Agent-based Models and Complex Networks in Risk Management

by

M. Schiegl[1], R.R. Cerchiara[2], C. Lam[3], N. K. Madhav[3], A. J. Mata[4], R. V. McKean[5]

Abstract

In the last decades, the natural sciences have developed models and methods, such as agent-based models or complex networks, to describe and understand complex systems. These methods have begun to spread to actuarial literature and the insurance industry in the early years of the current century. The non-life insurance section (ASTIN) of the International Actuarial Association (IAA) installed the working party "Agent-Based Models, Networks and Cellular Automata in Risk Management" (ANCRM) to shed light on this phenomenon in the actuarial context. The application of these methods in the insurance industry has been discussed for some years very critically. The task of the ANCRM working party is to give a structured overview of scientific contributions to develop a deeper understanding of and new ideas for application in risk evaluation and risk aggregation on P&C and Health insurance. This article intends to create a starting point for future studies of the evolution of these models and is the final report of the ASTIN ANCRM working party.

Keywords: Risk Management, Agent-based Models, Complex Networks, P&C, Health.

Corresponding author: Magda Schiegl, magda.schiegl@haw-landshut.de

[1] University of Applied Sciences Landshut, Am Lurzenhof 1, D-84036 Landshut, Germany.
[2] Università della Calabria, Arcavacata di Rende (CS), 87036, Italy.
[3] Metabiota, Inc., San Francisco, CA 94104, USA
[4] MatBlas Ltd., London, E14 9RP, UK
[5] New York, USA.

Introduction

Agent-based models (ABM), cellular automata (CA), a special case of ABMs, and complex networks (CN) have been developed as models and methods in the natural sciences to describe and understand complex systems. In the last decades, the models have been applied in the socio-economic context as well and documented in a vast amount of scientific literature. The potential use of these models in the insurance industry, in particular within the context of risk management and solvency assessment, has been discussed over the last few years. One of the main tasks of the ASTIN working party ANCRM is to provide a structured overview of scientific contributions in these fields with a clear focus on Property and Casualty (P&C) as well as Health insurance.

The working party searched for relevant articles in 25 pre-selected peer-reviewed journals (see Appendix 1) on the three search terms:

- agent-based models,
- complex (or neuronal) networks and
- cellular automata (CA)

in the context of "risk", "risk management" and "insurance".

Additional articles proposed by the members of the working party have been added to the list of literature. We focus on articles published between 2005 and 2017, a timespan we consider reasonable for when the application of ABMs and CNs in actuarial literature and the insurance industry may have started. We excluded macroeconomic models. We screened several hundred articles and selected those for subsequent evaluation which could be of relevance for an insurance company in the non-life or health business. We decided on a relatively extensive framework of articles because the usefulness and impact of an article depending on the experience, education, duties and responsibility of the reader – a very wide field, especially in the insurance industry. In the following text we put careful attention to those articles we consider as especially relevant for the insurance business.

This search and screening finally resulted in approximately 100 articles that have been analysed and evaluated by members of the working group. The distribution of the articles by specific topics is as follows: 83% are P&C, and 17% are health publications. More than half of the P&C publications relate to CN (58%), some 18% with both, ABMs and CNs, and 24% only with ABMs. In the health insurance sector, we find a very different distribution by topic: ABMs are the most frequent (56%) topics while CNs are relevant for 43% of the cases. Our search also includes four review papers: Three for P&C and one for Health. Although a very limited number of original papers were published in actuarial journals, all three of the P&C review papers [1–3] appeared in actuarial journals, published in the years 2012 and 2013. This relatively late point in time compared to much earlier state of the art contributions in the subjects under investigations somehow reflects the sceptical or complacent attitude of the actuarial practitioners and academics towards these topics. The health review paper [4] was published even later in 2017 in a biological journal. A summary of these review papers is found in the sections which follow.

The members of the working party used the following textbooks as an introduction and for general questions concerning the topics of the work [5–8]. They might be a good introduction for interested readers as well. Of course, there are many other appropriate textbooks in a rapidly growing literature on those topics.

Agent Based Models

Agent-based models (ABMs) consist of a large number of agents. The agents are defined on a micro level and interact in a rule-based manner with each other. In this way they determine the behaviour and dynamics of the whole (global) system on the macro level. Every single agent is characterized by:

- Internal degrees of freedom (internal parameters): The individual properties that characterise every single agent. The properties can change over time (dynamic model).
- Autonomy: The agent's actions depend on its internal parameters (properties) and the state of the environment. The environment is either given externally (external parameters) or is defined via statistical quantities of the other agents, e.g. the mean value of the nearest neighbours or the total ensemble of agents.
- Mobility: Every agent is mobile and can act on the micro-level. Its actions can be reactive (reacting to other agents, the environment, or both) or proactive (influencing other agents, the environment, or both). The interactions between the agents are rule-based and can change over time. The principle of locality is valid: The influence of one single agent is small compared to the total system.

The total system (macro-level) is analysed to derive the results of the ABM. Statistical methods are utilized to investigate the behaviour and properties of the macro system. CAs are special cases of ABMs: The agents live on a grid and can only interact with their nearest neighbours on the grid. Therefore the agent's local coordinates are two dimensional, discrete, and static. CAs are by definition time discrete models with recursive quantities depending only on values of the present and the last time step. Therefore agents' actions are time synchronously and possess the Markov property.

A large proportion of articles relating to ABMs are found in the area of banking, finance, and financial markets. Common topics found in the papers analysed include: testing strategies in volatile environments or under endogenous shocks, collective market dynamics such as herding, bubbles, and crashes; stylised facts of markets including but not limited to asymmetric return distributions, volatility clustering, fat tails and the autocorrelation of time series. We find only two papers that cover classical ASTIN topics: one by Ingram, Tayler, Thompson [9] and the other by Haer, Botzen, Moel, Aerts [10], both implementing and discussing strategic games. Given the relevance for the insurance industry, we start by summarizing these two articles, and we later include other relevant articles.

The first paper, the paper by Ingram et al. [9] describes and implements a strategic game between two types of agents: 30 insurance companies and one bank. Competing with each other, the 30 insurance companies build the insurance market. Every insurance agent decides on one of four strategies and follows that strategy if it becomes non-profitable. The strategies, entitled "Pragmatist"; "Conservator"; "Maximiser"; "Manager", and the decisions of the insurance companies are based on the "theory of plural rationality". The relations between strategies, decisions and the "theory of plural rationality" are described and discussed in detail in [9]. Depending on its financial situation and the situation of the insurance market (statistics of the other 29 agents) the insurance agent can change its strategy. Every insurance agent aims to maximize its prosperity in terms of cash balance, the amount of investment and return. During the game, the development of the insurance market and the bank is observed and documented. The game is implemented as a Monte Carlo simulation. As a result of the numerical calculation, a cyclic behaviour of the insurance market is observed. We think that in principle, the model of Ingram et al. is not only appropriate for insurance companies but at its heart, can be interpreted as a macroeconomic model.

The second paper by Haer et al. [10] describes and implements a strategic game as well: adaptive human decision making under flood risk scenarios is analysed. The ABM consists of two types of agents, the households and the insurance companies. Households can act in the following way: purchase the insurance, cancel the coverage and implement loss reduction by the help of technical risk management devices. The insurance companies can: set premiums, give discounts, and collect and pay-out claims. Increased climate change and flood risk characterize the environment. The environment is modelled via an external formula and enters the game from the outside. The authors conclude that human adaptive behaviour has an enormous impact on the expectation of flood claims. They find that insurance premium discounts are an important incentive to correct for non-adaptive behavior and therefore reduce flood risk. The interaction between the households and insurance companies is, therefore, decisive for the insurability of flood risk.

Most ABMs presented in other articles within our search criteria simulate a typical market situation with buyers and sellers following specific trading strategies to reproduce stylised facts of markets or collective market dynamics as herding, bubble formation and other psychological effects observed in real world-markets. Stylised facts of the market are for instance: asymmetric return distributions, fat tail distributions, volatility clustering and autocorrelations in time series. An early representative of this type of models is [11]. It implements a strategic market game in numerical simulations with buyers and sellers as agents that can change their opinion about the stock. In contrast to this, some very simplistic models can replicate stylised facts of the markets. Maymin [12] introduces a parameter-free trading strategy that generates market time series with fat tails and frequent crashes: Two consecutive market moves determine the choice of buy and sell strategy. The strategy is simplistic and comparable to a random walk but generates a much more realistic time series. Qiu et al. [13] use a CA model with two types of agents, fundamentalists and imitators as traders, living on a two-dimensional lattice with nearest neighbour interaction only. As a result, the authors find mean-reverting market prices, volatility clustering and fat tail distributions. The paper contains a review of CA market models and summarizes 2007 and earlier papers on stylized facts of the market. The main research problem is to identify the connection between the single aspects of the model and the produced stylized facts. In [14] , a three-type trader ABM is implemented. The three groups of traders are: 1) smart traders predicting prices from former time series and private information, 2) loss-averse traders knowing only the former price time series and 3) noisy traders investing randomly. In this case, the following market stylised facts are reproduced: asymmetry of return distributions, auto-correlation of return volatility and the cross-correlation between return volatility and trading volume. The paper by Zhang & Wang [15] starts with a thorough review of the previous literature in the field of modelling price returns in financial markets. The main criticism is that the methods for measuring autocorrelation and cross-correlation of nonlinear and non-stationary time series are not sufficiently developed. Therefore, the authors focus on autocorrelation within and cross-correlation between financial time series and suggest an improved method of measurement. Additionally, an ABM is developed to reproduce these effects as observed in real financial time series.

The following papers investigate psychological effects by the help of ABMs and shed light on the collective effects of market dynamics. Lamba & Seaman [16] construct an ABM with a herding effect. They look at heterogeneous agents with tolerance for belonging to a minority. They find typical stylised facts such as excess kurtosis and fat tails in price returns and volatility clustering. Similar results are presented and discussed in [17] where social groups form in a random cluster process as a model for the formation of social circles in real life. The implemented ABM shows an equilibrium distribution of the social groups. Stylised market effects are also found in Park [18] with a quite sophisticated ABM consisting of two types of trading agents that show psychological behaviour with regards to their risk aversion. The ABM also includes an adaptive belief system about the market price depending on the type of trader. Doblas Madrid [19] introduces an ABM of speculative bubble formation where rational agents buy overvalued assets. The treatment is entirely analytical without numerical simulation. The paper starts with an overview and evaluation of theories of asset price bubbles. These articles neglect the effects of speculation, the traders' beliefs, and the bursting of bubbles.

Besides the trading-focused papers, we find the second group of papers having systemic risk and market stability as topics, mostly within a banking environment and use trading models. They share a holistic view of the system and its behaviour. Szafarz [20] sheds light on the development of crises in efficient, financial markets. To this end, the author develops an ABM with two types of rational traders, short-term speculators and long-term fundamentalists. The key conclusion is that efficient markets are more volatile with a few speculators than with many speculators. In [21], the risk perception strategies of financial institutions with typical banking properties are investigated. An ABM is compared to a general equilibrium approach. The authors find that the perception of risk attitudes can increase the vulnerability of the system to external shocks. The impact of the Value at Risk (VaR) threshold risk management strategy on market stability is simulated by the use of a two-type trader (fundamentalists and technical traders) ABM in [22]. They conclude that the VaR system tends to destabilize the market: when a sufficient number of traders reach their VaR limit and are forced to sell their stocks, leading to market disruptions and high price volatilities. Poledna et al. [23] compare, with a sophisticated multi-agent ABM, three different regulation strategies of a credit market. One is the Basel II strategy, the second is the unregulated case, and finally, a strategy that seems to be superior under the given conditions. Lin [24] develops a more general and very technical approach to systemic risk with a risk aggregation algorithm. The paper also provides a brief introduction to Bayesian networks.

Furthermore, we find some papers with a more theoretical or general approach to ABMs: the very early paper by Leombruni and Richiardi [25] discusses the applicability of ABMs in the economy in general. Two important points of the discussion are the interpretation of ABM simulations and the estimations for the model parameters. Hooten [26] presents a hierarchical Bayesian framework for formal statistical ABMs using only binary data. A series of two very technical papers are published by Satinover & Sornette [27, 28], which delivers a meta-analysis of ABM games, namely the “Minority Game”, the “Majority Game” and the “Dollar Game”. They develop a new metric for comparing different games to real-world financial time series. Additionally, a method for generating predictors for ABMs is designed and applied to real-time series.

Complex Networks

A complex network (or a graph) is a set of nodes connected by links. In graph theory, the expressions “vertices” are common for nodes and “edges” for links. Complex networks (CN) are a general and powerful concept to describe and analyse complex interactions. Some examples for nodes and their links to other nodes are people and friendships or other connections among them, airports and flights between them, banks and debts, and reinsurers and treaties. The number of links of a node is defined as the degree. The degree distribution of the nodes is a key characteristic of a network. Two especially important types of networks are the “small-world” and the “scale-free” networks. In a small-world network (almost) every node is connected to a very high number of other nodes. In a scale-free network, there are a few very strongly connected nodes and many weakly connected nodes. In this case, the degree distribution obeys a power law, which explains the name. Networks can be static or dynamic and directed (one-directional) or undirected (bi-directional). For a bipartite graph there exist two different sets of nodes.

Similarly to ABMs, most of the CN papers apply to financial and banking environments. We identify three major subjects within the papers: systemic risk such as contagion spread or regulatory topics, the construction and examination of risk measures for complex networks and supply chain topics. Additionally, we report in this section on papers that use ABMs as well as CNs.

The first P&C review papers dealing with both ABMs and CNs are a series of two papers by Parodi [1, 2]. The first paper [1] relates to “statistical learning”, seemingly synonym for “machine learning”. It aims to draw parallels between artificial intelligence learning in computational science and typical actuarial methods. In line with this context, an agent is very generally defined as an object that exhibits intelligent behaviour. One distinguishes supervised learning from unsupervised learning. Supervised learning is a two-stage process: the training phase is followed by the testing phase. The author concludes that familiar, actuarial, data-based problems such as pricing, reserving, and capital modelling are examples of supervised learning. He gives a large variety of examples such as regularised regression, GLMs or neural networks and its applications to actuarial data. Unsupervised learning means finding structures in data without a training set. On the actuarial side, the author connects this to examples like clustering techniques and association rules. The second paper [2] deals with “uncertain knowledge” in the form of data that are uncertain themselves and the need for integrating soft expert knowledge. Parodi concludes that “uncertain and soft knowledge can be dealt with most successfully in a Bayesian context”. He introduces multi-agent systems and strategic games as models for collective behaviour but without giving examples or citing relevant papers in this area. Parodi also points out that he is very sceptical about the usefulness of these models for applications in decision making and regulation due to research that is still lacking, and the complexity of the models. He is more optimistic about using ABMs as scenario generators.
The second P&C review paper by Allan et al. [3] has its focus on enterprise risk management (ERM) and the tackling of companies with their risk appetite. Several

reviews of relevant literature from the practitioner’s point of view include the following topics: risk appetite, emerging risks and systemic risks. The paper incudes concise introductions to and evaluations of CAs, ABMs, and Bayesian networks.

We find four papers considering traditional insurance topics: The paper by Kley, Klüppelberg and Reinert [29] studies the diversification of large, endogenous, Pareto-tailed claims under different reinsurance network scenarios. A bipartite graph models the sharing of the losses among the reinsurers. The authors obtain asymptotic results for the VaR and conditional tail expectation risk measures. The authors also investigate the amount of uninsured losses to be covered by society in each scenario. Another paper with a focus on the reinsurance market is by Kanno [30]. The paper accesses the interconnectedness between reinsurers and insurers in the global non-life insurance market and its contribution to systemic contagion default risk. Centrality measures are used for network analysis. The authors argue that in the case of an initially defaulted reinsurer with a large network centrality contagious defaults will proceed. Otherwise, small or medium-sized reinsurers do not affect the stability of the network. The default analysis of real-world data after the global financial crisis shows the occurrence of many stand-alone defaults and only one contagious default within the global reinsurance network. Stress test results based on a hypothetical severe stress scenario also show that the possibility of contagious defaults in the future is generally low.
Bayesian networks as a framework for operational risk management are the topic of the paper by Cowell, Varrall and Yoon [31]. The paper includes an overview of operational risk models as well as a comprehensive introduction to Bayesian networks. A Bayesian network for modelling various risk factors and their combination into an overall loss distribution is proposed. An example of insurance fraud risk arising from a commercial fire insurance portfolio demonstrates that Bayesian networks can quantify operational risk with potential applications such as allocation of risk capital and scenario testing. One of the strengths of Bayesian networks highlighted in the paper is the ease of incorporating expert opinions, but, according to the authors, this can also be a disadvantage for supervisory capital approval that requires objective standards due to its high subjective content.
The fourth insurance paper is written by Hsu, Lin and Yang [32]. The authors demonstrate a neural network approach in calculating individual medical expenditures by combining two different neural network methods: Self Organised Maps (SOM, an unsupervised learning method) and Back Propagation Network (BPN, a supervised learning method) to improve predictive power in risk-adjustment models. Risk adjustment is the most effective strategy for reduction of “cream-skimming”, the fact that health plants seek out only the most profitable patients. The authors apply their method to the Taiwan National Health Insurance scheme. Predictive performance metrics are compared to the neural network method and other, traditional risk adjustment models to quantify the better fit of the first one.

Most of the CN papers are related to modelling holistic or systemic risk in the banking and economic environment. Several of them use CN as a medium to structure a stock market according to a network [33–35] for risk management, asset allocation, or optimal trading strategies, and herding and avalanche dynamics [36]. Davis et al. [37] use the asymptotic behaviour of stochastic networks for pricing of large credit risk portfolios. Acemoglu et al. [38] investigate how interconnections in microeconomic, idiosyncratic shocks can lead to aggregate fluctuations.

The majority of the papers use CN to model the systemic risk of a network of banks, such as contagion [39–41], default propagation across the banking system, or the impact of scenarios of stress, shock, or both [42–44]. Typically, the nodes of the CN are banks modelled by a simplified balance sheet and realistic interaction rules between them ([45], [40], [46]). In Amini et al. [47] , the magnitude of contagion in large banking networks is quantified analytically by an asymptotic fraction of defaults, in terms of network characteristics. The authors show that institutions with high contribution to network instability tend to have both large connectivity and a large fraction of contagious links. Several papers concentrate on specific regional banking networks such as: US [48], Germany [49], Venezuela [44] and Austria [40]. Papers often use traditional degree- or centrality measures of CN theory to quantify systemic risk [50], [51], [52]. It is known from the theory of complex systems that threshold functions can be used to detect phase transitions in random networks in the limit of a large network size. In this sense, Caccioli et al. [53] define a model with a critical threshold for leverage leading to a stability – instability transition of financial networks and similarly, [45] identifies a threshold for the shock's magnitude where sharp transition for contagion spreading happens to a large part of the network system. In addition to studies on stability transition threshold, other papers give regulatory advice for strategies in regards to network contagion defaults impact under different network structures [54], [55], [48] , [56], [57].

Due to the special features of complex networks, the traditional risk measures have to be adjusted. We find several papers with the main aim to construct appropriate risk measures, especially for the systemic risk of financial networks. Generally, centrality measures aim to measure the node's importance for different dimensions in relation to the entire network. These dimensions can be degree distribution, which is used when the number of links is important, distance reduction between other nodes in the network, or dynamic processes on the network. Cont [58] introduces a risk measure, the Contagion Index, which identifies the systemically important nodes under the condition of stress scenarios. The paper applies the Contagion Index to the Brazil banking system. Another measure targeting to find systemically important nodes in a network is the DeptRank introduced in Batiston [59]. DeptRank is a centrality measure that relates to every node the fraction of the potential systemic loss caused by this node in case of its default. All nodes in the network are taken into account recursively. The DeptRank measure is applied to an US FED dataset. The authors suggest that the debate on institutions being "too-big-to-fail" should shift to the more important issue "too-central-to-fail". Other papers such as [60] as well as [29, 61] examine and adjust the traditional risk measure VaR for assessing systemic risk on networks. Kley et al. [29, 61] derive analytical expressions for systemic risk measures in the asymptotic case of large markets. The influence of large, exogenous, Pareto-tailed losses on a network is modelled. The newly developed risk measures are based on VaR and Conditional Tail Expectation. They allow the quantification of the influence of individual institutions (nodes) on the risk of the whole system (network). Silva et al. [62] define the risk measure called "impact susceptibility index". The index indicates whether an institution is vulnerable or not. The paper constructs a financial stability monitoring tool and applies it to the financial market of Brazil. A detailed comparison of different systemic risk measures is also included in this paper. We also find stability measures of systems that are inspired by systems dynamics as the determination of the eigenvalues of a stability matrix [53].

A few papers highlight the systemic risk of supply chain networks. This topic is of special importance for industrial insurance of technological products and services. Blöchl et al. [63] describe the flows of goods and services between the sectors of the economy as weighted directed networks. They introduce the measures “random walk centrality” and “counting betweenness” to focus on the special properties of those networks and study the propagation and impact of supply shocks to real economic networks. The two papers by Tang and Stanley [64, 65] construct theoretical risk models of complex interdependent and assembly supply chain networks, respectively. They study cascading failures, for instance, based on production capability losses and redistribution strategies for failed loads propagation. They use numerical simulations of networks and simulate stochastic processes on the networks to derive their results. Xu [66] introduces a special weighted network analysis method to study international services trade and to analyse influence among services trade network of relations.
The following papers address very special CN topics: [67] quantitatively evaluate the influence and power of directors by analysing US corporate governance network. [68] deals with the extension of reduced form auctions in finance. Chatrabgoun [69] applies minimum information vine models for dependence structures in financial data.

Finally, we find papers that incorporate both, ABMs and CNs in one single framework. Their common feature is that at least a part of the rules of interaction between the agents is replaced by a network structure describing the interplay of different agents. Most of the papers are found on the analysis of systemic risk. Aymanns and Georg [70] analyse the financial stability of banking systems under different investment strategies, Georg [71] deals with contagion and common shocks on banking networks, and [72] studies the impact of Basel III regulation on the stability the banking network. In [73], an ABM is operating on a financial network to reduce the systemic risk of a financial network. The proposed systemic risk tax leads to a self-organised restructuring of the financial network. A more general view of the economic system as a whole is taken in [74] and [75], where networks model the interdependence of the different types of economic agents as well as geographical relations. A theoretical, simulation-based approach of behavioural finance on the diffusion of cooperation between agents is demonstrated in [76]. The agents “live” on a network and repeatedly play the Prisoner’s Dilemma. The combination of ABMs and networks is typical for health models (see next section), for instance [77]. Here, the epidemic spread is modelled via an ABM and the agents are connected on networks defining both transmission pathways and social contacts.

Health Models

Infectious diseases have been one of the greatest threats to the human population and can lead to tremendous impacts on public health, local and global economy, and society in general. From an insurance perspective, there can be a domino effect on other classes of business related to pandemics and epidemics. For example, business interruption and financial losses may result from the absence of a significant proportion of the workforce due to illness in a given period of time. Mathematical modelling is a powerful and ethical method to better understand infectious disease dynamics, including transmission mechanisms and intervention strategies. Some of the most successful granular epidemiological models include ABMs [4, 78–81], network models [82–88], and CA models [89–92]. In this systematic review of the related literature, we summarize and discuss the various applications and evaluations across the three aforementioned types of epidemiological models.

ABMs are simulation models that produce population-level, realistic scenarios arising from the behaviour and activities of every individual in the modelled population. The attributes of each individual, some of which vary temporally including spatial location and activities, contribute to the social and spatial contact patterns which impact disease transmission and dynamics. For example, vulnerable populations, such as infants or individuals who are engaged in high risk professions, would typically experience higher severity in the course of the epidemic. Census data, travel behaviour surveys, contact surveys, and workplace surveys form the basis of the socio-demographic structure of the synthetic population and individuals' physical activities [79, 80].

The assumption of the transmission pathway also helps identify the agents' contact or distance structure needed for the model. Diseases having close contacts or aerosols as the dominant pathway of transmission can utilize distance categories between individuals such as intimate, personal, social, and public distance to model the realistic minimum distance and contact duration required for disease transmission [80]. To add intricacy and complexity to the model, commuting and travelling patterns of the agents can be incorporated by using gravity models or transportation networks [78]. With the attributes and interaction of agents, infection occurrences are probabilistically simulated to occur in probable conditions given the transmission and frequency criteria of the modelled disease [78–81].

ABMs integrate population-specific socio-demographic and behavioural differences to simulate realistic heterogeneity in disease timeline and outcome that are distinctive to the population characteristics. The main obstacle for the use of ABMs is the difficulty to obtain high-confidence, detailed data in many regions of the world, which limits its use to mostly localized small epidemic modelling [78]. However, the use of small epidemic modelling also highlights the advantage of using ABMs since outbreaks are often subject to chance [4].

A type of ABM that specializes in modelling spatial dispersion and extinction processes is the CA model. A cell state is incorporated in each cell, with the cell state updating at every time step according to a set of probabilistic transition rules dependent on the current state of the cell and its neighbouring cells [89–92].

The grid cell structure of CA provides a framework to model spatial processes such as population migration and urbanization that could drive changes in transmission dynamics in disease spread. To quantify the effect of migration, Sun uses a CA model with probabilistic transition rules that model the occurrence and dynamics of infections and migration (through infecting and colonizing a neighbouring cell) in a susceptible population [91]. Results indicate that the migration rate works in combination with the infection rate, where the change in migration rate at different infection rate level will cause either persistence or extinction of disease and changes the epidemic outcome.

The strength of CA lies in its "heuristic, transparent, and flexible rules" [90]. Spatial diffusion processes such as wind dispersal [89] or urbanization [90] that can be formulated into scalable transition functions are well-described using CA models. Challenges come from modelling infection or dispersion processes that involve complex dispersion patterns that are not limited to the status of the neighbouring cells or proximity to the closest infected agent.

Network models, a class of models that can be implemented at individual or subpopulation level, feature model process outcomes that are dependent on the underlying contact network, such as modelling diseases that transmit through social contacts. Studies have shown (see for instance [85]) that humans interact in a structured way appropriately modelled by a scale-free network. The impact of this contact heterogeneity is significant on the spread of epidemics [85]. Ma illustrates and quantitatively compares four common network topologies: random, scale-free, small-world, and meta-random networks under four different vaccination strategies using two scenarios [84]. Although the paper does not intend to simulate any historical epidemics, results highlight that network topology significantly impacts modelled disease dynamics, more so than vaccine interventions.
Depending on the characteristics of the disease, disease transmission pathway, and the contact structure of the species, different network types and structure can be used to simulate realistic disease spread and intervention dynamics.
In modelling human epidemics, most network structures used are undirected since humans tend to interact in the same network such as school or work, repeatedly throughout the epidemics. However, for processes such as trade flows of plant pathogens spreading in commercial plant transport from growers to wholesalers to retailers, a directed network structure is more appropriate [83]. This also highlights a benefit that the network connection can be simply relational without any distance defined between the two entities. Lawyer measured the potential of individual airports for pandemic spread over the world airline network and showed that the potential is highly correlated with the number of edges instead of the distance between nodes [87, 93]. However, local density and mobility also play a role in influencing the dynamics of the disease spread.
Dynamic networks, network structures with connections that update temporally, can model the time-varying nature of human behaviour as opposed to a static network. Dynamic networks are similar to static networks, where both have the structure of nodes and edges. However, dynamic networks have additional probabilistic transitional rules that define the evolution behaviour of the networks for each time step [86, 88]. Individual demographic and behavioural characteristics can also be incorporated in the nodes to reflect the difference in disease transmission rate among low and high-risk individuals [86]. The evolution of the forming and dissolution of the connections can be based on behaviour surveillance surveys, estimated activities rate and connection per activities [82, 86].
Network models capture the impact on disease dynamics caused by the underlying social contact network, including the demographic and behavioural differences among individuals. This type of model is well-suited for diseases that require close contacts for transmission such as blood-borne diseases, sexually transmitted disease, and diseases that exhibit “super-spreading”, meaning that single individuals are responsible for infecting many more individuals than expected [94]. Social contact network data is often difficult to obtain due to the detailed level of an individual’s data and the evolving nature of social networks [86, 88]. Despite this limitation, a network framework is excellent in capturing the connected structure of transportation and mobility patterns that serve as a conduit for disease spread, such as the plant transport or airline transportation network [83, 87].
Epidemic modelling requires realistic simulation of the intricate interactions between the modelled pathogen, susceptible agents and the environment. The three types of models discussed in this paper provide strength in its specialized areas in modelling

epidemics. ABMs incorporate the most granular level of data for each agent in the model to simulate the heterogeneity in disease transmission within a population. CA models model the dynamic nature of spatial dispersal and migration process of the agents or pathogens that influence the spread of disease. Network models provide the network contact structure necessary within a population to model the transmission pathways of a pathogen. The choice of model strongly hinges on the characteristics of the pathogen and agent of interest. Hybrid approaches are commonly used to maximize the benefit of each model type and the available data granularity and sources. For example, ABMs can be incorporated into a meta-population model along with the airline transportation network both to benefit from the localized, realistic socio-demographic structure of the regional population and to produce reliable global estimates [78].

A major challenge across the models is the difficulty in data collection for model input and output validations. Detailed regional or longitudinal population studies are often unavailable in a standardized design for modelling and validating the heterogeneity in exposed risk and transmission among agents with different socio-demographic, environmental, or behavioural characteristics. However, population and global-level data from historical epidemics still serve as appropriate and suitable validation tools for different modelling purposes due to data limitations. Data limitations are also expected to be eased with the standardization of global surveillance data, electronic medical records, and other similar health and behavioural data collection agencies and aggregators. Combined with the advancement of computing and open source technology in the last decade and forward, producing large scale realistic global disease spread simulations is becoming more feasible and available to researchers. Furthermore, more quantitative modelling can be done to address current model limitations and provide more analysis of disease spread and intervention strategies to inform policy makers.

We have discussed the application of the three model types in the interest of disease modelling to quantify the impact on human and other species. The realistic structure of the environment and susceptible population produces model outcomes that increase our understanding of the risk attributes with its associated frequency and severity potential, which supports the development of insurance products. One tool for transforming disease model output into relevant insurance risk estimation and policy design is the exceedance probability (EP) curves, which shows the probability of exceeding a given level of an event or annual severity [95]. The inverse of EP is known as the return period which provides the probable maximum losses for a given recurrence time. Risk pooling and sovereign-level catastrophe insurance are types of risk transfer mechanism that can be used in managing pandemic risks. The use of risk transfer mechanism could provide timely and effective financial resources for impacted parties to respond to and recover from epidemic outbreaks.

Proposals and Outlook

Unlike the amount of literature on ABMs and complex networks available in the fields of banking and finance, only a limited amount of research is available in the field of non-life insurance. We find some evidence for a gap between academic research and industry applications of complex topics such as those presented in this review paper. The working party thoroughly believes that there are numerous possibilities for collaborative research between industry practitioners and academics in the implementation of realistic, complex models and validation of assumptions within the risk management and solvency framework. The following is a non-exhaustive list of potential topics that, in our opinion, are worth further investigating:

- The risk quantification of real-world networks in insurance can be used in lines of business that are very dependent on network structures, for instance, supply chain risks, cyber risks, and industrial insurance in general.
- Holistic risk management of insurance companies or lines of business: testing typical insurance strategies under an ABM environment. For instance, pricing strategies given a typical set of customer relations, including specific claim management and reserve strategies.
- Understanding the risk attributes with associated severity potential of the different population which supports the creation and development of novel insurance products.
- The connection between reinsurance networks and ABMs.
- The testing of supervisory and regulatory strategies on an ABM simulated insurance market.
- Shedding more light on phase transitions and regime switches on ABMs in dependence on the parameters of the models.
- Generate a deeper understanding of risk aggregation schemes: for example, the connection between dependent risks, network structures and copulae.

Acknowledgements

.
We would like to thank F. Cuypers for very fruitful discussion and for initiating the working party ANCRM. We are very grateful to the ASTIN section of the International Actuarial Association (IAA) for hosting our working party ANCRM.

Appendix 1

Journals

- Annals of Actuarial Science
- Astin Bulletin
- British Actuarial Journal
- CAS Monograph Series
- Econometrica
- European Actuarial Journal
- Finance and Stochastics
- IEEE Transactions on Evolutionary computation
- Insurance: Mathematics and Economics
- International Journal of Theoretical and Applied Finance
- Journal of Banking and Finance
- Journal of Economic Theory
- Journal of Risk
- Journal of Risk and Insurance
- Journal of the American Statistical Association
- Journal of the Royal Statistical Society: Series C
- Mathematical Finance
- North American Actuarial Journal
- Phys. Rev. E
- Physica A
- Quantitative Finance
- Risk
- Risk Analysis
- Scandinavian Actuarial Journal
- Variance

References

1. Parodi P (2012) Computational Intelligence With Applications To General Insurance: A Review: I – The Role Of Statistical Learning. Ann. actuar. sci. 6(2): 307–343. doi: 10.1017/S1748499512000036
2. Parodi P (2012) Computational Intelligence With Applications To General Insurance: A Review: II. Dealing With Uncertain Knowledge. Ann. actuar. sci. 6(2): 344–380. doi: 10.1017/S1748499512000048
3. Allan N, Cantle N, Godfrey P et al. (2013) A Review Of The Use Of Complex Systems Applied To Risk Appetite And Emerging Risks In ERM Practice: Recommendations For Practical Tools To Help Risk Professionals Tackle The Problems Of Risk Appetite And Emerging Risk. Br. Actuar. J. 18(1): 163–234. doi: 10.1017/S135732171200030X
4. Willem L, Verelst F, Bilcke J et al. (2017) Lessons From A Decade Of Individual-Based Models For Infectious Disease Transmission: A Systematic Review (2006-2015). BMC Infect Dis 11(17): 612. doi: 10.1186/s12879-017-2699-8
5. Thurner S, Hanel R, Klimek P (2018) Introduction to the theory of complex systems, First edition. Oxford University Press, Oxford
6. Schweitzer F (2007) Browning Agents and Active Particles: Collective Dynamics in the Natural and Social Sciences. Springer Series in Synergetics. Springer-Verlag Berlin Heidelberg, Berlin, Heidelberg
7. Newman MEJ (2016) Networks: An introduction, Reprinted. Oxford University Press, Oxford
8. Barabási A-L (2016) Network science. Cambridge University Press, Cambridge
9. Ingram, D., Tayler, P., Thompson, M. (2012) Invited Discussion Paper: Surprise, Surprise From Neoclassical Economics To E-Life. ASTIN Bulletin 42(2): 389–411
10. Haer T, Botzen WJW, Moel H de et al. (2017) Integrating Household Risk Mitigation Behavior in Flood Risk Analysis: An Agent-Based Model Approach. Risk Analysis 37(10): 1977–1992. doi: 10.1111/risa.12740
11. Bovier, Anton, Černý, Jirí, Hryniv O (2006) The Opinion Game: Stock Price Evolution From Microscopic Market Modeling. Int. J. Theor. Appl. Finan. 09(01): 91–111. doi: 10.1142/S0219024906003421
12. Maymin PZ (2011) The Minimal Model Of Financial Complexity. Quantitative Finance 11(9): 1371–1378. doi: 10.1080/14697681003709447
13. Qiu G, Kandhai D, Sloot PMA (2007) Understanding The Complex Dynamics Of Stock Markets Through Cellular Automata. Phys. Rev. E 75(4): 170. doi: 10.1103/PhysRevE.75.046116
14. Shimokawa T, Suzuki K, Misawa T (2007) An Agent-Based Approach To Financial Stylized Facts. Physica A: Statistical Mechanics and its Applications 379(1): 207–225. doi: 10.1016/j.physa.2006.12.014
15. Zhang W, Wang J (2017) Nonlinear Stochastic Exclusion Financial Dynamics Modeling And Time-Dependent Intrinsic Detrended Cross-Correlation. Physica A: Statistical Mechanics and its Applications 482: 29–41. doi: 10.1016/j.physa.2017.04.033
16. Lamba H, Seaman TIM (2008) Market Statistics Of A Psychology-Based Heterogeneous Agent Model. Int. J. Theor. Appl. Finan. 11(07): 717–737. doi: 10.1142/S0219024908005019
17. Cai H, Chen K (2009) A Random Cluster Process Approach To Collective Market Dynamics with Local Interactions. Int. J. Theor. Appl. Finan. 12(02): 251–266. doi: 10.1142/S0219024909005178
18. Park B-J (2014) Time-Varying, Heterogeneous Risk Aversion And Dynamics Of Asset Prices Among Boundedly Rational Agents. Journal of Banking & Finance 43: 150–159. doi: 10.1016/j.jbankfin.2014.03.009
19. Doblas Madrid A (2012) A Robust Model of Bubbles With Multidimensional Uncertainty. Econometrica 80(5): 1845–1893. doi: 10.3982/ECTA7887

20. Szafarz A (2012) Financial Crises In Efficient Markets: How Fundamentalists Fuel Volatility. SSRN Journal 36(1): 105–111. doi: 10.2139/ssrn.1689723
21. Kaszowska J, Santos JL (2014) The Role of Risk Perception in the Systemic Risk Generation and Amplification: Agent-Based Approach.". ACRN Journal of Finance and Risk Perspectives 3(4): 146–170
22. Llacay B, Peffer G (2017) Impact Of Value-At-Risk Models On Market Stability. Journal of Economic Dynamics and Control 82: 223–256. doi: 10.1016/j.jedc.2017.07.002
23. Poledna S, Thurner S, Farmer JD et al. (2014) Leverage-Induced Systemic Risk Under Basle II And Other Credit Risk Policies. Journal of Banking & Finance 42: 199–212. doi: 10.1016/j.jbankfin.2014.01.038
24. Lin P, Neil M, Fenton N (2014) Risk Aggregation In The Presence Of Discrete Causally Connected Random Variables. Ann. actuar. sci. 8(2): 298–319. doi: 10.1017/S1748499514000098
25. Leombruni R, Richiardi M (2005) Why Are Economists Sceptical About Agent-Based Simulations? Physica A: Statistical Mechanics and its Applications 355(1): 103–109. doi: 10.1016/j.physa.2005.02.072
26. Hooten MB, Wikle CK (2010) Statistical Agent-Based Models for Discrete Spatio-Temporal Systems. Journal of the American Statistical Association 105(489): 236–248. doi: 10.1198/jasa.2009.tm09036
27. Satinover JB, Sornette D (2012) Cycles, Determinism And Persistence In Agent-Based Games And Financial Time-Series: Part II. Quantitative Finance 12(7): 1065–1078. doi: 10.1080/14697688.2012.670260
28. Satinover JB, Sornette D (2012) Cycles, Determinism And Persistence In Agent-Based Games And Financial Time-Series: Part I. Quantitative Finance 12(7): 1051–1064. doi: 10.1080/14697688.2012.670260
29. Kley O, Klüppelberg C, Reinert G (2016) Risk in a Large Claims Insurance Market with Bipartite Graph Structure. Operations Research 64(5): 1159–1176. doi: 10.1287/opre.2016.1502
30. Kanno M (2015) The Network Structure and Systemic Risk in the Global Non-Life Insurance Market. SSRN Journal 67: 38–53. doi: 10.2139/ssrn.2617946
31. Cowell RG, Verrall RJ, Yoon YK (2007) Modeling Operational Risk With Bayesian Networks. Journal of Risk & Insurance 74(4): 795–827. doi: 10.1111/j.1539-6975.2007.00235.x
32. Hsu S, Lin C, Yang Y (2008) Integrating Neural Networks for Risk-Adjustment Models. Journal of Risk & Insurance 75(3): 617–642. doi: 10.1111/j.1539-6975.2008.00277.x
33. Materassi D, Innocenti G (2009) Unveiling The Connectivity Structure Of Financial Networks Via High-Frequency Analysis. Physica A: Statistical Mechanics and its Applications 388(18): 3866–3878. doi: 10.1016/j.physa.2009.06.003
34. Namaki A, Shirazi AH, Raei R et al. (2011) Network Analysis Of A Financial Market Based On Genuine Correlation And Threshold Method. Physica A: Statistical Mechanics and its Applications 390(21-22): 3835–3841. doi: 10.1016/j.physa.2011.06.033
35. Xia L, You D, Jiang X et al. (2018) Comparison Between Global Financial Crisis And Local Stock Disaster On Top Of Chinese Stock Network. Physica A: Statistical Mechanics and its Applications 490: 222–230. doi: 10.1016/j.physa.2017.08.005
36. Biondo AE, Pluchino A, Rapisarda A et al. (2013) Reducing Financial Avalanches By Random Investments. Phys. Rev. E 88(6): 777–780. doi: 10.1103/PhysRevE.88.062814
37. Davis MHA, Esparragoza-Rodriguez JC (2007) Large Portfolio Credit Risk Modeling. Int. J. Theor. Appl. Finan. 10(04): 653–678. doi: 10.1142/S0219024907004378
38. Acemoglu, D., Carvalho, V.M., Ozdaglar, A., Tahbaz-Salehi, A. (2012) The Network Origins of Aggregate Fluctuations. Econometrica 80(5): 1977–2016. doi: 10.3982/ECTA9623
39. Acemoglu D, Malekian A, Ozdaglar A (2016) Network Security And Contagion. Journal of Economic Theory 30(166): 536–585. doi: 10.1016/j.jet.2016.09.009
40. Caccioli F, Farmer JD, Foti N et al. (2015) Overlapping Portfolios, Contagion, And Financial Stability. Journal of Economic Dynamics and Control 51: 50–63. doi: 10.1016/j.jedc.2014.09.041

41. Kley O, Klüppelberg C, Reichel L (2014) Systemic risk through contagion in a core-periphery structured banking network
42. Lenzu S, Tedeschi G (2012) Systemic Risk On Different Interbank Network Topologies. Physica A: Statistical Mechanics and its Applications 391(18): 4331–4341. doi: 10.1016/j.physa.2012.03.035
43. Hurd TR, Cellai D, Melnik S et al. (2016) Double Cascade Model Of Financial Crises. Int. J. Theor. Appl. Finan. 19(05): 165–190. doi: 10.1142/S0219024916500412
44. Kenett DY, Levy Carciente S, Avakian A et al. (2015) Dynamical Macroprudential Stress Testing Using Network Theory. SSRN Journal 59: 164–181. doi: 10.2139/ssrn.2648467
45. Amini H, Cont R, Minca A (2012) Stress Testing The Resilience Of Financial Networks. Int. J. Theor. Appl. Finan. 15(01): 125–131. doi: 10.1142/S0219024911006504
46. Ding D, Han L, Yin L (2017) Systemic Risk And Dynamics Of Contagion: A Duplex Inter-Bank Network. Quantitative Finance 17(9): 1435–1445. doi: 10.1080/14697688.2016.1274046
47. Amini H, Cont R, Minca A (2016) Resilience To Contagion In Financial Networks. Mathematical Finance 26(2): 329–365. doi: 10.1111/mafi.12051
48. Kuzubaş TU, Saltoğlu B, Sever C (2016) Systemic Risk And Heterogeneous Leverage In Banking Networks. Physica A: Statistical Mechanics and its Applications 462: 358–375. doi: 10.1016/j.physa.2016.06.085
49. Anand K, Craig B, Peter G von (2015) Filling In The Blanks: Network Structure And Interbank Contagion. Quantitative Finance 15(4): 625–636. doi: 10.1080/14697688.2014.968195
50. Bargigli L, Gallegati M (2011) Random Digraphs With Given Expected Degree Sequences: A Model For Economic Networks. Journal of Economic Behavior & Organization 78(3): 396–411. doi: 10.1016/j.jebo.2011.01.022
51. Minoiu C, Kang C, Subrahmanian VS et al. (2013) Does Financial Connectedness Predict Crises? SSRN Journal 15(4): 607–624. doi: 10.2139/ssrn.2326429
52. Teteryatnikova M (2014) Systemic Risk In Banking Networks: Advantages Of "Tiered" Banking Systems. Journal of Economic Dynamics and Control 47: 186–210. doi: 10.1016/j.jedc.2014.08.007
53. Caccioli F, Shrestha MK, Moore C et al. (2012) Stability Analysis of Financial Contagion Due to Overlapping Portfolios. SSRN Journal 46: 233–245. doi: 10.2139/ssrn.2176080
54. Papadimitriou T, Gogas P, Tabak BM (2013) Complex Networks And Banking Systems Supervision. Physica A: Statistical Mechanics and its Applications 392(19): 4429–4434. doi: 10.1016/j.physa.2013.05.013
55. Gaffeo E, Molinari M (2015) Interbank Contagion and Resolution Procedures: Inspecting the Mechanism. SSRN Journal 15(4): 637–652. doi: 10.2139/ssrn.2340154
56. Leduc MV, Thurner S (2017) Incentivizing Resilience in Financial Networks. SSRN Journal 82: 44–66. doi: 10.2139/ssrn.2794371
57. Caux R de, McGroarty F, Brede M (2017) The Evolution Of Risk And Bailout Strategy In Banking Systems. Physica A: Statistical Mechanics and its Applications 468: 109–118. doi: 10.1016/j.physa.2016.10.005
58. Cont R, Moussa A, Santos EBe (2010) Network Structure and Systemic Risk in Banking Systems. SSRN Journal. doi: 10.2139/ssrn.1733528
59. Battiston S, Puliga M, Kaushik R et al. (2012) DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk. Scientific Reports 2: 541. doi: 10.1038/srep00541
60. Bluhm M, Krahnen JP (2014) Systemic Risk in an Interconnected Banking System with Endogenous Asset Markets. SSRN Journal 13: 75–94. doi: 10.2139/ssrn.2421265
61. Kley O, Klüppelberg C, Reinert G (2018) Conditional Risk Measures In A Bipartite Market Structur. Scandinavian Actuarial Journal 2018(4): 328–355. doi: 10.1080/03461238.2017.1350203
62. Silva TC, Souza SRS, Tabak BM (2017) Monitoring Vulnerability And Impact Diffusion In Financial Networks. Journal of Economic Dynamics and Control 76: 109–135. doi: 10.1016/j.jedc.2017.01.001

63. Blöchl F, Theis FJ, Vega-Redondo F et al. (2011) Vertex centralities in input-output networks reveal the structure of modern economies. Phys. Rev. E 83(4): 422. doi: 10.1103/PhysRevE.83.046127
64. Tang L, Jing K, He J et al. (2016) Complex Interdependent Supply Chain Networks: Cascading Failure And Robustness. Physica A: Statistical Mechanics and its Applications 443: 58–69. doi: 10.1016/j.physa.2015.09.082
65. Tang L, Jing K, He J et al. (2016) Robustness Of Assembly Supply Chain Networks By Considering Risk Propagation And Cascading Failure. Physica A: Statistical Mechanics and its Applications 459: 129–139. doi: 10.1016/j.physa.2016.04.030
66. Xu H, Cheng L (2016) The QAP Weighted Network Analysis Method And Its Application In International Services Trade. Physica A: Statistical Mechanics and its Applications 448: 91–101. doi: 10.1016/j.physa.2015.12.094
67. Huang X, Vodenska I, Wang F et al. (2011) Identifying influential directors in the United States corporate governance network. Phys. Rev. E 84(4): 425. doi: 10.1103/PhysRevE.84.046101
68. Che, Y.K, Kim J., Mierendorff, K. (2013) Generalized Reduced-Form Auctions: A Network-Flow Approach. Econometrica 81(6): 2487–2520. doi: 10.3982/ECTA11405
69. Chatrabgoun O, Hosseinian-Far A, Chang V et al. (2018) Approximating non-Gaussian Bayesian networks using minimum information vine model with applications in financial modelling. Journal of Computational Science 24: 266–276. doi: 10.1016/j.jocs.2017.09.002
70. Aymanns C, Georg C-P (2015) Contagious Synchronization and Endogenous Network Formation in Financial Networks. Journal of Banking & Finance 50: 273–285. doi: 10.2139/ssrn.2474293
71. Georg C-P (2013) The effect of the interbank network structure on contagion and common shocks. Journal of Banking & Finance 37(7): 2216–2228. doi: 10.1016/j.jbankfin.2013.02.032
72. Krug S, Lengnick M, Wohltmann H-W (2015) The impact of Basel III on financial (in)stability: an agent-based credit network approach. Quantitative Finance 15(12): 1917–1932. doi: 10.1080/14697688.2014.999701
73. Poledna S, Thurner S (2016) Elimination Of Systemic Risk In Financial Networks By Means Of A Systemic Risk Transaction Tax. Quantitative Finance 16(10): 1599–1613. doi: 10.1080/14697688.2016.1156146
74. Vitali S, Battiston S, Gallegati M (2016) Financial Fragility And Distress Propagation In A Network Of Regions. SSRN Journal 62: 56–75. doi: 10.2139/ssrn.2152697
75. Ikeda Y, Aoyama H, Iyetomi H et al. (2007) Response Of Firm Agent Network To Exogenous Shock. Physica A: Statistical Mechanics and its Applications 382(1): 138–148. doi: 10.1016/j.physa.2007.02.016
76. Boyer T, Jonard N (2014) Imitation And Efficient Contagion. Journal of Economic Behavior & Organization 100: 20–32. doi: 10.1016/j.jebo.2014.01.009
77. Frias-Martinez E, Williamson G, Frias-Martinez V (2011) An Agent-Based Model of Epidemic Spread Using Human Mobility and Social Network Information. In: IEEE Third International Conference on Privacy, Security, Risk and Trust (PASSAT), 2011 and 2011 IEEE Third International Conference on Social Computing (SocialCom): 9 - 11 Oct. 2011, Boston, Massachusetts, USA ; proceedings ; [including workshop papers. IEEE, Piscataway, NJ, pp 57–64
78. Ajelli M, Gonçalves B, Balcan D et al. (2010) Comparing Large-Scale Computational Approaches To Epidemic Modeling: Agent-Based Versus Structured Metapopulation Models. BMC Infect Dis 29(10): 190. doi: 10.1186/1471-2334-10-190
79. Smieszek T, Balmer M, Hattendorf J et al. (2011) Reconstructing The 2003/2004 H3N2 Influenza Epidemic In Switzerland With A Spatially Explicit, Individual-Based Model. BMC Infect Dis 9(11): 115. doi: 10.1186/1471-2334-11-115

80. Yang Y, Atkinson PM, Ettema D (2011) Analysis Of CDC Social Control Measures Using An Agent-Based Simulation Of An Influenza Epidemic In A City. BMC Infect Dis 18(11): 199. doi: 10.1186/1471-2334-11-199
81. Tully S, Cojocaru M, Bauch CT (2013) Coevolution Of Risk Perception, Sexual Behaviour, And HIV Transmission In An Agent-Based Model. Journal of Theoretical Biology 21(337): 125–132. doi: 10.1016/j.jtbi.2013.08.014
82. Perisic A, Bauch CT (2009) A Simulation Analysis To Characterize The Dynamics Of Vaccinating Behaviour On Contact Networks. BMC Infect Dis 28(9): 77. doi: 10.1186/1471-2334-9-77
83. Moslonka-Lefebvre M, Harwood T, Jeger MJ et al. (2012) SIS Along A Continuum (SIS C) Epidemiological Modelling And Control Of Diseases On Directed Trade Networks. Mathematical Biosciences 31(236): 44–52. doi: 10.1016/j.mbs.2012.01.004
84. Ma J, van den Driessche P, Willeboordse FH (2013) The Importance Of Contact Network Topology For The Success Of Vaccination Strategies. Journal of Theoretical Biology 21(325): 12–21. doi: 10.1016/j.jtbi.2013.01.006
85. Sherborne N, Blyuss KB, Kiss IZ (2016) Compact Pairwise Models For Epidemics With Multiple Infectious Stages On Degree Heterogeneous And Clustered Networks. Journal of Theoretical Biology 21(407): 387–400. doi: 10.1016/j.jtbi.2016.07.015
86. Fu R, Gutfraind A, Brandeau ML (2016) Modeling a dynamic bi-layer contact network of injection drug users and the spread of blood-borne infections. Mathematical Biosciences 31(273): 102–113. doi: 10.1016/j.mbs.2016.01.003
87. Lawyer G (2016) Measuring The Potential Of Individual Airports For Pandemic Spread Over The World Airline Network. BMC Infect Dis 9(16): 70. doi: 10.1186/s12879-016-1350-4
88. Rizzo A, Pedalino B, Porfiri M (2016) A network model for Ebola spreading. Journal of Theoretical Biology 7(394): 212–222. doi: 10.1016/j.jtbi.2016.01.015
89. Cannas SA, Marco DE, Montemurro MA (2006) Long Range Dispersal And Spatial Pattern Formation In Biological Invasions. Mathematical Biosciences 31(203): 155–170. doi: 10.1016/j.mbs.2006.06.005
90. Zhang P, Atkinson PM (2008) Modelling The Effect Of Urbanization On The Transmission Of An Infectious Disease. Mathematical Biosciences 31(211): 166–185. doi: 10.1016/j.mbs.2007.10.007
91. Sun G-Q, Liu Q-X, Jin Z et al. (2010) Influence Of Infection Rate And Migration On Extinction Of Disease In Spatial Epidemics. Journal of Theoretical Biology 7(264): 95–103. doi: 10.1016/j.jtbi.2010.01.006
92. Davies KJ, Green JEF, Bean NG et al. (2014) On The Derivation Of Approximations To Cellular Automata Models And The Assumption Of Independence. Mathematical Biosciences 31(253): 63–71. doi: 10.1016/j.mbs.2014.04.004
93. Brockmann D, Helbing D (2013) The hidden geometry of complex, network-driven contagion phenomena. Science 342(6164): 1337–1342. doi: 10.1126/science.1245200
94. Lloyd-Smith JO, Schreiber SJ, Kopp PE et al. (2005) Superspreading and the effect of individual variation on disease emergence. Nature 438(7066): 355–359. doi: 10.1038/nature04153
95. Madhav, N., Oppenheim, B., Gallivan, M., Mulembakani, P., Rubin, E., et. al. Pandemics: Risks, Impacts, and Mitigation. In: Disease Control Priorities (third edition): Volume 9, Disease Control Priorities, edited by D. T. Jamison, H. Gelband, S. Horton, P. Jha, R. Laxminarayan, C. N. Mock, R. Nugent.