Array designs optimized for both low-frequency NAH and high-frequency Beamforming

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Abstract [425] Near-field Acoustical Holography (NAH) can perform source location with high spatial resolution even at low frequencies by measuring very close to the sound source and by reconstructing part of the evanescent near-field. But a measurement grid with less than half wavelength spacing is required, and the measurement area should cover the main radiating regions to avoid windowing effects. These requirements make the method impractical at higher frequencies, typically above 3-5 kHz. At those higher frequencies, however, Beamforming can provide good resolution with typically 40-90 measurement points, because it is possible to use irregular arrays at intermediate measurement distances. Various array designs exist that can provide good suppression of ghost images up to frequencies, where the average element spacing is much larger than half a wavelength. The problem to be addressed here is that it is not practical to have to use two different arrays to perform the two types of measurement. The desired solution is to take two measurements with the same array: One for NAH at a short measurement distance and one for Beamforming at a somewhat longer distance. The paper deals with the problems of how to design an array that has good performance for both of these two measurements and how to perform NAH processing on irregular array measurements. The combined measurement technique using only one array seems to offer a very efficient way of performing broadband noise source location.

1 INTRODUCTION

Figure 1 shows a rough comparison of the resolutions on the source plane - R_{BF} and R_{NAH} - that can be obtained with Beamforming (BF) and with Near-field Acoustical Holography (NAH), respectively. The resolution is here defined as the smallest distance between two incoherent monopoles of equal strength on the source plane that allows then to be separated in a source map produced with the method in consideration. For Beamforming the resolution is roughly, [1],

$$R_{BF} \approx 1.22 \frac{L}{D} \lambda \tag{1}$$

where L is the measurement distance, D is the array diameter and λ is wavelength. Since typically the focusing capabilities of irregular Beamformer arrays require that all array microphones are exposed almost equally to any monopole on the source plane, the measurement distance is usually required to be not smaller than the array diameter. As a consequence, the resolution cannot be smaller than around one wavelength, which is poor at low frequencies. For NAH the resolution R_{NAH} is approximately half a wavelength at high frequencies, but at low frequencies it never gets poorer than approximately the measurement distance L, which can be as small as the spacing in the regular grid of measurement points, [2]. This much better low-frequency resolution is obtained

because NAH can measure very close using a regular grid, and because it has the capability of reconstructing some of the evanescent waves that decay exponentially away from the source. Thus, in many cases NAH is needed at the low frequencies to obtain an acceptable resolution. At high frequencies the difference is much smaller, and often an acceptable resolution can be obtained with Beamforming, but using much fewer measurement positions than required with NAH.
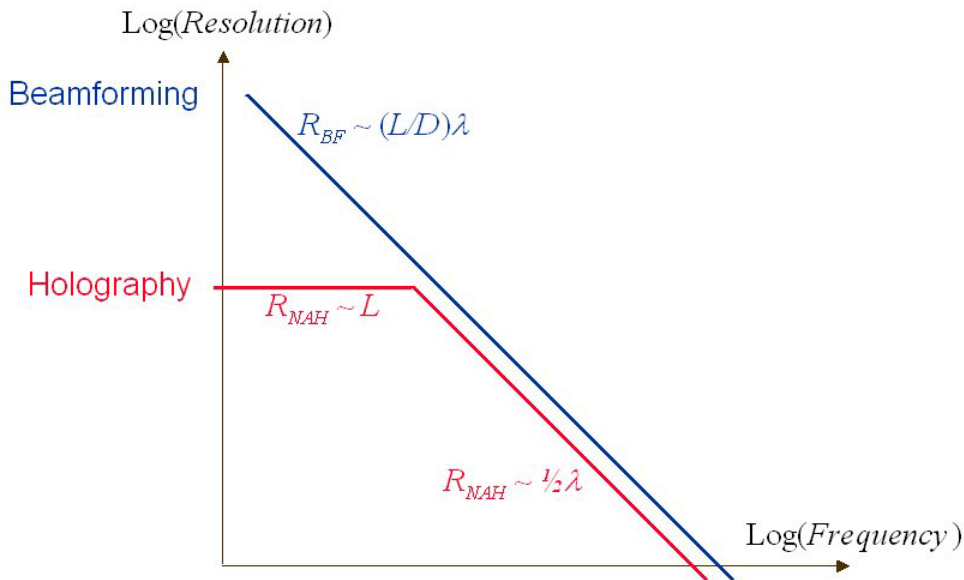

Figure 1: *Resolution of Holography (NAH) and Beamforming*

A combined measurement technique using NAH at low frequencies and Beamforming at high frequencies therefore seems to provide the better of two worlds. However, traditional NAH requires a regular grid array that completely covers the sound source, while Beamforming provides optimal high-frequency performance with an irregular array that can be smaller than the sound source. A need for repeated change between two different arrays would not be practical, but fortunately the new SONAH technique for NAH calculations can operate with irregular arrays, and also it allows measurement with arrays smaller than the source without severe spatial windowing effects, [3].
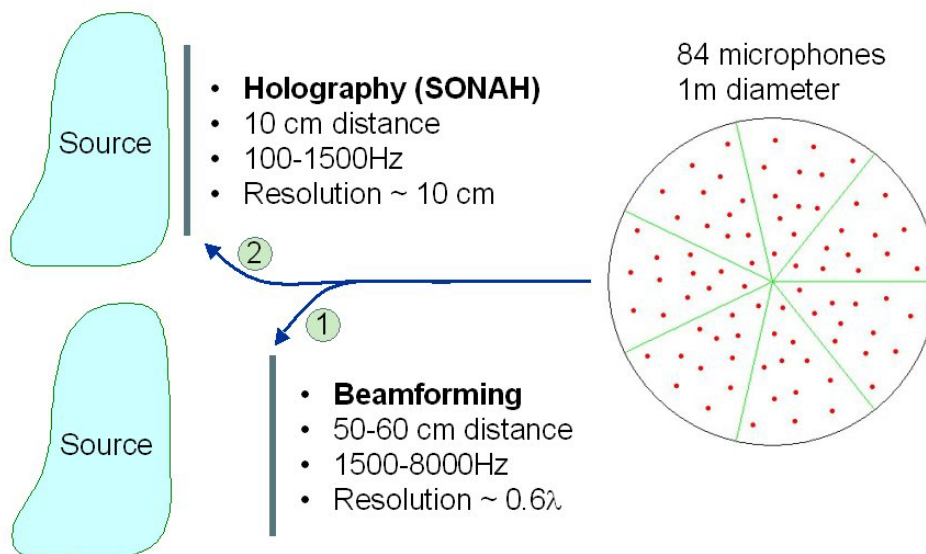

Figure 2: *Principle of the combined SONAH and Beamforming technique based on two measurements with the same array*

The principle of the combined measurement technique is illustrated in Figure 2, using a new so-called "Pizza Array" design, which will be explained further in the following chapter. Based on two recordings taken with the same array at two different distances, a high-resolution source map can be obtained over a very wide frequency range. The measurement distance shown for Beamforming is very small – only around half the array diameter. Simulations and practical measurements to be described in the present paper show that with for example the irregular Pizza Array of Figure 2 with rather uniform element density the Beamforming processing works well down to that distance.

2 ARRAY DESIGNS FOR THE COMBINED MEASUREMENT TECHNIQUE

Considering first the Beamforming application, it is well known that irregular arrays provide potentially superior performance in terms of low side-lobe level over a very wide frequency band, i.e. up to frequencies where the average microphone spacing is much larger than half a wavelength, [1]. The best performance is typically achieved, if the set of two-dimensional spatial sampling intervals is non-redundant, i.e. the spacing vectors between all pairs of microphones are all different. In references [4] and [1] an optimization technique was introduced to adjust the microphone positions in such a way that the Maximum Side-lobe Level (MSL) is minimized over a chosen frequency range. The MSL is here defined on the basis of the so-called Array Pattern, [1], i.e. in connection with a Delay-And-Sum Beamforming method focused at infinite distance. Since typically the MSL has many local minima when seen as a function of the design variables, an iterative optimization algorithm will usually stop at a local minimum close to the starting point. Many starting point are therefore needed to find a "good solution". Such starting points can for example be generated using random number generators to "scan" a certain "space of geometries".
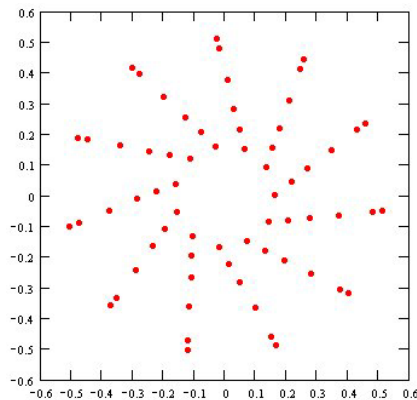

Figure 3: *Typical Wheel Array geometry optimized for Beamforming application*

In references [4] and [1] the optimized array geometries were typically Wheel Arrays consisting of an odd number of identical line arrays arranged as spokes in a wheel, see Figure 3. The odd number of spokes is chosen to avoid redundant spatial 2D sampling intervals. The optimization for low MSL ensures good suppression of ghost images over a wide frequency range, when the array is used at sufficiently long measurement distances, typically down to distances equal to the array diameter. If the distance becomes much smaller than that, the rather non-uniform density of the microphones across the Wheel Array area starts to have the effect that different points on the source plane will get very different exposure from the array. In that case, a more uniform density might be better, and numerical simulations have confirmed that hypothesis – specifically for the so-called Pizza Arrays to be described in the following. When the same array has to be used also for near-

field holography measurements at very small measurement distances, a more uniform density is even more necessary. This will be treated in more detail subsequently.

Various irregular array designs have been published that exhibit a more uniform density of the microphones over the array area and still maintain low side-lobe level over a wide frequency band, for example the spiral array of [5] and the Packed Logarithmic Spiral array, [6]. These arrays, however, lack the rotational symmetry of the Wheel Array that allows a modular construction and that can be exploited to reduce the number of geometric variables in a numerical optimization to minimize the MSL. Therefore, the Pizza Array geometry was developed. Figure 4 shows a Packed Logarithmic Spiral array with 60 elements, a Pizza Array with 60 elements and a Pizza Array with 84 elements. For all three arrays the diameter is approximately 1 meter. The Pizza Arrays maintain the rotational symmetry of the Wheel Arrays, but angularly limited slices each replace the small line arrays of the wheel. Each one of the identical slices contains in this case 12 elements in an irregular pattern, which has been optimized to minimize the MSL of the array.
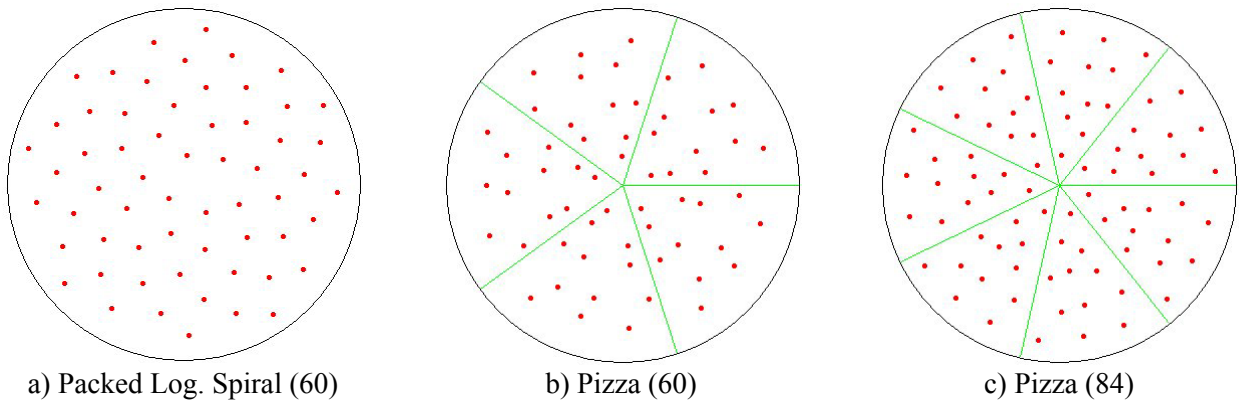

| a) Packed Log. Spiral (60) | b) Pizza (60) | c) Pizza (84) |

Figure 4: *Three different irregular array geometries with uniform element density. The enclosing circle around all three arrays has a diameter of 1.2 meter, so the array diameters are all around 1 meter.*
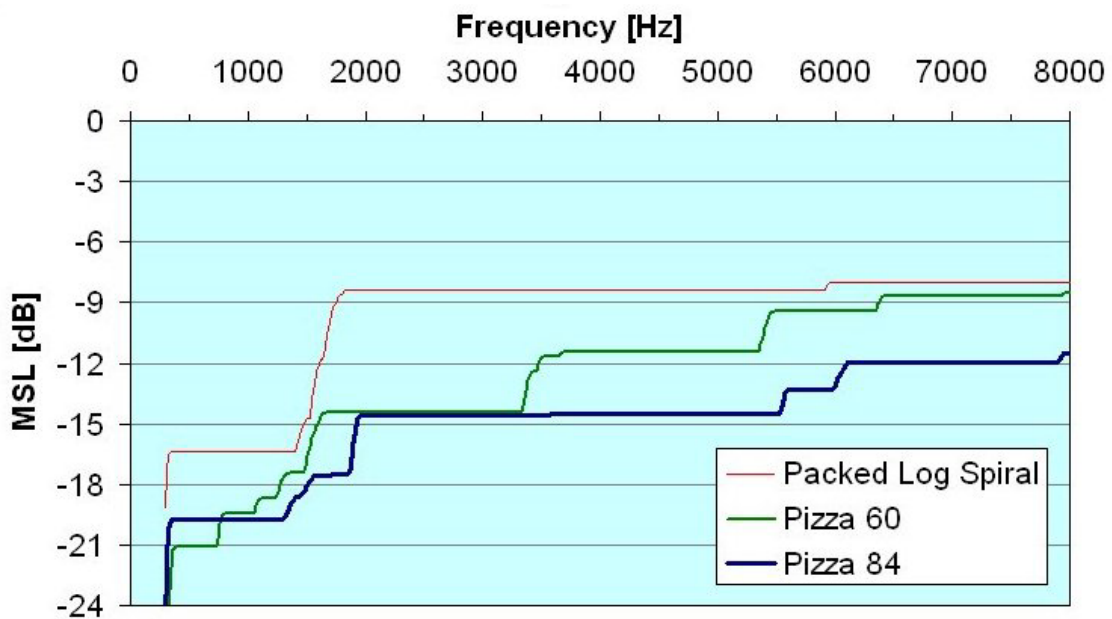

Figure 5: *Maximum Side-lobe Level (MSL) for the three different irregular array geometries of Figure 4. The focusing of the array is here restricted to be within 30° from the array axis.*

Figure 5 shows the Maximum Side-lobe Level (MSL) as a function of frequency for the three array geometries of Figure 4, assuming focusing of the array to be within 30° from the array axis. If free focusing is required (i.e. up to 90° from the array axis), then the numbers on the frequency axis have to be multiplied by a factor of 0.75, [1]. Clearly, the 84-element array has very low side-lobe level at frequencies below approximately 2000 Hz, which for free focusing angle would be 1500 Hz. With the array very close to the noise source, as required for holography processing, free focusing angle has to be considered, because waves will be incident from all sides. The 1500 Hz limit turns out to be just a little bit below the frequency, where the average spacing between the elements of the array is half a wavelength. The average element spacing is approximately 10 cm.

Optimization of the Pizza Array geometries in Figure 4 has been performed by adjusting (using a MiniMax optimization program) the coordinates of the elements in a single slice in such a way that the maximum MSL is minimized over the frequency range of interest. In this process a penalty was put on the MSL up to 1500 Hz for the 84-element array and up to 1200 Hz for the 60-element array. This turns out to help maintain the uniform element distribution and therefore the possibility of using the array for holography at frequencies with less than half wavelength average spacing.

2.1 Simulation of Beamforming Measurements at a Small Source Distance

Some simulated measurements were performed to investigate, how well the above three arrays would perform with Beamforming from a measurement distance of 0.6 meter, i.e. a bit more than half the array diameter. The results are shown in Figure 6 for the case of 5 uncorrelated monopoles of equal strength at 8000 Hz. The Beamforming calculations have been performed with the Cross-spectral algorithm (with exclusion of Auto-spectra) described in reference [1], focused on the source plane at 0.6 meter distance. For all three plots the displayed dynamic range is 10 dB, and as expected from Figure 5, the two 60-element arrays have very comparable performance at 8 kHz with a small advantage to the Pizza Array. The 84-element Pizza Array has 3 dB lower side-lobe level at 8 kHz, and therefore there are no visible ghost images in Figure 6. The MSL values are seen to be slightly higher at the very short measurement distance than for the infinite focus distance represented in Figure 5 – typically around 2 dB higher. The 8 kHz data presented in Figure 6 are not entirely representative for the relative performance of the three arrays over the full frequency range. If we look instead at 3 kHz, then according to Figure 4 the 60-element Pizza Array has approximately 6 dB lower side-lobe level than the 60-element Packed Logarithmic Spiral.
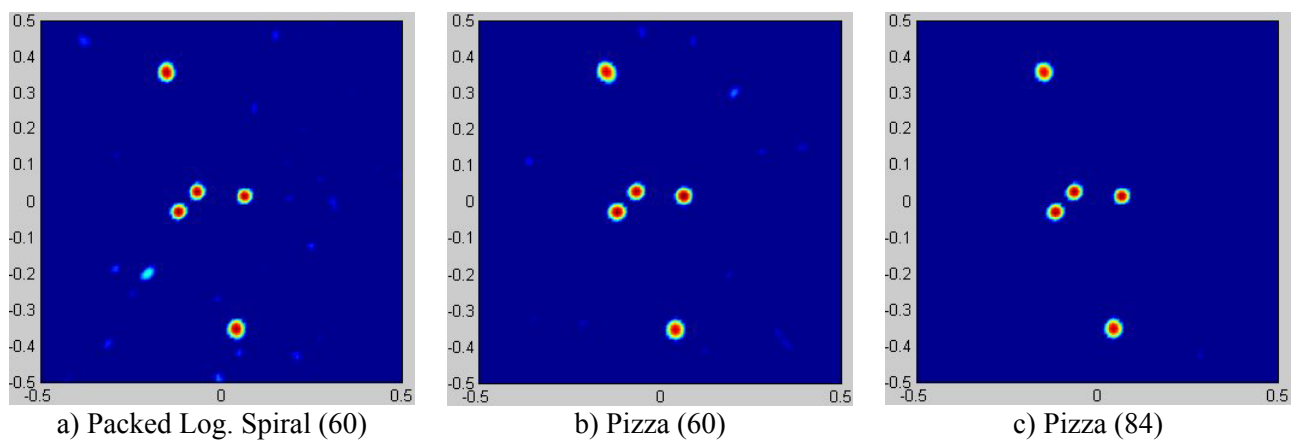

| a) Packed Log. Spiral (60) | b) Pizza (60) | c) Pizza (84) |

Figure 6: *Simulated measurements on 5 monopoles at 8 kHz and at 60 cm measurement distance with the three array designs shown in Figure 4. The displayed range is in any case 10 dB.*

The following consideration illustrates the advantage of Beamforming over NAH for source location at high frequencies: If the maps in Figure 6 should have been produced with traditional NAH, then a measurement grid with dimensions around 1.2 x 1.2 meter should have been used with a grid spacing around 2 cm, leading to approximately 3600 measurement positions!

2.2 Numerical Simulations to Clarify the Suitability of the Arrays for Holography

Another series of simulations were performed to investigate the frequency ranges over which the three arrays of Figure 4 and the Wheel Array of Figure 3 are suited for SONAH holography measurements. In Near-field Holography a complete reconstruction of the entire near field is attempted over a 3D region around the measurement area. This is possible only if the spatial samples of the sound field taken by the array microphones provide at least a complete representation of the pressure field over the area covered by the array. So from the available spatial samples it must be possible to reconstruct (interpolate) the sound pressure across the measurement area. This can be done by the SONAH algorithm, which is used also for the reconstruction of the sound field on parallel planes and of other sound field descriptors than the sound pressure.

The problem of reconstructing a (2D) band-limited signal from irregular samples has been treated quite extensively in the literature; see for example reference [7]. In order that the reconstruction can be performed in a numerically stable way, it is necessary that the distribution of the sampling (measurement) points exhibit some degree of uniform density across the sampling area. Such a criterion was used in the design of the Pizza Arrays.
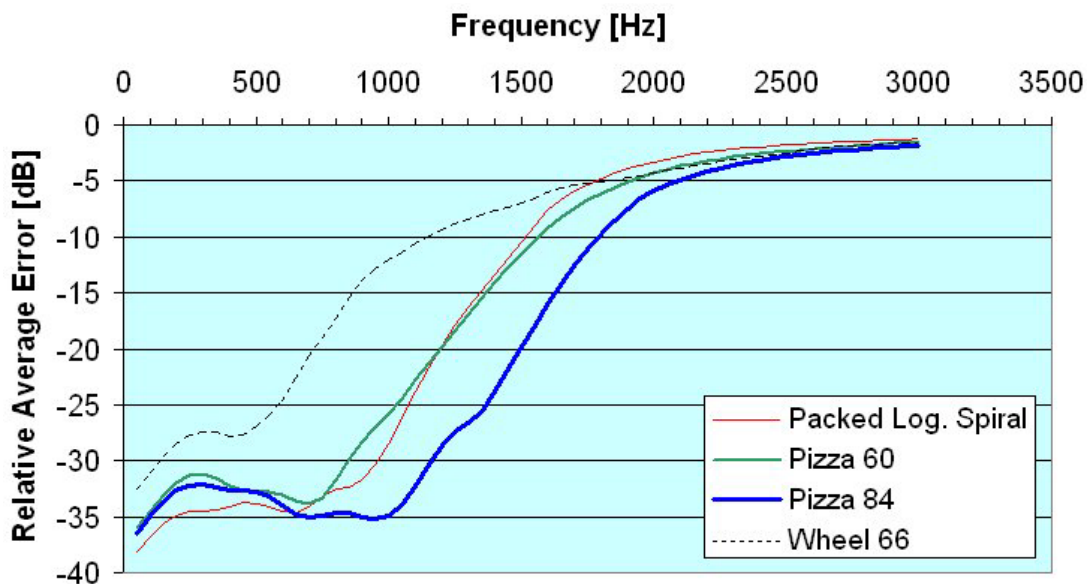

Figure 7: *Comparison of Relative Average Interpolation Error for the three arrays in Figure 4 and the Wheel Array of Figure 3. The error is averaged over a set of monopole point sources at 30 cm distance.*

For the numerical simulations, a set of (eight) monopole point sources at 30 cm distance from the array was used. All point sources were inside an area of the same size as the arrays. For each frequency and each point source, the complex pressure was calculated at the array microphone positions, and SONAH was then applied to calculate the sound pressure over a dense grid of points inside the measurement area in the measurement plane. For this a 40 dB dynamic range was used, [3]. The interpolated pressure from each monopole was then compared with the known pressure from the same monopole, and the Relative Average Error was estimated at each frequency as the

ratio between a sum of squared errors and a corresponding sum of squared true pressure values. The summation was in both cases over all interpolation points and all sources.

Figure 7 gives a comparison of the Relative Average Interpolation Errors obtained with the four different arrays. Clearly, the 84-element optimized Pizza Array can represent the sound field over the array area up to around the previously mentioned 1500 Hz, while the 60-element Pizza Array provides acceptable accuracy only up to around 1200 Hz. This actually means that the two Pizza Arrays apply over the same frequency ranges as regular arrays with the same average element spacing. The 60-element Packed Logarithmic Spiral is seen to perform much like the 60-element Pizza Array in connection with SONAH calculations. But as expected, the 66-element Wheel Array performs much poorer then the three arrays with more uniform element density. The acceptable interpolation accuracy (20dB suppression of errors) is achieved only up to approximately 700 Hz. The explanation is that the array elements are more bunched with relatively large spacing between the groups.

3 MEASUREMENTS

In order to test the performance of the 60-element Pizza Array of Figure 4, measurements were taken at 12 cm distance from a small loudspeaker for SONAH processing, and at 50 cm distance for Beamforming processing. Figure 8 shows a picture of the set-up taken from behind the array, looking towards to source. The small loudspeaker can be seen just to the left of and below the center of the array. The excitation was Pink random noise.

The results from Beamforming and SONAH processing, respectively, of the two measurements in the 250 Hz and 500 Hz 1/3-octave bands are given in Figure 9. Clearly, the measurement at 12 cm distance and with SONAH processing provides far better resolution at the low frequencies, which should be also expected according to Figure 1. The bend on the resolution curve for NAH in Figure 1 is in this case at approximately 1500 Hz.

Figure 8: *Picture of the measurement set-up. The small loudspeaker can be seen behind the 60-element Pizza Array, just below and to the left of the center.*
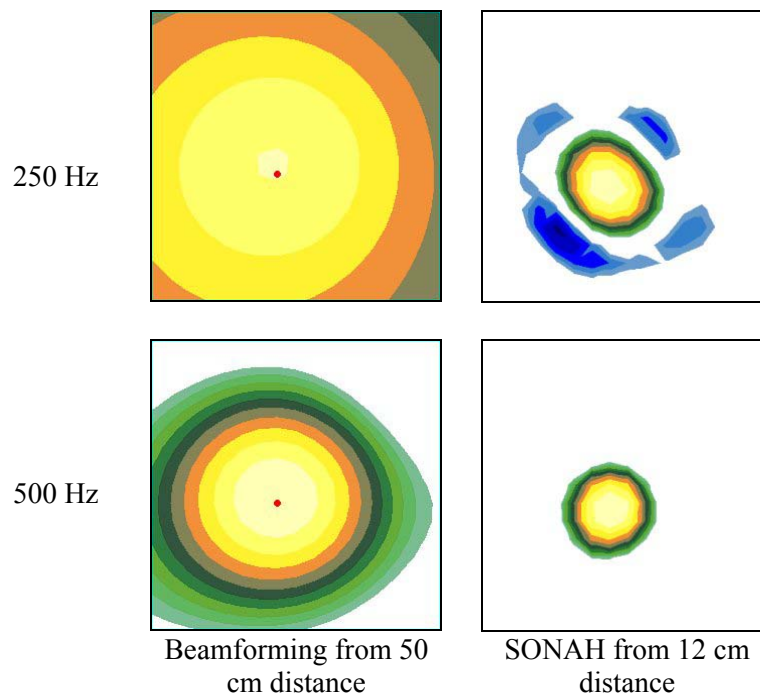
250 Hz		
500 Hz		
	Beamforming from 50 cm distance	SONAH from 12 cm distance

Figure 9: *Results from the measurements taken with the set-up in Figure 8. The contour plots all cover a 1m x 1m area and have 10 dB dynamic range. The SONAH plots show Sound Intensity, which can be inwards/negative (blue colors).*

4 CONCLUSIONS

A new combined array measurement technique has been presented that allows Near-field Acoustical Holography and Beamforming to be performed with the same array. This combination can provide high-resolution noise source location over a very broad frequency range based on two recordings with the array at two different distances from the source. The key elements in the presented solution are the use of SONAH for the holography calculation and the use of a specially designed irregular array with uniform element distribution. The optimized Pizza Array is an example of an applicable array with very high performance particularly for the Beamforming part. Numerical simulations and an actual measurement presented in the paper confirm the strengths of the combined method and of the Pizza Array design.

5 REFERENCES

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