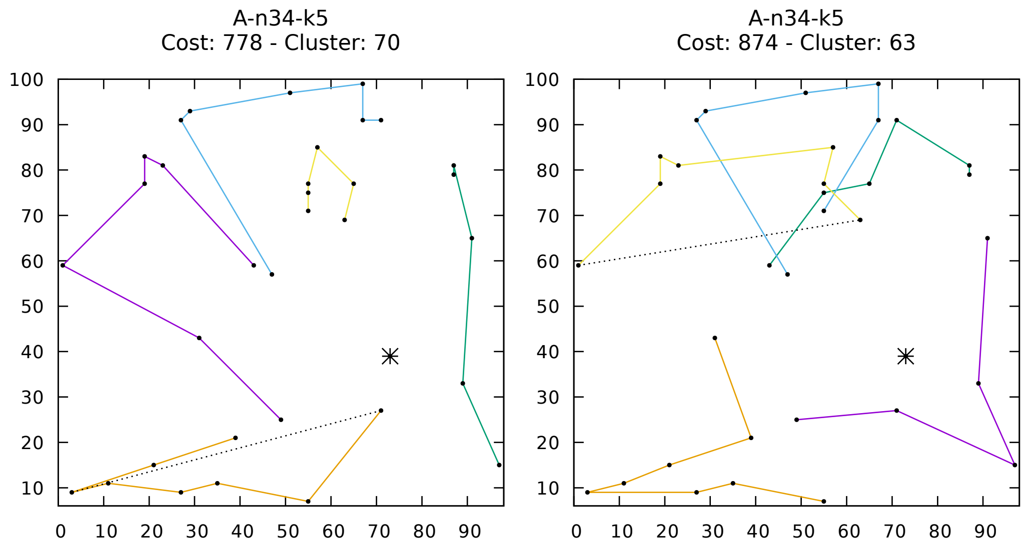
- For 10 out of the 85 VRP instances tested, NSGA-II was not able to find a Pareto front solution containing other solution than the one that minimizes cost, and that regardless of the clustering criterion used. This means that for these instances it is not possible to improve the visual metrics of VRP solutions by adding a second clustering optimization objective.

- By using MSC and MSSC our bi-objective model is very often able to improve the visual attractiveness metrics of the minimum cost solution. In average, the visual attractiveness metrics were improved in all groups of routes by the use of the MSC and MSSC clustering objective, except for metric $cross$ in group B instances and $comp_r^1$ for group E instances. The average improvements reach up to 7.60% for $comp_r^1$ in instances of group P, 14.40% for $comp_r^2$ in instances of group B, 33.57% for $prox_r$ in instances of group P, and 37.26% in instances of group A.

- MSSC seems to be the most effective clustering criterion for improving the visual attractiveness of VRP solutions. The obtained Pareto frontier solutions improved the $comp_1^r$ metrics in approx. 64.6% of the instances, the $comp_2^r$ in approx. 93.8%, the $prox_r$ in approx. 75.4%, and $cross$ in 50.9% of the cases.

- The DMin clustering criterion appears to be the least successful for improving the visualization metrics on average. Figure 2 presents a pair of Pareto solutions obtained by NSGA-II for instance A-n33-k4 using DMin as clustering objective. The solution in Figure 2(a) corresponds to the minimum cost solution. The reader can observe that the VRP solution obtained with DMin minimization is more compact in terms of the maximum distance between two customers in the same route. However, a drawback of the DMin criterion is that it might produce routes in which customers from different routes are close to each other, a phenomenon known as the *dissection effect* in the clustering literature (see e.g. [54]). This is due to the fact that the DMin criterion is seldom affected by the grouping of two close customers. Consequently, it is indifferent to the DMin criterion if they are grouped together or not in the optimal Dmin solution. This may lead to several inter-route crossings as observed in the Pareto solution illustrated in Figure 2(b).

Figure 2: A pair of Pareto solutions for instance A-n34-k5 for DMin clustering criterion. The maximum distance found among two customers in the same route are indicated by dotted lines in the solutions.



(a) Solution of minimum traveling cost

(b) A Pareto Solution

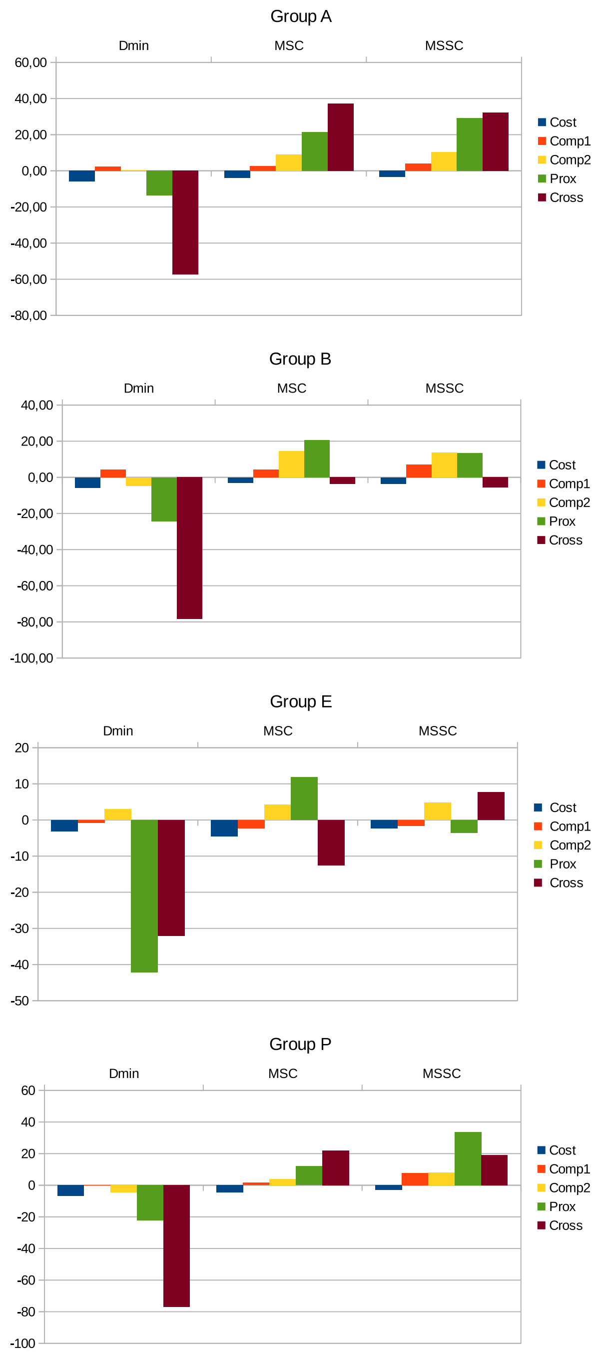
- The *cross* metric is particularly difficult to be improved for intances of the group E and P. When inspecting these instances, one often finds clusters that are clearly defined, which in turns makes the minimum-cost solutions naturally well clustered.

5.4. Traveling costs

We next check the effect of the quest for better visual attractiveness metrics values in the solution routing costs. Figure 3 presents the average increments in the routing costs of the Pareto solutions with respect to the optimal VRP solution. Besides, we show the average gains (or average deterioration) regarding the visualization metric values also with respect to the optimal VRP. The bar graphs in the figure are separated by clustering criterion and VRP group instance.

We can observe from the plots that, except for DMin, the average visualization gains yielded by the clustering objectives are almost always superior to losses in the routing costs. This is indeed a limited conclusion which considers routing costs and visualization attractiveness as equally important,

Figure 3: Average deviations of the routing costs and visualization metrics with respect to the optimal VRP solution

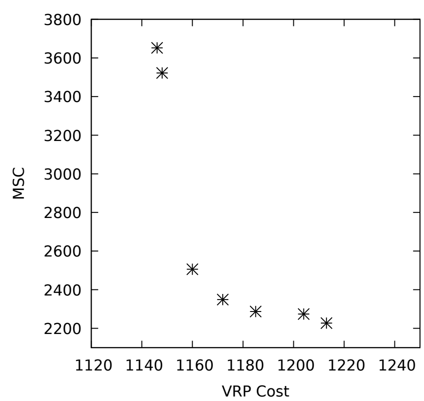


which is not the case for a vast amount of VRP applications. Yet, it is interesting to remark that improving visualization metrics, particularly with MSC and MSSC clustering, does not imply large increases of routing costs – they never exceeded 4% in average for the tested group instances.

5.5. Effectiveness results

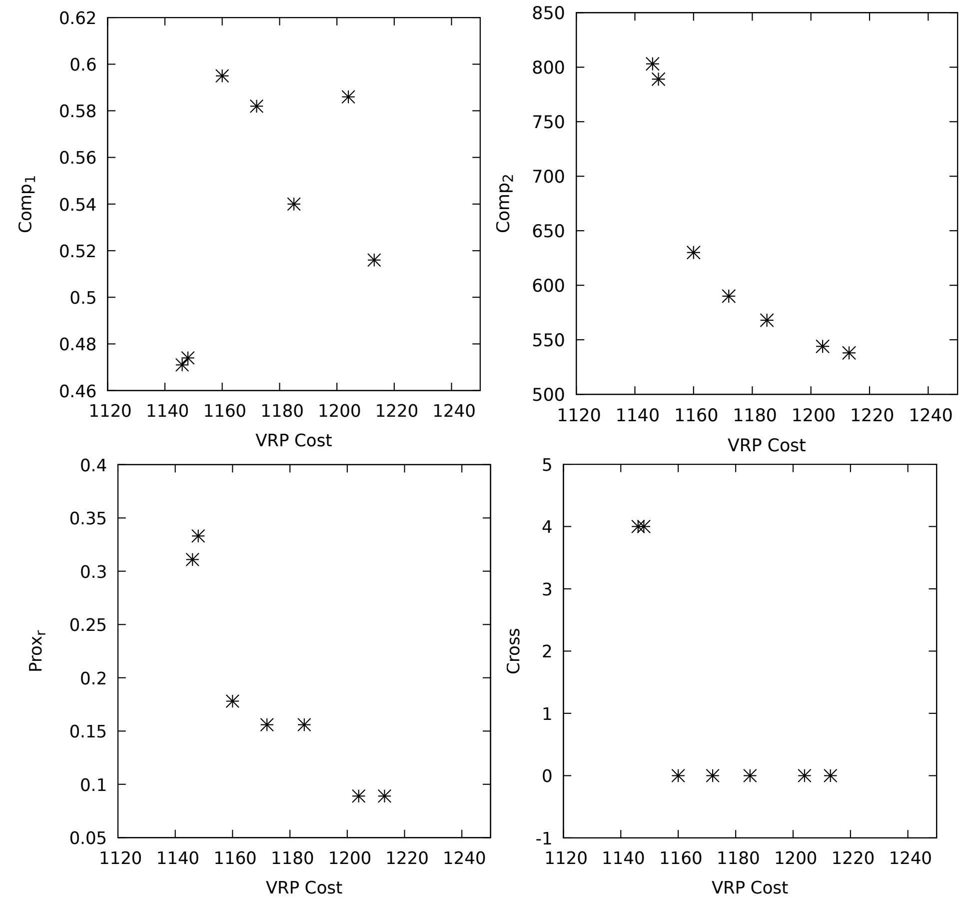
In order to assess in an integrated way the effectiveness on improving the visualization attractiveness of routes by using a clustering objective into VRP models, we analyze the hyper-volumes (see e.g. [55, 56] for details about hyper-volume computation) of the Pareto solutions obtained by the NSGA-II heuristic with each clustering criterion. For example, Figure 4 illustrates the Pareto frontier obtained for instance A-n45-k7 using the MSC clustering criterion.

Figure 4: Pareto frontier for instance A-n45-k7 obtained by the NSGA-II algorithm using MSC as clustering criterion.



As we aim to assess the effectiveness of the obtained Pareto frontiers regarding their visual attractiveness, we changed the original Pareto front space, that is, the two-dimensional objective function space composed by (5) and (6) as shown in Figure 4, to that of (5) and the visualization metric under consideration. Figure 5 shows the same Pareto front solutions plotted in Figure 4, now translated to the spaces of the VRP routing costs and each of the visualization metrics: $comp_1$, $comp_2$, $prox_r$ and $cross$.

Figure 5: Pareto frontier solutions of Figure 4 in the objective function space composed by the VRP costs and each of the visualization metrics.



Hypervolumes are computed from these transformed spaces and compared regarding each visualization metric, and taking into consideration the solutions obtained by NSGA-II using each one of the tested clustering criteria. Hypervolumes are computed with respect to reference points that correspond to the worst obtained routing cost and visualization metric value found across the analysed solutions. Figure 6 illustrates, for instance A-n45-k7 and $comp_2$, the hyper-volumes of the projected solutions obtained by NSGA-II considering DMin, MSC and MSSC. The reference points are the upper rightmost points exhibited in the plots. We note from the figure that MSC is the criterion that yields the largest hyper-volume among the compared models, which means that it is the most effective clustering criterion for instance A-n45-k7 regarding $comp_2$.

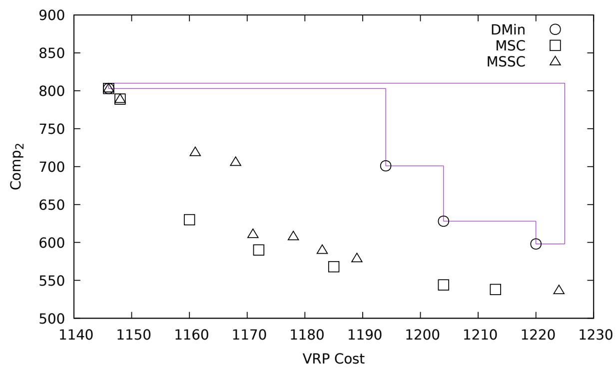
Tables (5)-(8) present the computed hyper-volumes regarding each clustering criterion. Note that we have omitted from our analysis the instances for which the sole Pareto front solution found by the NSGA-II heuristic using any of the three clustering criteria corresponds to the minimum-cost solution. We observe in the tables that the VRP model with the DMin criterion is almost always surpassed or equated by model with the MSC and MSSC criteria. By specifically contrasting the last two, we notice that MSSC is more effective for the $prox_r$ and $cross$ metrics, and largely better regarding the $comp_2$ metric. Regarding the $comp_1$ metric, MSSC and MSC present similar performance – MSC is superior for 10 instances while MSSC is superior for 13. The Pareto frontiers obtained by NSGA-II with these two criteria have equal hypervolumes regarding cross for other 38 instances.

Table 5: Hypervolume for instances of group A

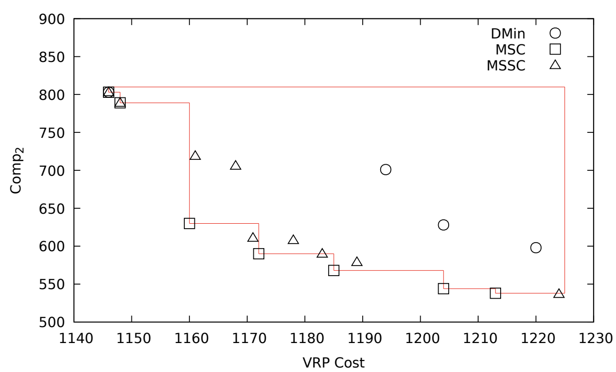
	DMin				MSC				MSSC			
Instance	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$
A-n32-k5	83	3114	79	**77**	83	7291	83	**77**	**84**	**9084**	**84**	**77**
A-n33-k5	**72**	3014	73	**187**	71	4645	**75**	182	**72**	**4960**	**75**	186
A-n33-k6	**72**	4760	83	**280**	**72**	5240	83	**280**	**72**	**6763**	**86**	**280**
A-n34-k5	112	11558	113	**582**	109	10910	116	**582**	**116**	**14063**	**118**	**582**
A-n36-k5	103	7038	112	**484**	**104**	**11067**	**119**	475	**104**	10791	**119**	392
A-n37-k5	103	7262	112	156	104	11149	117	166	**108**	**11908**	**121**	**194**
A-n37-k6	68	2114	69	**260**	**70**	5148	**75**	244	**70**	**7175**	74	**260**
A-n38-k5	**58**	7524	61	**228**	**58**	**8649**	64	**228**	**58**	8623	**65**	**228**
A-n39-k5	**77**	**1936**	**68**	**65**	75	65	65	**65**	75	105	65	**65**
A-n39-k6	159	6384	164	231	**161**	20914	**176**	**290**	159	**21889**	173	289
A-n44-k6	190	54900	237	1440	190	54900	237	1440	**196**	**59560**	**241**	**1581**
A-n45-k7	**96**	4924	84	162	**96**	**15068**	**92**	**339**	**96**	12843	90	318
A-n46-k7	**81**	**5818**	**75**	126	**81**	4523	**75**	**143**	**81**	5718	**75**	123
A-n48-k7	91	3061	90	**258**	91	4597	90	**258**	**92**	**5981**	**93**	**258**
A-n53-k7	**42**	545	**40**	**51**	**42**	530	**40**	49	**42**	**606**	**40**	**51**
A-n54-k7	**104**	1697	**102**	241	103	276	101	188	103	**2471**	101	**278**
A-n55-k9	**117**	8346	123	535	**117**	18011	126	635	**117**	**19088**	**129**	**637**
A-n60-k9	68	358	65	192	67	64	64	192	**69**	**1899**	**69**	**228**
A-n61-k9	32	713	31	**31**	**33**	**753**	**32**	**31**	**33**	**753**	31	**31**
A-n62-k8	112	1840	110	416	111	3146	110	**430**	**114**	**13636**	**117**	416
A-n63-k10	120	12320	119	365	**126**	18623	124	**616**	118	**20672**	**130**	567
A-n63-k9	**30**	232	**29**	**58**	**30**	232	**29**	**58**	**30**	**464**	**29**	**58**
A-n64-k9	197	11562	187	985	**199**	29369	**201**	**1262**	194	**32744**	199	1239
A-n65-k9	**56**	**870**	**52**	**150**	55	821	**52**	**150**	55	800	**52**	**150**
A-n69-k9	38	37	**38**	**74**	39	347	**38**	**74**	**40**	**855**	**38**	**74**
A-n80-k10	159	7498	144	654	**162**	12075	147	674	155	**21991**	**158**	**863**

Finally, Figure 7 presents a smoothed histogram for the number of times a Pareto frontier with a given number of Pareto solutions was obtained by

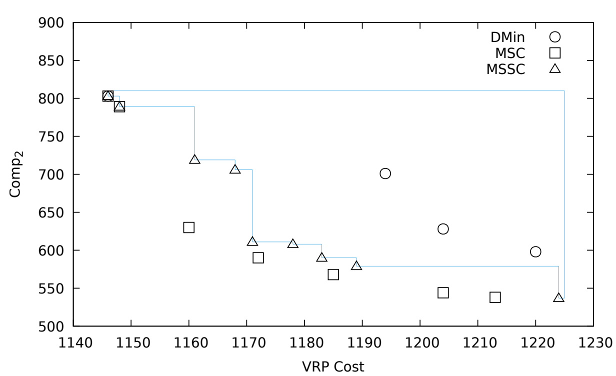
Figure 6: Hypervolumes of NSGA-II solutions for instance A-n45-k7 regarding each tested clustering criteria with respect to $comp_2$ metric.



(a) Hypervolume of NSGA-II solutions using DMin as clustering criterion regarding $comp_2$.



(b) Hypervolume of NSGA-II solutions using MSC as clustering criterion regarding $comp_2$.



(c) Hypervolume of NSGA-II solutions using MSSC as clustering criterion regarding $comp_2$.

Table 6: Hypervolume for instances of group B

	DMin				MSC				MSSC			
Instance	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$
B-n31-k5	**13**	108	**13**	**36**	**13**	108	13	**36**	**13**	**115**	**13**	**36**
B-n34-k5	**133**	21168	137	**1638**	131	**27281**	**150**	**1638**	130	25408	147	**1638**
B-n35-k5	**44**	2447	47	**43**	**44**	**13120**	**51**	**43**	**44**	12896	50	**43**
B-n38-k6	**38**	2733	38	**72**	**38**	3491	**39**	**72**	**38**	**3705**	**39**	**72**
B-n39-k5	**55**	1425	52	196	**55**	1821	**54**	229	**55**	**1826**	**54**	**234**
B-n41-k6	**30**	1624	**31**	**87**	**30**	1624	**31**	**87**	**30**	**2274**	**31**	**87**
B-n43-k6	**80**	3267	78	**257**	**80**	8754	79	219	**79**	**10767**	**83**	**244**
B-n44-k7	**86**	7352	81	264	**86**	8527	82	**317**	**86**	**9147**	**83**	**315**
B-n45-k5	**79**	8954	86	**444**	**79**	8954	**88**	**444**	**79**	**9830**	**87**	**444**
B-n50-k7	**37**	805	36	245	**37**	805	37	245	**37**	**2599**	**39**	**274**
B-n50-k8	**67**	6414	73	236	**67**	6704	72	195	**67**	**10770**	**77**	**357**
B-n52-k7	**50**	2238	54	226	**50**	**5084**	**56**	**238**	**50**	4145	**56**	**196**
B-n56-k7	**49**	1936	47	**264**	**49**	**3973**	**48**	**264**	**49**	3898	47	**264**
B-n57-k9	**110**	21293	114	891	109	28767	118	999	109	**31017**	**120**	**1065**
B-n63-k10	**169**	6561	177	1590	**169**	4452	177	1658	**169**	**8494**	**178**	**1762**
B-n64-k9	**115**	**4560**	**118**	**912**	114	**4560**	**118**	**912**	114	**4560**	**118**	**912**
B-n66-k9	242	122640	295	2880	**253**	**146698**	**309**	3581	245	122640	303	**3636**
B-n67-k10	**120**	5198	125	248	119	7714	125	360	119	**7963**	**126**	**392**
B-n68-k9	**102**	6125	96	424	**102**	13702	**100**	484	101	**14313**	98	**536**
B-n78-k10	**121**	4337	118	560	**121**	**17501**	**123**	**815**	120	12741	119	644

Table 7: Hypervolume for instances of group E

	DMin				MSC				MSSC			
Instance	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$
E-n22-k4	**21**	147	21	**21**	**21**	**381**	**22**	**21**	**21**	**381**	**22**	**21**
E-n23-k3	98	7064	112	**376**	**102**	6862	**114**	**376**	100	**7819**	113	**376**
E-n30-k3	**21**	1121	**23**	38	**21**	1121	**23**	38	**21**	**1124**	**23**	**46**
E-n33-k4	35	2539	36	68	35	2031	36	81	**36**	**3733**	**39**	**94**
E-n51-k5	**7**	**70**	**7**	**14**	**7**	**70**	**7**	**14**	**7**	**70**	**7**	**14**
E-n76-k14	**38**	**252**	**36**	**36**	**38**	**252**	**36**	**36**	**38**	**252**	**36**	**36**
E-n76-k7	89	2604	92	**252**	91	3383	93	**252**	**96**	**4717**	**94**	**252**
E-n76-k8	2	135	2	4	40	10803	51	195	**60**	**17440**	**77**	**348**
E-n101-k14	1	1	1	1	**33**	**6699**	**40**	**132**	10	1370	11	40
E-n101-k8	71	6307	58	742	**147**	**33387**	**146**	**2014**	112	30110	116	1649

Table 8: Hypervolume for instances of group P

	DMin				MSC				MSSC			
Instance	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$	$comp_r^1$	$comp_r^2$	$prox_r$	$cross$
P-n16-k8	**9**	**49**	**8**	14	**9**	**49**	**8**	**14**	**9**	**49**	**8**	**14**
P-n19-k2	**25**	72	24	24	**25**	78	24	**25**	**25**	**126**	**26**	**24**
P-n20-k2	**26**	368	24	24	24	202	24	25	**24**	**698**	**26**	**47**
P-n22-k8	**34**	448	**33**	53	32	**514**	**33**	33	32	448	**33**	**64**
P-n40-k5	37	52	37	37	**39**	411	**38**	**37**	38	**447**	**38**	**37**
P-n45-k5	**8**	7	**7**	7	**8**	**79**	7	7	**8**	**7**	**7**	**7**
P-n50-k7	**66**	1428	56	212	**66**	3066	**59**	**212**	**66**	**3179**	**59**	**212**
P-n50-k10	151	17556	150	396	151	17556	150	396	**154**	**19088**	**154**	**553**
P-n51-k10	**4**	4	**4**	4	**4**	10	**4**	**5**	**4**	**10**	**4**	**5**
P-n55-k10	**44**	154	38	44	**44**	**210**	38	40	**44**	**966**	**39**	**54**
P-n55-k15	**227**	**54707**	**277**	**3178**	**227**	**54707**	**277**	**3178**	**227**	**54707**	**277**	**3178**
P-n55-k8	**92**	120	85	**166**	**92**	749	85	**166**	**92**	**3803**	**88**	**166**
P-n55-k7	59	530	57	**56**	62	1475	59	**56**	**63**	**1707**	**61**	**56**
P-n60-k10	**65**	472	58	**116**	**65**	1148	**60**	**116**	**65**	**1253**	**60**	**116**
P-n60-k15	**47**	**1288**	**49**	**92**	46	**1288**	**49**	**92**	**46**	**1288**	**49**	**92**
P-n65-k10	**36**	168	35	**102**	35	278	34	**102**	**36**	**1120**	**36**	**102**
P-n70-k10	**92**	**9292**	**101**	**552**	**92**	**9292**	**101**	**552**	**92**	**9292**	**101**	**552**
P-n76-k4	**33**	57	27	**27**	**33**	27	27	**27**	**33**	**965**	**28**	**27**
P-n76-k5	**10**	**14**	10	**20**	**10**	10	**10**	**20**	**10**	10	**10**	**20**
P-n101-k4	70	9598	58	**168**	**76**	16344	62	**168**	70	**19249**	**64**	**168**

NSGA-II across the 85 instances of groups A-B-E-P. The figure illustrates three smoothed curves, one for each clustering criterion used by the NSGA-II heuristic. We can observe the Pareto frontier obtained with the DMin criterion often contains less solutions than those obtained with the MSC and MSSC criterion. Yet, the later appears to be the clustering criterion yielding the most populated Pareto frontiers.

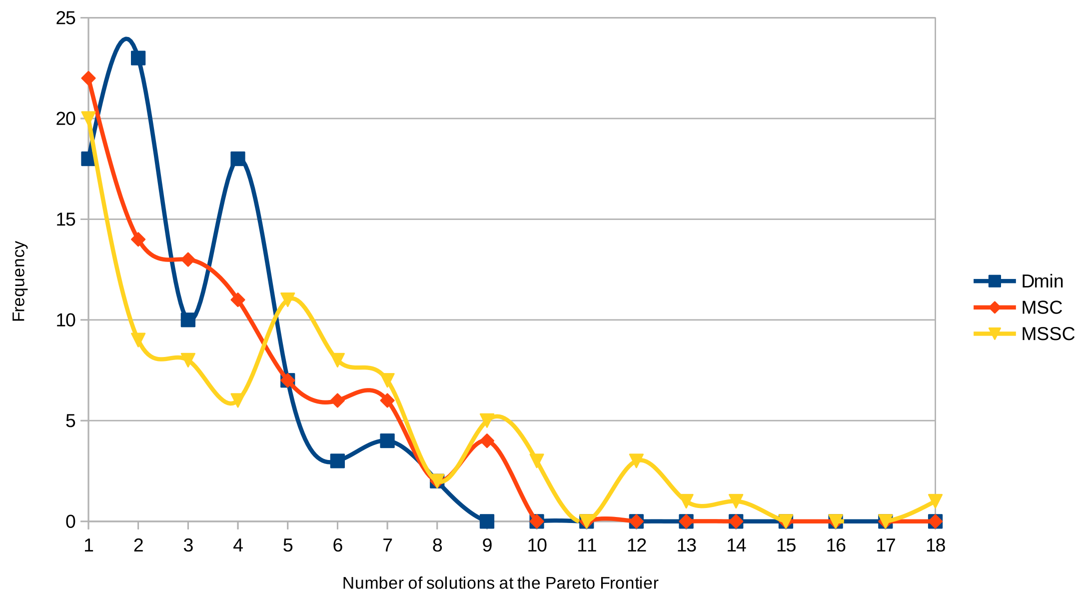
5.6. Experiments with a real-world network

In this section, we assess the visual attractiveness of VRP solutions obtained by NSGA-II on a real-world street network. In that case, the routing costs are no longer equivalent to the Euclidean distances between customers, as we considered for the A-B-E-P instances, but rather to the smallest travel time between customers in the network. Thus, we intend to evaluate our bi-objective proposed model in a more realistic testing scenario.

As underlying topology for our tests, we used the Washington D.C. network from the 9th DIMACS Implementation Challenge [1], which consists of a network with 9559 nodes and 14909 edges.

[1] available at `http://users.diag.uniroma1.it/challenge9/data/tiger/`

Figure 7: Frequency of the number of solutions at the Pareto frontier obtained by each of the clustering criteria



We have created three categories of instances from the D.C. road network, with 30, 50 and 75 randomly selected nodes that represent customer locations plus the depot. For each one of these quantities, we create three distinct instances with $K = 3, 4$ and 5 vehicles. We consider a unitary demand for the customers, and vehicle capacities of $\lceil \frac{n}{k} \rceil$ so that all the vehicles are used.

Table 9 presents the results of our NSGA-II on optimizing the proposed bi-objective VRP model with each one the clustering criteria: Dmin, MSC and MSSC. For this set of experiments, the evolutionary algorithm was halted after 800 generations. The illustration of all the obtained Pareto front solutions for this experiment can be also found at `https://github.com/diegorlima/CVRP-bi-objective`.

Table 9: Visualization metrics results for DC network instances

	$comp_r^1$			$comp_r^2$			$prox_r$			$cross$		
Instance	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC	Dmin	MSC	MSSC
DC-n30-k3	+4.45%	+0.98%	+28.79%	+10.36%	+7.34%	+2.61%	+20.00%	+50.00%	+53.33%	*	*	*
DC-n30-k4	+4.22%	+0.69%	+6.39%	+10.18%	+6.58%	+9.16%	+16.67%	+17.50%	+37.50%	*	*	*
DC-n30-k5	+5.74%	-6.51%	-3.10%	+1.27%	+8.52%	+8.56%	+25.00%	+40.00%	+43.75%	*	*	*
DC-n50-k3	-2.20%	+0.15%	-1.28%	-2.03%	+1.83%	+1.83%	-12.50%	+25.00%	+25.00%	*	*	*
DC-n50-k4	-3.53%	+4.16%	+7.20%	-6.86%	+18.75%	+2.37%	-44.44%	+40.00%	+34.72%	*	*	*
DC-n50-k5	+0.79%	+4.66%	-3.34%	+10.81%	+4.57%	+1.89%	+25.00%	+16.67%	+21.43%	+75.00%	*	*
DC-n75-k3	-	+1.19%	-9.40%	-	+2.08%	+1.92%	-	+22.22%	+50.00%	-	*	*
DC-n75-k4	-1.33%	+0.42%	-1.95%	-19.36%	+5.94%	+1.70%	-11.54%	+17.86%	+20.37%	-100.00%	+100.00%	*
DC-n75-k5	+0.68%	-0.72%	+2.86%	-13.32%	+4.10%	+6.46%	-16.67%	-42.86%	+27.47%	*	*	+85.71%
AVG	+1.1%	+1.44%	+3.66%	-1.12	+6.63%	+4.06%	-0.19%	+20.71%	+34.84%	-12.5%	+100.00%	+85.71%

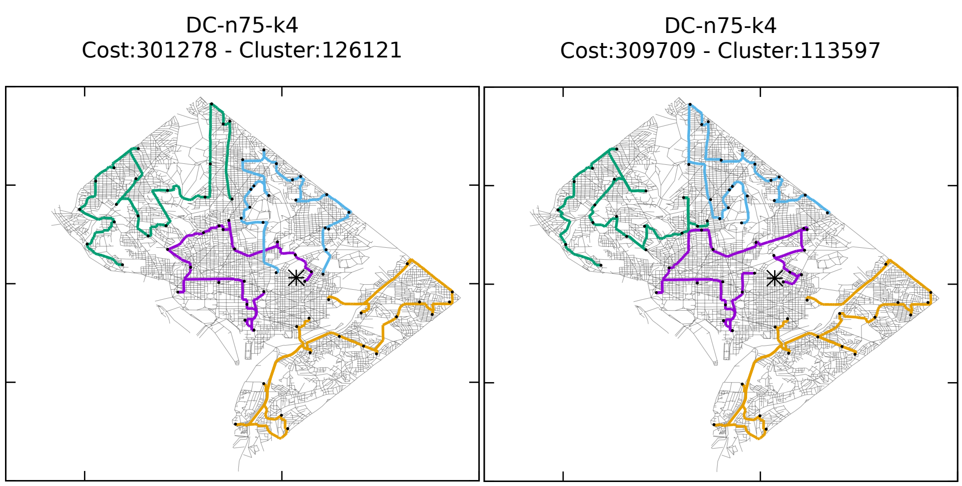
We remark from the table that:

- For all the tested instances, by using the MSC and MSSC criteria, the NSGA-II was always able to find a solution in the Pareto frontier other than the solution of minimum traveling cost. This result was somehow expected as the customers tend to be more spread in the same route by using travelling times instead of geographical distances (e.g. customers connected by a highway may be distant in the space albeit quickly reachable one from the other).

- By using the clustering criteria as second objective, our bi-objective model appears to very often improve the considered visual attractiveness metrics. In particular, the MSC and MSSC criteria were always able to improve those metrics in average.

  We illustrate in Figures 8 and 9 a pair of Pareto front solutions obtained by NSGA-II for instance DC-n75-k4 and instance DC-n75-k5 using the MSC and MSSC clustering objectives, respectively. The solutions in the left correspond to the solutions of minimum traveling cost.

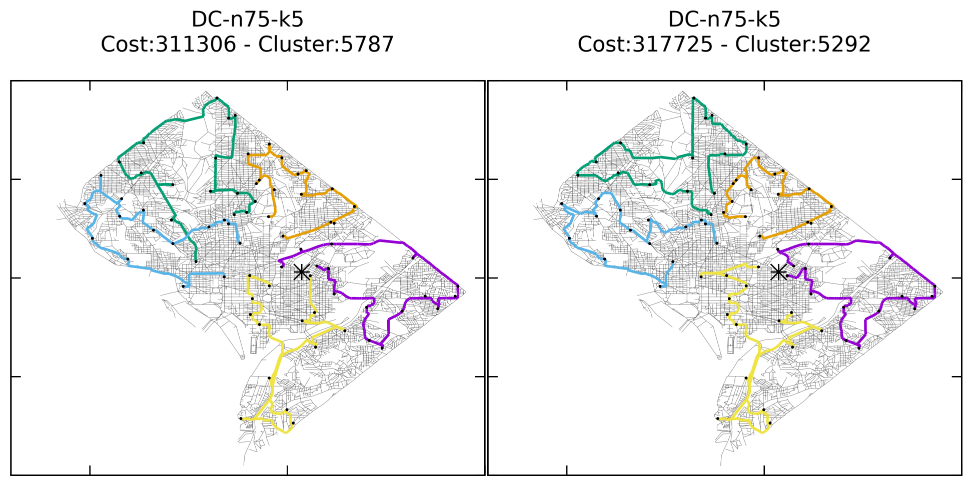
Figure 8: A pair of Pareto solutions for instance DC-n75-k4 for the MSC criterion



(a) Solution of minimum traveling cost (b) A Pareto solution

- Again MSSC seems to be the most suited clustering criterion for improving visualization metrics, while DMin again appears to not be the

Figure 9: A pair of Pareto solutions for instance DC-n75-k5 for the MSSC criterion



(a) Solution of minimum traveling cost

(b) A Pareto solution

least suited with approximately 50% of success rate. Average improvements by MSSC attained 34.84% for the $prox_r$ metric.

6. Concluding remarks

This article introduced a bi-objective vehicle routing problem with simultaneous minimization of traveling costs and clustering criteria, as a proxy to characterize routing solutions that are cost effective and visually attractive. We have introduced a compact two-index vehicle-flow model and a NSGA-II metaheuristic algorithm to approximate its Pareto frontier. By means of an extensive computational campaign, we assess the impact of three clustering criteria in producing visually attractive and cost-effective solutions: diameter minimization, min-sum of cliques, and minimum sum-of-squares. Our results suggest that the latter two clustering objectives are best to produce good-quality solutions according to the visual attractiveness metrics found in the literature while keeping the traveling costs low. Moreover, our metaheuristic is general and has the potential to be applied to other variants of vehicle routing problems. As an avenue of future research, we believe that extending this work to problems with time windows, or to location-routing problems would be worthy of investigation. Another potential avenue of research would be to investigate the use of other objectives to enforce the routes to be of low complexity, which as we explained, cannot be enforced by clustering objectives alone.

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