Layered Modeling of Porous Structures with Voronoi Diagrams

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ABSTRACT

Existing computer-aided geometric design methods are inadequate to model and design 3D porous structures. A layer-based approach with 2D Voronoi tessellation is proposed to reduce the 3D challenge to a 2D problem. We define each Voronoi generating point to be either solid or void point and organize the points into concentric clusters for designing small 2D shape regions. Dynamic 2D shape regions are achieved by specifying the motion of each generating point in the 2D plane. When expanding the time-dimension of these regions in the third dimension of the 2D plane, small pieces of 3D structures are defined. Moreover, when these regions undergo topological variations, small pieces of 3D branching structures are obtained. A technique on symmetry in 2D plane is utilized as a simple method to expand these structures to variations of large 3D branching and porous structures. The proposed method is a layer-based model as each horizontal cross-section of a final structure is a layer of region pattern specified by the dynamic generating points. The aim and novelty of the method presented are to provide a concise and flexible means of solid modeling for creating new porous structures.

Keywords: Geometric modeling, 2D tessellation, 3D structure, user interaction, symmetry.

1. INTRODUCTION

1.1 Motivation

3D porous structures with networks of interconnected pores facilitate fluid, molecules, particles and even biological cells to enter or pass through. These structures at the same time maintain the mechanical strength for load bearing [17-18]. However, existing computer-aided geometric design (CAGD) methods [7] are inadequate to model and manually design these structures. For instance, the boundary representation (B-rep) is inflexible for representing the changes of surface patch connectivity during conceptual design. The constructive solid geometry (CSG) method is suitable to represent truss-like porous structures with hierarchies of primitive solids. However, the hierarchies and their primitive solids are difficult to be flexibly modified in general. As a result, generic porous structures with complex surfaces and architectures are difficult to be created with CSG. Although the voxel representation is able to approximate all kinds of solid geometry, the manual addition and deletion of voxel element are tedious local operations. Even with virtual sculpturing interfaces, the assisted manipulations on voxels could only facilitate local shape modifications. Thus, the voxel representation is difficult to create novel porous structures, unless the user has a very clear final structure in mind. At present, the direct input, design and display of complex solid objects in the 3D space are difficult [16]. The difficulty is even exacerbated for 3D porous structures which have hidden internal surfaces and architectures. In this study, a layer-based approach with 2D Voronoi tessellation is proposed to reduce the 3D challenge to a 2D problem.

1.2 Terminology

For clarity, some terms are defined to describe the terminology appeared in this paper. First, all the Voronoi generating points are used for 2D tessellations and lie on the same horizontal 2D plane, unless otherwise specified. The attribute of each generating point is defined to be either solid or void. Each Voronoi cell is a defined to have the same attribute as its associated generating point. The connected solid cells are considered as a 2D region, which is simply called a region in the proposed method. The tessellation edges are only used to define the shape of each Voronoi cell. Some of them are the edges of a region.

Moreover, the point pattern refers to the organization of all generating points in the 2D plane. The basic point pattern is defined as the organization of all generating points in the basic unit for symmetry. The tessellation pattern refers to the tessellation of a point pattern. The basic tessellation pattern refers to the tessellation of a basic point pattern. All tessellations addressed refer to Voronoi tessellations. The region pattern refers to the organization of region(s) resulted from the point pattern. The basic region pattern refers to the organization of region(s) resulted from the basic point pattern. The dynamic region pattern and dynamic basic region pattern refer to the time-varying region pattern and time-varying basic region pattern respectively. In this study, every point pattern, tessellation pattern or region pattern considered is within a finite square area.

In addition, porous structures are considered as the 3D structural solids having graph structures with inter-connected pores. Branching structures are considered as the 3D structural solids having tree structures without pores. When slicing porous or branching structures, layers of 2D region(s) are obtained. Some layers are in the non-manifold configurations, in each of which some 2D regions are touched in a single point. These layers are the critical layers (cross-sections) through each of which branching of a region just takes place. The term shape topology refers the connectivity of 2D regions in 2D cases and the connectivity of branches in 3D cases.

1.3 Overview

The main parts of the proposed layer-based method are outlined here. Firstly, a basic region pattern is modeled using 2D Voronoi diagrams. Concentric rings are used to organize the Voronoi generating points into point clusters to facilitate more complex designs of basic region pattern. Dynamic basic region patterns are achieved by specifying the motion of each generating point in the 2D plane. When expanding the time-dimension of these dynamic patterns in the third dimension of the 2D plane, small pieces of 3D structures are defined. Moreover, when these dynamic patterns undergo topological variations, small pieces of 3D branching or porous structures are obtained. A technique on symmetry in 2D plane is utilized as a simple method to expand the designed dynamic basic region pattern to 17 variations of dynamic region pattern. Hence, large 3D branching and porous structures with symmetry are achieved. The proposed method is a layer-based model since each horizontal cross-section of a final structure is a layer of region pattern specified by the dynamic generating points. Each resultant 3D structure is approximated as a stack of thin extruded slices and is implemented with the Spatial ACIS geometric modeler [1].

The organization of this paper is as follow: in Section 2, the related work is reviewed; in Section 3, the proposed modeling method is illustrated; in Sections 4 and 5, discussions and conclusion on this work are presented respectively.

2. RELATED WORK

The modeling of 2D region(s) with generic shape topology is a part of the proposed layer-based method. Actually, there are many standardized mathematical descriptions on static shape in the area of geometric modeling in CAGD [7]. In geometric modeling, there are currently two main paradigms representing the shapes of 2D region – boundary-based approaches and region-based approaches. However, most of these existing approaches are ineffective and inflexible to support the modeling and design of shapes with different topologies. For boundary-based models, loop(s) of edge segments in the form of parametric curves or polylines can be used to represent the boundary of 2D regions. However, the manipulations on the edge(s) in each loop can introduce invalid self-intersection(s). The maintenance on edge connectivity is needed when shape regions experience topological changes. For region-based models, the block-structured grids, region quadtree, Boolean operations on shape regions and level sets are usually used to represent the 2D regions directly. The block-structured grids and region quadtree representations are in the forms of fixed grids. The hierarchy of Boolean operations on shape regions comprises a set of fixed shape primitives. These models contain fixed components and hence are inflexible to support different shape topologies. Although the level set methods support shapes with various topologies and are popular in free-form shape modeling and deformation, the precise control on their shape models is complex to achieve. Dynamic cell decompositions are available for dynamic meshing with various cellular topologies [9,21]. However, they are difficult to use for geometric modeling and CAGD without a good graphical user interface.

With the afore-mentioned considerations, we propose flexible techniques on the solid modeling of layer-based 3D porous structures. A method with 2D Voronoi diagrams is proposed for the modeling and is implemented with the Quickhull algorithm [3]. A Voronoi diagram is a space decomposition method and is defined by the un-ordered point positions [2,5,15,19]. In this section, the addressed definition and properties of Voronoi diagrams are adopted from [19] for reference. Suppose that, a point set $S = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ in the d-dimensional space $\mathbf{R}^d$ is given. For each point $\mathbf{p}_i$, the cell $R(S; \mathbf{p}_i)$ defined by

$$R(S;\, \mathbf{p}_i) = \{\mathbf{p} \in \mathbf{R}^d \quad | \quad ||\mathbf{p} - \mathbf{p}_i|| < ||\mathbf{p} - \mathbf{p}_j|| \quad \text{for} \quad j \neq i\} \tag{2.1}$$

is named the Voronoi cell of $\mathbf{p}_i$. $||\ \mathbf{p} - \mathbf{q}\ ||$ represents the Euclidean-distance between the points $\mathbf{p}$ and $\mathbf{q}$. In other words, the cell $R(S;\, \mathbf{p}_i)$ consists of points which are closer to $\mathbf{p}_i$ than to any other point in S. The d-dimensional space $\mathbf{R}^d$ is partitioned into the n cells $R(S;\, \mathbf{p}_1)$, $R(S;\, \mathbf{p}_2)$, … , $R(S;\, \mathbf{p}_n)$. The partition is named the ordinary Voronoi diagram of S, and is denoted by VD(S). There is always only a single generating point lies inside each Voronoi cell. In $\mathbf{R}^2$, the boundary of two Voronoi cells is a line segment that is the perpendicular bisector of the associated two points. The boundary is named the Voronoi edge. The point where three or more Voronoi edges meet is named the Voronoi point. Usually, each Voronoi point in $\mathbf{R}^2$ is incident to three Voronoi cells. However, if four or more generating points are on a common circle and there is not any other generating point located inside the circle, there arises a Voronoi point that is incident to four or more Voronoi cells. This situation is named degeneracy, and S or VD(S) is named degenerate. The Voronoi cell $R(S;\, \mathbf{p}_i)$ is un-bounded, if and only if $\mathbf{p}_i$ is on the boundary of CH(S).

In the later part of this work, symmetry is utilized to generate region patterns. The group theory addressed the mathematics behind the symmetry [8]. Mathematically, a symmetry of a subset S of Euclidean space R^n is a rigid transformation in R^n that keeps S set-wise invariant [12]. In other words, a symmetry of a pattern is a rigid transformation that leaves the pattern unchanged [8]. In principle, the rigid transformations are the mirror reflection, rotation, translation, or their resultant motions. The set of all rigid transformations that are symmetries of a pattern has a group structure. It is named the symmetry group of the pattern [12]. There are exactly 17 symmetry types of wallpaper groups describing all infinite 2D symmetric patterns [8]. In fact, the wallpaper symmetry is useful to generate variations from a basic pattern unit [11,12]. In Voronoi diagrams, a point pattern and its tessellation pattern have the same type of symmetry [11]. With a symmetry type and a 2D grid of small pattern, 17 different expanded patterns are generated. Each expanded pattern is a collection of side-by-side combined units, which are the duplications of the original small pattern with mirrored and rotated transformations. Among different symmetry types, the ways of allocating the duplicating units are different. The special type names for wallpaper symmetry are initiated from crystallography and commonly used in symmetry as well. The term symmetry used in our approach refers to the wallpaper symmetry in the 2D plane.

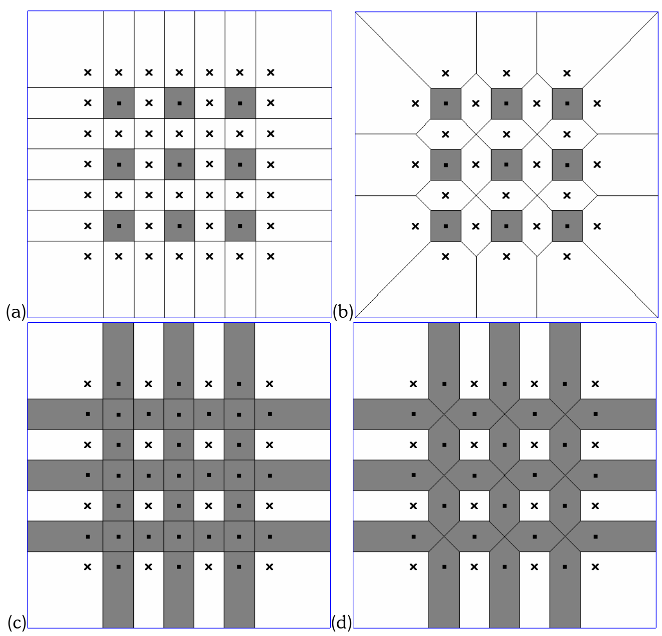

Fig. 1: Two basic region patterns are shown in (a)-(b) and (c)-(d). Two different basic tessellation patterns with their basic point patterns are demonstrated for each of the basic region patterns. The black dots and crosses denote the solid points and void points respectively. The solid regions and their constituting solid cells are in gray. The void regions and

their constituting void cells are in white. Each black straight line is a Voronoi edge. Each bounding box is the grid of basic unit for symmetry.

3. THE MODELING METHOD

3.1 Modeling of 2D Regions

We use the ordinary Voronoi diagram to achieve basic point patterns and basic tessellation patterns. The attribute of each generating point is defined to be either solid or void. Each Voronoi cell is defined to have the same attribute as its associated generating point. That is, a solid cell is defined as a Voronoi cell associated with a solid point. Also, a void cell is defined as a Voronoi cell associated with a void point. The connected solid cells are considered as a region. Therefore, a set of finite un-ordered solid and void points specifies a basic region pattern modeled in a collection of disjoint convex polygons. The basic region patterns are unambiguous but non-unique as demonstrated in Fig. 1. When the insertion or deletion operation on a generating point does not affect the modeled shape of a basic region pattern, the generating point is redundant in shape modeling. From the data structure point of view, the Voronoi diagram is a responsive and efficient structure admitting local changes of spatial decomposition [10]. Besides, the basic region patterns implicitly represent 2D holes, disconnected regions and regions in non-manifold configurations as demonstrated in Fig. 2. Also, they adaptively support different cellular topologies in Voronoi tessellations.

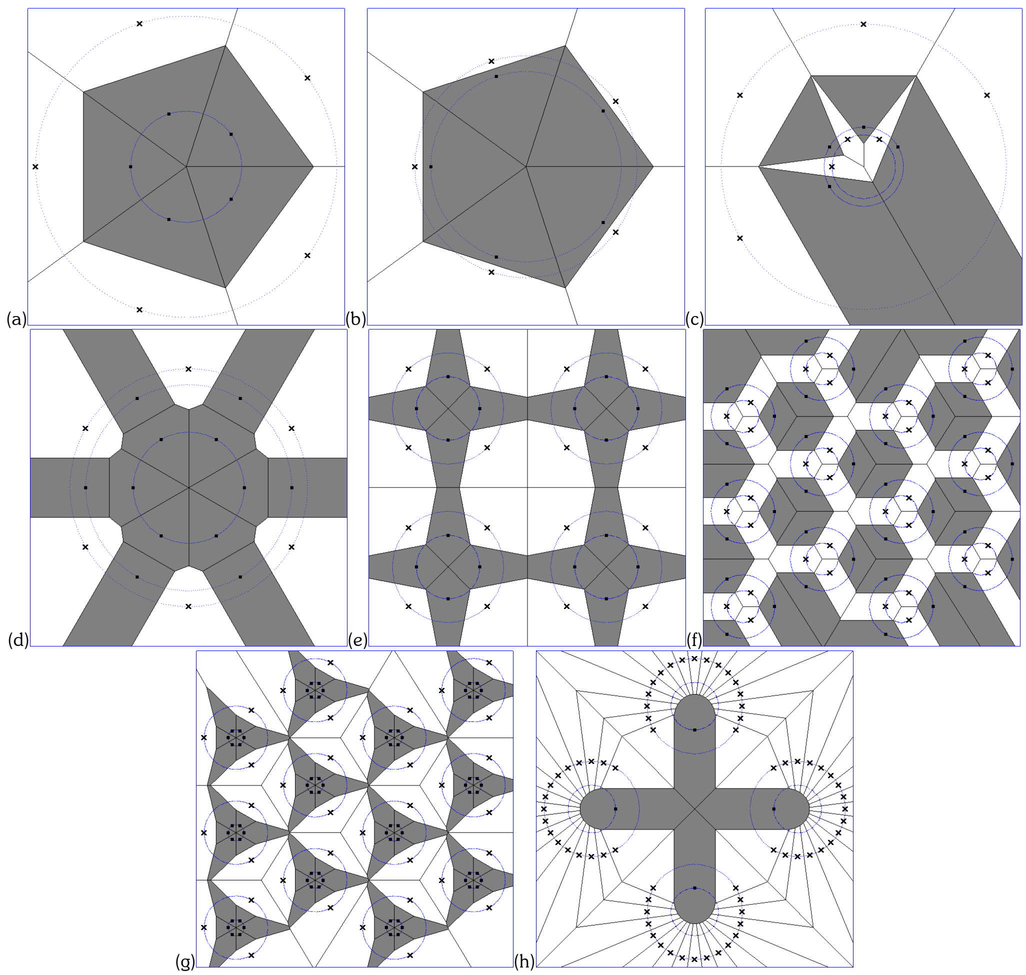

Fig. 2: Basic region patterns with single point cluster and multiple point clusters are shown respectively in (a)-(d) and (e)-(h) together with their concentric rings. The basic region patterns in (a) and (b) are the same, while the basic point and tessellation patterns in (a) and (b) are different.

3.2 Organizations of Point Clusters

Techniques using individual rings are proposed for designing regular colour patterns [4,11]. In our proposed method, rings are located on the 2D plane (e.g. evenly distributed) and each ring has a few solid or void points attached. The rings are organized into concentric groups with clusters of generating points facilitating complex designs of basic region pattern. In general, any finite point set with an arbitrary distribution can be attached to a set of concentric rings (i.e. a single point cluster) such that each point is attached to its own ring. However, this may not be a systematic strategy for designing generic basic region patterns. Actually, the single point cluster is suitable as a basic element for modeling relatively simple basic region patterns. The set of multiple point clusters is a collection of single point clusters for the modular organization of sub-regions. Some examples of basic region pattern generated with the proposed point cluster method are shown in Fig. 2. The design parameters for each point cluster are the locations of ring center, the ring radii and the phase angles (i.e. absolute angle) of the generating points on their rings.

The rings act as the physical "handles" in the form of geometric entities for manipulation and visualization. They provide alignments and low degree-of-freedoms for assigning the generating points on each ring. Moreover, they are allowed to be freely located in order to achieve high modeling flexibility, with only one condition on model validity. That is, there should not be any overlap between a solid point and a void point such that the solid or void attribute of any Voronoi cell can be determined. In our settings, whenever all the overlapping points have the same attribute, these points can be considered as a single point with that attribute.

3.3 Layer-based 3D Branching Structures

Dynamic basic region patterns are achieved by specifying the motion for each generating point of a basic region pattern in the 2D plane. Their shape topologies can be changed when the generating points move in the 2D plane. Any addition or deletion operation on the generating point(s) or any discontinuous motion of the generating points is prohibited. Thus, any abrupt change [11,13,15] in the dynamic basic region pattern is avoided. When expanding the time-dimension of these dynamic basic patterns in the third dimension of the 2D plane, small pieces of static 3D structures are generated. With variations in shape topology which these patterns undergo, small pieces of 3D branching or porous structures are obtained. The proposed layer-based model does not require explicit specifications on the surface construction and branching correspondence as required in the contour-based branching structures [9]. The model allows the design of 3D branching structures in the 2D plane.

The rings used in Sub-section 3.2 can undergo 2D transformations to facilitate group motions of the attached generating points [11] and achieve dynamic basic region patterns. However, to maintain the "transformed rings" still being as rings, these transformations are restricted to be rotation, radius oscillation, displacement of each ring's center, and their combinations. As a result, the group motions of generating points are likely to be in some special forms. For instance, circular movements and sinusoidal oscillations are resulted from the rotations and radius oscillations of rings respectively. To achieve high flexibility on motion specification, we do not limit to using rings for the motion design of generating points. Instead, straight lines, polylines or parametric curves in the 2D plane could be used for simplicity or generality reasons.

An example is shown in Fig. 3 to further illustrate the proposed 3D modeling method. A basic region pattern is firstly designed (Fig. 3b) in a non-manifold configuration. It corresponds to a critical layer (cross-section) of the final structure (approximated in Fig. 4) through which branching of a region just takes place. The rings are useful to align generating points and hence generate degenerate Voronoi tessellations for specifying the non-manifold configuration. A dynamic basic region pattern is specified by the designed basic region pattern with displacements of the generating points (indicated by an arrow representation adopted from [15] for dynamic Voronoi diagrams) in the 2D plane. When there is no arrow attached to a generating point, the point is static. During the motions, any collision between the points with different attributes must be avoided in order to maintain the model validity. All points start and stop their motions simultaneously. When all points are at their arrow heads, the represented basic region pattern is the top-most layer of the final 3D structure. When all points are at their arrow tails, the represented basic region pattern is the bottom-most layer of the final 3D structure. The location of a generating point on its associated arrow indicates the particular time when the point reaches that location. For instance, when a generating point is located at its arrow head, the point reaches that location at the end of its motion. In a dynamic basic region pattern, the locations of all points on their arrows must indicate the same time. As a result, this firstly designed pattern is guaranteed to be included in the final 3D structure. Fig. 3 shows when each generating point is on the middle of its associated arrow. As a result, the designed basic region pattern in non-manifold configuration is the middle horizontal layer of the final 3D structure (approximated in Fig. 4). In other words, branching takes place at the middle height of the structure in Fig. 4a (shown approximately).

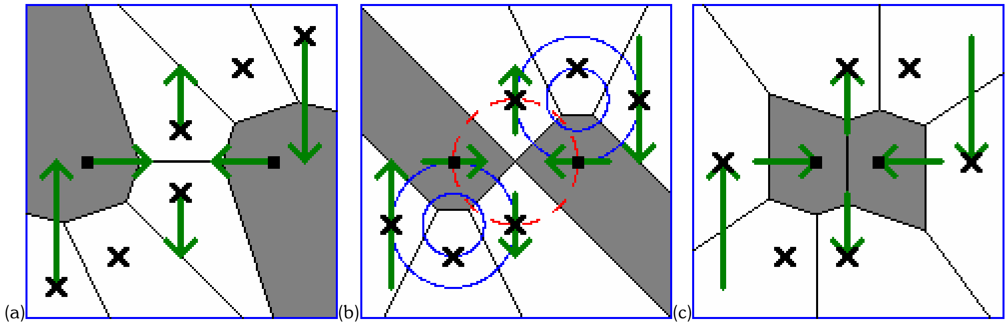

Fig. 3: A dynamic basic region pattern with varying shape topology is shown at the start, middle and end of motion in (a)-(c) respectively. The generating points have constant velocities and their displacements are indicated by arrows. The non-manifold configuration in (b) is constituted by the concentric arrangement of 2 solid points and 2 void points as indicated by the dash circle drawn purely for illustration purpose.

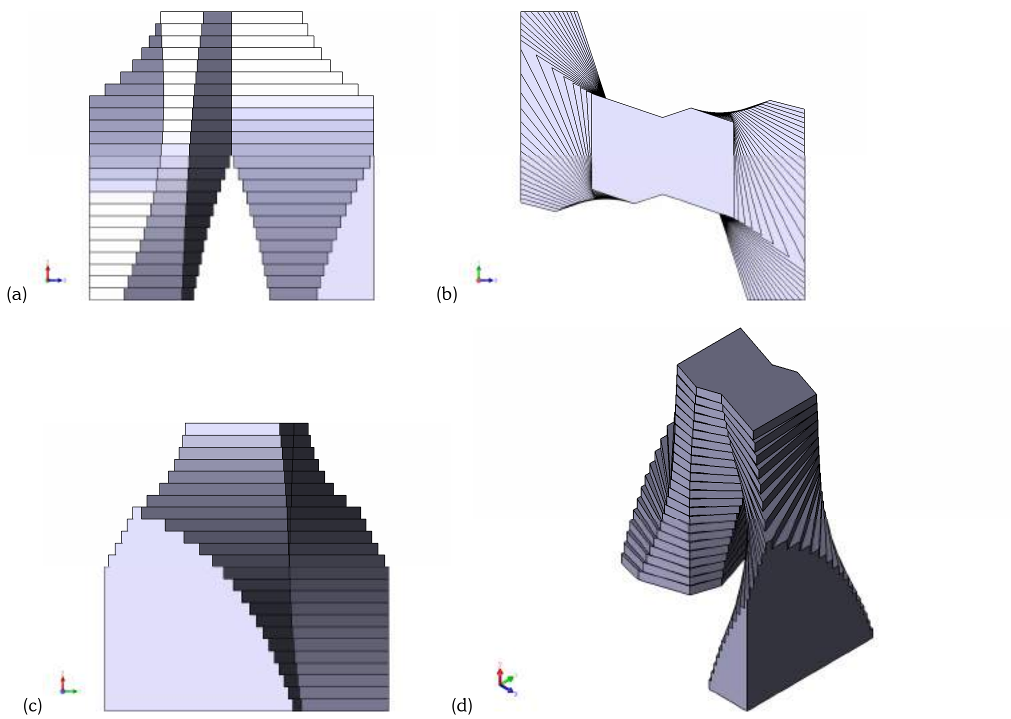

Fig. 4: A branching structure is shown in front, top, side and isometric views in (a)-(d) respectively. It is specified by the dynamic basic region pattern drawn in Fig. 3 with a given height in the z-direction.

3.4 3D Porous Structures with Symmetry

In Sub-section 3.3, we focused on the design of dynamic basic region patterns in a small unit area and achieved a small piece of 3D branching structures. To achieve large pieces of structure, a technique on symmetry is utilized as a simple method to expand the designed dynamic basic region pattern to 17 variations of dynamic region pattern. Dynamic region patterns are achieved by selecting a symmetry type and a dynamic basic point pattern located in the grid of basic unit. As examples, four different dynamic region patterns are generated with this method and shown in Fig. 5. These patterns have the same dynamic basic region pattern (identical to Fig. 3) but different symmetry types. Also, they specify the four 3D different branching structures shown in Fig. 6. The results demonstrate that even with the same dynamic basic region pattern, different 3D branching structures can be obtained for different symmetry types. However, some resultant structures can be invalid models with disconnected solids as shown in Fig. 6b and Fig. 6e. This indicates that mismatches exist between some dynamic basic point patterns and wallpaper symmetry types. Nevertheless, the modeling method is useful to create new 3D branching structures as demonstrated. A 3D porous structure (Fig. 7) can be made by stacking a transformed branching structure on top of the original valid branching structure (types *pmm* and *p4g* in Fig. 6). The transformations used are horizontal mirror (with an arbitrary mirror plane) and translation.

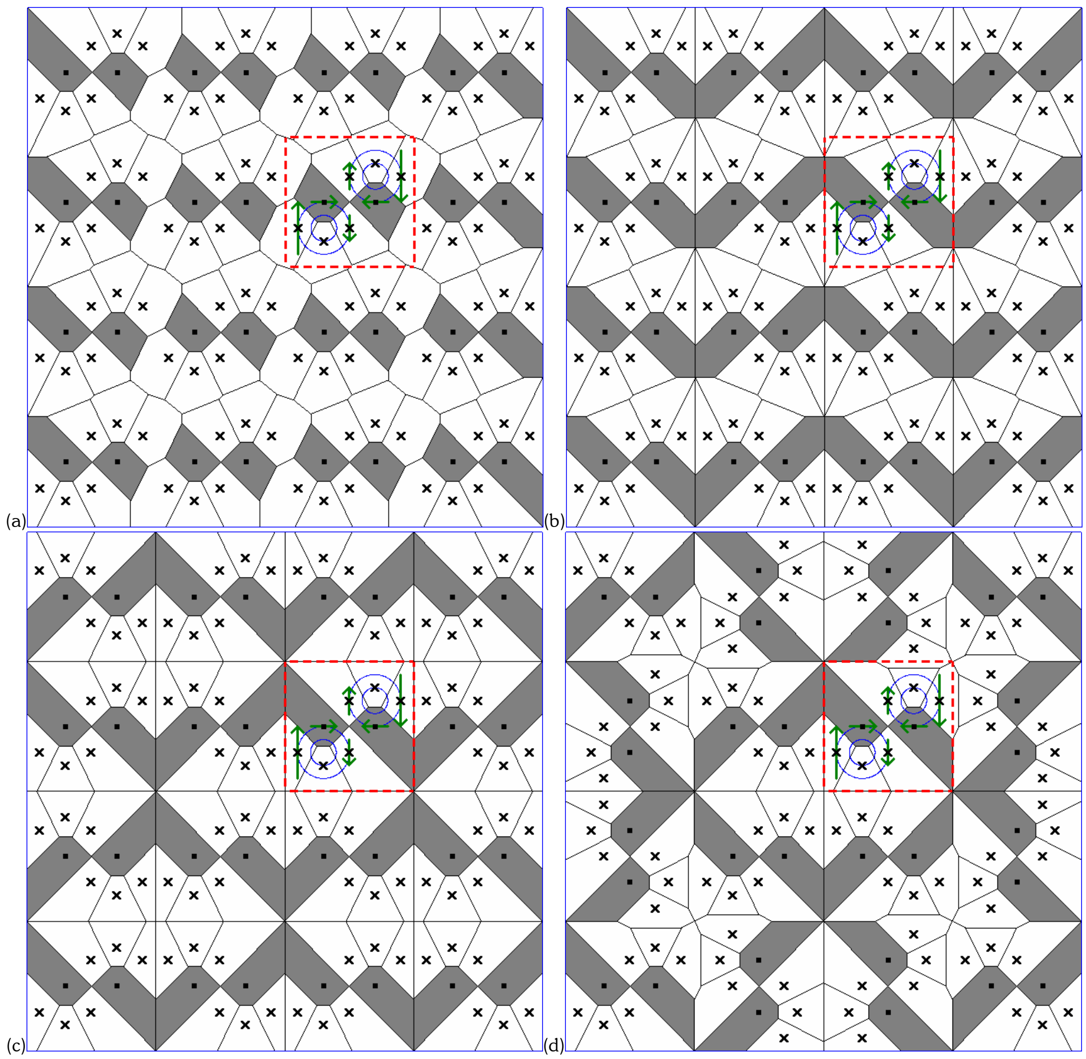

Fig. 5: Four dynamic region patterns with symmetry types *p1*, *pgg*, *pmm* and *p4g* are shown in (a)-(d) respectively. Each of them is within a finite square area indicated by the outer square bounding box. In the basic units for symmetry (dash squares), the basic point patterns and motion specifications are the same (identical to Fig. 3). The arrows illustrate the motions of generating points in the basic unit (as drawn) and its duplicating units.

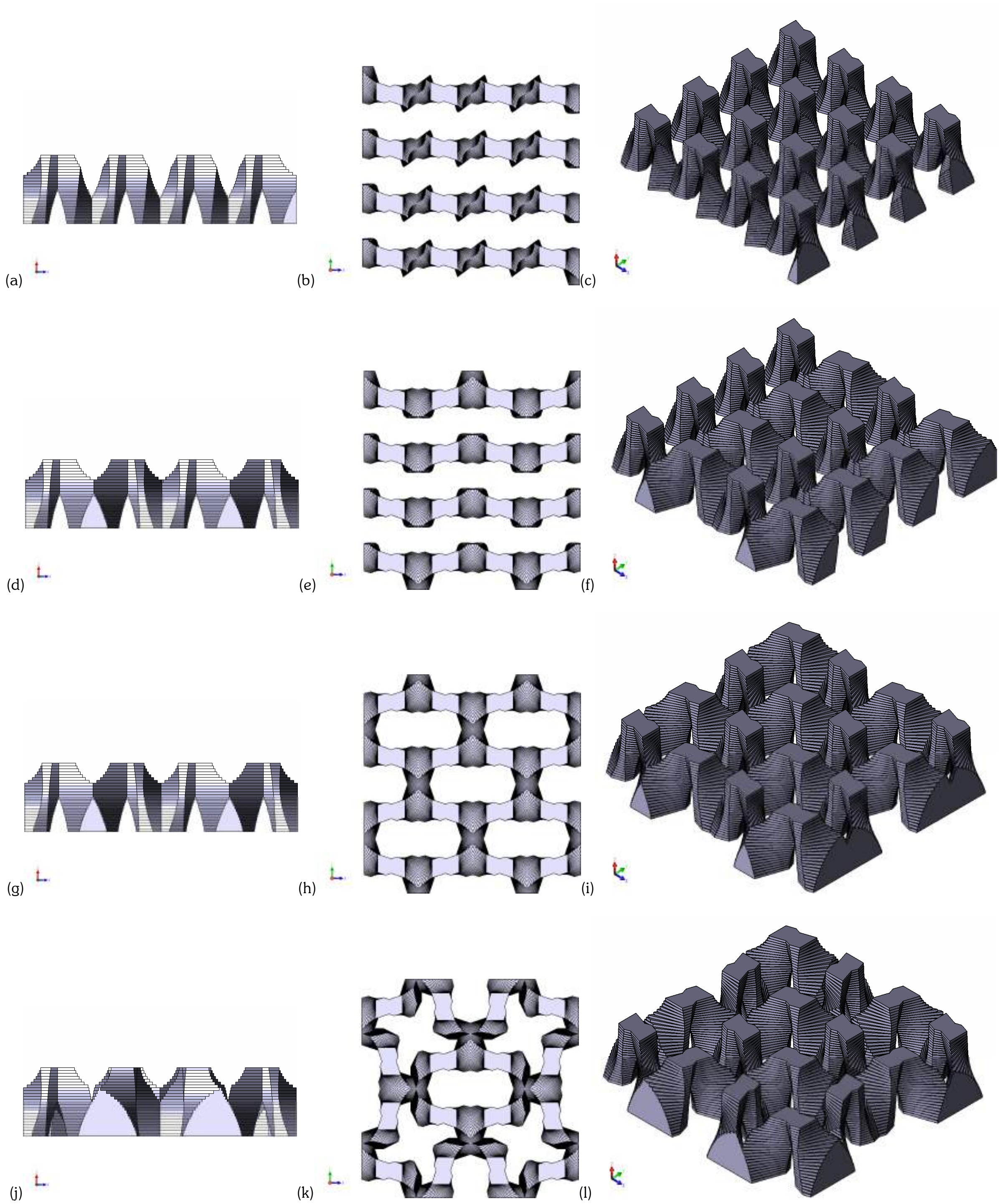

Fig. 6: Four branching structures corresponding to the dynamic region patterns with types *p1*, *pgg*, *pmm* and *p4g* (shown in Fig. 5) are shown in (a)-(c), (d)-(f), (g)-(i), and (j)-(l) respectively in front, top and isometric views. The structures are with the same given height in the z-direction. The structures with type *p1* and type *pgg* are invalid models with disconnected solids as shown in (b) and (e) respectively.

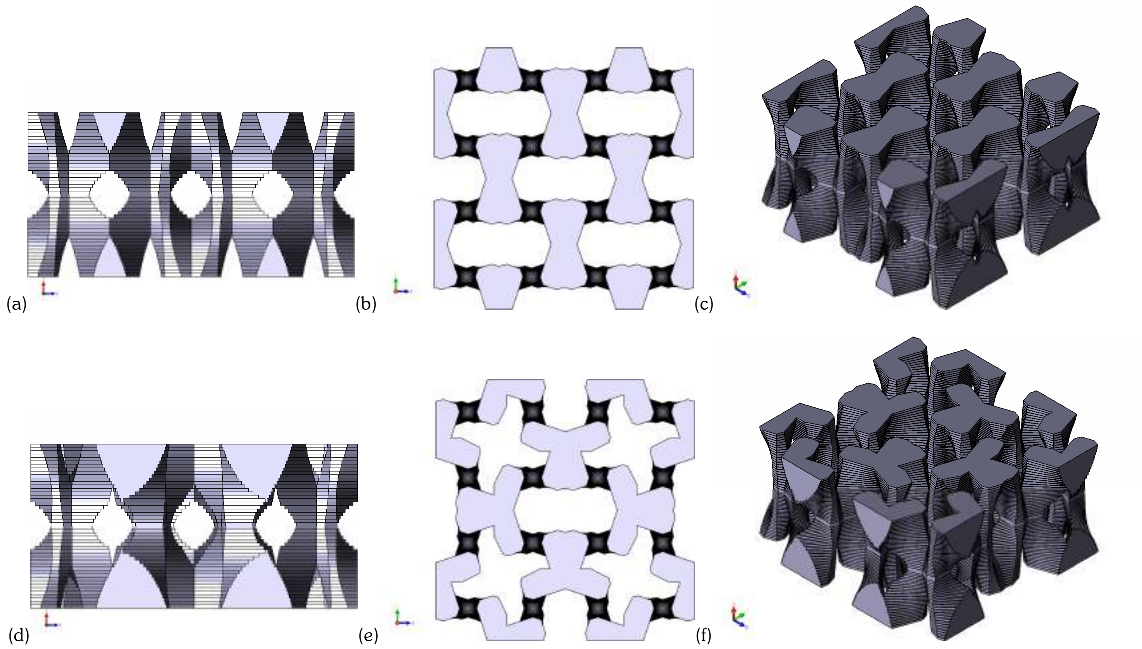

Fig. 7: Two porous structures are constructed with the valid branching structures (Fig. 6) with type *pmm* and type *p4g* shown in (a)-(c) and (d)-(f) respectively in front, top and isometric views.

4. DISCUSSIONS

In this study, we do not assume users to have detailed final structures in mind. The aim and novelty of our method are to provide a concise and flexible means of solid modeling with a few restrictions for creating new porous structures. Certain experience on designing the dynamic region patterns are required in order to achieve meaningful and valid 3D porous structures. Also, it is sometimes nontrivial to construct the basic point patterns for a desired tessellation. This problem is also discussed in [15]. Nevertheless, users can start with a pattern in a predefined library and proceed with modifications. With a generated solid model of porous structure, some geometric quantities could be calculated such as porosity, which is the ratio of the volume of solid region within a bounding volume to the volume of that bounding volume. Actually, the density and porosity are sometimes misused. With a given solid model, the density is a material property of the solid region while the porosity is a geometric property.

For future applications, this work could be used for scaffold design in tissue engineering (TE) [14,20]. Scaffolds are the architectures for cell seeding, cell migration, vascularization and tissue growth. In practice, there are several demanding specifications on scaffold geometry. For instances, bounding geometry, porosity, specific internal surface area, size of cell window opening, strut geometry, as well as the shape, size, orientation and connectivity of pores are the aspects of geometric specifications [17]. Different kinds, sites and stages of cells have different requirements on scaffold geometry in general. Conceptually, each specific structure in general has its unique geometric characteristics and hence physical functionality. However, the available scaffold shapes and architectures, which are usually simple truss-liked, are restricted by the limited modeling capability of current CAGD technologies. The proposed work may act as a design tool to satisfy this need. Also, it may facilitate some important future investigations. For example, outstanding scaffold shapes and architectures with representative functionality could be achieved and may serve as a new tool for the future TE researches and developments. To fabricate these structures, the layered manufacturing technologies [6] could be used which construct artifacts by depositing material layer-by-layer under computer control.

5. CONCLUSION

Existing computer-aided geometric design methods are inadequate to model and design 3D porous structures. A layer-based approach with 2D Voronoi tessellation is proposed to reduce the 3D challenge to a 2D problem. The proposed method is a layer-based model as each horizontal cross-section of a final structure is a layer of region pattern specified by a dynamic region pattern with varying shape topology. A dynamic region pattern is composed by a basic point pattern, arrows and a wallpaper symmetry type. The basic region patterns implicitly represent 2D holes, disconnected regions and regions in non-manifold configurations. They adaptively support different cellular topologies for dynamic region patterns. A basic region pattern is firstly designed in a non-manifold configuration. It corresponds to a critical layer (cross-section) of the final structure through which branching of a region just takes place. The rings are useful to align generating points and hence generate degenerate Voronoi tessellations for specifying non-manifold configurations. The aim and novelty of the method presented are to provide a concise and flexible means of solid modeling for creating new porous structures.

6. ACKNOWLEDGEMENTS

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