Estimation of 2-D Direction of Arrival with an Extended Correlation Matrix

Ferid Harabi, Hatem Changuel, Ali Gharsallah

feridsamia@yahoo.fr, Changuelh@yahoo.fr, ali.gharsallah@fstrnu.tn

groupe d'électronique, laboratoire de physique de la matiére molle, Faculty of sciences, Tunis,2092, Tunisia.

Abstract— **This paper presents a new subspace-based 2-D direction of arrival (DOA) estimation algorithm for narrow-band sources with high-resolution localization capabilities. DOA estimation is achieved by using the noise-subspace eigenvectors of a new extended correlation matrix (ECM). A 2-L shape antenna array is proposed. Unlike common planar and circular arrays, the novel antenna array with this special geometry requires no pair matching between the azimuth and elevation angle estimation also this key can remove the drawbacks of estimation-failure problems. The performance of the proposed approach is examined by a simulation study. The simulation results show a good estimate performance.**

Index Terms— **DOA estimation, extended correlation matrix, 2-L shape antenna array**

I. INTRODUCTION

Direction of arrival (DOA) estimation from the outputs of an array of sensors is an important and well-studied problem with many applications in radar, sonar, and wireless communications. Recent applications include array processing for wireless mobile communications at the base station for increasing the capacity and quality of these systems [1,4,5].

A large number of DOA estimation algorithms and analytical performance bounds have been developed. Beamformer method which scans the main beam of array antenna is the most fundamental technique. The other techniques based on eigenvalue decomposition of array input correlation matrix are the Min-Norm method, MUSIC (Multiple Signal Classification) [11], and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [12,13] have are attracted attention in the mobile communication system. The maximum likelihood estimator [2] provides optimum parameter estimation. Its computational complexity is extremely demanding. However, MUSIC is most well-known for its super-resolution capability and simplicity, in addition, it has less computational complexity than maximum likelihood methods.

The problem of estimating the two-dimensional directions of arrival, namely, the azimuth and elevation angles, of multiple sources was the topic of several researches [8,10].

In this paper, we present a new subspace-based 2-D direction of arrival estimation method based on the proprieties of an extended covariance matrix developed by Horn and Johnson in [14].

The DOA estimation performance of an array strongly depends on the number and locations of the array elements. A number of researchers have considered the design of arrays to achieve or optimize desired performance goals. Much of the array design literature is devoted to linear arrays [3]-[6]. For planar arrays, performance comparisons of some common array geometries are given in [7]-[9].

The number and locations of the elements in an array strongly affect the DOA estimation performance of the array system. That is why, in this paper we consider a 2-L shape array geometry design for "good" DOA estimation performance.

Tayem and Kwon in [15] have demonstrated the reason why the novel antenna array with this special geometry requires no pair matching between the azimuth and elevation angle estimation. They have also explained that this key can remove the drawbacks of estimation-failure problems, unlike common planar and circular arrays.

The rest of the paper is organized as follows. The data model is presented in section II. The new correlation matrix is presented in section III. Section IV presents the 2-D direction of arrival estimation algorithm. Section V shows simulation results and section VI makes conclusions.

II. DATA MODEL

Consider the two-L-shape uniform linear array (ULAs) in the x-z and the y-z planes shown in Fig.1 with inter-element equals d, using three array elements placed on the x, y and z axes. Each linear array consists of N elements. The element placed at the origin is common for referencing purposes.

Assume that K signals *s*(t) from far-field uncorrelated sources impinge on the array with the same wavelength λ. Such that k[th] source has an elevation angle θ_k and an azimuth angle ϕ_k, $k=1,...,K$.

We put the complex base-band representation of the signal received by the nth element of one subarray as $x_n(t)$ ($n = 1, 2. . . N$), the signal sources are far apart from the subarray.

The subarray output vector at the snapshot *t* is then given by:

$$X(t) = (x_1(t), x_2(t),...., x_N(t))^T$$
$$= \sum_{k=1}^{K} a(\theta_k, \phi_k)s_k(t) + \eta(t), \text{t=1,...,} P \qquad (1)$$

"*T* " is the transpose, $\eta(t)$ is an *N*-dimensional complex white noise vector with mean zero and covariance $\sigma^2 I$, *I* is the identity matrix of size *N* and $a(\theta_k, \phi_k)$ is the steering vector defined by :

$$a(\theta_k, \phi_k) = [(1, e^{-j\varphi_{k,2}}, ..., e^{-j\varphi_{k,N}})]^T \qquad (2)$$

with $\varphi_{k,n}$ depends on the position and the geometry of the subarray.
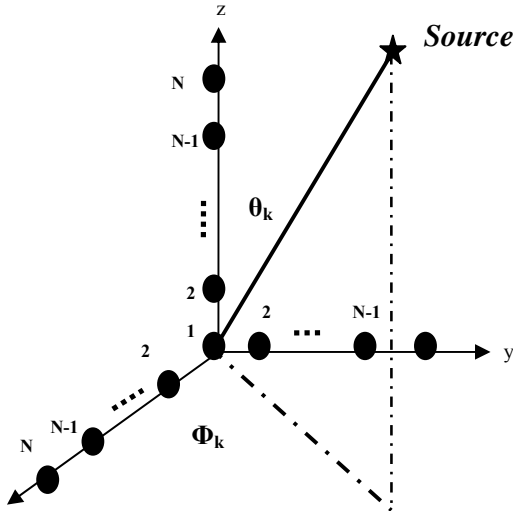

Fig.1: the 2-L-shape array configuration used for the joint azimuth and elevation (ϕ, θ) DOA estimation.

A- The steering vector of the z axes subarray:

Let $X_z(t)$ be the *Nx1* signal received at the linear subarray in the z axes at the snapshot *t*.

$$X_z(t) = (x_{1z}(t), x_{2z}(t),...., x_{Nz}(t))^T$$
$$= \sum_{k=1}^{K} a_z(\theta_k)s_k(t) + \eta_{kz}(t), \text{t=1,.....,} P \qquad (3)$$

where

$$a_z(\theta_k) = [(1, e^{-j\varphi_{zk,2}}, ..., e^{-j\varphi_{zk,N}})]^T \qquad (4)$$

and $\quad \varphi_{zk,n} = \dfrac{2\pi(n-1)d\cos\theta_k}{\lambda} \qquad (5)$

θ_k is the elevation angle of the kth source signal. $\eta_{kz}(t)$ is the additive White Gaussian noise of the kth source signal in the z axes at the snapshot *t*.

B- The steering vector of the x axes subarray:

Let $X_x(t)$ be the *Nx1* signal received at the linear subarray in the x axes at the snapshot *t*.

$$X_x(t) = (x_{1x}(t), x_{2x}(t),.... x_{Nx}(t))^T$$
$$= \sum_{k=1}^{K} a_x(\theta_k, \phi_{kx})s_k(t) + \eta_{kx}(t), \text{t=1,.....,} P \qquad (6)$$

where

$$a_x(\theta_k, \phi_{kx}) = [1, e^{-j\varphi_{xk,2}}, ..., e^{-j\varphi_{xk,N}})]^T \qquad (7)$$

and $\quad \varphi_{xk,n} = \dfrac{2\pi(n-1)d\sin\theta_k\cos\phi_{k,x}}{\lambda} \qquad (8)$

C- The steering vector of the y axes subarray:

Let $X_y(t)$ be the *Nx1* signal received at the linear subarray in the y axes at the snapshot *t*.

$$X_y(t) = (x_{1y}(t), x_{2y}(t),...., x_{Ny}(t))^T$$
$$= \sum_{k=1}^{K} a_y(\theta_k, \phi_{ky})s_k(t) + \eta_{ky}(t), \text{t=1,.....,} P \qquad (9)$$

where

$$a_y(\theta_k, \phi_{ky}) = [1, e^{-j\varphi_{yk,2}}, ..., e^{-j\varphi_{yk,N}}]^T \qquad (10)$$

$$\text{and} \quad \varphi_{yk,n} = \frac{2\pi(n-1)d\sin\theta_k\sin\phi_{k,y}}{\lambda} \qquad (11)$$

III. PREVIOUS WORK

III.1. The MUSIC algorithm:

Let us review the procedure of the MUSIC algorithm [11]. The correlation matrix of sensor observations $X(t)$ is calculated:

$$R = X(t).X(t)^H \qquad (12)$$

where H represents a conjugate transpose. In practice, only a sample covariance matrix is available, i.e., an estimate of R based on a finite number (P) of data samples or snapshots

$$\hat{R} = \frac{1}{P}\sum_{j=1}^{P} X(t_j).X(t_j)^H \qquad (13)$$

Then, obtain the eigenvalues decomposition $\hat{R} = V \Lambda V^H$, $V = [v_1, ..., v_N]$, $\Lambda = diag[y_1, ..., y_N]$, where $\mathbf{v}_k$ is an eigenvector (N- dimensional column vector) and y_k is the eigenvalue of $\mathbf{v}_k$ sorted as $y_1 \geq ... \geq y_K$. The K points where the function $U(\theta) = \sum_{k=K+1}^{N} |v_k^H a(\theta)|^2$ approaches zero correspond to the directions $\theta_1,..., \theta_K$ of the source signals.

If the number K of sources is smaller than the number N of sensors, all the signal components are represented in the signal subspace spanned by the first K eigenvectors $v_1,...,v_N$, and the remaining $N-K$ eigenvectors $v_{K+1},...,v_N$ represents the noise subspace. The signal subspace and the noise subspace are orthogonal to each other since they are spanned by different eigenvectors.

The subspace spanned by the K steering vectors $a(\theta_1),..., a(\theta_K)$ is also the signal subspace. When θ coincides with one of the source directions $\theta_1,..., \theta_K$, the steering vector $\mathbf{a}(\theta)$ and the noise subspace $\mathbf{v}_{K+1},...,\mathbf{v_N}$ are orthogonal, and therefore $U(\theta)$ approaches zero. This is why source directions can be estimated using $U(\theta)$. For the noise subspace to exist, the number N of sensors should be larger than the number K of sources. Thus, the MUSIC algorithm is applicable for mixtures of up to $N - 1$ signals.

III.2. The new extended correlation matrix:

In [14] a new correlation matrix is presented as:

$$G^\varphi = \hat{R} + m \, a(\varphi)a(\varphi)^H \qquad (14)$$

where $\hat{R}$ is given in (13), $a(\varphi)$ is $1 \times N$ array response vector for direction φ given in (2) and m is a positive scalar. The matrix G^φ is also positive definite and if μ_k and e_k denote eigenvalues and eigenvectors of G^φ respectively, k=1,...,N,
Then it can be shown that while m in (14) is a positive scalar we have [14]:

$$\mu_k \geq y_k \qquad \text{k=1,...,N} \qquad \forall\varphi \qquad (15)$$

Another important property of G^φ is that when φ in (14) is set to one of the source directions, i.e., $\varphi = \theta_k$ for some $1 \leq ki \leq K$, then it can be shown that [14]

$$\mu_k \geq y_k \qquad k = 1,..., K$$
$$\mu_k = y_k = \sigma_n^2 \qquad k = K+1,..., N \qquad (16)$$

IV. THE NEW DIRECTION OF ARRIVAL ESTIMATION ALGORITHM

We can note that except the actual source directions no other value of φ have the properties showed in (15) and (16). The property stated in (16) does not depend on the value of scalar m of (14) explicitly and will hold while m is positive. Based on the above proprieties of the extended covariance matrix G^φ and the 2-L shape arrays antenna an algorithm for estimating the direction of sources can be formed with the following steps:

A- Estimate of the elevation angle using the array in the z axes:

1- From P data samples compute $\hat{R}_z$ using the steering vector calculated in (4).

2- Compute $\hat{y}_{kz}$, k=1,...,N, the eigenvalues of $\hat{R}_z$ that are in decreasing order.

3- Choose a positive m and compute G_z^φ for all possible values of φ.

4- For each value of φ, compute μ_{kz}^φ, k=1,...,N the eigenvalues of G_z^φ and calculate the function $D_z(\varphi)$ as:

$$D_z(\varphi) = \frac{1}{\displaystyle\sum_{k=K+1}^{N}\left(\mu_{kz}^{\varphi} - \hat{y}_{kz}\right)} \qquad (17)$$

5- Find all the values of φ that maximize $D_z(\varphi)$. Elevation angles of sources $\theta_{1,\dots}$, θ_K are those values of φ that correspond to K largest maximums.

B- Estimate of the x and y axes array components of the azimuth angle:

1- Using elevation angles of sources estimated previously and the steering vector of the signal received at the antenna subarray placed on the x axes calculated in (6), we compute $\hat{R}_x$ from N data samples.

2- Compute $\hat{y}_{kx}$, k=1,…,N, the eigenvalues of $\hat{R}_x$ that are in decreasing order.

3- Choose a positive m and compute G_x^{φ} for all possible values of φ.

4- For each value of φ, compute μ_{kx}^{φ}, k=1,…,N, the eigenvalues of G_x^{φ} and calculate the function $D_x(\varphi)$ in (18).

$$D_x(\varphi) = \frac{1}{\displaystyle\sum_{k=K+1}^{N}\left(\mu_{kx}^{\varphi} - \hat{y}_{kx}\right)} \qquad (18)$$

5- Find all the values of φ that maximize $D_x(\varphi)$. The x axes array component of the azimuth angles of sources $\hat{\phi}_{1x},\dots,\hat{\phi}_{Kx}$ are those values of φ that correspond to K largest maximums.

In the same way, we estimate the y axes array components of azimuth angles $\hat{\phi}_{1y},\dots,\hat{\phi}_{Ky}$ using the signal received at the antenna subarray placed on the y axes using the steering vector calculated in (10).
The azimuth angle estimation $\hat{\phi}_k$ can be written [15] as:

$$\hat{\phi}_k = \begin{cases} \frac{1}{2}(\hat{\phi}_{kx} + \hat{\phi}_{ky}) & if \quad both \quad \hat{\phi}_{kx} \ and \ \hat{\phi}_{ky} \ are \ real \\\\ \hat{\phi}_{kx} & if \quad \hat{\phi}_{ky} \ is \ complex \\\\ \hat{\phi}_{ky} & if \quad \hat{\phi}_{kx} \ is \ complex \\\\ Failure & if \quad both \quad are \quad complex \end{cases} \qquad (19)$$

V. Simulation results

Computer simulations have been conducted to evaluate the 2-D DOA estimation performance of the proposed method. The parameters used in the simulation are as follows:

The sensors displacement d is taken to be half the wave length of the signal waves. We assume one single direct source K=1, with direction of arrival DOA (θ,ϕ). The additive noise is white Gaussian processes. The number of snapshots per trial is P=200 and 1000 independent trials in total are used.

Figs.2 and 3 show the histogram plots for the joint elevation and azimuth angles, respectively, for a single source with DOA located at (70°,40°) by using the proposed method of the 2-L shape array. We notice in each case that the method gives close elevation and azimuth DOA estimation and the clear peaks appear around (70°) and (40°), respectively.
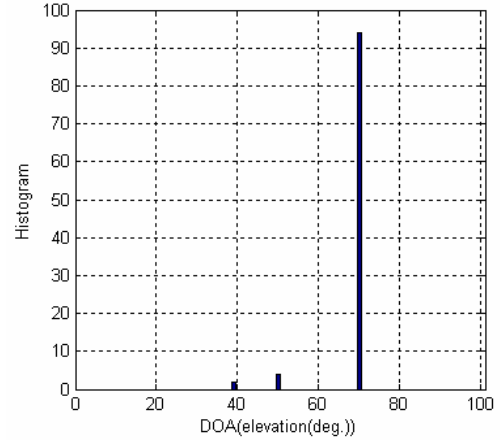

Fig.2: histogram of elevation angle for a single source located at (70°,40°), SNR = 10 dB by the proposed method with 5 elements on the z axes subarray
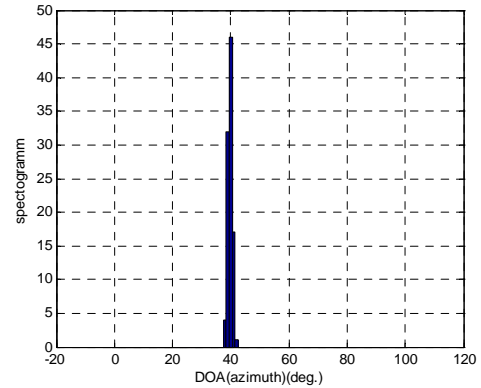

Fig.3: histogram of azimuth angle for a single source located at (70°,40°), SNR = 10 dB by the proposed method with 5 elements on the z axes subarray

Figure 4 prove the performance of the proposed method and shows the spectrogram of the elevation and the azimuth angles for a single source located at (90°,10°) with SNR=10 dB and 5 elements on each subarray.

The 2-L shape configuration shows zero number of estimation failures for any pair of incident DOAs (θ, ϕ) and all trials. The reasons why the proposed antenna array configuration shows no failures are discussed in [15].
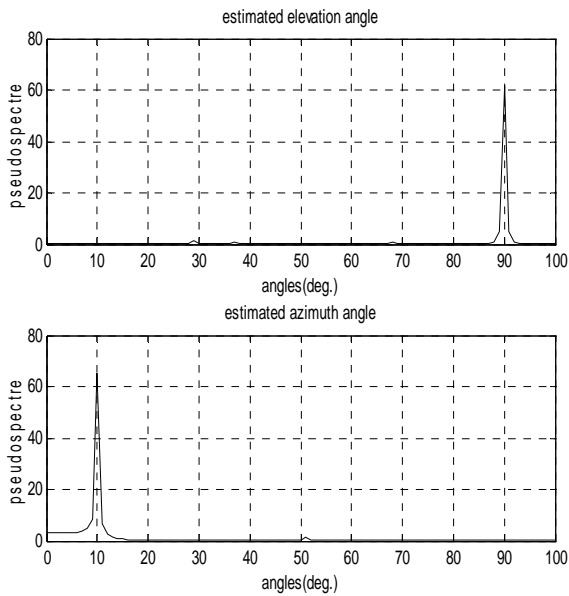

Fig..4. Pseudo spectre of elevation and azimuth angles for a single source located at (90°,10°), SNR = 10 dB by the proposed method with 5 elements on each subarray
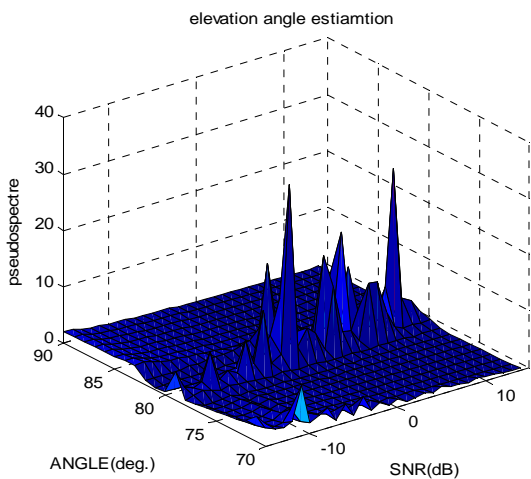

Fig.5. Elevation angle estimation with the proposed method of a single source located at (80°,40°) versus SNRs with 5 elements on the z axes subarray
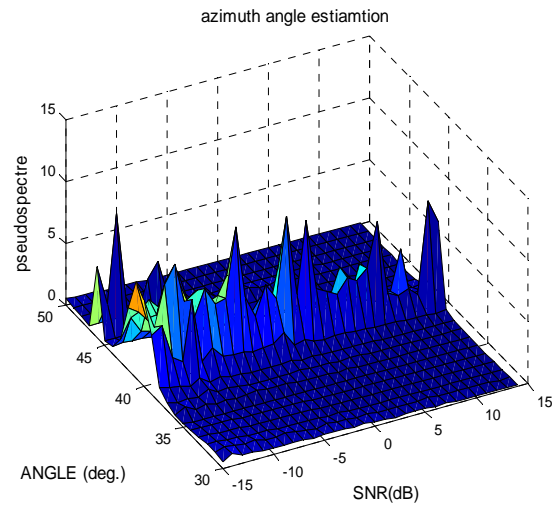

Fig.6. Azimuth angle estimation with the proposed method of a single source located at (80°,40°) versus SNRs with 5 elements on each subarray.

Figs. 5 and 6 show the spectrogram of elevation and azimuth angles, respectively versus SNRs in dB for a single source located at (80,40°). While comparing it to results of MUSIC method form figs 6 and 7 with the 2-L shape array, we observe that it gives good performances with low SNRs for the angle of elevation and we note a success of estimation in low SNRs for the azimuth angle better than MUSIC.
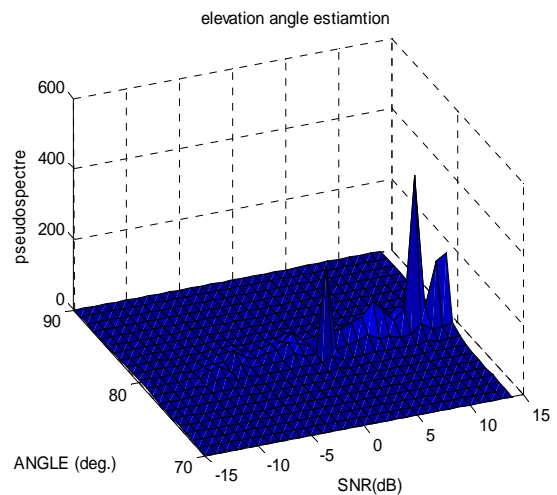

Fig .7. Elevation angle estimation with MUSIC method for a single source located at (80°,40°) versus SNRs with 5 elements on the z axes subarray.
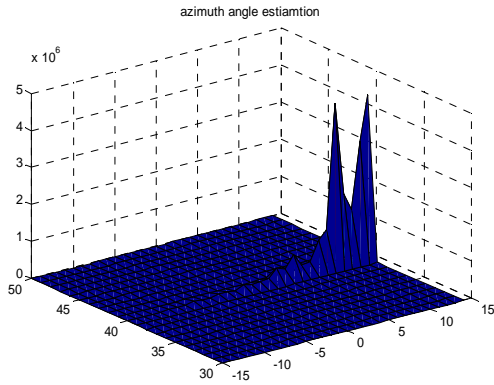
Fig .8. Azimuth angle estimation with MUSIC method for a single source located at (80°,40°) versus SNRs with 5 elements on each subarray.

The root mean square error (RMSE) of the proposed DOA estimation scheme was compared with that of the MUSIC and Propagator algorithms, using the 2-L shape array configuration in Fig.1. The RMSE for the joint DOA estimation is defined as:

$$RMSE = \sqrt{E[(\hat{\theta}_k - \theta_k) + (\hat{\phi}_k - \phi_k)]} \qquad (20)$$

where k represents the source index, $E[X]$ denotes the expectation of a random variable X, and $(\hat{\theta}_k, \hat{\phi}_k)$ are the pair of the elevation and azimuth angle estimates.

Fig. 9 shows the RMSE values of the joint elevation θ and azimuth ϕ DOA estimation versus the SNR in dB for the proposed method, MUSIC and Propagator method, respectively, when source signal arrive form direction located at (90°,70°). The total number of elements is 15 elements for the 2-L shape configuration for all methods. (five elements in each subarray).
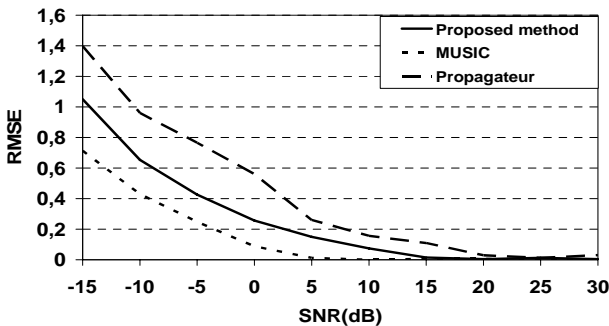

Fig.9. RMSE for a single source located at (90°,70°) with a 2-Lshape array antenna versus SNRs

We observe that the proposed method is 5.1 dB better than the Propagator method with the 2-L shape array and ~3dB less well than the MUSIC method with the same configuration.

VI. CONCLUSIONS

A new method based on an extended correlation matrix with a 2-L shape antenna array configuration was proposed for the 2-D azimuth and elevation angle estimation problem. The proposed 2-D DOA estimation scheme shows a significant improvement. The proposed algorithm is quite general and can be applied to all array configurations. In other words, the proposed scheme does not require any pair matching for the 2-D DOA estimation problems, and the proposed algorithm shows no estimation failure for any pair of azimuth and elevation angles. Simulation results prove that the proposed scheme reduces the estimation error of both the azimuth and elevation angles and it proves a good performance at low SNRs.

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