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## Combined networked switching output feedback control with $\mathcal{D}$ -region stability for performance improvement

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In this article, a combined networked switching output feedback control scheme, with a  $\mathcal{D}$ -region stability performance improvement module is presented. The network induced time delays, that are considered to be time varying and integer multiples of the sampling period, are being embedded in the system model, by state augmentation. The resulting model of the overall networked closed-loop system is switching, with the current measured round-trip time delay acting as the switching rule. Based on this modelling approach, a Linear Matrix Inequality (LMI) tuned switching output feedback controller is designed. The proposed approach establishes robustness against time delays and is able to guarantee the overall stability of the switching closed-loop system. Integrated in the controlled synthesis phase, an LMI tuned performance improvement module is being introduced, based on  $\mathcal{D}$ -region stability. Multiple simulation results are being presented that prove the efficacy of the proposed scheme.

**Keywords:** networked controlled systems; switching output feedback;  $\mathcal{D}$ -region stability; LMI

### 1. Introduction

Networked Controlled System (NCS) architectures are becoming dominant due to the recent developments in the communication capabilities and in the improvements of the networks' infrastructure (Chen, Cao, Cheng, Xiao, & Sun, 2010; Sipahi, Niculescu, Abdallah, Michiels, & Gu, 2011; Zhang, Branicky, & Phillips, 2001). These networks are being affected by various issues (Heemels, Teel, Woow, & Nes, 2011), stemming from the need to exchange information over a shared communication link (Halevi & Ray, 1988; Puccinelli & Haengi, 2005), while special effort should be paid to design proper control schemes, able to adjust their settings to account for possible peculiarities encountered in typical real-time control applications and mainly present robustness against the networked induced time delays, which deteriorate the overall performance of the closed-loop system and can even drive it to instability (Tzes, Nikolakopoulos, & Koutroulis, 2005; Yang, 2006).

In the relative literature, there have been multiple theoretical and experimental approaches, in the field of control design and stability analysis (Walsh, Ye, & Bushnell, 2002; Zhang et al., 2001). In most of the cases the adopted approaches have proposed static or switching control schemes that can take into account many issues of an NCS, such as: (a) varying time delays (Mahmoud, 2009; Shu, Lam, & Xiong, 2010), (b) data-packet losses (Sun & Qin, 2011; Yu, Wang, & Chu, 2005), (c) data-packet reordering (Leung,

Li, & Yang, 2007; Li, Zhang, & Cai, 2009), and (d) data-packet transmission scheduling (Kyung, Suck, & Hyung, 2005). Most of these approaches, based their main contribution in proposing control schemes that guarantee the overall stability of the closed-loop system, with respect to the previous (a)–(b) problems, without focusing in overall system's closed-loop performance and with the switching control schemes depicting superiority against the static ones.

The main novelty of this article stems from the combination of the control synthesis approach for switched networked controlled systems, with the principle of  $\mathcal{D}$ -region stability. More specifically a novel LMI-based  $\mathcal{D}$ -region stability criterion will be proposed, which will be able to guarantee at the same time both overall stability for the switched closed-loop system and significant performance improvement. Robust pole clustering has been widely discussed over the past few decades, see for example Arzelier, Bernussou, and Garcia (1993), Garcia, Daafouz, and Bernussou (1996), Garcia and Bernussou (1995), Chou, Ho, and Horng (1991), Bachelier, Peaucelle, and Arzelier (2002), Chen and Lin (2004), Lee, Park, Joo, and Lin (2012) and the references therein. The resulting dual scheme guarantees the overall stability of the closed-loop system, under arbitrary switching sequences, while providing significant improvement in the overall performance, such as faster response and settling times. More specifically, an output

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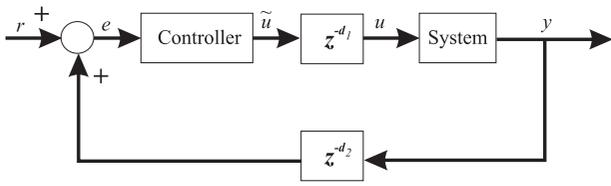


Figure 1. Model representation of a time delayed LTI system.

feedback scheme is proposed for each switching subsystem that assures the asymptotic stability of the system, while satisfying certain pole placement requirements. The proposed control scheme is tuned based on LMIs, and provides a flexible approach for improving the performance of the control design.

The rest of the article is structured as it follows. In Section 2, the system modelling with the state augmentation is presented. In Section 3, the switching controller synthesis scheme is being presented, followed by Section 4, where the tuning of the  $\mathfrak{D}$ -region stability performance improvement module is analysed. Finally, simulation results are being presented in Section 5, followed by the conclusions in Section 6.

**Notations:** Standard notations have been utilised throughout this article.  $\mathbf{L}^T$  and  $\mathbf{L}^{-1}$  are the transpose and the inverse of the matrix  $\mathbf{L}$ , respectively.  $\mathbf{I}$  denotes the  $n \times n$  identity matrix,  $\mathbf{L} > 0$  ( $\mathbf{L} < 0$ ) denotes that  $\mathbf{L}$  is positive definite (negative definite). Finally, let  $\mathfrak{R}^+$  and  $\mathbb{Z}^+$  denote, respectively, the non-negative real numbers and the finite set of integers  $\{1, \dots, Z\}$ .

## 2. System modelling

Consider the networked system setup depicted in Figure 1. The discrete time Linear Time Invariant (LTI) dynamics of the system model can be described as:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \quad (1)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k), \quad (2)$$

where  $\mathbf{x}(k) \in \mathfrak{R}^n$ ,  $\mathbf{u}(k) \in \mathfrak{R}^m$ ,  $\mathbf{y}(k) \in \mathfrak{R}^n$ , and  $k, n, m \in \mathbb{Z}^+$ .

In a networked controlled system, the feedback loop is being closed over a common communication channel, without any real-time characteristics. Due to the sharing of this communication channel among multiple users, various delays and of variable length can occur. These networked induced time delays, in a networked controlled system, can be categorised as delays from controller to actuator (feed forward delays) and from sensor to controller (feedback delays), while it should be noted that these delays almost always differ among them. Although the values of these delays are random and depend on the traffic, the packet losses, the collisions, etc. in the communication network,

it is straightforward to measure their values by performing a time stamping process. In this approach, both the remote controller and the process are time synchronised, while all the exchanged data packets (in both directions) are being time stamped, which means that the contained valuable information (control signal or sensory feedback) is being extended by appending the current time stamp. In the case of time stamped data packets and for extracting the elapsed time that the transmission lasted, a simple subtraction, upon reception of the data packet, between the current time and the appended time stamp, would provide a direct indication of the time delay. This procedure allows to have a complete and precise measurement of the real networked induced time delays, in both the feed forward and the feedback loops, while experimental verifications of this approach can be located in Tzes, Nikolakopoulos, and Koutroulis (2003) and Nikolakopoulos, Panousopoulou, and Tzes (2008).

Without a generality loss, it can be assumed that these random and real valued time delays are bounded, which means that assumptions, in the form of maximum bounds, on the network induced maximum communication delay (worst case scenario) can be posed a priori. For mathematically formulating this problem, the random and bounded time delays from the sensor to the controller are notated as  $d_1 \in \mathfrak{R}^+$  and from the controller to the sensor as  $d_2 \in \mathfrak{R}^+$ , as it has been also indicated in Figure 1. In a zero-latency environment ( $d_1 = d_2 = 0$ : immediate transmission conditions), the utilised controller corresponds to that of a static output feedback, or  $\tilde{\mathbf{u}}(k) = \mathbf{K}\mathbf{e}(k) = \mathbf{K}(\mathbf{r}(k) - \mathbf{y}(k))$ . However, owing to the networked environment, transmission delays should be considered: (a) to the applied control signal; feed forward (actuation) path delay  $\mathbf{u}(k) = \tilde{\mathbf{u}}(k - d_1)$ , and (b) to the feedback (sensory) path delay in the reverse transmission;  $\mathbf{e}(k) = \mathbf{r}(k) - \mathbf{y}(k - d_2)$ .

Let  $\tau(k) = d_1(k) + d_2(k)$ , with  $\tau \in \mathfrak{R}^+$ , be the time varying overall round trip delay, at sampled instant  $k$ , and  $\tau(k) \in [0, \bar{\lambda}]$ ,  $\bar{\lambda} \in \mathfrak{R}^+$ , with  $\bar{\lambda}$  the *a priori* known bound on the maximum time delay that the system can encounter. Moreover,  $r_s(k)$  is being introduced, which is a bounded integer sequence with  $0 \leq r_s(k) \leq D \leq \infty$ , and  $D \in \mathbb{Z}^+$ . The switching mode selection signal  $r_s(k)$  can be defined as a sequence of integer multiples or:

$$r_s(k) \in [0, 1, \dots, D], \quad (3)$$

with  $D$  to represent the maximum expected delay, measured as integer multiples of the adopted sampling time  $T_s \in \mathbb{Z}^+$ , which is equal to the number of discrete states that the switching signal  $r_s$  can take. The entries in Equation (3) are defined as it follows:

$$r_s(k) = \begin{cases} \lceil \frac{\tau(k)}{T_s} \rceil, & \forall \tau(k) > T_s \\ T_s, & \forall \tau(k) < T_s \end{cases} \quad (4)$$

where  $\lceil(\cdot)\rceil$  denotes the upper ceiling operation.

The mode-dependent switching  $K_{r_s}$  state feedback control law, at the sampling instant  $k$  can be defined as:

$$\mathbf{u}(k) = \mathbf{K}_{r_s} \mathbf{C} \mathbf{x}(k - \mathbf{r}_s(k)). \quad (5)$$

Based on the previous analysis, the state vector  $\mathbf{x}(k)$  is augmented to include all the delayed terms as follows:

$$\tilde{\mathbf{x}}(k) = [\mathbf{x}(k)^T, \mathbf{x}(k-1)^T, \dots, \mathbf{x}(k-D)^T]^T \quad (6)$$

with  $\tilde{\mathbf{x}} \in \mathfrak{R}^{D+1 \times D+1}$ , while the dynamics of the system in Equation (1) at sample time  $k$  are being transformed as:

$$\begin{aligned} \tilde{\mathbf{x}}(k+1) &= \tilde{\mathbf{A}} \tilde{\mathbf{x}}(k) + \tilde{\mathbf{B}} \mathbf{u}(k), \\ \mathbf{y}(k) &= \tilde{\mathbf{C}}_{r_s} \tilde{\mathbf{x}}(k), \end{aligned} \quad (7)$$

where the augmented version of the state space matrices defined as:

$$\begin{aligned} \tilde{\mathbf{A}} &= \begin{bmatrix} \mathbf{A} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{I} \end{bmatrix}, & \tilde{\mathbf{B}} &= \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}, \\ \tilde{\mathbf{C}}_{r_s} &= [\mathbf{0} \dots \mathbf{0} \mathbf{C} \mathbf{0} \dots \mathbf{0}], \end{aligned} \quad (8)$$

while  $\mathbf{C}$  corresponds to the matrix term that it is found on the  $(r_s + 1)^{\text{th}}$  column of the  $\tilde{\mathbf{C}}_{r_s}(k)$  and  $\tilde{\mathbf{A}} \in \mathfrak{R}^{(D+1) \times (D+1)}$ ,  $\tilde{\mathbf{B}} \in \mathfrak{R}^{(D+1) \times m}$ ,  $\tilde{\mathbf{C}}_{r_s} \in \mathfrak{R}^{n \times (D+1)}$ .

The resulting closed-loop system is switched, since the mode selection signal  $r_s(k)$  and thus the feedback term  $\mathbf{K}_{r_s} \mathbf{C}$  in Equation (5) is of a time varying nature (Xiao, Hassibi, & How, 2000). From Equations (5) and (8), this mode-dependent switching state feedback closed-loop discrete time system can be formulated as:

$$\begin{aligned} \tilde{\mathbf{x}}(k+1) &= (\tilde{\mathbf{A}} + \tilde{\mathbf{B}} \mathbf{K}_{r_s} \tilde{\mathbf{C}}_{r_s}) \tilde{\mathbf{x}}(k), \\ \mathbf{y}(k) &= \tilde{\mathbf{C}}_{r_s} \tilde{\mathbf{x}}(k), \end{aligned} \quad (9)$$

and the closed-loop matrix  $\tilde{\mathbf{A}} + \tilde{\mathbf{B}} \mathbf{K}_{r_s} \tilde{\mathbf{C}}_{r_s}$  can switch in any of the  $D + 1$   $i$ -vertices, or  $\mathbf{A}_i = \tilde{\mathbf{A}} + \tilde{\mathbf{B}} \mathbf{K}_i \tilde{\mathbf{C}}_i$ , and therefore conditions are sought for the stabilisation of the switched system:

$$\begin{aligned} \tilde{\mathbf{x}}(k+1) &= \mathbf{A}_i \tilde{\mathbf{x}}(k), \\ \text{with } i &: \mathbb{Z}^+ \rightarrow I = 0, 1, \dots, D, \end{aligned} \quad (10)$$

where the notation  $r_s(k) : \mathbb{Z}^+ \rightarrow I = 0, 1, \dots, D$  will be considered to be the switching function (varying time delay) in the following analysis. It should be noted that the equations in Equation (9) are in the form of an output

feedback control problem, even the fact that initially state feedback control in Equation (5) has been considered.

### 3. Switching controller synthesis

For the synthesis of the switching controller, the measurement of the round trip latency time  $r_s(k) = d_1(k) + d_2(k)$ , for calculating the index of the switched-state is a mandatory information and as it has been mentioned before this measurement can be directly obtained by time stamping in all the data packets from the controller to the plant and vice versa. This time stamping has no effect on the network's bandwidth as the control commands are being described by a few bytes and the time information can be easily added in the transmitted and received data packets.

In this section, the aim will be to design an overall controller that will be able to stabilise the networked controlled system described in Equation (10) under an arbitrary latency time  $r_s(k)$ . The system description in Equation (10) indicates that for a specific delay, the overall networked controlled system can be described by a specific augmented state-space representation. However, for assuring global stability, a proper control scheme should be designed that will be able to guarantee the stability of each of the switching systems (based on specific  $r_s(k)$ ), as well as all the arbitrary transitions from one mode of Equation (10) to another one. For such a demand, the overall system containing all the possible switching states of Equation (10), based on all the possible variations of the corresponding latency time should be considered. The overall set of the switching systems can be mathematically formulated as it follows:

$$\tilde{\mathbf{x}}(k+1) = \sum_{i=0}^D \xi_i(k) \mathbf{A}_i \tilde{\mathbf{x}}(k), \quad (11)$$

where

$$\xi(k) = [\xi_0(k), \dots, \xi_D(k)]^T \text{ and } \xi_i = \begin{cases} 1, & \text{mode} = \mathbf{A}_i \\ 0, & \text{mode} \neq \mathbf{A}_i \end{cases}. \quad (12)$$

Such a system representation indicates the set of all the possible  $D$  switchings that the network system can execute, while it should be noted that for a specific time delay the system description is being provided by Equation (10).

The problem of calculating a switching output feedback controller  $\mathbf{K}_{r_s}$  or with an equivalent simpler notation  $\mathbf{K}_i$  ( $r_s = i$ ), which will be able to guarantee the stability of the closed-loop switching system in Equation (11), under arbitrary and time varying switching, can be transformed into calculating  $\mathbf{P}_i > 0$  and  $\mathbf{K}_i$  matrices  $\forall i \in I$  such that

(Daafouz, Riedinger, & Iung, 2002):

$$\begin{bmatrix} \mathbf{P}_i & \mathbf{A}_i^T \mathbf{P}_j \\ \mathbf{P}_j \mathbf{A}_i & \mathbf{P}_j \end{bmatrix} > 0, \quad \forall (i, j) \in I \times I,$$

or finding  $r_s + 1$  symmetric positive definite matrices  $\mathbf{S}_i > 0$ ,  $\mathbf{G}_i > 0$ , and  $\mathbf{U}_i, \mathbf{V}_i$  matrices,  $\forall i \in I$ , that satisfy the following LMI:

$$\begin{bmatrix} \mathbf{G}_i + \mathbf{G}_i^T - \mathbf{S}_i & (\tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\tilde{\mathbf{C}}_{r_s})^T \\ \tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\tilde{\mathbf{C}}_{r_s} & \mathbf{S}_j \end{bmatrix} > 0, \quad \forall (i, j) \in I \times I, \tag{13}$$

and

$$\mathbf{V}_i \mathbf{C}_i = \mathbf{C}_i \mathbf{G}_i, \quad \forall i \in I. \tag{14}$$

Then the switching output feedback gains  $\mathbf{K}_{r_s}(k)$  can be calculated as:

$$\mathbf{K}_i = \mathbf{U}_i \mathbf{V}_i^{-1} \tag{15}$$

with  $\mathbf{V}_i$  provided by Equation (14) as:

$$\mathbf{V}_i = \mathbf{C}_i \mathbf{G}_i \mathbf{C}_i^T [\mathbf{C}_i \mathbf{C}_i^T]^{-1}, \tag{16}$$

where  $\mathbf{C}_i$  is of full row rank. Based on these  $\mathbf{S}_i > 0$  matrices, it is possible to calculate a positive Lyapunov function of the following form:

$$V(k, \tilde{\mathbf{x}}_k) = \tilde{\mathbf{x}}_k^T \left( \sum_{i=1}^{D+1} \xi_i(k) \mathbf{S}_i^{-1} \right) \tilde{\mathbf{x}}_k \tag{17}$$

whose difference:

$$\Delta V(k, \tilde{\mathbf{x}}_k) = V(k + 1, \tilde{\mathbf{x}}(k + 1)) - V(k, \tilde{\mathbf{x}}(k)) \tag{18}$$

decreased along all  $\tilde{\mathbf{x}}(k)$  solutions of the switched system in Equation (11), thus ensuring asymptotic stability of the system, under any arbitrary switching.

#### 4. $\mathcal{D}$ -Region pole placement for performance improvement

Let  $\mathcal{D}$  be a subregion in the discrete plane. A dynamical system  $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$  is called  $\mathcal{D}$ -stable if its all poles lie in  $\mathcal{D}$ , or all of the eigenvalues of the matrix  $\mathbf{A}$  lie also in  $\mathcal{D}$ , while  $\mathbf{A}$  is called  $\mathcal{D}$ -stable. When  $\mathcal{D}$  is the full unitary circle, this notion reduces to asymptotic stability, which can be characterised in LMI terms by the Lyapunov Theory (Chilali & Gahinet, 1996). A general LMI region in the complex plane is any subset  $\mathcal{D}$  where symmetric matrices  $\mathbf{L} = [\lambda_{kl}] \in \Re^{m \times m}$  and  $\mathbf{M} = [\mu_{kl}] \in \Re^{m \times m}$  exist, such

that

$$\mathcal{D} = \{z \in \mathbf{S} : \mathbf{L} + z\mathbf{M} + \bar{z}\mathbf{M}^T < 0\}, \tag{19}$$

where the  $\bar{\cdot}$  symbol to represent the complex conjugate, and the following notation:

$$[\lambda_{kl}]_{1 \leq k, l \leq m} := \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{1m} \\ \vdots & \ddots & \vdots \\ \lambda_{m1} & \cdots & \lambda_{mm} \end{bmatrix} < 0.$$

The matrix valued function:

$$f_{\mathcal{D}}(z) = \mathbf{L} + z\mathbf{M} + \bar{z}\mathbf{M}^T \tag{20}$$

is called the characteristic function of  $\mathcal{D}$ , and can be formulated also as:

$$f_{\mathcal{D}}(z) = [\lambda_{kl} + \mu_{kl}z + \mu_{lk}\bar{z}]_{1 \leq k, l \leq m}, \tag{21}$$

where for the general case of a disk centred at  $(-q, 0)$ , with a radius  $r$ ,  $\mathbf{L}$  and  $\mathbf{M}$  are being defined as ( $m = 2$ ):

$$\mathbf{L} = \begin{bmatrix} -r & q \\ q & -r \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$

and the characteristic function results in (Chilali, Gahinet, & Apkarian, 1999):

$$f_{\mathcal{D}}(z) = \begin{bmatrix} -r & q + z \\ q + \bar{z} & -r \end{bmatrix} < 0. \tag{22}$$

The pole location in a given LMI region can be characterised in terms of the  $m \times m$  block matrix  $M_{\mathcal{D}}(\mathbf{A}, X)$  (Chilali & Gahinet, 1996), which can be considered as the output of the application of the matrix valued function in Equation (21) on the system's state-space matrix  $\mathbf{A}_i$ .

**Theorem 4.1:** *The matrix  $\mathbf{A}_i$  is  $\mathcal{D}$ -stable, if and only if there exists a symmetric matrix  $\mathbf{X}$  such that:*

$$M_{\mathcal{D}}(\mathbf{A}, \mathbf{X}) < 0, \quad \mathbf{X} > 0. \tag{23}$$

*For the switching system in Equation (10), the  $\mathcal{D}$ -stability is formulated as:*

$$[\lambda_{kl}\mathbf{P} + \mu_{kl}\mathbf{A}_i\mathbf{P} + \mu_{lk}\mathbf{P}(\mathbf{A}_i)^T]_{1 \leq k, l \leq m} < 0. \tag{24}$$

*Or in an LMI form, the switching system  $\mathbf{A}_i$  is  $\mathcal{D}$ -stable, with  $\mathcal{D}$  defined in Equation (22) if and only if there exist*

matrices  $\mathbf{K}_i \in \mathbb{S}^+, i \in \mathbb{Z}^+$ , satisfying the following LMIs:

$$L = \begin{bmatrix} -r\tilde{\mathbf{A}} & \tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i \\ (\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)^T & -r\tilde{\mathbf{A}} \end{bmatrix} < 0, \forall i \in I. \tag{25}$$

**Proof:** See Theorem 2.2 in Chilali and Gahinet (1996), for the closed-loop system described in Equation (9).  $\square$

For a prescribed performance as it has been indicated in Equation (26) and an overall system stability for the switching closed-loop system in Equation (9), the proposed problem consists of finding switching output-feedback gains  $\mathbf{K}_i$  that:

- places the closed-loop poles in an *a priori* defined LMI stability region  $\mathcal{D}$  with the characteristic function (22);
- guarantees the overall stability of the switching system in Equation (9), under arbitrary time delays and mode-dependent switchings.

**Theorem 4.2:** *The stability of the mode-dependent switching closed-loop matrix  $\mathbf{A}_i$  is guaranteed and it is  $\mathcal{D}$ -stable, with  $\mathcal{D}$  defined in Equation (22), if and only if there exist matrices  $\mathbf{G}_i, \mathbf{U}_i \in \mathbb{S}^+, i \in \mathbb{Z}^+$ , satisfying the following LMIs:*

$$\begin{bmatrix} -r\mathbf{G}_i & \tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i \\ (\tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i)^T & -r\mathbf{G}_i \end{bmatrix} < 0, \forall i \in I.$$

**Proof:** By replacing in Equation (24) the analytical closed-loop description of the system in Equation (9), the following formulation is being obtained:

$$[\lambda_{kl}\mathbf{P}_i + \mu_{kl}(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)\mathbf{P}_i + \mu_{lk}\mathbf{P}_i(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)^T]_{1 \leq k, l \leq m} < 0. \tag{26}$$

By utilising the characteristic function  $f_{\mathcal{D}}$  of Equation (22) in Equation (26), the following inequalities are being derived:

$$(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)\mathbf{P}_i < 0 \tag{27}$$

$$\mathbf{P}_i(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)^T < 0 \tag{28}$$

$$-r\tilde{\mathbf{P}}_i < 0. \tag{29}$$

Recalling from Equation (14) that  $\mathbf{V}_i\mathbf{C}_i = \mathbf{C}_i\mathbf{G}_i$ , or  $\mathbf{V}_i = \mathbf{C}_i\mathbf{G}_i\mathbf{C}_i^T[\mathbf{C}_i\mathbf{C}_i^T]^{-1}$  and Equation (15) the following formulation can be derived:

$$\mathbf{K}_i\mathbf{C}_i\mathbf{G}_i = \mathbf{K}_i\mathbf{V}_i\mathbf{C}_i = \mathbf{U}_i\mathbf{V}_i^{-1}\mathbf{V}_i\mathbf{C}_i = \mathbf{U}_i\mathbf{C}_i \tag{30}$$

and by setting  $\mathbf{P}_i = \mathbf{G}_i$ , the LMI in Equation (27), after some basic mathematics and substitution of Equation (30) is being formulated as:

$$(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)\mathbf{P}_i = \tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i < 0. \tag{31}$$

The LMI in Equation (28), with  $\mathbf{P}_i = \mathbf{G}_i$ , can be transformed as:

$$\mathbf{G}_i(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)^T = \mathbf{G}_i\tilde{\mathbf{A}}^T + \mathbf{G}_i\mathbf{C}_i^T\mathbf{K}_i^T\tilde{\mathbf{B}}^T < 0. \tag{32}$$

By transposing Equation (30), it is obtained that:

$$(\mathbf{U}_i\mathbf{C}_i)^T = \mathbf{G}_i\mathbf{C}_i^T\mathbf{K}^T \tag{33}$$

and the final step is to replace Equation (33) into Equation (32) to obtain:

$$\mathbf{G}_i(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{K}_i\mathbf{C}_i)^T = \mathbf{G}_i\tilde{\mathbf{A}}^T + (\tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i)^T. \tag{34}$$

By utilising equations in Equations (31) and (34) the following LMIs can be obtained:

$$\begin{bmatrix} -r\mathbf{G}_i & \tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i \\ (\tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i)^T & -r\mathbf{G}_i \end{bmatrix} < 0, \forall i \in I$$

and this completes the proof.  $\square$

To conclude this section, it is being summarised that for overall switching stability and  $\mathcal{D}$ -stable stability for performance improvement, positive definite matrices  $\mathbf{S}_i, \mathbf{G}_i$  should exist that satisfy the following LMIs:

$$\begin{bmatrix} \mathbf{G}_i + \mathbf{G}_i^T - \mathbf{S}_i & (\tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\tilde{\mathbf{C}}_{r_s})^T \\ \tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\tilde{\mathbf{C}}_{r_s} & \mathbf{S}_j \end{bmatrix} > 0, \tag{35}$$

$$\begin{bmatrix} -r\mathbf{G}_i & \tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i \\ (\tilde{\mathbf{A}}\mathbf{G}_i + \tilde{\mathbf{B}}\mathbf{U}_i\mathbf{C}_i)^T & -r\mathbf{G}_i \end{bmatrix} < 0, \forall (i, j) \in I \times I,$$

with  $\mathbf{K}_i$  provided by Equations (14) and (15).

By the establishment of the current switching output feedback control scheme with  $\mathcal{D}$ -region performance improvement, as it will be presented in the following section, a controller can be designed that will be able to guarantee the stability of the networked controlled system, while introducing a criterion for significant performance improvement. The current approach is not focusing on the selection of an optimal  $\mathcal{D}$  region since it is assumed that this region has been *a priori* set. In general it can be stated that by achieving smaller  $\mathcal{D}$  radiuses, faster responses can be also achieved. However, since the tuning of the switching controller gains, as it has been presented, it is a solution of two coupled LMIs in Equation (35) there is not always a feasible solution to this problem, while the smaller the  $\mathcal{D}$  radius becomes, the harder and more computationally tedious is to find a proper LMI solution. For dealing with this

issue a generic engineering approach can be followed by starting with a large  $\mathcal{D}$  radius and progressively reducing as along the corresponding LMIs in Equation (35) are being satisfied, which indicates a bound on the performance improvement. However, there is still on going research in the field and on the way that the required  $\mathcal{D}$  region can be theoretically defined as it can be addressed in Lee and Lee (1987), Mao and Chu (2009), Montagner, Leite, Oliveira, and Peres (2006), where some explicit formulas have been derived, but these issues are out of the scope of the presented research and will not be addressed further.

## 5. Simulation results

All simulations have been carried out on the nonlinear quadrotor attitude dynamics presented in Bouabdallah and Siegwart (2007), which take into account all aerodynamic forces and moments acting on the rotations subsystem of the quadrotor including the hub forces, the rolling moments, and the variation in the aerodynamic coefficients due to the relative motion of the quadrotor inside the atmosphere. It must be noted that separate design of the attitude controller is a valid process since the attitude and translational dynamics of the quadrotor can be decoupled (Alexis, Nikolakopoulos, & Tzes, 2011; Bouabdallah, Noth, & Siegwart, 2004).

The quadrotor attitude nonlinear dynamics have been linearised around  $\mathbf{x}^\circ = [0, 0, 0, 0, 0, 0]^T$  and assuming a sampling time  $T_s = 0.001$ , the discrete time state matrices for the model under study take the following values:

$$\mathbf{A} = \begin{bmatrix} 1 & 0.001 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.001 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0.0021 & 0 & 0 & 0 \\ 0 & 0.0433 & 0 & 0 & 0 \\ 0 & 0 & 0.0021 & 0 & 0 \\ 0 & 0 & 0.0433 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0.10201 & 0 \end{bmatrix}$$

For the presented simulation results, it should be recalled that all the data transmissions and the calculation of the controllers gains were event triggered and time stamped. Moreover, it is assumed that the round-trip delay at the  $k$ -sampling time instant can always be measured.

According to the presented scheme in Section 4, the first stage in the design of the switching controller with  $\mathcal{D}$ -region performance should be the definition of the estimated

maximum time delay that can be encountered in the adopted communication network. Based on this *a priori* information the augmented system model representation will be formulated according to the augmentation procedure depicted in Equations (7) and (8). By having the state-space representation or the equivalent switching system formulated for all the corresponding time delays that can be encountered and the desired  $\mathcal{D}$ -region, a solution to the LMIs in Equation (34) should be evaluated. In case that these LMIs are feasible then the gains for the switching controller can be calculated from Equations (14) and (15). For improving further the performance of the overall networked controlled system, smaller  $\mathcal{D}$ -regions could be evaluated by repeating the previous control design process (feasibility check of the corresponding LMIs). From a practical implementation point of view, and after the calculation of the gains for the switching output feedback controller, the control scheme can be implemented in the form of a look-up table, where for a specific round trip measured time delay a corresponding gain matrix should be selected as the active controller.

For the first simulation results, a time varying latency time has been considered, bounded from  $D = 2$ , which results in a switching system with  $i \in [0, 1, 2]$ , while the  $\mathcal{D}$ -stability region, for performance improvement, has been selected as  $r = 0.85$ . The adopted bound in the time delay assumes that the maximum round-trip communication delay is 0.003 sec, allowing every arbitrary switching. Based on the presented methodology, the gains for the switching feedback controller have been tuned as:

$$\mathbf{K}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0193 & -0.0633 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0193 & -0.0633 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0082 & -0.0269 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0182 & -0.1511 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0182 & -0.1511 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0077 & -0.0642 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0257 & -0.3532 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0257 & -0.3532 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0109 & -0.1500 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The initial states of the quadrotor were  $x^0 = [0.1, 0, 0.2, 0, 0.3, 0]$ . In Figure 2, the responses of the quadrotors' states ( $\phi, \theta, \psi$  angles and the corresponding accelerations) based on the novel LMI proposed scheme are depicted in red colour. The response of the quadrotor based on only the controller synthesis approach, described by Equation (13)

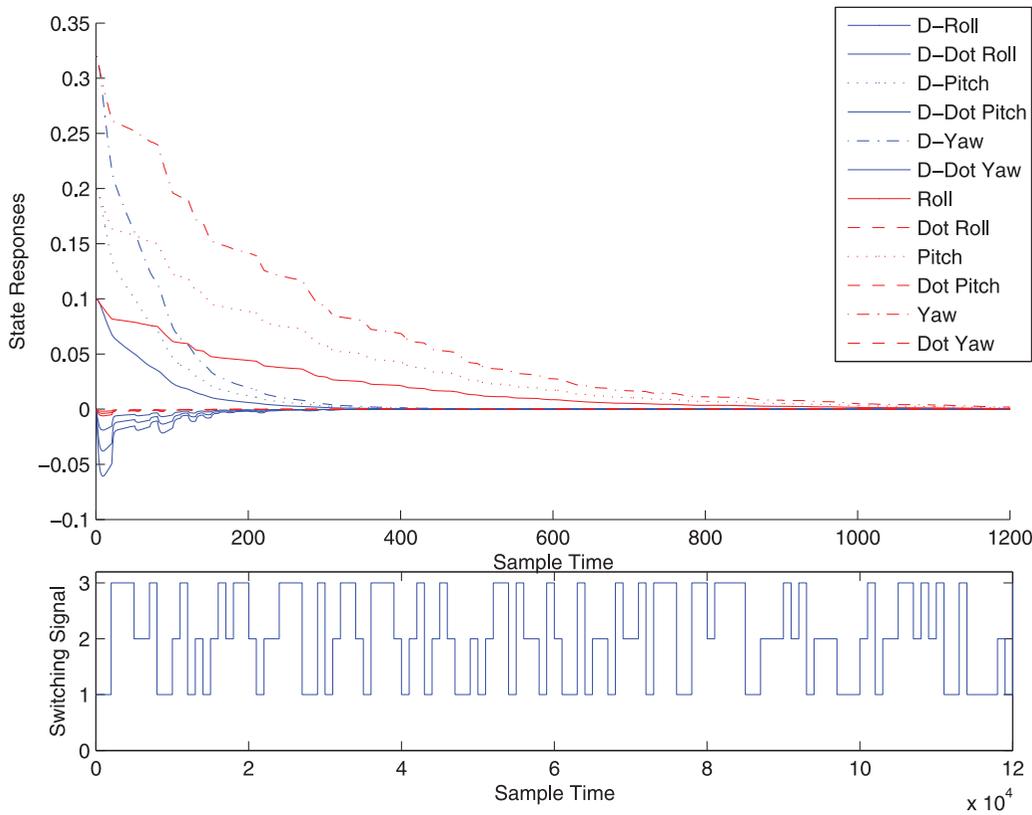


Figure 2. Comparison of the quadrotor’s state responses with and without the  $\mathcal{D}$ -stability region performance improvement ( $D = 3, r = 0.85$ ).

as in Daafouz et al. (2002), is also being displayed in the same Figure 2 by red coloured lines. Moreover, random simulated time delays (with a period of 1 sec) have been considered, while the time delay-based switching among the controllers has also been depicted in the lower part of this figure. From comparing the obtained responses, it is obvious that the proposed switched control synthesis approach, with combined performance  $\mathcal{D}$ -stability region, achieves much faster settling times, while assuring for the overall stability of the system against switchings due to time delays.

In the second simulation results, greater time delays have been considered, with  $D = 4$ , which results in a switching system for  $i \in [0, 1, 2, 3, 4]$ , while in this case, the  $\mathcal{D}$ -stability region, for performance improvement, has been selected as  $r = 0.9$ . The adopted bound in the time delay assumes that the maximum round-trip communication delay is 0.004 sec, with arbitrary switching. For this case, the gains for the switching feedback controller have been tuned as:

$$\mathbf{K}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0181 & -0.1160 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0181 & -0.1160 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0077 & -0.0493 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0161 & -0.1512 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0161 & -0.1512 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0068 & -0.0642 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0165 & -0.1845 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0165 & -0.1845 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0070 & -0.0784 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0177 & -0.2194 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0177 & -0.2194 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0075 & -0.0932 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0195 & -0.2607 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0195 & -0.2607 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0083 & -0.1107 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The initial states of the quadrotor were  $x^0 = [0.1, 0, 0.15, 0, 0.2, 0]$ . In Figure 3, the responses of the quadrotors’

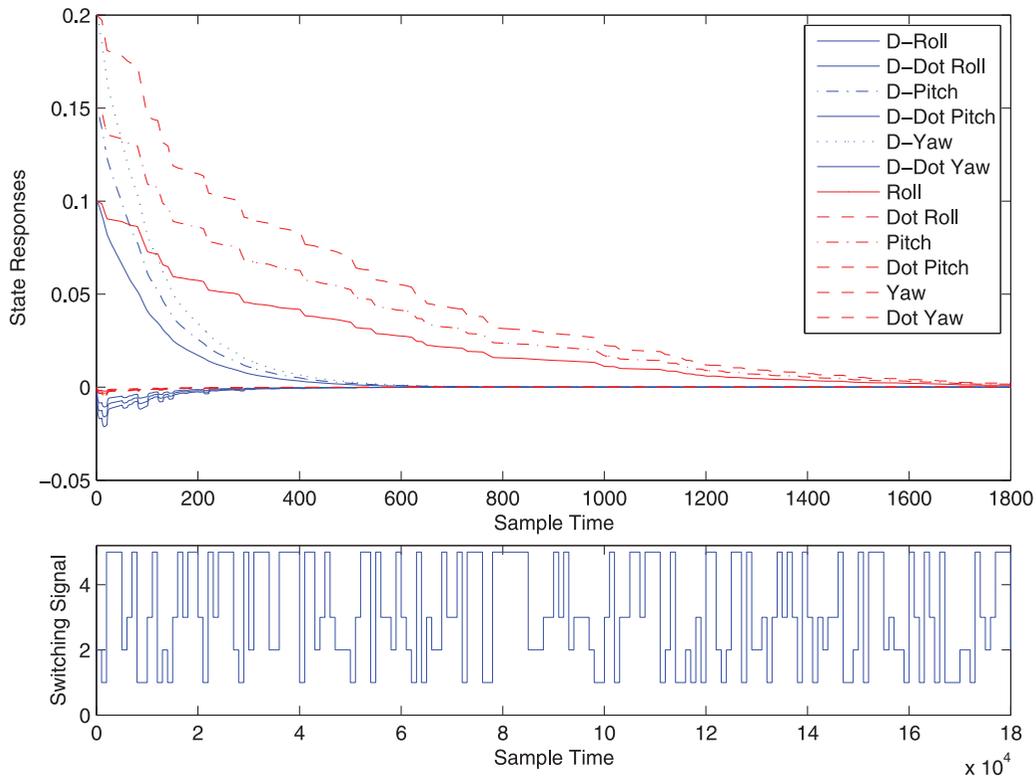


Figure 3. Comparison of the quadrotor's state responses with and without the  $\mathcal{D}$ -stability region performance improvement ( $D = 5$ ,  $r = 0.9$ ).

states ( $\phi$ ,  $\theta$ ,  $\psi$  angles and the corresponding accelerations) based on the proposed LMI switching control scheme are depicted in blue colour. As it has been observed in the previous examined case, it is obvious that based on the control synthesis approach from Equation (13) in Daafouz et al. (2002), depicted with red-coloured lines, the settling time is much slower than the responses obtained with the combined stability and performance control synthesis (blue-coloured lines). In general it has been observed that the larger the communication latency time is, the worse the performance of the control synthesis scheme is (without the performance improvement). By selecting appropriate  $\mathcal{D}$ -stability region, the resulting control scheme is able to: (a) assure the overall stability of the switching system and (b) achieve fast responses.

From a complexity point of view, Table 1 summarises the necessary iterations for calculating the solutions to the

Table 1. LMI iterations complexity.

D	Switched control synthesis	$\mathcal{D}$ -switched control synthesis
1	8	10
2	24	25
3	36	39
4	49	53
5	63	68

LMIs in Equations (13) and (37)–(38) correspondingly. The results have been obtained by the utilisation of the Matlab's LMI Toolbox, for various number of utilised switched systems  $D$ . According to Table 1, it is clear that the addition of the  $\mathcal{D}$ -stability region performance improvement, retains the complexity of the proposed algorithm and has the same computational efficiency.

## 6. Conclusions

In this article, a combined networked switching output feedback control scheme, with a  $\mathcal{D}$ -region stability performance improvement module, has been presented. The proposed scheme has been applied for the case of time varying and multiples of the sampling period induced delays, which act as the switching signal. Based on this modelling approach, a Linear Matrix Inequality (LMI) tuned switching output feedback controller has been applied that allowed robustness against time delays and was able to guarantee the overall stability of the switching closed-loop system. Integrated in the controlled synthesis phase, an LMI tuned performance improvement module has been introduced, based on  $\mathcal{D}$ -region stability. Part of the future objectives in the recommended research framework is the experimental evaluation of the suggested control architecture.

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