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Generalized vehicle routing problem: A survey of solution methods and variants

Honoré Randriamaromisy [1*], Harrimann Ramanantsoa [2], André Totohasina [3]
[1] High School Mixed Sadjooavato, Antsiranana II, Madagascar
[2] Higher Institute of Technology of Antsiranana, Antsiranana, Madagascar
[3] Higher Normal School for Technical Education, University of Antsiranana, Antsiranana, Madagascar

* Corresponding Author: **Honoré Randriamaromisy**

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Abstract
Generalized Vehicle Routing Problem(GVRP) is a NP-hard combinatorial problem. It consists of finding a set of routes for a number of vehicles with limited capacity on a graph with the vertices partitioned into clusters with given demands such that exactly one vertex from each cluster is visited. Since there is no survey on this problem, this paper can review all variants studied and the solution method used to solve it. This article provides an up-to-date overview of the research papers addressed on this difficult problem.

Keywords: Cluster, generalized vehicle routing problem, generalized traveling salesman problem, Metaheuristics

1. Introduction

Vehicle Routing Problems (VRP) constitute a class of Combinatorial Optimization and Integer Programming Problems (COP and IPP), the most studied in the field of Operational Research (OR), Transportation and distribution (eg transportation of goods and people and urban logistics, green logistics). It was proposed first by Dantzig and Ramser in 1959 (Dantzig and Ramser (1959) [13] under the name « Truck Dispatching Problem » in which they modelled how a fleet of homogeneous trucks could serve the demand for oil of a number of gas stations from a central hub and with a minimum travelled distance.

VRP is a generalization of Travelling Salesman Problem (TSP). TSP is a NP-hard combinatorial problem, so the VRP is NP-hard too and very complex to solve. It aims to determine a set of least cost delivery routes from a depot to a set of geographically scattered customers, subject to a set of constraints in order to minimize the cost of visits.

To model a more realistic problems and due to the development of communication and technologies, several versions of the Vehicle Routing problems have been proposed and studied in the literature by adding a constraint or taking into account other characteristics of the problem. One of the most studied and the most popular are Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing problem with Times Windows (VRPTW), Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD), Multi-Depot Vehicle Routing Problem (MDVRP), Stochastic Vehicle Routing Problem, Dynamic vehicle Routing Problem (DVRP), Locating vehicle Routing Problem (LVRP), Open Vehicle Routing Problem (OVRP) and Green Vehicle Routing Problem (GreenVRP).

The Generalized Vehicle Routing Problem (GVRP) is a variant of VRP introduced by Ghiani and Improta, Ghiani and Improta (2000) [20] in 2000. It is a generalization of VRP and consists of finding a set of routes for a number of vehicles with limited capacities on graphs with vertices (nodes) partitioned into clusters with given demands such that the total cost of travel is minimized. All demands in each cluster are considered as satisfied by visiting exactly one client(nodes) in the cluster.

In the literature, the are a large number of surveys on VRP and its variants, among them Braekers *et al*. (2016) [12] who presented a review of the classification of VRP.

Goel and Maini (2017) [21] presented a survey on solution methodologies on VRP and Kumar and Panneerselvam (2012) [38], Han and Wang (2018) [26] review the variants of VRP. A MDVRP survey has been conducted by Sharma and Monika in Sharma and Monika (2015) [70]. This article by Prodhon and Prins (2014) [63] present a survey on LRP and the recent contribution before 2014 and one other concerns the variant and extensions (Drexl and Schneider, 2015) [16]. A survey on DVRP has been done in Ritzinger *et al*. (2016) [67]. A systematic review on SVRP has also been conducted in this article Berhan *et al*. (2014) [6].

Although GVRP can model many variants of VRP and has many applications in the real world R. Baldacci and Laporte (2008) [64], it has been considered by very few authors in the literature and to the best of our knowledge there is no survey on GVRP so far. Therefore this paper aims to review on GVRP and provide an up-to-date overview of the researches on this problem and its variants.

The remainder of this paper is organized as follows. First, we give the definition and formulation of GVRP in Section 2. Then, we present in Section 3, a literature review on GVRP in which we start with an overview of Generalized Traveling Salesman Problem(GTSP) in Section 3.1. We give a non-exhaustive literature review on GTSP but do not discuss about it. We continue in section 3.2 where we give a literature revue on the solutions approach of the GVRP and its variants. After that, Section 4 is dedicated to discuss the results of the survey. Finally, we conclude the survey in Section 5.

Definition and formulation of GVRP

The formal definition of GVRP is given as follows. Let $G = (V, E)$ be a graph, where V is the vertex set and E is the edge set. $V = v_0, v_1, ..., v_{n-1}$ is the set of n vertices that can be visited, and vertex v_0 is the depot containing m homogeneous vehicles with capacity Q. The set $C = C_0, C_1, ..., C_{K-1}$ is the set of K clusters. Each cluster C_i except C_0, which contains only the depot, has a demand D_i. Each cluster includes a number of vertices of V, and every vertex in V belongs to exactly one cluster. For each $v_i \in V$, let $\alpha(i)$ be the cluster that contains a vertex v_i. The term $D(S) = {}^{\mathrm{P}}{}_{i|C} i \subseteq_S D_i$ is used to represent the total demand in set S which is a subset of V. A length c_{ij} is associated with each edge of $E = (v_i, v_j) : v_i, v_j \in E, i < j$. The GVRP consists in finding m vehicle routes such that :

1. Each route begins and ends at the depot;
2. Each route visits exactly one vertex of each cluster and visits it only once;
3. The demand served by each route does not exceed the vehicle capacity Q;
4. The total cost is minimized.

The GVRP is clearly NP-hard problem since it reduces to a CVRP when each cluster includes only one vertex or to a GTSP when the capacity constraints are relaxed and the number of vehicles is $m = 1$. The problem can be formulated using an integer linear programming formulations like in (Kara and Bektas, 2003) [32], (Pop *et al*., 2012) [53] and (Bektas *et al*., 2011). The most popular is the two index flow formulations presented by (Bektas *et al*., 2011) [54]. Another formulations possible is set covering formulations (see Yuan *et al*. (2021) [79].

FIGURE 1 presents an optimal solution of the GVRP's instance decribed by Ghiani and Improta (2000) [20]. This solution is obtained by the formulation presented in (Pop *et al*., 2012) [53]. In this instance, the number of vehicles is $m = 4$ with common capacity $Q = 15$. The number of vertices is $n = 50$, which is partitionned into $K = 25$ clusters.

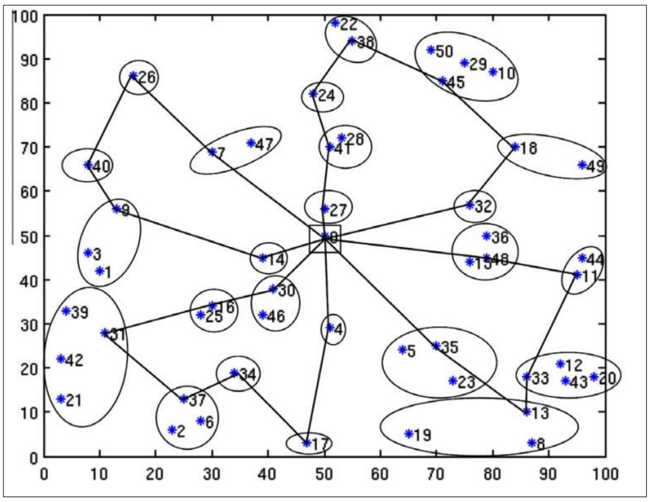

Fig 1: The optimal solution of the GVRP instance described by Ghiani and Improta (2000) [20]

Literature review on GVRP

A related work: An Overview of GTSP

GTSP is a particular case of GVRP. We think it is important to make an overview of the research on this class of problems even if it is not exhaustive because the literature on the GTSP is rich. This problem is a generalization of the most popular combinatorial problem « Traveling Salesman Problem ». It was proposed first in 1969, independently by Srivastava *et al*. (1969) [74] and Henry-Labordere (1969) [27], and it was solved by exact method called « Dynammic Programming ».

Table 1 represent articles addressed on the GTSP. In this tabular, we give an up to date contribution on this problem. It may be useful to conceive an algorithm for solving GVRP.

LAPORTE *et al*. (1987) [39] solved exactly the asymmetrical GVRP (A-GTSP) by Branch-and-Bound

Algorithm. They solved the instances of GTSP up to 100 nodes and 8 clusters to optimality. (Noon and Bean, 1991) [46] used a Lagrangian based approach to solve the same problem. Fischetti *et al*. (1997) [19] proposed a Branch-and-Cut algorithms for symmetric GTSP

A hybrid genetic algorithms with several local searches has been proposed by Potvin and Ladurantaye (2001) [60] for GTSP. (Huang *et al*., 2005) [30] proposed, respectively, a novel chromosome's encoding called « Generalized Chromosome » and « Hybrid Chromosome » for solving the GTSP. Snyder and Daskin (Snyder and Daskin, 2006) [73] proposed to solve this problem with an another Genetic Algorithm using a random key encoding, which assures that solutions constructed by crossover or mutation are feasible. The Genetic Algorithm was coupled with local search improvement, namely, a swap procedure and a 2-opt neighborhood search, yielding a Memetic Algorithm. Computational results show the efficiency of their algorithm in terms of solution quality and computation time.

Silberholz and Golden (Silberholz and Golden, 2007) [72] proposed another Genetic Algorithm with several new features, including isolated initial populations and a new reproduction mechanism, based upon the TSP ordered crossover operator. This new mechanism was called mrOX, for modified rotational ordered crossover. Local improvement procedures combined with this mechanism, yielding again a Memetic Algorithm, permit to obtain very good results on large new instances. This algorithm can be considered as the most competitive algorithm published before 2012. A Particle Swarm Optimization based algorithm was also recently developed in this paper Shi *et al*. (2007) [71]. Bontoux *et al*. (2010) [10] proposed a memetic algorithm.

Table 1: Solutions approach and variants of GTSP

		GTSP variants		**Solution method**	
Year	**References**	**Name**	**Objective**	**Exact method and software**	**Heuristic and Metaheuristic approach**
1969	Srivastava *et al*. (1969) [74]	GTSP	SHC	Dynamic programming	
1969	Henry-Labordere (1969) [27]	GTSP	SHC	Dynamic Programming	
1987	LAPORTE *et al*. (1987) [39]	A-GTSP	SHC	Branch and Bound	
1991	Noon and Bean (1991) [46]	A-GTSP	SHC	Lagragian based Approach	
1997	Fischetti *et al*. (1997) [19]	GTSP	SHC	Branch and Cut	
1998	Renaud and Boctor (1998) [65]	GTSP	SHC		Composite heuristic
2001	Potvin and Ladurantaye (2001) [60]	GTSP	SHC		Hybrid Genetic Algorithm
2004	Wu *et al*. (2004) [74]	GTSP	SHC		Generalized Chromosome GA
2005	Huang *et al*. (2005) [30]	GTSP	SHC		Hybrid Chromosome GA
2006	Snyder and Daskin (2006) [73]	GTSP	SHC		Genetic Algorithm
2007	Silberholz and Golden (2007) [72]	GTSP	SHC		Genetic Algorithm
2007	Shi *et al*. (2007) [71]	GTSP	SHC	X	Particle Swarm Optimization
2008	Gutin *et al*. (2008) [22]	A-GTSP	SHC		Memetic Algorithm
2008	Hu and Raidl (2008) [29]	GTSP	SHC		Local Search+VNS
2010	Bontoux *et al*. (2010) [10]	GTSP	SHC		Memetic Algorithm
2010	Matei and Pop (2010) [42]	GTSP	SHC		Genetic Algorithm
2010	Bontoux *et al*. (2010) [10]	GTSP	SHC		Memetic Algorithm
2010	Pop *et al*. (2010a) [54]	GTSP	SHC		Local-Global Approach+GA
2010	Pop *et al*. (2010a) [54]	GTSP	SHC		Local-Global Approach+GA
2011	Karapetyan and Gutin (2011) [33]	GTSP	SHC		Lin-Kernighan Heuristic
2015	Pourhassan and Neumann (2015) [61]	GTSP	SHC		Variable Neighborhood Search
2017	Pintea *et al*. (2017) [51]	GTSP	SHC		Reinforcing Ant Colony System(RACS)
2017	L.Smith and Imeson (2017) [41]	GTSP	SHC		Large Neighborhood Search
2018	Zia *et al*. (2018) [80]	E-GTSP	SHC		Transformation to TSP
2018	Pourhassan and Neumann (2018) [62]	GTSP	SHC		Local Search+ Evolutionary Algorithm
		GTSP variants		**Solution method**	
Year	**References**	**Name**	**Objective**	**Exact method and software**	**Heuristic and Metaheuristic approach**
2019	Krari *et al*. (2019) [36]	GTSP	SHC		
2020	Schmidt and Irnich (2020) [70]	GTSP	SHC		Iterated Local Search
2020	Yuan *et al*. (2020) [78]	GTSPTW	SHC	Branch and Cut	
2020	Krari *et al*. (2020) [37]	GTSP	SHC		Memetic Algorithm
2020	Derya *et al*. (2020) [14]	SGTSP	MTP	CPLEX 12.6	

Their main contribution is the introduction of the « Large Neighboorhood Crossover Operator » inspired from *DropStar procedure*. The Local Searches are also applied to improve the quality solutions of individuals, both for the initial population and for every child obtained from the Crossover Operator. Four Local searches are used and managed like this : when a new individual is introduced into the population during the initialization phase, 2-opt, 3-opt, Lin-Kernighan procedure are successively applied in this order. When a new child is computed with the crossover operator, one of the following local search scheme are applied with probability 0,5 :2-opt, 3-opt, Move or only Lin-Kernighan.

Hu and Raidl (2008) [29] have presented two hierarchical approaches for solving the GTSP: The cluster-based approach, which uses a permutation on different clusters in the upper level and finds the best node selection for that permutation on the lower level, and the node-based approach, which selects a node for each cluster and then works on finding the best permutation of the chosen nodes. Combining the two hierarchical approaches, they have also presented a variable neighbourhood search (VNS) algorithm for solving the GTSP. In (Pourhassan and Neumann, 2018) [62], the authors contribute to the theoretical understanding of local search methods and simple evolutionary algorithms based on these hierarchical approaches for GTSP.

A modified Ant Colony System, called Reinforcing Ant Colony System, have been proposed by Pintea *et al*. (2017) [51] for GTSP. Two new rules were proposed : a new pheromon rule and a new pheromon evaporation.

The first variant of GTSP including the time window(GTSPTW) was studied first by Yuan *et al*. (2020) [78]. Two integer linear programming(ILP) formulations for GTSPTW are provided as well as several problem-specific valid inequalities. A branch-and-cut algorithm is developed in which the inequalities are separated dynamically. To reduce the computation time, an initial upper bound is provided by a simple and fast heuristic. Different sets of instances characterized by their time window structures have been proposed. Experimental results show that the algorithm is effective and instances including up to 30 clusters can be solved to optimality within one hour.

Krari *et al*. (2020) [36] proposed a memetic algorithm with a novel local search procedure: Breakout Local Search(BLS). BLS is a metaheuristic based on Iterated Local Search(ILS). It consist of discovering the search space by moving from a neighborhood to a new one via perturbation when the algorithm meets a local optimum. Three types of perturbation are presented : direct perturbation, recency based perturbation and random perturbation. The results shows the effectiveness of the algorithm compared with other methods. In 2019, The autors proposed a novel preprocessing reduction method for GTSP (Krari *et al*., 2019) [37].

Recently, a variant of the GTSP, called Selective GTSP(SGTSP), has been studied by Derya *et al*. (2020) [14]. In this problem, nodes are clustered into groups and neither all clusters or all nodes need to be visited, but only one node within each cluster may be visited. The constraint « a cluster may be do not visit » is not allowed in the classical GTSP. This problem is also different in that a profit is associated with each node and collected only if it is visited. Eight mixed integer linear programming have been proposed for Selective GTSP. 4608 computational runs on CPLEX 12.6 were conducted and 90% test instances were solved to optimality by using all formulations.

Several researchers suggested the transformation of GTSP to TSP (Noon and Bean, 1993) [47], (Laporte and Semet, 1999) [40], (Ben-Arieh *et al*., 2003), (Zia *et al*., 2018) [80], because there exist many efficient exacts, heuristics and metaheuristics algorithms for the TSP. This idea is limited since we must have an exact solutions from obtained TSP instance, otherwise it may lead to an infeasible GTSP solution.

Several local search heuristics and neighborhood structures were studied and proposed for the GTSP : Large Neighborhood Search Heuristic (L.Smith and Imeson, 2017) [41]; Variable Neighborhood Search (Pourhassan and Neumann, 2015) [61]; Composite Heuristic (Renaud and Boctor, 1998) [65]; Lin-Kernighan (Karapetyan and Gutin, 2011) [33], Local search and neighborhood (Hu and Raidl, 2008) [29], (Karapetyan and Gutin, 2012) [34]; Iterated Local Search and New Neighborhood (Schmidt and Irnich, 2020) [70].

To conclude this section, we can say that the literature on the GTSP is rich because this lierature revue is not exhaustive but representative. The recent contribution on the GTSP have been presented. We hope that this overwiew help a future searcher in this domain.

Solutions approaches and variants of GVRP

GVRP is a recent variant of VRP. It was proposed by Ghiani and Improta in this paper Ghiani and Improta (2000) [20]. They conducted a theory and methodology research in which they proved that GVRP can be transformed efficiently into Capacitated Arc Routing Problem(CARP) for which an exact algorithm and several approximate procedures are reported in literature. This transformation into CARP was the only approach to solve GVRP. They applied the CARPET heuristic (Hertz *et al*., 1996) [28] to experiment their work numerically. They proposed the first problem test of GVRP with 50 nodes partionned in 22 clusters. This instance is obtained by taking VRP test problem 7 from Araque *et al*. (1994).

In 2003, Kara and Bektas (2003) [32] proposed the first integer linear programming formulation(ILPF) for GVRP with a polynomially increasing number of binary variables and constraints. Their model reduce to ILPFs of Generalized multiple travelling salesman problem(GmTSP), generalized Travelling Salesman Problem(GTSP), and CVRP. Two problem tests from Araque *et al*. (1994) [3] have been used to test the proposed formulations. They experimented the model by using CPLEX 6.0. As result, the problems are solved to optimality. In 2012, Pop *et al*. (2012) [53] introduced two new formulations simular. The first is called *node formulation*, simular with the formulations in Kara and Bektas (2003) [32] but produces a stronger lower bound and the second is *a flow formulation*. Using CPLEX 12.2, they solved one instance from Ghiani and Improta (2000) [20] and obtained the same result as in Kara and Bektas (2003) [32]. However, there has seen no study comparison between the two formulations. At the same time, Bektas *et al*. (2011) [4] proposed four new formulations for GVRP. Contrary to Pop *et al*. (2012) [53], the authors conducted an experimental study to compare their formulations. They concluded that the best one was an undirected two-index flow based on an exponential number of constraints. They also proposed a branch-and-cut algorithm for each formulation in which the upper bounds has been calculated by a Large Neighborhood Search(LNS) heuristic.

Pop *et al*. (2009) [52] proposed Ant Colony System(ACS) algorithm to solve GVRP. For that purpose, the artificial ants construct vehicles routes by successively choosing exactly one node from each cluster to visit until each cluster has been visited. Whenever the choice of another node from a cluster must take into consideration two aspects : the pheromon tails associated with each edge at time *t* and the visibility of the node. They also proposed a genetic algorithm combined with local-global approach to solve the same problem (Pop *et al*., 2010b) [55]. They then proposed several local searches (Pop and Horvat-Marc, 2012) [53] and heuristics (Pop *et al*., 2011) [51] to solve the GVRP.

In his thesis, HA (2012) [25] introduces a version of the GVRP where the number of vehicles can be constant or variable. He presented a new formulation based on two-comodity flow model. He also developped an Branch-and-Cut algorithm and a hybrid metaheuristic combining Greedy Randomized Adaptive Search Procedure(GRASP) and the Evolutionary Local Search(ELS). The metaheuristic aims to provide a good upper bound for the exact method proposed. The main component of this algorithm is *a split,concat and mutate procedure*. *The split procedure* converts a giant tour based on the clusters and encoded as TSP tour in a GVRP solution. *The concat procedure* concatenates GVRP routes into a giant tour and *the mutate procedure* swaps randomly the position of two vertices in the giant tour. Six local searches which include *One-point move*, *Or-Opt move*, *Three-point move*, *Two-point move*, *Two-opt move* and *Two-adjacent move*, have been integrated, in this order, to improve the solution. All these moves operate within and between routes. Computational experiment shows that the exact method can solve instances up to 121 vertices and 51 clusters. The metaheuristic gives a high-quality solution in a reasonable computational time.

In (Afsar *et al*., 2013) [1], The GVRP with flexible fleet size have been studied and solved by an exact and heuristic method. The exact method is based on column generation algorithm. Two methaheuristics derived from Iterated Local Search (ILS) have also been proposed. The result showed that the exact method provides good upper and lower bonds, whereas the metaheuristics find a solution with small optimality gap in a few seconds.

Ha *et al*. (2014) [25] solved the same problem using Branch-and-Cut algorithm and a hybrid of the Greedy Randomized Adaptive Search Procedure and the Evolutionary Local Search proposed in Ha *et al*. (2012).

Moccia *et al*. (2012) [44] considered the variant with time windows(GVRPTW). An incremental tabu search has been developed for solving it with a new neighborhood structure.

In the context of E-commerce, Rodriguez *et al*. (2017) [68] have studied a special case where time windows(TWs) within the same cluster do not overlap. They were inspired by the trunk delivery system and called this problem VRP with Roaming Delivery Locations (VRPRDL). They developed construction and improvement heuristics, and their results proved the advantage of using trunk delivery. The construction heuristic is inspired by the family of GRASP and the improvement heuristic implements a variable neighborhood search using destroy and recreate paradigm. Three different destroy-recreateneighborhood were used in their heuristics. Following this work, VRPRDL was formulated as a set-partitioning problem and a branch-andprice algorithm was proposed in (Ozbaygin *et al*., 2017) [48]. This algorithm is able to deal with a hybrid delivery strategy combining trunk delivery and home delivery, in which case the TWs within a cluster are no longer nonoverlapping.

Always in the context of E-commerce, Tilk *et al*. (2020) [75] introduced the Vehicle Routing problem with Delivery Options(VRPDO). This problem is clearly a generalization of the vehicle routing problem with time windows and the generalized vehicle routing problem with time windows(Moccia *et al*., 2012) [44] in which each customer request is represented by one or several delivery options. The delivery options of a customer differ within the designated location and delivery time window. Exactly one delivery option for each customer has to be selected. The VRPDO extends the GVRPTW by two important real-world aspects : First, customers can individually prioritize their different delivery options beforehand, and the overall customer satisfaction level is taken into account for a given service level that must be achieved. Second, some delivery options may share a common location. The capacity of these locations is limited, in particular in densely populated areas of cities where space is scarce and expensive. For finding an optimal set of routes, both extensions lead to a nontrivial interdependence problem, where modifying one route can make another route infeasible regarding location capacity or required service level. The objective of the VRPDO is to minimize the overall cost while ensuring a minimum customer satisfaction level as well as not violating location capacity restrictions. Tilk *et al*. (2020) [75] proposed a branch-cut-price algorithm to tackle this generalized problem.

Dumez *et al*. (2021a) [17] define the VRPDO which integrates several types of delivery locations and they proposed a Large Neighborhodd Search heuristic for finding an optimal set of routes with minimal cost. Then they proposed a hybrid metaheuristic combining LNS and an exact method for VRPDO Dumez *et al*. (2021b) [18].

An another variant called Distance Constrained Generalized Vehicle Routing Problem(DCGVRP) has been studied for the first time by Markus Mattila in Mattila (2016) [43]. The main characteristics of the problem are : unlimited vehicle fleet, limited vehicle capacity, arrangement of customers into clusters, and an upper limit on each route length. He presented a mathematical formulation of DCGVRP in which they use two commodity and single commodity flow formulations to describe the constraints on vehicle capacity and maximum route length. Furthermore, he proposed a heuristic algorithm in which the main components include split procedure and solving a set partitioning model.

A variant with stochastic demands has been considered first by Biesinger *et al*. (2015) [9]. The GVRP with stochastic demand combines the GVRP and the vehicle routing problem with stochastic demands(VRPSD), where the exact demands of the nodes are not known beforehand. The authors proposed a variable neignhborhood search(VNS) to minimize the expected tour length through all clusters of this NP-hard problem. They used a permutation encoding of the clusters (giant tour) and considered the preventive restocking strategy. This strategy will allow the vehicle to restock before it potentially runs of goods. Dynamic programming is used to evaluate the exact solution, but in order to reduce the time consuming, they proposed a multi-level evaluation scheme (ML-ES) of the solutions. The farthest insertion and solving GTSP relaxation problem are two different algorithms considered for finding the initial solution. 1-Shift, 2-Opt, and Or-Opt are the neighborhood structures included within the VNS. The experimental results show that the proposed ML-ES is able to reduce drastically the overall run-time of the algorithm. It is also essential to tackle larger instances. The VNS proposed is able to find optimal or nearoptimal solution in much shorter time. Pop *et al*. (2014) [52] proposed the VNS for the classical GVRP.

In 2016, Biesinger *et al*. (2016) [7] developed exact algorithm for GVRPSD. This algorithm is based on decomposition and Branch-and-Cut. The results show that their approach is effective for solving smaller instances up to about 40 nodes and 13 clusters. Two years later, the same authors published an article (Biesinger *et al*., 2018) [8] where they presented a genetic algorithm for the GVRPSD called Genetic Algorithm with Solution Archive (GASA). They reused all the strategy used in Biesinger *et al*. (2015) [9] but the main component of this approach is the solution archive.

Posada *et al*. (2018) [59] and Sabo *et al*. (2020) [69] proposed a mixed integer formulation for the selective vehicle routing problem(SVRP). This is a variant of the GVRP presented first by Posada *et al*. (2018) [59]. In this problem, certain nodes can belong to more than one cluster. Sabo *et al*. (2020) [69] experimented the model using CPLEX 12.4. Analyzing the results of the experimentation on the Posada *et al*. (2018) [59] instances, the proposed model outperforms the existing models, providing lower average gaps, the shortest running time, and the largest number of achieved optimal solutions. In the case of the GVRP instances, if it allowed the computer to run for more than 1800 s, it were able to obtain 11 optimal solutions out of 12 instances and the obtained average gap was 0.59%. In addition, they proposed new instance of SVRP.

Recently, Yuan *et al*. (2021) [79] studied the generalized vehicle routing problem with time windows (GVRPTW) and proposed a heuristic based on the exact method called « Column generation ». The time window are associated with each vertex during which the visit must take place if the vertex is selected. Their objective is to find a set of routes such that the total traveling cost is minimized, exactly one vertex per cluster is visited, and the capacity and time window constraints are respected. The GVRPTW has been modelled by using the set covering formulation. This formulation has been used to provide their heuristic method. The proposed method combines several components that include a construction heuristic, a route optimization procedure, a local search and the generation of negative reduced cost. They solved several benchmarck up to 120 clusters efficiently with high-quality of solution.

Table 2

		GVRP variant			Solution method	
Year	**References**	**Name**	**Objective**	**Particularity attribut**	**Exact method and software**	**Heuristic and Metaheuristic approach**
2000	Ghiani and Improta (2000) [20]	GVRP	shortest routes	indentical vehicles, symmetric	X	CARPET heuristic based on tabu search
2003	Kara and Bektas (2003) [32]	GVRP	Shortest routes	indentical vehicles, directed graph	CPLEX 6.0 (softaware)	X

Table 3: Solutions approach and variants of GVRP

		GVRP variants			Solution methods	
2008	R. Baldacci and Laporte (2008) [64]	GVRP	Shortest routes	indentical vehicles, directed graph	X	X
2009	Pop *et al.* (2009) [51]	GVRP	Shortest routes	indentical vehicles, directed graph	X	Ant Colony System
2010	Pop *et al.* (2010b) [55]	GVRP	Shortest routes	indentical vehicles, directed graph	X	Genetic Algorithm
2011	Pop *et al.* (2011) [54]	GVRP	Shortest routes	Identical vehicle	X	Several Heuristic
2011	Bektas *et al.* (2011) [4]	GVRP	Shortest routes	Identical vehicle	Branch and Cut; CPLEX 10.0	Large Neighborhood Search
2012	Pop and Horvat-Marc (2012) [53]	GVRP	Shortest routes	Identical vehicle	X	Local Search
2012	Pop *et al.* (2012) [53]	GVRP	Shortest routes	directed graph, identical vehicles	CPLEX 12.2	X
2012	HA (2012)	GVRP	Shortest routes	Number of vehicles is variable	Branch and Cut; CPLEX 10.4	GRASP+ELS
2012	Moccia *et al.* (2012) [44]	GVRPTW	Shortest routes	Time window	X	Tabu Search
2013	Afsar *et al.* (2013) [1]	Flexible-GVRP	Shortest routes	flexible fleet size	Column Generation	Iterated Local Search
2014	Ha *et al.* (2014)	Flexible-GVRP	Shortest routes	flexible fleet size	Branch and Cut	GRASP+ELS
2014	Pop *et al.* (2014) [52]	GVRP	Shortest routes	classical	X	Variable Neighborhood Search
2015	Biesinger *et al.* (2015) [8]	GVRPSD	Shortest routes	stochastic demand	X	Variable Neighborhood Search
2016	Biesinger *et al.* (2016) [75]	GVRPSD	Shortest routes	stochastic demand	Integer L-shaped method, CPLEX 12.6	X
2016	Mattila (2016) [43]	DCGVRP	Shortest routes	Distance constrained, unlimited vehicle fleet	X	Heuristic
2017	Rodriguez *et al.* (2017) [68]	VRPRDL	Shortest routes	Delivery Locations	X	Heuristic
2017	Ozbaygin *et al.* (2017) [48]	VRPRDL	Shortest routes	Delivery Locations	Branch and Price	X
2018	Posada *et al.* (2018) [59]	SVRP	Shortest routes	Certain clusters are not disjoint	CPLEX	X
2018	Biesinger *et al.* (2018) [8]	GVRPSD	Shortest routes	stochastic demand	X	Genetic Algorithm
2020	Sabo *et al.* (2020) [69]	SVRP	Shortest routes	Certain clusters are not disjoint	CPLEX 12.4	X
2020	Tilk *et al.* (2020) [75]	VRPDO	Shortest routes	Delivery options	Branch and Cut and Price	X
2021	Yuan *et al.* (2021) [79]	GVRPTW	Shortest routes	time windows	X	A column generation based heuristic
2021	Dumez *et al.* (2021a) [17]	VRPDO	Shortest routes	Delivery options	X	Large Neighborhood Search
2021	Dumez *et al.* (2021b) [18]	VRPDO	Shortest routes	Delivery options	X	hybrid Large Neighborhood Search and exact method

Discussion and results

GVRP is a generalization of VRP and it is a recent since it has proposed first by Ghiani and Improta in 2000 [20]. Since there, we have selected 26 papers that addresses on this problem. Although this problem has many applications and can model many variants of VRP, there is a few attention on this variant of the VRP compared to the other. Note that in this survey, we have included journal paper, conference paper, improceedings, discussion paper, technical report and thesis.

FIGURE 2 represent the evolution of the paper published up to 2021. We see that the interest of researchers for this problem and its variants stardted 10 years later the first publication. Sure enough, between 2010 and 2015 there are 10 articles addressed on GVRP compared to 4 artciles since 2000 up to 2010. After 2015 there are 12 articles has been published. Based on this survey, table 2 represent the works that addressed on the GVRP and its variants. The first column contains the year of the publication followed by the article's references. The GVRP variant combine three columns that contain the objective function, the name of the variant, and the particularity of the variant. We show that with respect to the objective function, to the best of our knowledge, all the variant consider the minimization of the distance travelled (shortest routes). The remaining columns represent the solution methods used in the literature.

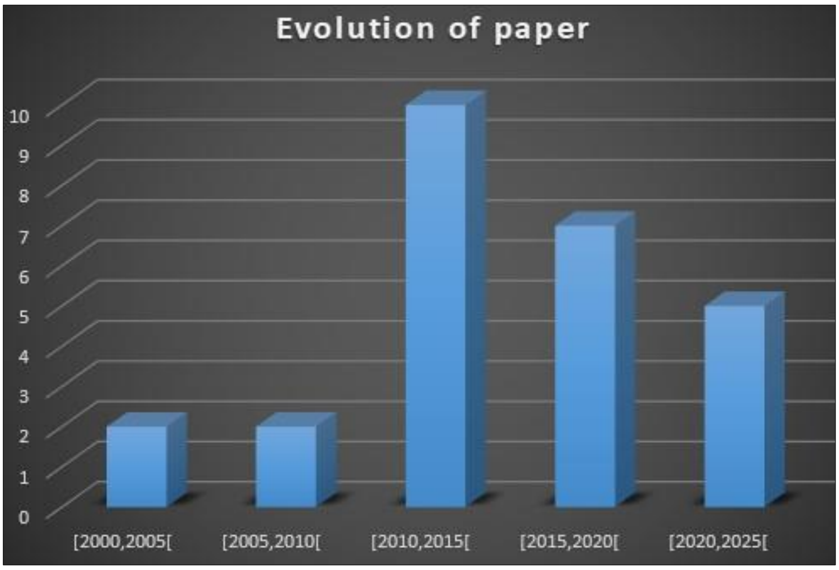

Fig 2: Evolution of the paper on GVRP and its variants

Figure 3 shows the distribution of the paper based on the solution methods classified into exact and approximative approaches (heuristic and metaheuristic). As we saw in the table 2, the majority of the work solved the problem by the heuristic and metaheuristic with 70% of proportion. Sure enough, the GVRP is a NP-hard problem and finding the exact solution is difficult in less time. Thus the searchers consider the metaheuristic to find a near-optimal solution of a small and larger instances in less time. FIGURE 4 classify the solutions technic and give a list of all methods used in the literature.

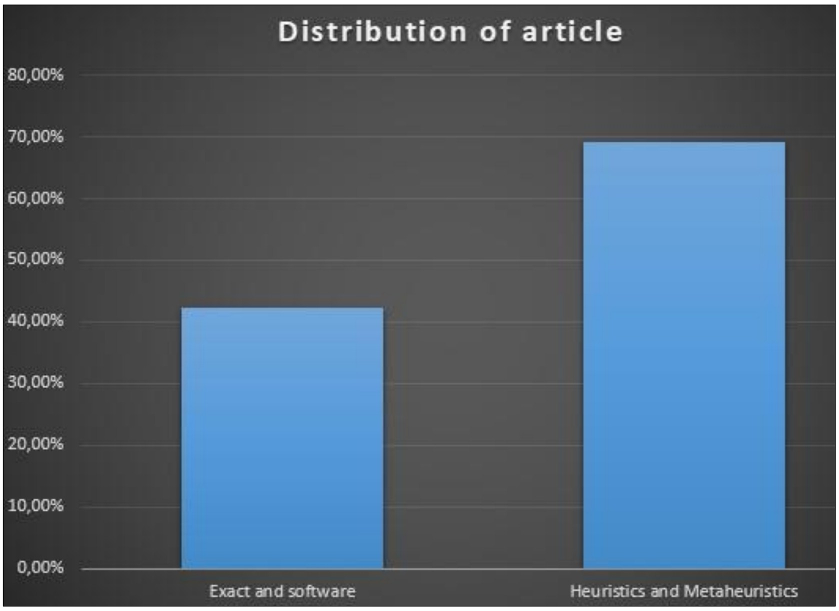

Fig 3: Exact method VS Metaheuristic approach

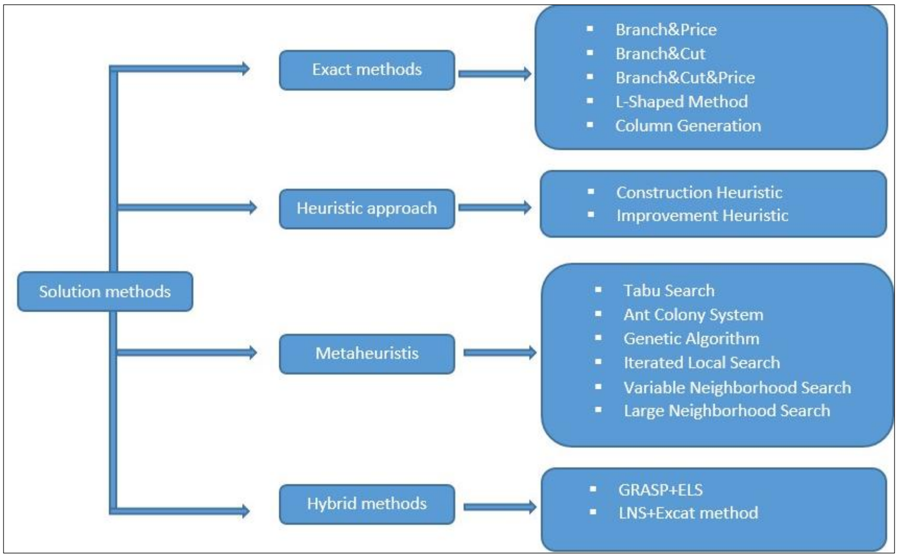

Fig 4: Classification of the solution methods on GVRP

With respect to the GVRP variants, the survey show that the classical variant still the most studied (42%). The study include transformation into another variants of VRP, formulations, applications and solution method. GVRPTW variants (VRPDO and VRPRDL) take the second place with 27% of proportion and followed by the stochastic variant(GVRPSD)(11%). DCVRP, SVRP, flexible-GVRP are the least studied.FIGURE 5 represent the distribution of te=he GVRP variants.

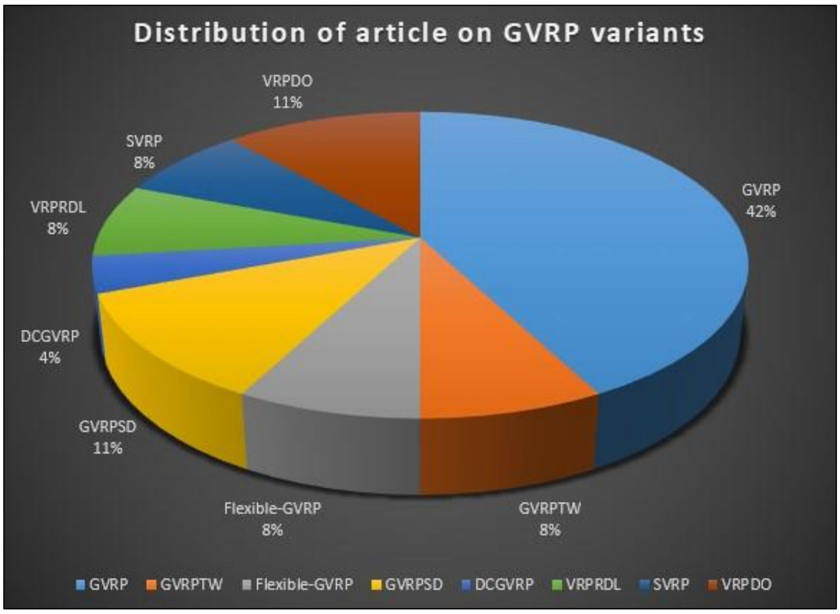

Fig 5: The variants of GVRP in the literature

Conclusion and perspectives

In this paper, we have made a survey on the Generalized Vehicle Routing Problem.This survey focused on its variants and the solution methods used in the literature. We have recensed 26 articles that adressed with this problem. It means that, this variant is not yet very considered in compared with other variants like Capacitated vehicle Routing problem, Vehicle Routing Problem with Time Windows, Multi Depot Vehicle Routing Problem, Stochastic Vehicle Routing Problem.

On the one hand, with respect to variants of this problem we showed that there are principally 8. The classical GVRP still the most studied compared to other even if another variants have been proposed and studied recently including GVRPSD, GVRPTW, DCVRP, VRPDO. On the other hand, with respect to the solution methods, since the problem is NP-hard, heuristic and metaheuristic is the dominant methods used to solve this problem and its variants including hybrid metaheuristic, genetic algorithm, tabu search, variable neighborhood search, iterated local search.

As a perspective, we cite here the possible future research on the GVRP and its variants.

1. Taking account another VRP's domain or attribute (Braekers *et al*., 2016) [12] and consider a new variants of this problem and proposing a solution method and formulation. For example, Multi Depot Generalized Vehicle Routing Problem (MDGVRP), Generalized Locating Routing Problem (GLRP), Green Generalized vehicle Routing Problem (GreenGVRP), Open Generalized Vehicle Routing problem (OGVRP),

Dynamic Generalized vehicle Routing Problem.
2. Applying and exploring the capacity of another metaheuristic inspired by nature to solve GVRP and its variants. Now, There are many metaheuristics in the literature that include Particle Swarm optimization (Peng *et al*., 2017) [50], (Peng *et al*., 2018) [49], Cuckoo Search (Rezaeipanah *et al*., 2019) [66], Firefly Algorithm (Jamil *et al*., 2021) [31], Bat Algorithm (Amalia *et al*., 2020) [2] (see this survey Boussaid *et al*. (2013)) [11]. Adding to that, many new metaheuristics have been proposed after this survey, among them include Whale Optimization Algorithm (Dewi and Utama, 2021) [15], Butterfly Optimization Algorithm (Utama *et al*., 2020) [76], Chicken Swarm Optimization (Niazy *et al*., 2020) [45]; Grey Wolf Optimization (Korayem *et al*., 2015) [35], etc.

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