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Which GARCH model is best for Value-at-Risk?

Abstract

The purpose of this thesis is to identify the best volatility model for Value-at-Risk (VaR) estimations. We estimate 1 % and 5 % VaR figures for Nordic indices and stocks by using two symmetrical and two asymmetrical GARCH models under different error distributions. Out-of-sample volatility forecasts are produced using a 500 day rolling window estimation on data covering January 2007 to December 2014. The VaR estimates are thereafter evaluated through Kupiec's test and Christoffersen's test in order to find the best model. The results suggest that asymmetrical models perform better than symmetrical models albeit the simple ARCH is often good enough for 1 % VaR estimates.

KEYWORDS: Value-at-Risk, ARCH/GARCH forecasting, Backtesting, Kupiec test, Christoffersen test.

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1 Introduction

Risk and uncertainty on financial assets has always played an integral part in financial theory and practice. A customary gauge for the riskiness of an asset is the standard deviation of returns, in finance known as *volatility*. Markowitz (1952) mentioned the benefits of diversification as a mean for mitigating portfolio risk and followed up with the conclusion that more risk is inevitable in order to obtain higher expected returns in his classic book on portfolio theory (Markowitz, 1959).

The distribution of returns has also been of great interest for researchers. Mandelbroit (1963) noted that large price changes tend to be followed by large changes – of either sign – and small changes tend to be followed by small changes. His findings are today known as volatility clustering – something that Schwert (1989) noted is more prominent during times of economic recession. Fama (1965) found similar results as Mandelbroit and observed “fat tails” in the unconditional distribution of price changes, in contrast to what is expected from a Gaussian distribution. Researchers have for a lengthy period tried to fit returns to various distributions, and Praetz (1972), followed by Blattberg & Gonedes (1974), found that stock returns described by Student’s t distribution are better than by symmetric-stable distributions such as the Normal distribution.

Financial risk has indeed been an inherent interest for the general as well as the professional investor. Since the investment bank J.P Morgan began publishing RiskMetrics in 1994, a methodology to measure potential losses at the trading desk, the concept of value at risk (VaR) has become a widespread measure of market risk. Today, financial institutions are obliged to report VaR estimates according to the Basel III framework presented by The Basel Committee on Banking Supervision as an attempt to strengthen the banks capability to deal with financial stress. Although criticized; see e.g. Nwogugu (2006), VaR is one of the primary components determining the banks daily capital requirements and it can be a difficult task selecting the appropriate methodology since different methods lead to different capital requirements (Dardac & Grigore, 2011). The methodology used in this thesis will be presented in the second section.

One way of modelling volatility, a fundamental component in VaR estimates, is to use the Auto Regressive Conditional Heteroscedasticity (ARCH) model by Engle

(1982), later generalized independently by Bollerslev (1986) and Taylor (1986). The ARCH models capture the characteristic of volatility clustering and are today the most popular way of parameterizing this dependence (Teräsvirta, 2006). Although risk *management* for portfolios requires multivariate GARCH models, univariate models can serve as tools for risk *measurement* (Andersen et. al, 2007), as well as providing accurate volatility forecasts (Andersen & Bollerslev, 1998).

Today, the number of extensions to the original GARCH model is vast. A thorough survey by Poon & Granger (2003) finds that GARCH generally dominates ARCH. However, asymmetric models, such as the exponential GARCH by Nelson (1991) and GJR-GARCH by Glosten et. al (1993), tend to perform better than the original GARCH. A comprehensive study by Hansen & Lunde (2005) comparing a large number of models concludes that GARCH dominates other models on forecasting volatility for exchange rates but that models incorporating leverage effects are more suitable for stocks. Similar conclusions are presented by Köksal (2009) and by Hung-Chun & Jui-Cheng (2010). The models tested in this thesis are the ARCH, GARCH, EGARCH and GJR-GARCH.

The error term in the financial time series modelled by GARCH, nonetheless needs to be assumed and Bollerslev (1987) proposed the Student's t distribution rather than the Normal distribution originally assumed by Engle (1982) and Bollerslev (1986). Nelson (1991) suggested another density function that takes fat tails into account, namely the Generalized Error Distribution (GED) by Harvey (1981). However, a study by Hung-Chun & Jui-Cheng (2010) concludes that the error distribution does not significantly improve volatility forecasting using GARCH after testing different distributions e.g. the skewed generalized t (SGT) distribution by Theodossiou (1998). Wilhelmsson (2006) nonetheless, finds that allowing for leptokurtic error distributions leads to significant improvements compared to the Normal distribution. The distributions used in this thesis are the Normal distribution, Student's t distribution and the Generalized Error Distribution.

Hence, the purpose of this thesis is to answer the question:

Which volatility model, given different assumptions of the error distribution, provides the best VaR estimates for a selection of Nordic equity assets?

1.1 Earlier research

Earlier studies using GARCH volatility forecasts in VaR estimates fail to provide a definite answer on which model is the best. Yet, the use of GARCH in VaR has been extensive and the need for research continues to be of interest. Vlaar (2000) tested the GARCH model under different distribution assumptions on Dutch bond portfolios and concluded that the GARCH model under the Normal distribution dominates the common practice of using historical simulation models.

Brooks & Persand (2003) tested the effect of asymmetries on VaR estimations for a selection of Southeast Asian stock indices. They found that models ignoring asymmetrical effects in returns lead to inappropriately small VaR estimates compared to models taking the asymmetries in account. Angelidis et. al (2004) found no clearly superior model but concluded that leptokurtic distributions outperformed the Normal distribution – especially for the ARCH model – and that the estimation window length had an influence on the VaR estimates for a selection of major stock indices. In a more recent study, Orhan & Köksal (2012) concluded that the ARCH model and leptokurtic error distributions yielded the best results for VaR estimations after testing a wide range of volatility models. Thus, the need for continued testing is of great interest to financial risk managers searching for the optimal volatility forecasting model.

2 Theoretical framework

The theoretical framework in this thesis will firstly consider the volatility models that are used to perform forecasts. Secondly, the assumed distributions for the error terms will be presented followed by a third part covering the theory of VaR and its implications. Lastly, the tests used for evaluating the VaR forecasts will be presented and explained.

Returns will throughout this thesis be defined as

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = (ln\, P_t - ln\, P_{t-1}) \tag{1}$$

where r_t is the daily return and P_t is the price at time t.[1]

2.1 Volatility models

Volatility is defined as the standard deviation of the daily returns and is commonly denoted as σ. However, this thesis focuses mainly on the conditional volatility of the daily returns, i.e. σ_t. Nevertheless, the mean has to be addressed and in order to capture the conditional volatility with the models below, the mean of the daily returns is assumed to be zero, i.e. $E(r_t) = 0$. Returns are in financial econometrics commonly defined as a MA(1)-process as shown by Equation (2).

$$r_t = \theta\varepsilon_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is the error term. Hence, the returns have been *de-meaned* and the residuals will further on be given as

$$\varepsilon_t = \sigma_t Z_t \tag{3}$$

where Z_t is a sequence of independently and identically distributed random variables with zero mean and variance one. Different assumptions on the distribution of Z_t will be discussed below.

The Auto Regressive Conditional Heteroscedasticity model (ARCH), presented by Engle (1982), was the first model to capture time varying variance of returns. ARCH is defined as

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{4}$$

under the assumption that ω and α_i are strictly positive but $\alpha_i < 1$ and where q is the number of lags taken in account. Hence, using one lag results in the ARCH(1) model which states that today's conditional variance of the return is equal to a constant plus yesterday's squared return. A 1-step ahead forecast is given by

$$\sigma_{t+1}^2 = \omega + \alpha_1 \varepsilon_t^2. \tag{5}$$

[1] For illustrative reasons, the returns have been multiplied by 100 in order to be expressed as a percentage.

Bollerslev (1986), developed the Generalized ARCH model – GARCH – by adding the lagged variance as shown by

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{6}$$

where ω, α_i and β_j being positive for all i and j is sufficient to guarantee positive conditional variance.

Subsequently, a GARCH(1,1) model is obtained by setting $p = q = 1$ which results in the following equation for an 1-step ahead forecast:

$$\sigma_{t+1}^2 = \omega + \alpha_1 \varepsilon_t^2 + \beta_1 \sigma_t^2. \tag{7}$$

The assumption $\alpha_1 + \beta_1 < 1$ is sufficient to guarantee positive conditional variance.

A GARCH(1,1) model therefore states that today's volatility depends on a constant plus yesterday's squared return and yesterday's conditional variance. The unconditional variance is given by

$$\sigma^2 = \frac{\omega}{1 - (\alpha_1 + \beta_1)}. \tag{8}$$

The terms $\alpha_1 + \beta_1$ determines the time it takes for the variance forecast to converge to the unconditional variance in an l-step ahead forecast.

The exponential GARCH, or the EGARCH, introduced by Nelson (1991) differs from other GARCH models as it models the logarithm of the conditional variance. The model is asymmetric in the sense that it takes the impact of negative innovations in account unlike the GARCH model. The EGARCH includes a multiplicative dummy variable in order to check whether negative shocks are statistically significant as Nelson (1991) noted that negative shocks give rise to larger volatility than positive shocks. The EGARCH is given by

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \left(\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \sqrt{\frac{2}{\pi}} \right) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-i}}{\sigma_{t-i}} I_{t-k} + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2) \tag{9}$$

The term $\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - \sqrt{\frac{2}{\pi}} = |e_{t-i}| - \sqrt{\frac{2}{\pi}}$ is the expected value of the absolute value of a normal random variable,$|e_{t-i}|$, minus its expectation, so the shock has a mean of zero. I_{t-k} is a dummy variable indicating whether the return is positive or negative. The indicator is formally expressed as

$$I_t = \begin{cases} 1 & if\ r_t < 0 \\ 0 & if\ r_t \geq 0 \end{cases}. \tag{10}$$

The term e_{t-i}, is also a mean zero shock and the final term is the lagged log variance. Given the asymmetric structure of the EGARCH, the two shocks behave differently as the first term yields a symmetric rise in the log variance whereas the second term produces an asymmetric effect. The parameter γ_k is restricted to be < 0 and represents the rise in volatility following negative shocks. Since the EGARCH models the logarithm of the variance, the conditional volatility can never be negative and the assumption of positive parameters is no longer necessary (Sheppard, 2013).

By letting $p = r = q = 1$, the EGARCH(1,1) is obtained and subsequently the 1-step ahead forecast is expressed as

$$\ln(\sigma_{t+1}^2) = \omega + \alpha_1 \left(\left|\frac{\varepsilon_t}{\sigma_t}\right| - \sqrt{\frac{2}{\pi}} \right) + \gamma_1 \frac{\varepsilon_t}{\sigma_t} I_t + \beta_1 \ln(\sigma_t^2). \tag{11}$$

Glosten et. al (1993) introduced another asymmetric model that takes the sign in front of the return in account. The GJR-GARCH includes a similar multiplicative dummy variable as in the EGARCH; see Equation (10). The GJR-GARCH can be written as

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i} + \sum_{k=1}^{r} \gamma_k \varepsilon_{t-k}^2 I_{t-k} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{12}$$

Sheppard (2013). If $I_t = 1$, the model includes the effect of the negative lagged return expressed by γ_k. The GJR-GARCH(1,1) is obtained by letting $p = r = q = 1$ and the 1-step ahead forecast is therefore expressed as

$$\sigma_{t+1}^2 = \omega + \alpha_1 \varepsilon_t^2 + \gamma_1 \varepsilon_t^2 I_t + \beta_1 \sigma_t^2. \tag{13}$$

2.2 Distributions

For the models to fully function, the error term has to have zero mean i.e.

$$\varepsilon_t \sim N(0,1)$$

where the error term in this case is normally distributed with zero mean and variance one. The density function for the Normal distribution is

$$f(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}} e^{(-\frac{(\varepsilon_t-\mu)^2}{2\sigma^2})} \tag{14}$$

where μ constitute the mean and σ is the standard deviation.

The fatter tails, frequently observed in stock returns, are allowed for in the Student's t distribution assumed by Bollerslev (1987) which is given by the density function

$$f(\varepsilon) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\Gamma\left(\frac{\upsilon}{2}\right)\sqrt{\pi(\nu-2)\sigma_t^2}}(1+\frac{\varepsilon_t^2}{(\nu-2)\sigma_t^2})^{-\frac{(\upsilon+1)}{2}} \tag{15}$$

where υ is the degrees of freedom and we assume $\upsilon > 2$. Γ(·) is the gamma function,

$$\Gamma(\varepsilon) = \int_0^\infty t^{\varepsilon-1}e^{-t}\,dt. \tag{16}$$

The t distribution converges to the Normal distribution as $\upsilon \to \infty$.

The Generalized Error Distribution is useful since it can easily transform a Normal density function into a leptokurtic or platykurtic distribution by altering β in Equation (17). Its density function is given by

$$f(\varepsilon) = \frac{\beta}{2\sigma\Gamma\left(\frac{1}{\beta}\right)}\exp\left\{-\left(\frac{|\varepsilon-\mu|}{\sigma}\right)^\beta\right\}. \tag{17}$$

The GED assumes a symmetrical shape and is equal to the Normal distribution when $\beta = 2$.

2.3 Value-at-Risk

VaR can be viewed as a gauge that summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence (Jorion, 2007). More formally, a (α)VaR is expressed as

$$\Pr(L > VaR) = \alpha \tag{18}$$

where L is the loss on a given day and α is the significance level. VaR is therefore a *quantile* in the distribution of profit and loss that is expected to be exceeded only with a certain probability, formally expressed as

$$p = \int_{-\infty}^{-VaR(p)} f_q(x)dx. \tag{19}$$

Throughout this thesis, the VaR figures will be given using a 1 % and 5 % significance level, i.e. 1 % and 5 % VaR estimates will be presented.

VaR is computed using the conditional volatility of returns multiplied by the quantile of a given probability distribution, e.g. the Normal distribution as shown in Equation (20):

$$(\alpha)VaR = -\sigma_t \phi_\alpha \tag{20}$$

where ϕ_α in the Normal distribution is equal to -2,33 for a 1 % VaR and -1,65 for a 5 % VaR. Thus, VaR is presented as a positive number.

2.4 Backtesting VaR

Finding suitable forecast models for VaR estimates requires a method for evaluating the predictions ex-post. The VaR estimates in this thesis will be evaluated using two tests: an unconditional and a conditional test of coverage originally developed by Kupiec (1995) and Christoffersen (1998) respectively.

Henceforth, daily returns will be labelled according to Equation (21) in order to define whether the daily return exceeded the VaR estimate or not. The indicator variable is constructed as

$$\eta_t = \begin{cases} 1 & if\ r_t < -VaR \\ 0 & if\ r_t \geq -VaR \end{cases} \tag{21}$$

where 1 indicates a violation and 0 indicates a return less than the VaR. The violations are thereafter summed and divided by the total number of out-of-sample VaR estimates with the intention of obtaining the empirical size.

2.4.1 Kupiec's test

Kupiec's test was developed to test whether the empirical proportion of violations congregate with the nominal proportion specified by the VaR significance level. Kupiec (1995) suggests a likelihood ratio test constructed as in Equation (22) below.

$$LR_{uc} = 2\ln\left[\left(1-\frac{\mathrm{F}}{\mathrm{T}}\right)^{\mathrm{T-F}}\left(\frac{\mathrm{F}}{\mathrm{T}}\right)\mathrm{F}\right] - 2\ln[(1-\mathrm{p})^{\mathrm{T-F}}\mathrm{p}^{\mathrm{F}}] \tag{22}$$

where T is the number of out-of-sample estimates and F the observed number of violations. Hence, F/T is the empirical VaR size which follows the binominal distribution so $\mathrm{F} \sim \mathrm{B}(\mathrm{T},\mathrm{p})$. LR_{uc} follows the chi-square distribution with one degree of freedom, i.e. $LR_{uc} \sim \chi^2{}_{(1)}$, under the null hypothesis which states that $\mathrm{F}/\mathrm{T} = \mathrm{p}$. Hence, a rejection of the null hypothesis implies that the empirical VaR size is significantly different from the stated VaR significance level, i.e. the nominal size.

2.4.2 Christoffersen's test of independence

Ideally, a violation today does not reveal any information about the likelihood of a violation tomorrow, i.e. the violations occur independently of each other. A disadvantage with Kupiec's test is its ability detect whether the violations occur independently or clustered in a sequence. Christoffersen (1998) developed a test to detect clusters of violations. The advantage with the Christoffersen test of independence is its deference to the conditionality in the volatility forecasts. Good volatility forecasts ought to respond to periods of high and low volatility and subsequently adjust its predictions accordingly after the volatility clusters.

The probability of two subsequent violations are therefore defined as

$$p_{ij} = \mathrm{P}(\eta_t = i|\eta_{t-1} = j) \tag{23}$$

where η is either 0 or 1 as in Equation (21). Independence of violations is therefore defined as violations that do not occur in two subsequent days. A drawback with this test is arguably the definition of independence as a violation today followed by a violation the day after tomorrow is not detected in this test. Christoffersen (1998) nonetheless, suggests as likelihood ratio test of conditional coverage, shown in Equation (25) below.

$$LR_{ind} = -2\ln[(1-p)^{T-F}p^{F}] + 2\ln[1-\pi_{01})^{\eta_{00}}\pi_{01}^{\eta_{01}}(1-\pi_{11})^{\eta_{10}}\pi_{11}^{\eta_{11}} \tag{24}$$

where η_{ij} is the number of observations with the value i followed by j for $i, j = 0, 1$ and

$$\pi_{ij} = \frac{\eta_{ij}}{\sum_j \eta_{ij}} \tag{25}$$

are the corresponding probabilities. $LR_{ind} \sim \chi^2_{(1)}$ under the null hypothesis which states that the violations are independently distributed. Hence, a rejection of the null hypothesis infers that the violations are clustered and consequently not independent.

3 Data

The data in this thesis is collected from *Nasdaq OMX* and contains closing price data on the Swedish index OMXS30, the Danish index OMXC20 as well as a selection of two stocks from each country; see Table 1 below. The daily closing price data starts on January 1st 2007 extending to December 1st 2014 leading to a total number of 1992 trading days in Sweden and 1981 trading days in Denmark.

Company	Sector
H&M	Retail
Volvo	Heavy equipment
Carlsberg	Beverages
Maersk	Shipping

Table 1 - Description of company sectors

The selected companies comprise some of the most frequently traded equities in Sweden and Denmark. The Danish index OMXC20 is heavily influenced by chemical companies and retail firms whereas the Swedish index OMXS30 is largely comprised by heavy industrial companies as well as a prominent financial sector. Swedish OMXS30 and Danish OMXC20 are relatively dissimilar in their compositions and together make up a somewhat fair picture of the Nordic equity markets. Descriptive statistics for the return series are displayed below in Table 2.

Index/Equity	OMXS30	H&M	VOLVO	OMXC20	CARL	MAERSK
Mean	0,011	0,030	-0,005	0,027	0,007	0,006
St.d.	1,540	1,657	2,455	1,428	2,297	2,230
Min	-7,513	-10,248	-15,377	-11,723	-19,213	-13,918
Max	9,865	9,549	15,128	9,496	14,547	12,292
Skewness	0,093	0,028	0,014	-0,225	-0,352	0,089
Kurtosis	7,307	6,989	6,372	9,411	12,267	6,516
Jarque-Bera	1534,5	1315,0	939,1	3390,4	7102,7	1017,9

Table 2 - Descriptive statistics for the period 2007-01-01 to 2014-12-01

The indices and stocks exhibits leptokurtic characteristics as the returns reveal excess kurtosis as well as centred mean of zero. This indication is further strengthened by the large Jarque-Bera statistics which suggests the returns are incongruent with the Normal distribution. Swedish OMXS30 displays a slight positive skewness whereas the Danish OMXC20 exhibits a slight negative skewness. Carlsberg displays the largest kurtosis of 12,267 and Volvo the smallest kurtosis of 6,372 – both equities are well over a kurtosis of 3 which is implied by the Normal distribution. The plotted returns for the two indices are displayed below in Figure 1 & 2.[2]

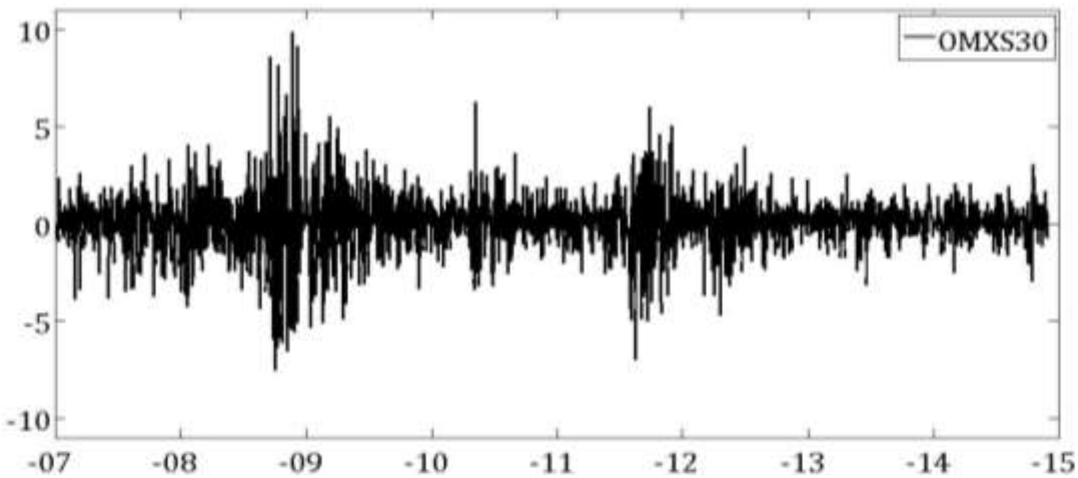

Figure 1 – Plotted daily returns for OMXS30. The data exhibits volatility clustering.

[2] Plotted returns for the equities are found in the appendix.

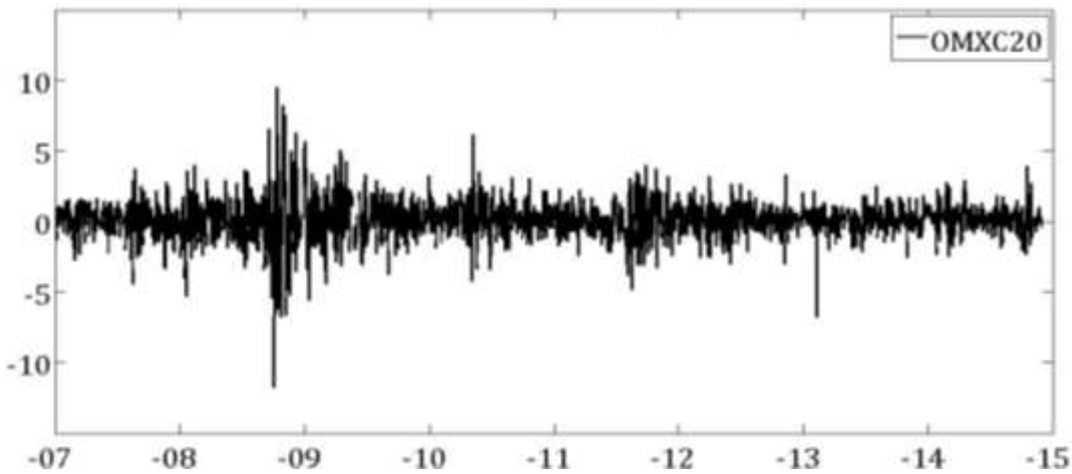

Figure 2 – Plotted daily returns for OMXC20. The data exhibits volatility clustering.

Figure 1 & 2 shows the plotted returns and the returns appear stationary but exhibits distinct volatility clusters. Hence, the variance cannot be assumed to be constant over time and estimations by ARCH/GARCH models appear appropriate. The initial volatility in the figures is mainly explained by the financial crisis of 2007-2009. The volatility cluster in the latter part of the figures is associated with the European debt crisis of 2012 which led to tumble in the financial markets.

4 Methodology

The data set containing a given number of observations has been divided into an estimation window and a test window. The first 500 observations have been used to estimate the GARCH parameters and the remaining observations are used as an out-of-sample testing window for the 1-day VaR estimates, i.e. the estimation window length: $W_L = 500$. The GARCH parameters are subsequently updated throughout the data set using rolling window estimation instead of being held constant over time. This is made in order to achieve flexibility in the parameters.

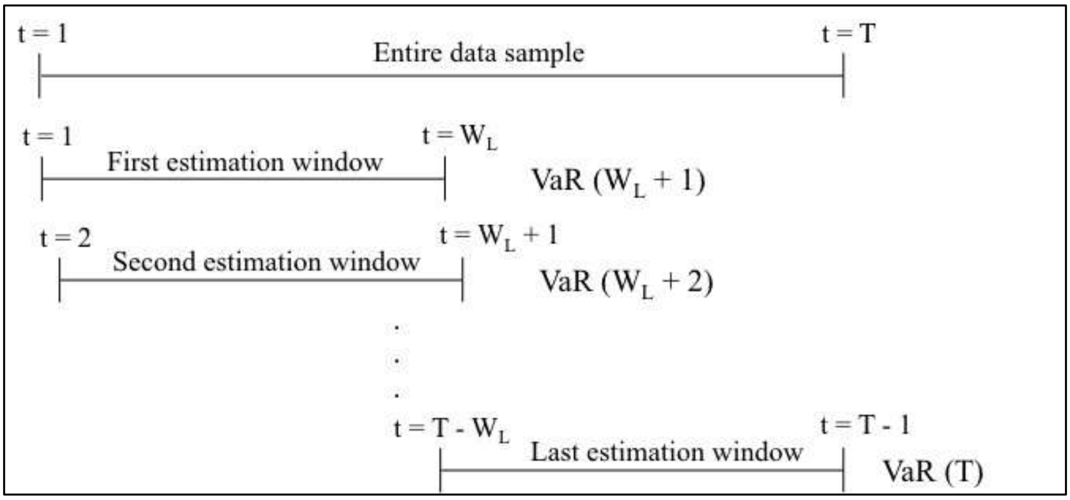

Figure 3 - Rolling window estimation

At time $t = W_L$, an out-of-sample forecast is made for time $W_L + 1$ by using the first 500 observations. The volatility forecasts are used to calculate the VaR estimate for time $W_L + 1$ and the estimate is thereafter compared to the actual return for that day – leading to either a violation or not. The same procedure is then repeated for a forecast for time $W_L + 2$ when the first observation is dropped whilst the actual return for time $W_L + 1$ observation is added. Thus, the window moves forward step by step until the end of the data set as illustrated by Figure (3) above.

All operations have been performed using the programming software Matlab version 2014a with the MFE Toolbox. The out-of-sample VaR estimates are calculated using a 1 % and 5 % significance level which corresponds to an expected violation every 100th and 20th trading day respectively.

5 Results

The VaR estimates produced by the volatility models are evaluated by Kupiec's test of unconditional coverage and Christoffersen's test of independence. The tests are evaluated on the 5 % significance level, hence the null hypothesis is rejected and the model subsequently discarded, if the p-value is below five percent. Table 3 displays the models that yielded the VaR estimates closest to the stated nominal size. In many cases, more than one model and distribution for each equity/index passed both tests. Hence, the models in Table 3 are not necessarily significantly better than other models but nonetheless performed the most accurate VaR estimates with this data and time period.[3]

Index/Equity	1%	5%
OMXS30	GARCH - t	GJR - N, t, GED
H&M	ARCH - t	EGARCH - t
Volvo	GJR - N	-
OMXC20	ARCH - t	EGARCH - N
Carlsberg	ARCH - GED	EGARCH - GED
Maersk	GJR - N, t	GJR - GED

Table 3 – Most accurate models that also passed Christoffersen's test

A striking result is that the volatility models tend to *underestimate* the risk for 1 % nominal VaR estimates and *overestimate* the risk for 5 % nominal VaR estimates. All models except one yielded an empirical size larger than 1 % for the examined data. Albeit, the empirical sizes were not necessarily significantly larger than the empirical size according to Kupiec's test. The ARCH model yielded the best results for half of the equities/indices on 1 % VaR estimates and the common denominator is that leptokurtic error distributions produced the best results for the ARCH model as well as for the GARCH model for OMXS30.

One explanation of the good performance of the ARCH model for 1 % VaR estimates could be that since the ARCH model excludes the lagged conditional variance and only models the lagged squared return, it responds faster to changes in the conditional variance. When a 'high volatility' cluster starts, the lagged squared return captures that change immediately and thus adapts faster than the GARCH model where the lagged conditional variance contributes to keeping the conditional variance at time t closer to its previous conditional variance, i.e. the previous 'low volatility' cluster.

The GJR-GARCH dominated the GARCH model for the Danish data but the t-distributed GARCH produced the best results for OMXS30. In the cases where GJR-GARCH yielded the best results, there was no error distribution that clearly dominated although the Normal distribution worked well for both equities.

For the 5 % VaR estimates however, the asymmetrical models thoroughly dominated the symmetrical models. The symmetrical models tended to fail

[3] Complete tables with test statistics and p-values are found in the appendix

Kupiec's test of conditional coverage and were subsequently discarded. No error distribution stands out as better than the other, although leptokurtic distributions appear to have a slight edge on the Normal distribution. Notably, not a single model yielded any acceptable VaR estimates for the Volvo data as the estimates could not pass Christoffersen's test of independence.

6 Conclusion

The purpose of this thesis has been to evaluate VaR estimates produced by various ARCH/GARCH forecasts, made under different error distributions, through a 500 day rolling window estimation. The estimates are based on a Swedish and Danish equity index as well as a selection of two frequently traded stocks from each country during the time period January 1st 2007 to December 1st 2014. Overall, no model is clearly superior, however asymmetrical models appear to outperform symmetrical models for 5 % VaR estimates after evaluation through Kupiec's unconditional coverage test and Christoffersen's test of independence. For 1 % VaR estimates, the ARCH model under leptokurtic distributions yields accurate results throughout the data set, albeit asymmetrical models yield acceptable results as well. Lastly, leptokurtic distributions appear to improve forecasting with symmetrical models but no overall dominating distribution could be found for all models in this thesis.

6.1 Recommendations for further studies

Further research is necessary and it would be a good idea to expand the number of models as well as test whether a skewed t distribution would yield better results than the models and distributions tested in this thesis. Finally, altering the estimation window length to see whether this improves the forecasts would be an interesting approach.

6.2 Recommendations for practitioners

The recommendations for practitioners are that conservative risk managers should use a simple ARCH model for VaR estimates since it almost always overestimates the risk. Asymmetrical models did nonetheless yield empirical VaR sizes closer to the nominal VaR size for most assets in this thesis, although they sometimes underestimated the risk. The models that passed both tests are summarized in tables that are found in the appendix.

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8 Appendix

The appendix includes plots over daily returns for the equities used in this thesis as well as the complete results with test statistics and p-values presented below. Finally, the models that passed Kupiec's and Christoffersen's test are summarized in two separate tables for facilitating purposes.

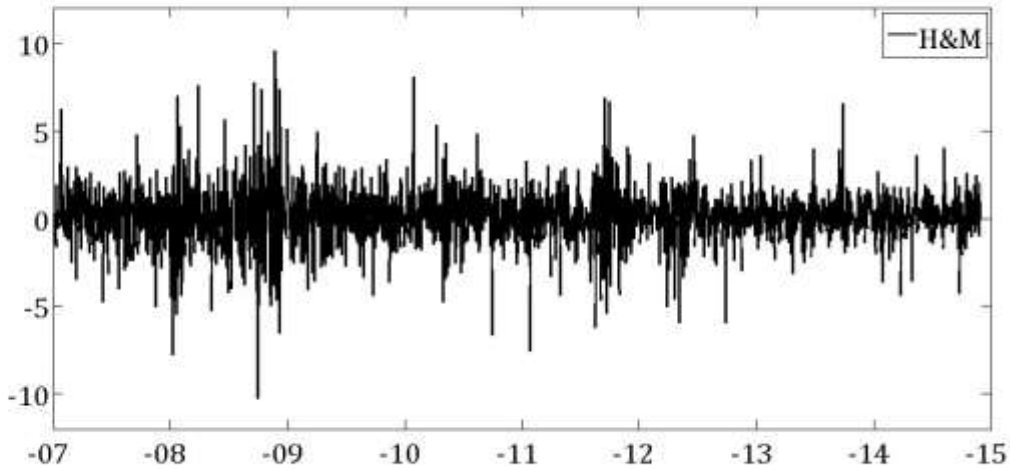

Figure 4 – Plotted daily returns for H&M. Exhibits of volatility clustering.

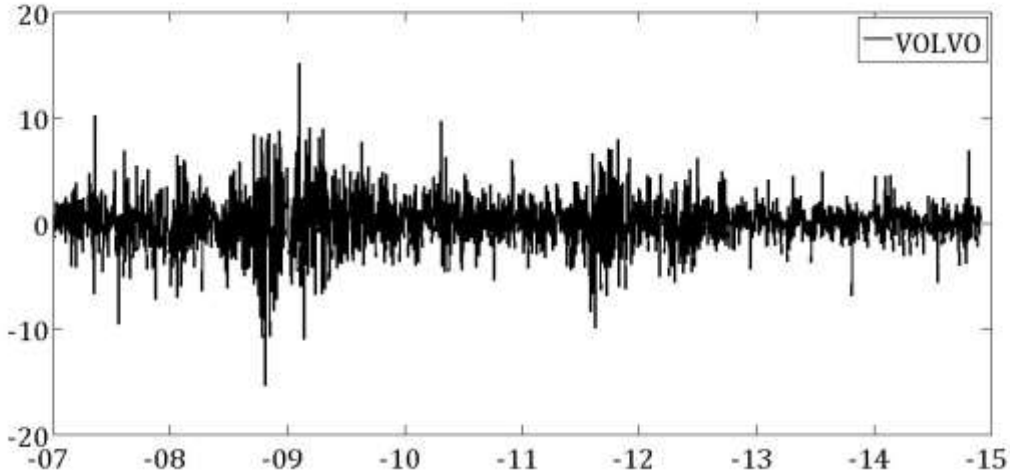

Figure 5 – Plotted daily returns for VOLVO. Exhibits of volatility clustering.

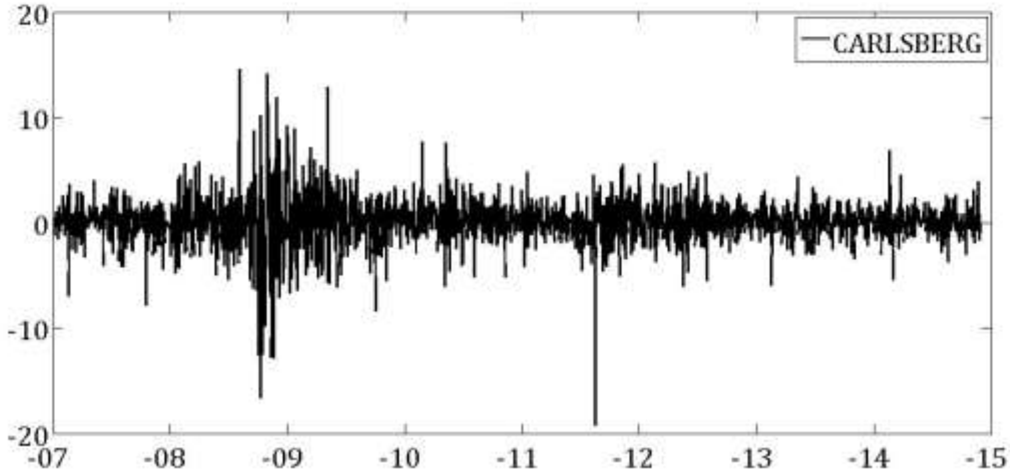

Figure 6 – Plotted daily returns for CARLSBERG. Exhibits of volatility clustering.

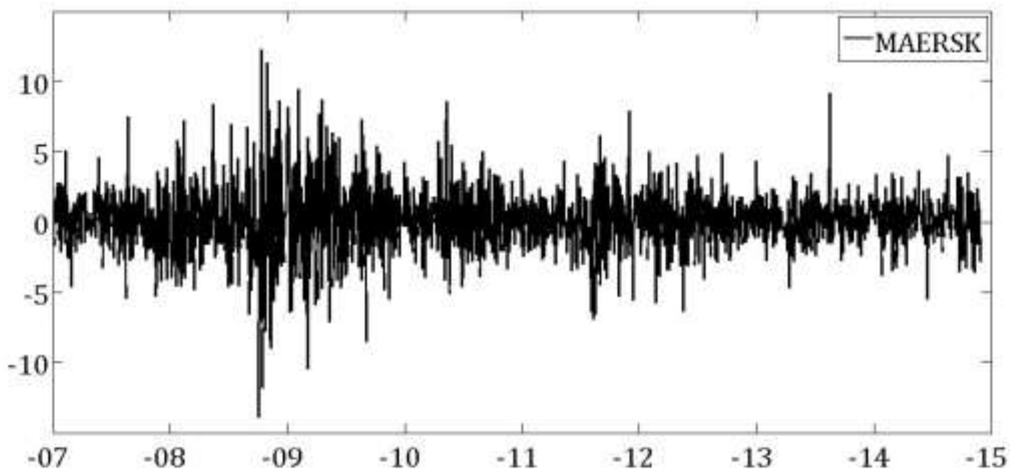

Figure 7 – Plotted daily returns for MAERSK. Exhibits of volatility clustering.

	ARCH					
OMXS30	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,41%	3,69%	1,34%	3,36%	1,41%	3,76%
Kupiec	2,238	5,886	1,593	9,544	2,238	5,271
	(0,135)	(0,015)	(0,207)	(0,002)	(0,135)	(0,023)
Christoffersen	1,075	5,680	1,221	7,451	1,075	5,363
	(0,300)	(0,017)	(0,269)	(0,006)	(0,300)	(0,021)

Table 4 – GARCH for OMXS30. Kupiec, Christoffersen statistics and p-values in parenthesis.

GARCH						
OMXS30	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,28%	5,30%	1,21%	5,37%	1,28%	5,37%
Kupiec	1,048	0,281	0,611	0,418	1,048	0,418
	(0,306)	(0,596)	(0,434)	(0,518)	(0,306)	(0,518)
Christoffersen	0,491	3,131	0,441	4,512	0,491	4,512
	(0,483)	(0,077)	(0,507)	(0,034)	(0,483)	(0,034)

Table 5 – GJR-GARCH for OMXS30. Kupiec, Christoffersen statistics and p-values in parenthesis.

EGARCH						
OMXS30	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,95%	6,11%	1,95%	5,84%	1,95%	6,04%
Kupiec	10,559	3,604	10,559	2,100	10,559	3,192
	(0,001)	(0,058)	(0,001)	(0,147)	(0,001)	(0,074)
Christoffersen	5,583	6,607	2,336	4,320	5,583	6,945
	(0,018)	(0,010)	(0,126)	(0,038)	(0,018)	(0,008)

Table 6 – EGARCH for OMXS30. Kupiec, Christoffersen statistics and p-values in parenthesis.

GJR-GARCH						
OMXS30	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,34%	5,30%	1,41%	5,30%	1,34%	5,30%
Kupiec	1,593	0,281	2,238	0,281	1,593	0,281
	(0,207)	(0,596)	(0,135)	(0,596)	(0,207)	(0,596)
Christoffersen	0,545	0,165	0,601	0,165	0,545	0,165
	(0,461)	(0,685)	(0,438)	(0,685)	(0,461)	(0,685)

Table 7 – ARCH for OMXS30. Kupiec, Christoffersen statistics and p-values in parenthesis.

ARCH						
HM B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,27%	3,15%	1,21%	2,68%	1,27%	3,15%
Kupiec	1,037	12,307	0,603	20,177	1,037	12,307
	(0,309)	(0,001)	(0,438)	(0,000)	(0,309)	(0,001)
Christoffersen	0,491	1,300	0,440	0,682	0,491	1,300
	(0,484)	(0,254)	(0,507)	(0,409)	(0,484)	(0,254)

Table 8 – GARCH for HM B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GARCH						
HM B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,41%	3,42%	1,41%	3,49%	1,47%	3,69%
Kupiec	2,222	8,799	2,222	8,025	2,961	5,941
	(0,136)	(0,003)	(0,136)	(0,005)	(0,085)	(0,015)
Christoffersen	0,600	0,808	0,600	0,705	0,942	0,441
	(0,439)	(0,369)	(0,439)	(0,401)	(0,332)	(0,507)

Table 9 – GJR-GARCH for HM B. Kupiec, Christoffersen statistics and p-values in parenthesis.

EGARCH						
HM B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	2,01%	4,42%	1,88%	4,69%	1,88%	4,56%
Kupiec	11,904	1,084	9,208	0,305	9,208	0,633
	(0,001)	(0,298)	(0,002)	(0,581)	(0,002)	(0,426)
Christoffersen	5,234	4,628	5,957	2,018	5,964	2,391
	(0,022)	(0,032)	(0,015)	(0,156)	(0,015)	(0,122)

Table 10 – EGARCH for HM B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GJR-GARCH						
HM B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,54%	3,62%	1,47%	3,82%	1,47%	3,95%
Kupiec	3,793	6,597	2,961	4,742	2,961	3,688
	(0,052)	(0,010)	(0,085)	(0,029)	(0,085)	(0,055)
Christoffersen	0,721	0,001	0,659	0,016	0,659	0,055
	(0,396)	(0,974)	(0,417)	(0,898)	(0,417)	(0,816)

Table 11 – ARCH for HM B. Kupiec, Christoffersen statistics and p-values in parenthesis.

ARCH						
VOLVO B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,07%	3,22%	1,07%	3,15%	1,07%	3,35%
Kupiec	0,077	11,366	0,077	12,307	0,077	9,613
	(0,781)	(0,001)	(0,781)	(0,001)	(0,781)	(0,002)
Christoffersen	6,606	8,267	6,606	8,690	6,606	7,464
	(0,010)	(0,004)	(0,010)	(0,003)	(0,010)	(0,006)

Table 12 – GARCH for VOLVO B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GARCH						
VOLVO B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,34%	3,75%	1,34%	4,09%	1,34%	3,95%
Kupiec	1,579	5,323	1,579	2,776	1,579	3,688
	(0,209)	(0,021)	(0,209)	(0,096)	(0,209)	(0,055)
Christoffersen	1,223	10,947	1,223	11,613	1,223	12,623
	(0,269)	(0,001)	(0,269)	(0,001)	(0,269)	(0,000)

Table 13 – GJR-GARCH for VOLVO B. Kupiec, Christoffersen statistics and p-values in parenthesis.

EGARCH						
VOLVO B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,74%	4,96%	2,01%	5,16%	1,81%	5,03%
Kupiec	6,804	0,005	11,904	0,081	7,968	0,002
	(0,009)	(0,943)	(0,001)	(0,777)	(0,005)	(0,962)
Christoffersen	3,036	6,328	2,135	7,446	2,790	8,192
	(0,081)	(0,012)	(0,144)	(0,006)	(0,095)	(0,004)

Table 14 – EGARCH for VOLVO B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GJR-GARCH						
VOLVO B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,21%	3,95%	1,34%	4,16%	1,27%	3,95%
Kupiec	0,603	3,688	1,579	2,371	1,037	3,688
	(0,438)	(0,055)	(0,209)	(0,124)	(0,309)	(0,055)
Christoffersen	0,440	6,848	0,544	5,829	0,491	6,848
	(0,507)	(0,009)	(0,461)	(0,016)	(0,484)	(0,009)

Table 15 – ARCH for VOLVO B. Kupiec, Christoffersen statistics and p-values in parenthesis.

ARCH						
OMXC20	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,08%	3,52%	1,01%	3,45%	1,08%	3,59%
Kupiec	0,099	7,587	0,003	8,341	0,099	6,873
	(0,753)	(0,006)	(0,954)	(0,004)	(0,753)	(0,009)
Christoffersen	0,351	6,814	0,308	7,186	0,351	6,455
	(0,554)	(0,009)	(0,579)	(0,007)	(0,554)	(0,011)

Table 16 – GARCH for OMXC20. Kupiec, Christoffersen statistics and p-values in parenthesis.

GARCH						
OMXC20	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,22%	3,72%	1,22%	3,86%	1,29%	3,72%
Kupiec	0,663	5,562	0,663	4,401	1,116	5,562
	(0,416)	(0,018)	(0,416)	(0,036)	(0,291)	(0,018)
Christoffersen	0,444	1,600	0,444	1,305	0,495	1,600
	(0,505)	(0,206)	(0,505)	(0,253)	(0,482)	(0,206)

Table 17 – GJR-GARCH for OMXC20. Kupiec, Christoffersen statistics and p-values in parenthesis.

EGARCH						
OMXC20	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,29%	4,94%	1,49%	5,14%	1,22%	5,07%
Kupiec	1,099	0,016	3,068	0,054	0,649	0,013
	(0,294)	(0,900)	(0,080)	(0,817)	(0,420)	(0,910)
Christoffersen	1,369	2,947	0,929	5,736	1,543	2,535
	(0,242)	(0,086)	(0,335)	(0,017)	(0,214)	(0,111)

Table 18 – EGARCH for OMXC20. Kupiec, Christoffersen statistics and p-values in parenthesis.

GJR-GARCH						
OMXC20	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,08%	3,92%	1,15%	4,06%	1,15%	3,86%
Kupiec	0,099	3,876	0,321	2,933	0,321	4,401
	(0,753)	(0,049)	(0,571)	(0,087)	(0,571)	(0,036)
Christoffersen	0,351	0,921	0,396	1,115	0,396	0,831
	(0,554)	(0,337)	(0,529)	(0,291)	(0,529)	(0,362)

Table 19 – ARCH for OMXC20. Kupiec, Christoffersen statistics and p-values in parenthesis.

ARCH						
CARL B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	0,88%	2,84%	0,88%	2,57%	1,01%	2,84%
Kupiec	0,233	17,191	0,233	22,312	0,003	17,191
	(0,629)	(0,000)	(0,629)	(0,000)	(0,961)	(0,000)
Christoffersen	2,693	0,487	2,693	0,880	2,171	0,487
	(0,101)	(0,485)	(0,101)	(0,348)	(0,141)	(0,485)

Table 20 – GARCH for CARL B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GARCH						
CARL B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,55%	3,78%	1,55%	3,58%	1,55%	3,78%
Kupiec	3,915	5,039	3,915	6,962	3,915	5,039
	(0,048)	(0,025)	(0,048)	(0,008)	(0,048)	(0,025)
Christoffersen	0,810	0,354	0,810	0,592	0,810	0,354
	(0,368)	(0,552)	(0,368)	(0,442)	(0,368)	(0,552)

Table 21 – GJR-GARCH for CARL B. Kupiec, Christoffersen statistics and p-values in parenthesis.

EGARCH						
CARL B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,35%	4,12%	1,42%	4,05%	1,49%	4,39%
Kupiec	1,656	2,569	2,313	2,992	3,068	1,212
	(0,198)	(0,109)	(0,128)	(0,084)	(0,080)	(0,271)
Christoffersen	1,212	0,821	1,066	0,932	0,932	0,451
	(0,271)	(0,365)	(0,302)	(0,334)	(0,334)	(0,502)

Table 22 – EGARCH for CARL B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GJR-GARCH						
CARL B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,35%	3,92%	1,42%	3,78%	1,42%	3,92%
Kupiec	1,656	3,944	2,313	5,039	2,313	3,944
	(0,198)	(0,047)	(0,128)	(0,025)	(0,128)	(0,047)
Christoffersen	1,212	0,231	1,066	0,354	1,066	0,231
	(0,271)	(0,631)	(0,302)	(0,552)	(0,302)	(0,631)

Table 23 – ARCH for CARL B. Kupiec, Christoffersen statistics and p-values in parenthesis.

ARCH						
MAERSK B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,22%	2,84%	1,22%	2,84%	1,22%	2,84%
Kupiec	0,649	17,191	0,649	17,191	0,649	17,191
	(0,420)	(0,000)	(0,420)	(0,000)	(0,420)	(0,000)
Christoffersen	0,443	0,035	0,443	0,035	0,443	0,035
	(0,506)	(0,853)	(0,506)	(0,853)	(0,506)	(0,853)

Table 24 – GARCH for MAERSK B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GARCH						
MAERSK B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,28%	3,85%	1,35%	3,92%	1,42%	3,85%
Kupiec	1,099	4,473	1,656	3,944	2,313	4,473
	(0,294)	(0,034)	(0,198)	(0,047)	(0,128)	(0,034)
Christoffersen	0,494	1,313	0,548	1,178	0,605	1,313
	(0,482)	(0,252)	(0,459)	(0,278)	(0,437)	(0,252)

Table 25 – GJR-GARCH for MAERSK B. Kupiec, Christoffersen statistics and p-values in parenthesis.

EGARCH						
MAERSK B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,42%	4,86%	1,42%	4,73%	1,35%	4,59%
Kupiec	2,313	0,060	2,313	0,237	1,656	0,534
	(0,128)	(0,806)	(0,128)	(0,626)	(0,198)	(0,465)
Christoffersen	1,066	6,921	1,066	5,429	1,212	8,416
	(0,302)	(0,009)	(0,302)	(0,020)	(0,271)	(0,004)

Table 26 – EGARCH for MAERSK B. Kupiec, Christoffersen statistics and p-values in parenthesis.

GJR-GARCH						
MAERSK B	Normal		Student's t		GED	
Nominal size	1,00%	5,00%	1,00%	5,00%	1,00%	5,00%
Empirical size	1,08%	3,85%	1,08%	3,92%	1,15%	3,98%
Kupiec	0,094	4,473	0,094	3,944	0,312	3,451
	(0,759)	(0,034)	(0,759)	(0,047)	(0,576)	(0,063)
Christoffersen	0,350	0,289	0,350	2,706	0,395	2,502
	(0,554)	(0,591)	(0,554)	(0,100)	(0,530)	(0,114)

Table 27 – ARCH for MAERSK B. Kupiec, Christoffersen statistics and p-values in parenthesis.

Index/Equity	1%
OMXS30	GARCH - N - t - GED, GJR - N - t - GED, ARCH - N - t -GED
HM B	GARCH - N - t - GED, GJR - N - t - GED, ARCH - N - t - GED
VOLVO B	GARCH - N - t -GED, GJR - N - t - GED
OMXC20	All models with all error distributions
CARL B	GJR - N - t -GED, EGARCH - N - t - GED, ARCH - N - t - GED
MAERSK B	All models with all error distributions

Table 28 – Models that passed Kupiec's and Christoffersen's test

Index/Equity	5%
OMXS30	GARCH - N, GJR - N - t - GED
HM B	GJR - GED, EGARCH - t - GED
VOLVO B	None of the models passed both tests
OMXC20	GJR - t, EGARCH - N - GED
CARL B	EGARCH - N - t - GED
MAERSK B	GJR - GED

Table 29 - Models that passed Kupiec's and Christoffersen's test