A Blind Multichannel Identification Algorithm Robust to Order Over Estimation

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Houcem Gazzah*,†, Phillip A. Regalia†, Jean-Pierre Delmas† and Karim Abed-Meraim‡

Abstract— **Active research in blind SIMO (Single Input Multiple Output) channel identification has led to a variety of second order statistics based algorithms, particularily the Subspace and the Linear Prediction approaches. The Subspace algorithm shows good performance when the channel output is corrupted by noise and available for a finite time duration. However, its performance is subject to exact knowledge of the channel order, which is not guaranteed by current order detection techniques. On the other hand, the Linear Prediction algorithm is sensitive to observation noise while its robustness to channel order over estimation is not always verified when the channel statistics are estimated. We propose a new second order statistics based blind channel identification algorithm which is truly robust to channel order over estimation i.e., able to accurately estimate the channel impulse response from a finite number of noisy channel measurements when the assumed order is arbitrarily greater than the exact channel order. Another interesting feature is that the identification performance can be enhanced by increasing a certain smoothing factor. Moreover, the proposed algorithm proves to clearly outperform the Linear Prediction algorithm. These facts are justified theoretically and verified through simulations.**

Keywords— **Blind channel identification and equalization, Second-order statistics algorithms, Order over estimation.**

I. Introduction

Blind identification of communication channels addresses those signal processing techniques that estimate the channel impulse response using solely its output statistics. Such an estimate may be fed to an equalization algorithm in order to restore the transmitted data. This obviates the need for training sequences, thereby achieving a much desired bandwidth gain. As Second Order Statistics (SOS) of the Baud sampled channel output do not contain information about the channel phase, early techniques [1], [2] exploited Higher Order Statistics (HOS) to achieve blind equalization. However, the channel needs to be observed for long durations before output HOS estimates are accurate enough to allow for reliable equalization. The proof that (the much easier to estimate) SOS of the cyclostationary oversampled output (this result was later extended to the multiple antenna case) does contain phase information of the channel renewed the hope of developing blind algorithms that can achieve equalization with relatively short data lengths. Since the first algorithm by Tong *et al* [3], a number of SOS based blind algorithms have been proposed. Among the more popular are the Subspace (SS) [4] and the Linear Prediction (LP) [5], [6] algorithms. The former achieves better performance but requires precise knowledge of the channel order which is a rather delicate and improbable task. The latter can handle an over estimated value of the channel order but its performance is very sensitive to observation noise. It has been pointed that its (claimed) robustness to channel order over estimation does not hold when SOS contain estimation errors [7], [8]. It was shown [6] that the LP algorithm can achieve acceptable performance when the assumed order equals that provided by an order detection criterion (the MDL and the AIC criteria [9]) over estimated by few (one or two) taps only. This behavior does not make the LP algorithm fully robust to order over estimation and, more importantly, does not dispense with the need to estimate the channel order prior to its response estimation. Another algorithm, similar in properties and performance to the LP algorithm, is the Outer Product Decomposition (OPD) algorithm [10].

In this paper, we develop a novel algorithm that combines advantages of both algorithms. It exhibits good performance at low SNR, while being robust to channel order over estimation. We emphasize that the proposed algorithm is truly robust to order over estimation as accurate identification is still achievable using estimated channel statistics. The proposed algorithm is based on a *shifted* version of the correlation matrix and the properties of the associated kernel. The algorithm does not require the computation of the correlation matrix pseudo-inverse as with LP and OPD algorithms, nor is the whole kernel necessary to achieve identification as with the SS algorithm. It is hence proved theoretically then verified through simulations that identification is possible when the channel order is arbitrarily over estimated and when the SOS are estimated from a finite sample size. This has the major advantage of allowing blind identification without prior detection of the channel order. The *a priori* knowledge of the propagation conditions (in the case of a multipath channel for example), in terms of channel delay spread, will be sufficient.

Shifted correlation matrices have previously [11], [12] been used to estimate ZF equalizers of arbitrary delays. In addition to an important computational complexity (especially, a singular value decomposition has to be performed twice), these equalizers are limited by the noise enhancement problem. Better equalization techniques (MMSE or Viterbi) require the channel response to be identified. The contribution of this work is not only to use *shifted* correlation matrices to perform blind identification, but also to show that this approach comes with performance and

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robustness advantages w.r.t. existing ones.

The paper is organized as follows. In Sec. II, we present the channel model and recall the main steps of the Subspace and the Linear Prediction algorithms. We point out, in particular, their (non) robustness to order over estimation. In Sec. III, we introduce a novel SOS based blind identification algorithm and prove its robustness to channel order over estimation. In Sec. IV, we rewrite the proposed algorithm using estimated statistics and prove that its robustness to channel order over estimation still holds under such circumstances. Simulation results are presented in Sec. V and commented on in Sec. VI. Concluding remarks appear in Sec. VII.

The following notations are used throughout the paper. Matrices (resp. vectors) are represented by bold or calligraphic upper case (resp. bold lower case) characters. Vectors are by default in column orientation, while T, H and $*$ stand for transpose, transconjugate and conjugate, respectively. $\mathbf{e}_{k,i}$ is the ith unit vector in $\mathcal{R}^k$. $\mathbf{0}_{a,b}$ is the $a \times b$ zero matrix. It is noted $\mathbf{0}$ when its dimension can be inferred from the context. $\mathbf{I}_a$ is the $a \times a$ identity matrix and $\mathbf{J}_a \stackrel{\text{def}}{=} \begin{bmatrix} 0 & & & \\ 1 & \ddots & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{bmatrix}$ is the $a \times a$ (down) shift matrix. $\|\cdot\|$ denotes the Euclidean norm. $\mathbf{A} \otimes \mathbf{B}$ is the Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$ defined such that its (i,j) block element is $a_{i,j}\mathbf{B}$. Vec$(.)$ is the *vectorization* operator that turns a matrix into a vector by stacking the columns of the matrix one below another.

II. Blind Identification of SIMO channels

A. The SIMO channel

It is common to model a fractionally spaced and/or multi-sensor receiver by a Single Input Multiple Output (SIMO) scheme as depicted in Fig. 1. A set of c filters are driven by a common scalar input $s(n)$. The SIMO channel order m is defined as the maximum among those of the different filters $\mathbf{h}^1 \cdots \mathbf{h}^c$. We define the c-dimensional taps $\mathbf{h}(k) \stackrel{\text{def}}{=} \left[h^1(k) \cdots h^c(k)\right]^T, k \in \{0, \cdots, m\}$, where $h^i(k)$ is the k-th tap of the i-th filter. The SIMO impulse response is defined as $\mathbf{h}_m \stackrel{\text{def}}{=} \left[\mathbf{h}^T(0) \cdots \mathbf{h}^T(m)\right]^T$. The noise corrupted output is the c-dimensional vector $\mathbf{y}(n) \stackrel{\text{def}}{=} \left[y^1(n) \cdots y^c(n)\right]^T$. The input-output relation is expressed as a multi-dimensional convolution $\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{b}(n) = \mathcal{T}(\mathbf{h}_m)\,\mathbf{s}_{m+1}(n) + \mathbf{b}(n)$ where $\mathcal{T}(\mathbf{h}_m) \stackrel{\text{def}}{=} [\mathbf{h}(0) \cdots \mathbf{h}(m)]$ and $\mathbf{s}_k(n) \stackrel{\text{def}}{=} [s(n) \cdots s(n-(k-1))]^T$ for any k.

To exploit the time invariant property of the SIMO channel, the channel output is observed over durations larger than a symbol period. We stack l successive samples into $\mathbf{y}_l^T(n) \stackrel{\text{def}}{=} \left[\mathbf{y}^T(n) \cdots \mathbf{y}^T(n-(l-1))\right]^T$, where l is called the *smoothing factor*. We have

$$\mathbf{y}_l(n) = \mathcal{T}_l(\mathbf{h}_m)\,\mathbf{s}_{l+m}(n) + \mathbf{b}_l(n)$$

where

$$\mathcal{T}_l(\mathbf{h}_m) \stackrel{\text{def}}{=} \begin{bmatrix} \mathcal{T}(\mathbf{h}_m) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathcal{T}(\mathbf{h}_m) & \cdots & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathcal{T}(\mathbf{h}_m) \end{bmatrix}$$

is the $cl \times (l+m)$ *Filtering Matrix*, $\mathbf{b}_l(n)$ is defined similarly as $\mathbf{y}_l(n)$ and $\mathbf{0}$ is the c-dimensional null vector.

The channel output SOS are completely described by the correlation matrix functions $\Gamma(k) \stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}(n+k)\mathbf{y}^H(n)\right], k \geq 0$. These SOS terms can be arranged in different ways as correlation matrices. We define the *standard* correlation matrix

$$\begin{aligned} \mathbf{R}_l &\stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}_l(n)\mathbf{y}_l^H(n)\right] \\ &= \begin{bmatrix} \Gamma(0) & \Gamma(1) & \ldots & \Gamma(l-1) \\ \Gamma^H(1) & \Gamma(0) & \ddots & \Gamma(l-2) \\ \vdots & \ddots & \ddots & \vdots \\ \Gamma^H(l-1) & \Gamma^H(l-2) & \ldots & \Gamma(0) \end{bmatrix} \end{aligned}$$

and the (down) *shifted* correlation matrix

$$\begin{aligned} \mathcal{R}_l &\stackrel{\text{def}}{=} \mathbf{E}\left[\mathbf{y}_l(n)\mathbf{y}_l^H(n-1)\right] \\ &= \begin{bmatrix} \Gamma(1) & \Gamma(2) & \ldots & \Gamma(l) \\ \Gamma(0) & \Gamma(1) & \ddots & \Gamma(l-1) \\ \vdots & \ddots & \ddots & \vdots \\ \Gamma(-l+2)) & \Gamma(-l+3) & \ldots & \Gamma(1) \end{bmatrix} \end{aligned}$$

For any other process $\mathbf{p}(n)$, we denote by $\Gamma^p(k)$, $\mathbf{R}_l^p$ and $\mathcal{R}_l^p$ the corresponding correlation matrices.

When the symbols are uncorrelated with the noise, the correlation matrices are given by $\mathbf{R}_l = \mathcal{T}_l(\mathbf{h}_m)\,\mathbf{R}_{l+m}^s\mathcal{T}_l^H(\mathbf{h}_m) + \mathbf{R}_l^b$ and $\mathcal{R}_l = \mathcal{T}_l(\mathbf{h}_m)\,\mathcal{R}_{l+m}^s\mathcal{T}_l^H(\mathbf{h}_m) + \mathcal{R}_l^b$. If the symbols $s(n)$ are uncorrelated then the correlation matrices are given by $\mathbf{R}_l = \sigma_s^2\mathcal{T}_l(\mathbf{h}_m)\,\mathcal{T}_l^H(\mathbf{h}_m) + \mathbf{R}_l^b$ and $\mathcal{R}_l = \sigma_s^2\mathcal{T}_l(\mathbf{h}_m)\,\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m) + \mathcal{R}_l^b$. If, in addition, the noise components are uncorrelated, then $\mathbf{R}_l^b = \sigma_b^2\mathbf{I}_{cl}$ and $\mathcal{R}_l^b = \sigma_b^2(\mathbf{J}_l \otimes \mathbf{I}_c)$. The above assumptions on the transmitted symbols and on noise will be maintained throughout the paper.

As the process $\mathbf{x}(n)$ is an m-th order moving average (MA) multivariate process then only $\Gamma(k), |k| \leq m$ are possibly non zero and the set $\{\Gamma(0), \cdots, \Gamma(m)\}$ contains all the SOS information of the channel output.

It is worth recalling here an important result [3] on the rank of the Sylvester matrix $\mathcal{T}_l(\mathbf{h}_m)$: $\mathcal{T}_l(\mathbf{h}_m)$ has full column rank if the channel is co-prime (the transfer functions of the channels $\mathbf{h}^{c'}$, $c' = 1, \ldots, c$, do not have zeros in common) and $l \geq m$.

B. Existing Algorithms and Robustness to Order Over Estimation

We now briefly recall the principal steps and properties of the most cited blind identification algorithms developed so far, the Subspace (SS) and the Linear Prediction (LP) algorithms. We particularly comment on their behavior when the channel order is over estimated. This feature is of practical interest because order detection criteria, such as the MDL and AIC criteria [9], are not reliable when the channel output is observed under practical conditions involving measurement noise and a limited time interval. However, over estimated values of the channel order are easy to obtain, especially with the AIC criterion, proved to asymptotically provide an over estimated channel order [9]. Alternatively, an over estimated value of the channel order can be obtained without resorting to order detection tests but simply from the *a priori* knowledge about the channel delay spread.

The SS algorithm exploits the fact that a (full column rank) filtering matrix $\mathcal{T}_l(\mathbf{h}_m)$ is uniquely (up to a scalar constant) determined by its left kernel. When the noise is spatially and temporally white, the latter is given by the noise subspace of the correlation matrix $\mathbf{R}_l$. An important feature of the SS algorithm is that the noise subspace of the exact correlation matrix is also the noise subspace of the empirical correlation matrix in the noiseless case. This allows for exact estimation of the channel impulse response when there is no observation noise [13] and is the reason why the SS algorithm significantly outperforms the other blind identification algorithms. However, the knowledge of the exact channel order is required to fully characterize the channel.

The LP approach is based on the proof by *Slock* [5] that the m-th order Moving Average (MA) SIMO output is also an m-th order autoregressive (AR) multivariate process whose innovation is proportional to the SIMO scalar input. Hence, the m-th order linear predictor, obtained by solving the Yule-Walker (YW) equations, is used to derive an m-th order Zero Forcing (ZF) zero delay equalizer. The channel impulse response is then derived from the equalizer expression and the SOS. As an m-th order AR process can also be regarded as an m'-th order AR process, $m' \geq m$, the LP algorithm was cited [5], [6] as robust w.r.t. channel order over estimation. However, as pointed out in [7], [8], this does not hold when the SOS are estimated and the channel order is arbitrarily over estimated. In fact, solving the YW equation requires the computation of the pseudo-inverse of the noise-free correlation matrix. The latter approximates a rank deficient matrix and the theoretical rank of the noise-free correlation matrix (that relative to the exact statistics case) needs to be exactly known to properly compute the pseudo-inverse matrix. When the order is over estimated, the noise subspace dimension is under estimated and some of its (small) eigenvalues are wrongly classified in the signal subspace, and hence are inverted, leading to the failure of the algorithm. Therefore, solving the blind identification problem is subordinate to solving the order detection problem.

Another algorithm, with performance similar to (or slightly better than) the LP algorithm, is the Outer Product Decomposition (OPD) [10]. Its robustness to order over estimation is not maintained in the estimated statistics case, for the same reason : the computation of the pseudo-inverse of the noise-free correlation matrix is required.

III. Exact Statistics Case

A. Theoretical Development

The proposed algorithm assumes knowledge of the correlation matrix $\mathcal{R}_l$. The noise power is the smallest eigenvalue of the Hermitian positive definite matrix $\mathbf{R}_l$ with multiplicity $cl - (l+m) = (c-1)l - m$. We have $\mathcal{R}_l^b = \sigma_b^2 (\mathbf{J}_l \otimes \mathbf{I}_c)$ and $\mathcal{R}_l - \mathcal{R}_l^b = \sigma_s^2 \mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)$.

Hypothesis $\mathbf{H}1$: The smoothing factor is no smaller than the channel order : $l \geq m$.

Under $\mathbf{H}1$, $\mathcal{T}_l(\mathbf{h}_m)$ is full column rank (throughout the paper, the channel $\mathbf{h}_m$ is assumed to be co-prime) so that $\operatorname{rank}(\mathcal{R}_l - \mathcal{R}_l^b) = \operatorname{rank}(\mathbf{J}_{l+m}) = l+m-1$. So there exist an orthogonal set $\{\mathbf{n}_{l,1}^{(i)}\}_{i=1,\cdots,w}$ (resp. $\{\mathbf{n}_{l,2}^{(i)}\}_{i=1,\cdots,w}$) of vectors in the right (resp. left) null space of $\mathcal{R}_l - \mathcal{R}_l^b$, where $w \stackrel{\text{def}}{=} (c-1)l - m + 1$. For every $i = 1, \cdots, w$, we have

$$\mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,1}^{(i)} = \mathbf{0}$$

and

$$\left(\mathbf{n}_{l,2}^{(i)}\right)^H \mathcal{T}_l(\mathbf{h}_m)\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m) = \mathbf{0}.$$

So,

$$\mathbf{J}_{l+m}\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,1}^{(i)} = \mathbf{0}$$

and

$$\mathbf{J}_{l+m}^T\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,2}^{(i)} = \mathbf{0}.$$

Consequently, there exist[1] $\alpha_1^{(i)}$ such that $\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,1}^{(i)} = \alpha_1^{(i)}\mathbf{e}_{l+m,l+m}$ and similarly, there exist $\alpha_2^{(i)}$ such that $\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,2}^{(i)} = \alpha_2^{(i)}\mathbf{e}_{l+m,1}$. The unknowns $\alpha_1^{(i)}$ and $\alpha_2^{(i)}$ can be determined (up to an unknown phase) from

$$\begin{aligned}
\mathbf{n}_{l,j}^{(i)^H}\left(\mathbf{R}_l - \mathbf{R}_l^b\right)\mathbf{n}_{l,j}^{(i)} &= \sigma_s^2 \mathbf{n}_{l,j}^{(i)^H}\mathcal{T}_l(\mathbf{h}_m)\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,j}^{(i)} \\
&= \sigma_s^2 \left\|\mathcal{T}_l^H(\mathbf{h}_m)\mathbf{n}_{l,j}^{(i)}\right\|^2 \\
&= \sigma_s^2 |\alpha_j^{(i)}|^2 \quad , j = 1,2
\end{aligned}$$

Consequently

$$\mathbf{g}_{l-1,l+m}^{(i)} \stackrel{\text{def}}{=} \frac{1}{\sigma_s\sqrt{\mathbf{n}_{l,1}^{(i)^H}\left(\mathbf{R}_l - \mathbf{R}_l^b\right)\mathbf{n}_{l,1}^{(i)}}}\mathbf{n}_{l,1}^{(i)^*} \qquad (1)$$

and

$$\mathbf{g}_{l-1,1}^{(i)} \stackrel{\text{def}}{=} \frac{1}{\sigma_s\sqrt{\mathbf{n}_{l,2}^{(i)^H}\left(\mathbf{R}_l - \mathbf{R}_l^b\right)\mathbf{n}_{l,2}^{(i)}}}\mathbf{n}_{l,2}^{(i)^*} \qquad (2)$$

verify $\mathcal{T}_l^T(\mathbf{h}_m)\mathbf{g}_{l-1,l+m}^{(i)} = \frac{\alpha_1^{(i)^*}}{|\alpha_1^{(i)}|}\mathbf{e}_{l+m,l+m}$ and $\mathcal{T}_l^T(\mathbf{h}_m)\mathbf{g}_{l-1,1}^{(i)} = \frac{\alpha_2^{(i)^*}}{|\alpha_2^{(i)}|}\mathbf{e}_{l+m,1}$ and hence are $(l-1)$ order ZF equalizers with maximum and minimum delay respectively, in that, in

[1] In fact, there exist orthonormal bases of the left and right null spaces of $\mathcal{R}_l - \mathcal{R}_l^b$ such that $\alpha_1^{(i)} = 0$ and $\alpha_2^{(i)} = 0$ for $i = 1, \cdots, w-1$. Hopefully, these bases are computed with a zero probability in the exact and estimated statistics cases.

the absence of noise, they restore the transmitted symbols with the exact amplitude and up to unknown phases. We have as many equalizers as vectors $\mathbf{n}_{l,1}^{(i)}$ and $\mathbf{n}_{l,2}^{(i)}$. The channel taps can be retrieved from the channel output statistics and any of the ZF equalizers since $\mathbf{h}(k) = \frac{1}{\sigma_s^2}\mathbf{E}\left(\mathbf{y}(n)s(n-k)^*\right) = \frac{1}{\sigma_s^2}\mathbf{E}\left(\mathbf{x}(n)\mathbf{x}_l^H(n-k)\right)\mathbf{g}_{l-1,1}^{(i)^*} = \frac{1}{\sigma_s^2}\mathbf{E}\left(\mathbf{x}(n)\mathbf{x}_l^H(n-k+l+m-1)\right)\mathbf{g}_{l-1,l+m}^{(i)^*}$ which can be rewritten as follows:

Based on $\mathbf{g}_{l-1,1}^{(i)}$, the channel response is

$$\mathbf{h}_m = \frac{1}{\sigma_s^2}\begin{bmatrix}\Gamma^x(0) & \cdots & \Gamma^x(l-1)\\ \vdots & & \vdots\\ \Gamma^x(m) & \cdots & \Gamma^x(l+m-1)\end{bmatrix}\mathbf{g}_{l-1,1}^{(i)^*} \quad (3)$$

Based on $\mathbf{g}_{l-1,l+m}^{(i)}$, the channel response is

$$\mathbf{h}_m = \frac{1}{\sigma_s^2}\begin{bmatrix}\Gamma^x(-l-m+1) & \cdots & \Gamma^x(-m)\\ \vdots & & \vdots\\ \Gamma^x(-l+1) & \cdots & \Gamma^x(0)\end{bmatrix}\mathbf{g}_{l-1,l+m}^{(i)^*} \quad (4)$$

This step is a generalization of a similar one proposed for the LP algorithm [6]. As equalizers are determined with a phase ambiguity, the channel response is determined up to a phase ambiguity as well.

Using the fact that $\mathbf{x}(n)$ is an m-th order MA process ($\Gamma(k) = \mathbf{0}$ if $|k| > m$), we rewrite (3) as follows

$$\mathbf{h}_m = \frac{1}{\sigma_s^2}\begin{bmatrix}\Gamma^x(0) & \cdots & \Gamma^x(m) & \mathbf{0} & \cdots\\ \vdots & & \vdots & & \\ \Gamma^x(m) & \mathbf{0} & \cdots & & \end{bmatrix}\mathbf{g}_{l-1,1}^{(i)^*}, \text{if } l > m$$

$$= \frac{1}{\sigma_s^2}\begin{bmatrix}\Gamma^x(0) & \cdots & \Gamma^x(m-1)\\ \Gamma^x(1) & \cdots & \Gamma^x(m)\\ \vdots & & \mathbf{0}\\ & & \vdots\\ \Gamma^x(m) & \mathbf{0} & \cdots\end{bmatrix}\mathbf{g}_{m-1,1}^{(i)^*}, \text{if } l = m$$

and we rewrite (4) as

$$\mathbf{h}_m = \frac{1}{\sigma_s^2}\begin{bmatrix} & \cdots & & \mathbf{0} & \Gamma^x(-m)\\ & & & & \vdots\\ \cdots & \mathbf{0} & \Gamma^x(-m+1) & \cdots & \Gamma^x(0)\end{bmatrix}\mathbf{g}_{l-1,l+m}^{(i)^*}, \text{if } l > m$$

$$= \frac{1}{\sigma_s^2}\begin{bmatrix}\cdots & \mathbf{0} & \Gamma^x(-m)\\ \vdots & & \vdots\\ \mathbf{0} & & \\ \Gamma^x(-m) & \cdots & \Gamma^x(-1)\\ \Gamma^x(-m+1) & \cdots & \Gamma^x(0)\end{bmatrix}\mathbf{g}_{m-1,2m}^{(i)^*}, \text{if } l = m$$

Here $\mathbf{0}$ is the $c \times c$ zero matrix.

B. Robustness to Order Over Estimation

We now prove an important feature of the proposed algorithm which is its ability to estimate the exact channel impulse response $\mathbf{h}_m$ when the channel order is over estimated. In fact, if we detect an order $m' > m$, the rank of $\mathcal{R}_l - \mathcal{R}_l^b$ is (over) estimated to be $l + m' + 1$. Any among the vectors $\mathbf{n}_{l,1}^{(i)}$ and $\mathbf{n}_{l,2}^{(i)}$ suffices to estimate the channel response following the above steps. If $\mathbf{g}_{l-1,l+m'}^{(i)}$ and $\mathbf{g}_{l-1,1}^{(i)}$ are constructed as indicated above, using (3), the algorithm attempts to compute $\frac{1}{\sigma_s^2}\begin{bmatrix}\Gamma^x(0) & \cdots & \Gamma^x(l-1)\\ \vdots & & \vdots\\ \Gamma^x(m) & \cdots & \Gamma^x(l+m-1)\\ \vdots & & \vdots\\ \Gamma^x(m') & \cdots & \Gamma^x(l+m'-1)\end{bmatrix}\mathbf{g}_{l-1,1}^{(i)^*} =$

$$\begin{bmatrix}\mathbf{h}_m\\ \frac{1}{\sigma_s^2}\begin{bmatrix}\Gamma^x(m+1) & \cdots & \Gamma^x(l+m)\\ \vdots & & \vdots\\ \Gamma^x(m') & \cdots & \Gamma^x(l+m'-1)\end{bmatrix}\mathbf{g}_{l-1,1}^{(i)^*}\end{bmatrix} = \begin{bmatrix}\mathbf{h}_m\\ \mathbf{0}_{c(m'-m),1}\end{bmatrix}$$

where we have used the fact that, because $\mathbf{x}(n)$ is an m-th order MA process, $\Gamma^x(k) = \mathbf{0}$, if $|k| > m$. Similarly, using (4), the algorithm attempts to compute $\begin{bmatrix}\mathbf{0}_{c(m'-m),1}\\ \mathbf{h}_m\end{bmatrix}$. Consequently, the channel response so estimated is a zero-padded version of the true channel response, and, hence, can be used for equalization purposes.

IV. Estimated Statistics Case

Because of the finite sample size, the estimate of $\mathcal{R}_l - \mathcal{R}_l^b$ may not be rank deficient. The vector $\mathbf{n}_{l,1}^{(i)}$ (resp. $\mathbf{n}_{l,2}^{(i)}$) is chosen to be the right (resp. left) singular vector associated with the i-th smallest singular value of the estimate $\hat{\mathcal{R}}_l$ of $\mathcal{R}_l$. They are no longer equivalent as they may not achieve perfect ZF equalization. The algorithm will be rewritten w.r.t. the estimated SOS case, in two ways (Sec. IV-A and Sec. IV-B). We prove in Sec. IV-C that, even though SOS may be estimated, robustness to order over estimation is still maintained.

A. Correlation Matching Criterion

Each among the vectors $\mathbf{n}_{l,1}^{(i)}$ and $\mathbf{n}_{l,2}^{(i)}$ leads, throughout the procedure described in Sec. III-A, to an estimate of the channel response (or of a zero-padded version if the detected channel order is over estimated). We need to introduce a criterion to select the *best* among the $2w$ candidates $\{\hat{\mathbf{h}}_m^{(i)}\}$ using the sole available (second order) information about the channel i.e., its output estimated covariance matrix. Hence, we compare $\mathcal{T}_l\left(\hat{\mathbf{h}}_m^{(i)}\right)\mathcal{T}_l^H\left(\hat{\mathbf{h}}_m^{(i)}\right)$ to $\hat{\mathbf{R}}_l - \widehat{\sigma_b^2}\mathbf{I}_{cl}$. We propose the following criterion, henceforth named the Correlation Matching Criterion (CMC),

$$\hat{\mathbf{h}}_m = \text{argmin}_i\left(\min_\beta\left\|\hat{\mathbf{R}}_l - \widehat{\sigma_b^2}\mathbf{I}_{cl} - \beta\mathcal{T}_l\left(\hat{\mathbf{h}}_m^{(i)}\right)\mathcal{T}_l^H\left(\hat{\mathbf{h}}_m^{(i)}\right)\right\|_M^2\right) \quad (5)$$

where $\|.\|_M$ stands for a matrix norm. β can be chosen to be σ_s^2 if the channel response needs to be approximated with a phase ambiguity i.e., up to a unitary complex constant.

In the case of phase and amplitude ambiguity, the algorithm can be simplified by modifying (1) and (2) to compute $\mathbf{g}^{(i)}_{l-1,l+m} \stackrel{\text{def}}{=} \mathbf{n}^{(i)^*}_{l,1}$ and $\mathbf{g}^{(i)}_{l-1,1} \stackrel{\text{def}}{=} \mathbf{n}^{(i)^*}_{l,2}$. The identification procedure continues as before. If we choose the Frobenius matrix norm defined as $\|\mathbf{A}\|_F \stackrel{\text{def}}{=} \|\text{Vec}\,(\mathbf{A})\|$ for any matrix $\mathbf{A}$, (5) is simplified as follows:

$$\hat{\mathbf{h}}_m = \text{argmin}_i \left(\left\| \hat{\mathbf{R}}_l - \widehat{\sigma^2_b}\mathbf{I}_{cl} \right\|^2_F - \frac{\left| \text{Vec}\left(\hat{\mathbf{R}}_l - \widehat{\sigma^2_b}\mathbf{I}_{cl}\right)^H \text{Vec}\left(\mathcal{T}_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\mathcal{T}^H_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\right)\right|^2}{\left\|\mathcal{T}_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\mathcal{T}^H_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\right\|^2_F} \right)$$

Finally,

$$\hat{\mathbf{h}}_m = \text{argmax}_i \frac{\left| \text{Vec}\left(\hat{\mathbf{R}}_l - \widehat{\sigma^2_b}\mathbf{I}_{cl}\right)^H \text{Vec}\left(\mathcal{T}_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\mathcal{T}^H_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\right)\right|}{\left\|\mathcal{T}_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\mathcal{T}^H_l\left(\hat{\mathbf{h}}^{(i)}_m\right)\right\|_F} \quad (6)$$

Note that this criterion tolerates channel order over estimation as $\mathcal{T}_l\left(\begin{bmatrix}\mathbf{0}_{cm_1,1}\\ \mathbf{h}_m \\ \mathbf{0}_{cm_2,1}\end{bmatrix}\right)\mathcal{T}^H_l\left(\begin{bmatrix}\mathbf{0}_{cm_1,1}\\ \mathbf{h}_m \\ \mathbf{0}_{cm_2,1}\end{bmatrix}\right) = \mathcal{T}_l\left(\mathbf{h}_m\right)\mathcal{T}^H_l\left(\mathbf{h}_m\right)$, for any m_1 and m_2.

B. Equalization Peak Criterion

We are interested here in introducing a new criterion on the equalizers that allows for selecting an equalizer *better* than those directly issued from the left and right singular vectors $\mathbf{n}^{(i)}_{l,j}$, $i = 1, \cdots, w$, $j = 1, 2$ while, at the same time, reducing the computational complexity. For the reasons mentioned above, the vectors $\mathbf{n}^{(i)^*}_{l,j}$, $i = 1, \cdots, w$, $j = 1, 2$ may not achieve perfect ZF equalization, and hence, are no longer equivalent, in the sense that their output SNRs depend directly on the values of the scalars $\alpha^{(i)}_j$. Moreover, each linear combination of equalizers with the same delay is another equalizer. The different ZF equalizers can be compared on the basis of the amplitude of the restored symbol. We thus refer to the following as Equalization Peak Criterion (EPC). It was introduced in [14] to improve the performance of the LP and the MRE (Mutually Referenced Equalizers) algorithms.

The combined channel-equalizer response is given by $\mathcal{T}^T_l\left(\mathbf{h}_m\right)\mathbf{n}^{(i)^*}_{l,j}$. Its norm approximates (the square of) the amplitude of the restored symbol, when the intersymbol interference is negligible. Let $\mathbf{N}_{l,j} \stackrel{\text{def}}{=} \left[\mathbf{n}^{(1)}_{l,j} \cdots \mathbf{n}^{(w)}_{l,j}\right]^*$ contain all estimated equalizers with zero delay if $j = 1$ and with (maximum) delay $l + m$ if $j = 2$. For all w-dimensional vectors $\mathbf{f}_j$, $\mathbf{N}_{l,j}\mathbf{f}_j$ is also an equalizer. The best choice of $\mathbf{f}_j$, in the sense of maximizing the equalizer's output SNR, is achieved by [14] $\text{argmax}_{\|\mathbf{f}\|=1}\left\|\mathcal{T}^T_l\left(\mathbf{h}_m\right)\mathbf{N}_{l,j}\mathbf{f}\right\|$. Hence, we select $\mathbf{f}_j$ as an eigenvector associated with the largest eigenvalue of $\mathbf{N}^H_{l,j}\left(\hat{\mathbf{R}}_l - \widehat{\sigma^2_b}\mathbf{I}_{cl}\right)^T\mathbf{N}_{l,j}$, i.e., with the largest eigenvalue of $\mathbf{N}^H_{l,j}\hat{\mathbf{R}}^T_l\mathbf{N}_{l,j}$. Let $\mathbf{n}_{l,j} \stackrel{\text{def}}{=} \mathbf{N}_{l,j}\mathbf{f}_j, j = 1, 2$ the so-computed equalizers. We finally select the ZF equalizer $\mathbf{n}_l \stackrel{\text{def}}{=} \text{argmax}_{j=1,2}\mathbf{n}^H_{l,j}\hat{\mathbf{R}}^T_l\mathbf{n}_{l,j}$ associated with the largest equalizer output SNR. We compute the equalizer $\mathbf{g}_{l-1} \stackrel{\text{def}}{=} \mathbf{n}_l$ or $\mathbf{g}_{l-1} \stackrel{\text{def}}{=} \frac{1}{\sigma_s\sqrt{\mathbf{n}^H_l\left(\hat{\mathbf{R}}_l - \widehat{\sigma^2_b}\mathbf{I}_{cl}\right)^T\mathbf{n}_l}}\mathbf{n}_l$ depending on whether we want to achieve identification with phase and amplitude ambiguity or with phase ambiguity only.

C. Robustness to Order Over Estimation

When the channel order is over estimated, the noise subspace dimension is (under)estimated to be $\hat{w} < w$, and the vectors $\mathbf{n}^{(\hat{w}+1)}_{l,j}, \cdots, \mathbf{n}^{(w)}_{l,j}, j = 1, 2$ are wrongly classified in the signal subspace. However, unlike the LP and OPD algorithms, the associated (small) singular values will not be inverted, and the algorithm will be able to provide estimates that well approximate zero-padded versions of the channel response. However, as the proposed algorithm has fewer vectors $\mathbf{n}^{(i)}_{l,j}, j = 1, 2$ available than actually exist, the set of the estimates $\hat{\mathbf{h}}^{(i)}_m$ is restricted and the identification error is higher than it would be if the exact order were known. This loss in performance can be compensated for by increasing the smoothing factor, and hence the number of candidate estimates.

D. The Algorithm

The algorithm can be summarized as follows:

1. Choose an order m superior to the exact channel order.
2. Choose a smoothing factor $l \geq m$.
3. Compute the estimate $\hat{\mathcal{R}}_l$ of $\mathcal{R}_l$.
4. Estimate the noise power $\widehat{\sigma^2_b}$ as the average of[2] the $(c-1)(l+1) - m$ smallest eigenvalues of $\hat{\mathbf{R}}_{l+1}$.
5. For $i = 1, \cdots, w$, $w \stackrel{\text{def}}{=} (c-1)l - m + 1$, compute the cl-dimensional left singular vector $\mathbf{n}^{(i)}_{l,1}$ and right singular vector $\mathbf{n}^{(i)}_{l,2}$ associated with the i-th smallest singular value of $\hat{\mathcal{R}}_l - \widehat{\sigma^2_b}\left(\mathbf{J}_l \otimes \mathbf{I}_c\right)$.
6. EPC,
   (a) Compute $\mathbf{f}_j, j = 1, 2$ as the eigenvector associated with the largest eigenvalue of $\left[\mathbf{n}^{(1)}_{l,j} \cdots \mathbf{n}^{(w)}_{l,j}\right]^T \hat{\mathbf{R}}^T_l \left[\mathbf{n}^{(1)}_{l,j} \cdots \mathbf{n}^{(w)}_{l,j}\right]^*$.
   (b) Let $\mathbf{n}_l \stackrel{\text{def}}{=} \left[\mathbf{n}^{(1)}_{l,1} \cdots \mathbf{n}^{(w)}_{l,1}\right]^* \mathbf{f}_1$ if $\mathbf{f}^H_1 \left[\mathbf{n}^{(1)}_{l,1} \cdots \mathbf{n}^{(w)}_{l,1}\right]^T \hat{\mathbf{R}}^T_l \left[\mathbf{n}^{(1)}_{l,1} \cdots \mathbf{n}^{(w)}_{l,1}\right]^* \mathbf{f}_1 \geq \mathbf{f}^H_2 \left[\mathbf{n}^{(1)}_{l,2} \cdots \mathbf{n}^{(w)}_{l,2}\right]^T \hat{\mathbf{R}}^T_l$ $\mathbf{n}_l \stackrel{\text{def}}{=} \left[\mathbf{n}^{(1)}_{l,2} \cdots \mathbf{n}^{(w)}_{l,2}\right]^* \mathbf{f}_2$ otherwise.
   (c) Compute $\mathbf{g}_{l-1} \stackrel{\text{def}}{=} \mathbf{n}_l$ (phase and amplitude ambiguity) or $\mathbf{g}_{l-1} \stackrel{\text{def}}{=} \frac{1}{\sigma_s\sqrt{\mathbf{n}^H_l\left(\hat{\mathbf{R}}_l - \widehat{\sigma^2_b}\mathbf{I}_{cl}\right)^T\mathbf{n}_l}}\mathbf{n}_l$ (phase ambiguity).
   (d) Deduce the channel estimate $\hat{\mathbf{h}}_m$ using (3) or (4) depending on $\mathbf{n}_l$ being a left or right singular vector.
7. CMC,
   (a) Construct the set $\{\mathbf{n}^{(i)}_l\} = \{\mathbf{n}^{(i)}_{l,1}\} \cup \{\mathbf{n}^{(i)}_{l,2}\}$.
   (b) For each $\mathbf{n}^{(i)}_l$, $i = 1, \cdots, 2w$, estimate the ZF equalizer

[2] We don't use $\hat{\mathcal{R}}_l$ as $\mathbf{R}_l - \sigma^2_b\mathbf{I}$ may be full rank (if l equals the exact channel order and $c = 2$) and hence, does not allow us to estimate the noise power.

$\mathbf{g}_{l-1}^{(i)} \stackrel{\text{def}}{=} \frac{1}{\sigma_s \sqrt{\mathbf{n}_l^{(i)H}\left(\hat{\mathbf{R}}_l - \widehat{\sigma_b^2}\mathbf{I}_{cl}\right)\mathbf{n}_l^{(i)}}}\mathbf{n}_l^{(i)*}$ (phase ambiguity) or $\mathbf{g}_{l-1}^{(i)} \stackrel{\text{def}}{=} \mathbf{n}_l^{(i)*}$ (phase and amplitude ambiguity).

(c) For each $\mathbf{g}_{l-1}^{(i)}$, deduce the estimate $\hat{\mathbf{h}}_m^{(i)}$ of the channel response using (3) or (4), depending on $\mathbf{n}_l^{(i)}$ being a left or right singular vector.

(d) Choose $\hat{\mathbf{h}}_m$ such that $\hat{\mathbf{h}}_m = \text{argmin}_i \left(\left\|\hat{\mathbf{R}}_l - \widehat{\sigma_b^2}\mathbf{I}_{cl} - \sigma_s^2 \mathcal{T}_l\left(\hat{\mathbf{h}}_m^{(i)}\right)\mathcal{T}_l^H\left(\hat{\mathbf{h}}_m^{(i)}\right)\right\|\right)$ (phase ambiguity), $\hat{\mathbf{h}}_m = \text{argmax}_i \frac{\left|\text{Vec}\left(\hat{\mathbf{R}}_l - \widehat{\sigma_b^2}\mathbf{I}_{cl}\right)^H \text{Vec}\left(\mathcal{T}_l\left(\hat{\mathbf{h}}_m^{(i)}\right)\mathcal{T}_l^H\left(\hat{\mathbf{h}}_m^{(i)}\right)\right)\right|}{\left\|\mathcal{T}_l\left(\hat{\mathbf{h}}_m^{(i)}\right)\mathcal{T}_l^H\left(\hat{\mathbf{h}}_m^{(i)}\right)\right\|_F}$ (phase and amplitude ambiguity).

V. Simulations

A set of simulations has been conducted to test the proposed algorithm w.r.t. different observation parameters (SNR, sample size, smoothing factor), and more particularly its robustness to order over estimation and its performance compared to existing SOS based blind algorithms, namely, the SS, LP and OPD algorithms.

With respect to the targeted applications (equalization of communication channels), the identification problem will be considered as perfectly solved whenever the solution matches the exact channel response up to an unknown complex factor and an unknown number of zero trailing terms. Hence, for an m'-th order channel estimate $\hat{\mathbf{h}}_{m'}$, with $m' \geq m$, we suggest the following identification error measure, inspired by that proposed in [15], which we will continue to call Mean Square Error (MSE)

$$\text{MSE}(\hat{\mathbf{h}}_{m'}) \stackrel{\text{def}}{=} \min_{\substack{\beta \\ m_1+m_2=m'-m}} \left(\frac{\left\|\hat{\mathbf{h}}_{m'} - \beta\begin{bmatrix}\mathbf{0}_{cm_1,1}\\ \mathbf{h}_m \\ \mathbf{0}_{cm_2,1}\end{bmatrix}\right\|}{\|\mathbf{h}_m\|}\right)^2$$

where β stands for a complex constant.

For the proposed algorithm, such an m'-th order channel estimate is expected to match, up to a constant, either $\begin{bmatrix}\mathbf{0}_{c(m'-m),1}\\ \mathbf{h}_m\end{bmatrix}$ or $\begin{bmatrix}\mathbf{h}_m\\ \mathbf{0}_{c(m'-m),1}\end{bmatrix}$. The above proposed error measure simplifies to the following : $\text{MSE}(\hat{\mathbf{h}}_{m'}) \stackrel{\text{def}}{=} \frac{1}{\|\mathbf{h}_m\|^2}\min\left(\min_\beta\left\|\hat{\mathbf{h}}_{m'} - \beta\begin{bmatrix}\mathbf{0}_{c(m'-m),1}\\ \mathbf{h}_m\end{bmatrix}\right\|^2, \min_\beta\left\|\hat{\mathbf{h}}_{m'} - \beta\begin{bmatrix}\mathbf{h}_m\\ \mathbf{0}_{c(m'-m),1}\end{bmatrix}\right\|^2\right)$. This can be proved to be equal to $\text{MSE}(\hat{\mathbf{h}}_{m'}) = 1 - \left(\frac{\max\left(\left|\left[\mathbf{0}_{1,c(m'-m)}\mathbf{h}_m^H\right]\hat{\mathbf{h}}_{m'}\right|, \left|\left[\mathbf{h}_m^H\mathbf{0}_{1,c(m'-m)}\right]\hat{\mathbf{h}}_{m'}\right|\right)}{\|\mathbf{h}_m\|\left\|\hat{\mathbf{h}}_{m'}\right\|}\right)^2$. This identification error was, each time, averaged over 100 Monte Carlo realizations.

We tested the proposed algorithm under the same conditions as in [4]. The SIMO channel coefficients are ($c = 4$ and $m = 4$) $\mathbf{h}(0) = [-0.049 + i\ 0.359 \quad 0.443 - i\ 0.0364 \quad -0.211 - i\ 0.322 \quad 0.417 + i\ 0.030]^T$, $\mathbf{h}(1) = [0.482 - i\ 0.569 \quad 1 \quad -0.199 + i\ 0.918 \quad 1]^T$, $\mathbf{h}(2) = [-0.556 + i\ 0.587 \quad 0.921 - i\ 0.194 \quad 1 \quad 0.873 + i\ 0.145]^T$, $\mathbf{h}(3) = [1 \quad 0.189 - i\ 0.208 \quad -0.284 - i\ 0.524 \quad 0.285 + i\ 0.309]^T$ and $\mathbf{h}(4) = [-0.171 + i\ 0.061 \quad -0.087 - i\ 0.054 \quad 0.136 - i\ 0.190 \quad -0.049 + i\ 0.161]^T$. The conditioning w.r.t. inversion of the processed correlation matrices is well described by the lowest non-zero singular value σ_{min}, given by $\sigma_{min}\left(\mathbf{R}_4 - \mathbf{R}_4^b\right) = 0.0642$ and $\sigma_{min}\left(\mathcal{R}_4 - \mathcal{R}_4^b\right) = 0.1985$. The SIMO channel is driven by a source of unit-variance i.i.d. 4-QAM symbols and corrupted by unit-variance AWG noise. The SNR is defined as $SNR \stackrel{\text{def}}{=} \frac{\mathbf{E}\left(\|\mathbf{x}(n)\|^2\right)}{\mathbf{E}\left(\|\mathbf{b}(n)\|^2\right)} = \frac{\sigma_s^2\|\mathbf{h}_m\|^2}{c\sigma_b^2}$.

Fig. 2.a and Fig. 2.b compare the proposed algorithm, with both the CMC and EPC criteria, to the SS, LP and OPD algorithms w.r.t. the number of channel observations and w.r.t. the SNR, respectively. As the SS, LP and OPD algorithms are not robust to order over estimation, this comparison is done assuming the exact order to be known. The proposed algorithm has better performance than the LP and OPD algorithms. Even though outperformed by the SS algorithm in this case, the proposed algorithm, interestingly, shows good performance at low SNR.

The more important issue of channel order over estimation is depicted in Fig. 3. As the SS, LP and OPD algorithms fail to identify the channel under such conditions, only results from the proposed algorithm are reported. For different over estimation values, the simulations show low estimation error, from 200 samples only. Fig. 3 shows also that this estimation error can be further lowered by increasing the smoothing factor. This is also illustrated in Fig. 4 and Fig. 5. This is especially useful when the order is over estimated as initial ($l = m'$) estimation errors can be high. However, for the EPC criterion, the smoothing factor should not be chosen excessively large. The best results are obtained with $l = m' + 2$ where m' is the assumed channel order.

To match more practical situations, we test the existing and the proposed algorithms with the channel response from [16, Table III] which corresponds to a three-ray, long delay (delays at 0, 0.5 and 3 baud periods) multipath channel. The SIMO channel coefficients are ($c = 4$ and $m = 5$) $\mathbf{h}(0) = [0.0222 - i\ 0.0031 \quad -0.1065 + i\ 0.0651 \quad 0.3757 - i\ 1.2429 \quad -0.7860 - i\ 0.4996]^T$, $\mathbf{h}(1) = [0.5236 - i\ 1.9480 \quad -0.9114 - i\ 0.9867 \quad 0.2682 - i\ 1.2279 \quad -0.2713 - i\ 0.8143]^T$, $\mathbf{h}(2) = [-0.0683 + i\ 0.0095 \quad 0.3268 - i\ 0.1998 \quad -0.1083 + i\ 0.4256 \quad 0.2297 + i\ 0.1934]^T$, $\mathbf{h}(3) = [0.0222 - i\ 0.0031 \quad -0.1065 + i\ 0.0651 \quad 0.0267 - i\ 0.2953 \quad -0.0658 - i\ 0.1874]^T$, $\mathbf{h}(4) = [-0.0812 - i\ 0.0977 \quad 0.1887 - i\ 0.1856 \quad -0.0902 + i\ 0.0914 \quad 0.1788 - i\ 0.0320]^T$ and $\mathbf{h}(5) = [0.0085 - i\ 0.0012 \quad -0.0406 + i\ 0.0249 \quad 0.0472 - i\ 0.0887 \quad -0.0955 - i\ 0.0133]^T$. The corresponding lowest non-zero singular values $\sigma_{min}\left(\mathbf{R}_5 - \mathbf{R}_5^b\right) = 0.0042$ and $\sigma_{min}\left(\mathcal{R}_5 - \mathcal{R}_5^b\right) = 0.0171$ indicate that the processed (shifted and standard) correlation matrices are rather poorly conditioned compared to those associated with the channel corresponding to Fig. 2 to Fig. 5. Simulations results relative to this channel are summarized in Fig. 6. It shows that only the proposed algorithm (with the EPC criterion) and the SS algorithm (but only in the exact order case) are able to achieve low identification errors. This is still true when the channel order is over estimated (by one tap). The fact that the CMC criterion behaves better in the over estimated case than in the exact order case is not meaningful as estimation errors are unpractical in both cases.

VI. Discussion

As shown through simulations (Sec. V), the proposed algorithm has performances intermediate between the SS and the LP algorithms when the exact channel order is known. Unlike the SS algorithm, it requires estimating the noise power, which leads to a supplementary estimation error. The SS algorithm is still the only one to exactly estimate the channel response from noiseless finite observation samples, contrary to the proposed algorithm, which, hence, is not deterministic. This explains the threshold observed in Fig. 2.b. The improved performance of the proposed algorithm w.r.t. the LP algorithm can be justified in different ways. First, the proposed algorithm uses singular vectors, while the LP algorithm explicitly (pseudo)-inverts the correlation matrix to solve the YW system. Second, like the LP algorithm, the proposed algorithm estimates a ZF equalizer prior to channel response estimation. However, unlike the LP algorithm, the proposed algorithm provides a set of estimates and hence has a better *chance* to achieve a lower estimation error. The number of *candidates* can be increased by increasing the smoothing factor, improving, as verified through simulations, the algorithm performance. The decrease in the estimation error when the smoothing factor increases, however, is only global (Fig. 3). This is due to the fact that the selection criteria, CMC and EPC, are both sub-optimal w.r.t. the MSE criterion. Hence, as verified through tests, they happen to select an estimate that does not achieve the lowest MSE on the channel response.

The CMC criterion shows performance slightly better than that obtained by EPC. We believe that this might be explained by the *local behavior* of the proposed channel estimation technique. Local behavior refers to the instantaneous performance of any estimation algorithm achieved in a single trial without any statistical averaging [17]. In fact, although the noise power is the same in all antenna sensors, the instantaneous noise realization is particular to each of them and thus weighting differently the sensor outputs leads to different *local* estimation results. In the CMC criterion, we select the best estimate among a set of channel estimates that have different local behaviors.

Notice that, as long as ZF equalization is concerned, the proposed algorithm can provide a set of minimum or maximum delay equalizer of any desired order, contrarily to the LP algorithm which provides only one (m-th order zero delay) equalizer.

While the proposed algorithm has been proved to be (truly) robust to order over estimation, its performance is still dependent on the conditioning of the *shifted* correlation matrix. This sensitivity to ill conditioning is a common drawback with the existing algorithms [18].

In fact, the proposed algorithm corrects a major drawback of the existing algorithms, which are unable to (accurately) estimate the channel response from a finite observation set when its order is over estimated. The proposed method ensures that any channel with good *diversity* (i.e., whose exact noise-free correlation matrix is well conditioned) can be well estimated from a finite observation set and with an assumed order arbitrarily greater than its exact order. However, if the channel has poor diversity, the performance of the proposed method, as well as the existing ones, will degrade. This happens, for example, when the channel response contains small tails [19], [7]. Effective order detection [20] was proposed and shown to be relevant in many situations. The issue of robustness to poor diversity remains a challenging one.

VII. Conclusion

We proposed a novel second order statistics based blind identification algorithm that is truly robust to channel order over estimation. By truly, we mean that the channel response can be *well* estimated when an arbitrarily over estimated value of the channel order is known and when a finite number of noise corrupted observation samples is available. This is qualified as *true robustness* in comparison with the Linear Prediction algorithm which, in some situations, is able to handle over estimated channel order obtained by statistical criteria, such as MDL and AIC. In addition, the proposed algorithm is shown to outperform the LP and OPD algorithms. Its performance can be enhanced by increasing the size of the processed correlation matrix (the smoothing factor) at a fixed observation size.

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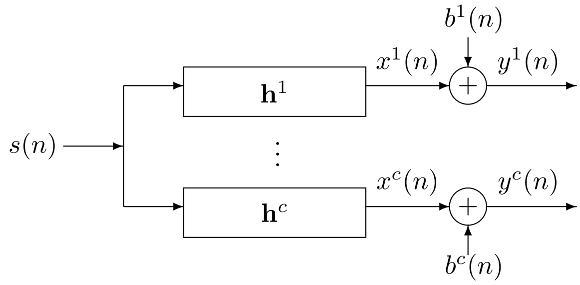
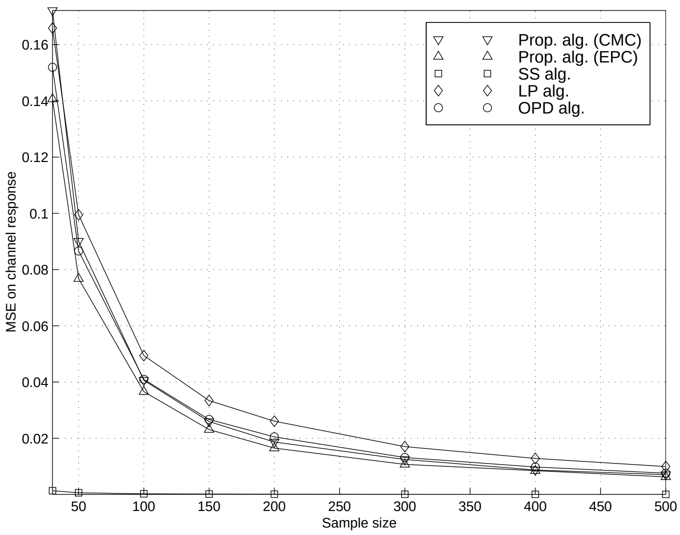
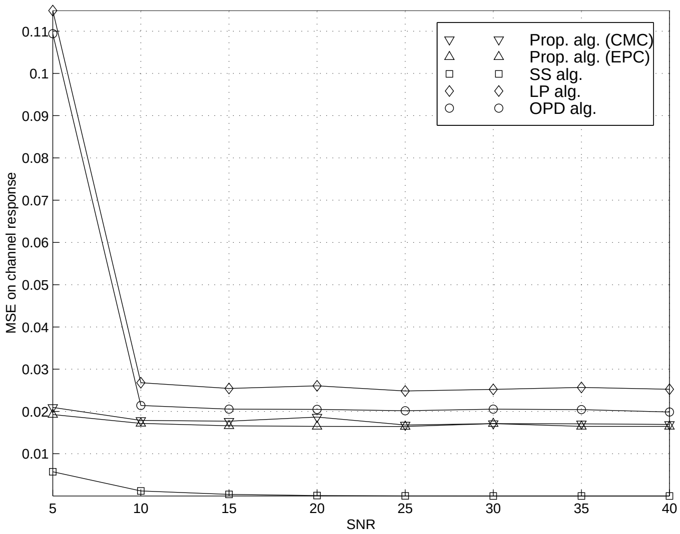


Fig. 1. Single input multiple output channel

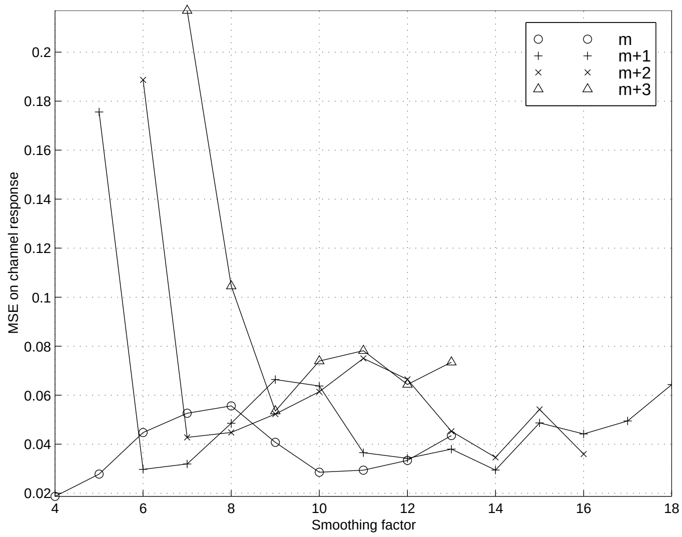


(a) $SNR = 20$ dB.

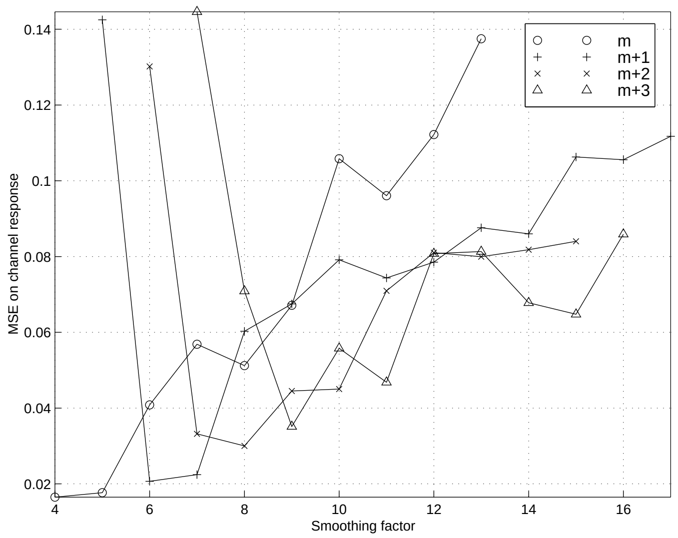


(b) Sample size= 200.

Fig. 2. Algorithms comparison. $l = m$. Exact order known.

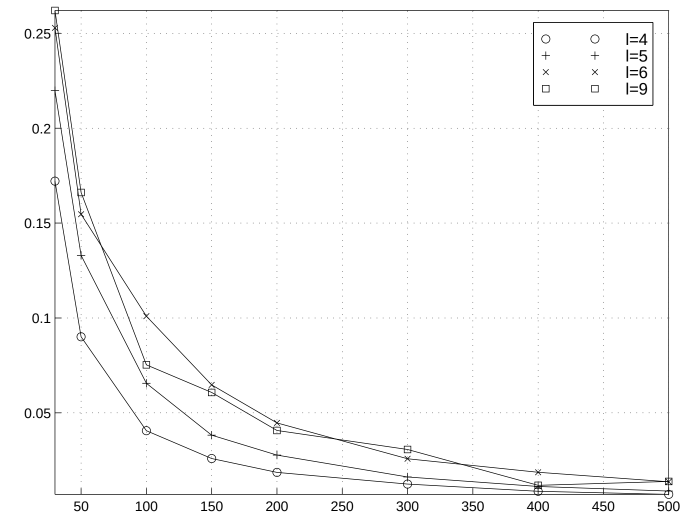


(a) CMC

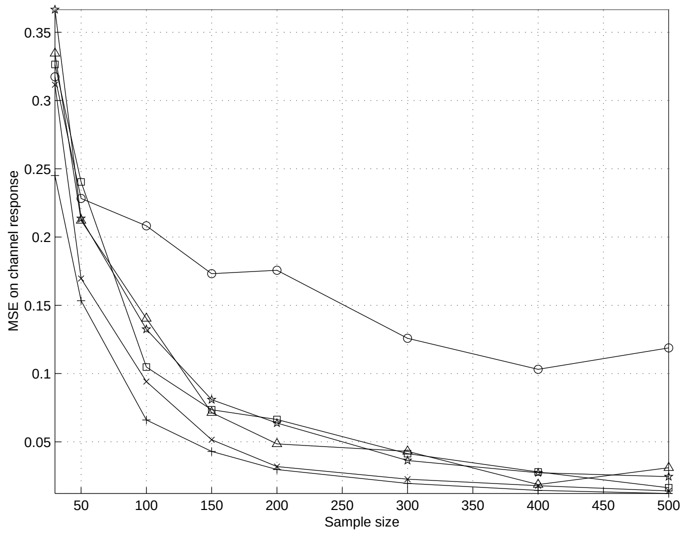


(b) EPC

Fig. 3. Channel order over estimation. The legend shows the assumed channel order. Sample size = 200, $SNR = 20$ dB.

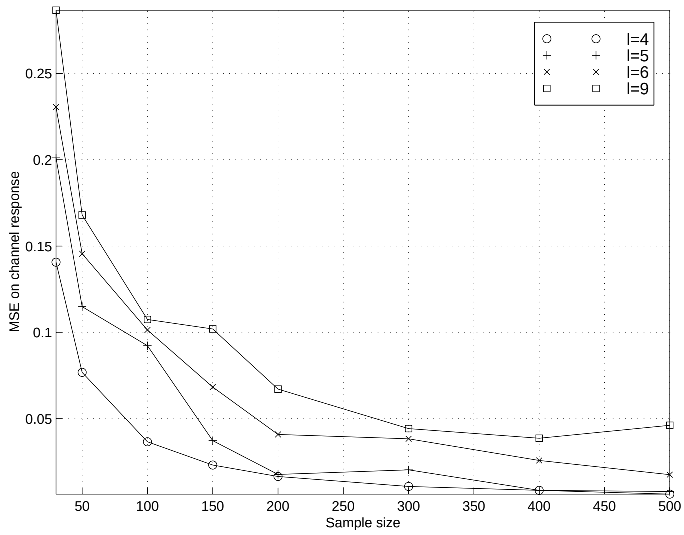


(a) Exact order

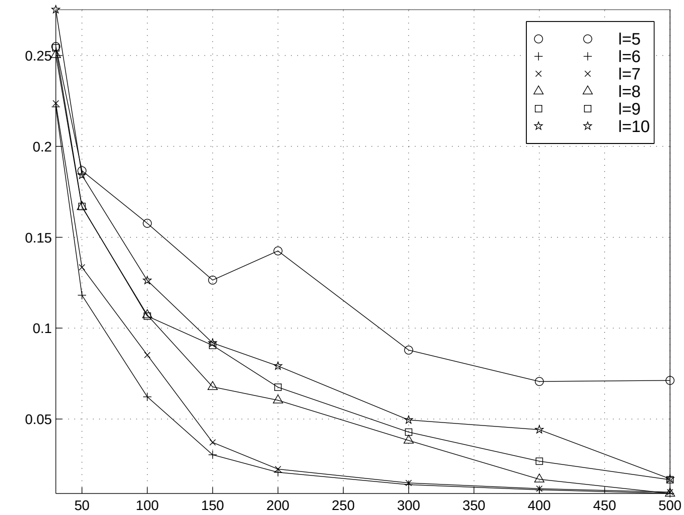


(b) Over estimated order (by one tap)

Fig. 4. Smoothing factor effect (CMC). $SNR = 20$



(a) Exact order



(b) Over estimated order (by one tap)

Fig. 5. Smoothing factor effect (EPC). $SNR = 20$

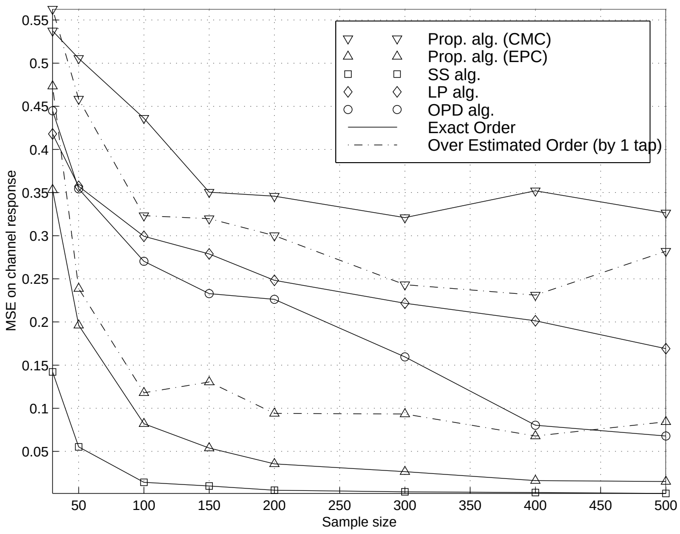


Fig. 6. Algorithms comparison. Badly conditioned channel. Exact and over estimated order.

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