Box-Rectangular Drawings of Plane Graphs
(Extended Abstract)

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Abstract. In this paper we introduce a new drawing style of a plane graph G, called a "box-rectangular drawing." It is defined to be a drawing of G on an integer grid such that every vertex is drawn as a rectangle, called a box, each edge is drawn as either a horizontal line segment or a vertical line segment, and the contour of each face is drawn as a rectangle. We establish a necessary and sufficient condition for the existence of a box-rectangular drawing of G. We also give a simple linear-time algorithm to find a box-rectangular drawing of G if it exists.
Keywords: Graph, Algorithm, Graph Drawing, Rectangular Drawing, Box-drawing, Box-rectangular drawing.

1 Introduction

Recently automatic drawings of graphs have created intense interest due to their broad applications, and as a consequence, a number of drawing styles and corresponding drawing algorithms have come out [DETT94]. Among different drawing styles, an "orthogonal drawing" has attracted much attention due to its beautiful applications in circuit layouts, database diagrams, entity-relationship diagrams, etc [B96, CP98, K96, T87]. An *orthogonal drawing* of a plane graph G is a drawing of G in which each vertex is drawn as a grid point on an integer grid and each edge is drawn as a sequence of alternate horizontal and vertical line segments along grid lines as illustrated in Fig. 1(a). Any plane graph with the maximum degree at most four has an orthogonal drawing. However, a plane graph with a vertex of degree 5 or more has no orthogonal drawing.

A *box-orthogonal drawing* of a plane graph G is a drawing of G on an integer grid such that each vertex is drawn as a rectangle, called a *box*, and each edge is drawn as a sequence of alternate horizontal and vertical line segments along grid lines, as illustrated in Fig. 1(b). Some of the boxes may be degenerated rectangles, i.e., points. A box-orthogonal drawing is a natural generalization

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of an ordinary orthogonal drawing, and moreover, any plane graph has a box-orthogonal drawing even if there is a vertex of degree 5 or more. Several results are known for box-orthogonal drawings [BK97, FKK96, PT98].

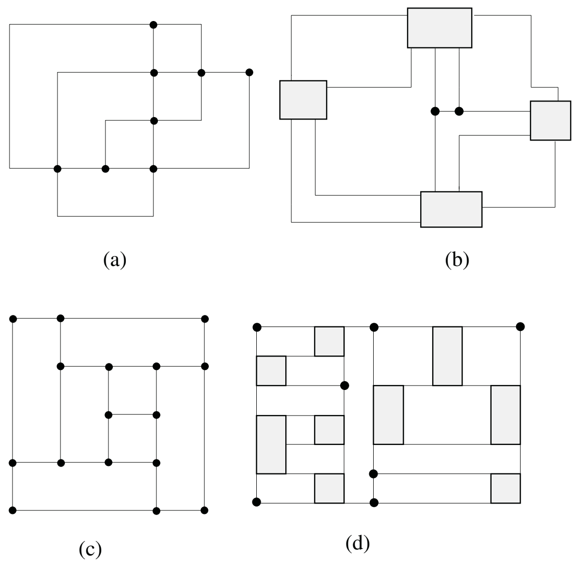

Fig. 1. (a) An orthogonal drawing, (b) a box-orthogonal drawing, (c) a rectangular drawing, and (d) a box-rectangular drawing.

An orthogonal drawing of a plane graph G is called a *rectangular drawing* of G if each edge of G is drawn as a straight line segment without bends and the contour of each face of G is drawn as a rectangle, as illustrated in Fig. 1(c). Since a rectangular drawing has practical applications in VLSI floorplanning, much attention has been paid to it [KK84, L90]. However, not every plane graph has a rectangular drawing. A necessary and sufficient condition for a plane graph G to have a rectangular drawing is known [T84], and several linear-time algorithms to find a rectangular drawing of G are also known [KH97, RNN98a].

Thus a box-orthogonal drawing is a generalization of an orthogonal drawing, while an orthogonal drawing is a generalization of a rectangular drawing. Hence an orthogonal drawing is an intermediate of a box-orthogonal drawing and a rectangular drawing. In this paper we introduce a new style of drawings as another intermediate of the two drawing styles. The new style is called a *box-rectangular drawing* and is formally defined as follows.

A *box-rectangular drawing* of a plane graph G is a drawing of G on an integer grid such that each vertex is drawn as a (possibly degenerated) rectangle, called a *box*, and the contour of each face is drawn as a rectangle, as illustrated in Fig. 1(d). If G has multiple edges or a vertex of degree 5 or more, then G has no rectangular drawing but may have a box-rectangular drawing. However, not every plane graph has a box-rectangular drawing. We will see in Section 2 that box-rectangular drawings have beautiful applications in floorplanning of MultiChip Modules (MCM) and in architectural floorplanning.

In this paper we establish a necessary and sufficient condition for the existence of a box-rectangular drawing of a plane graph, and give a linear-time algorithm to find a box-rectangular drawing if it exists. The sum of the width and the height of an integer grid required by a box-rectangular drawing is bounded by $m + 2$, where m is the number of edges in a given graph.

The rest of the paper is organized as follows. Section 2 describes some applications of box-rectangular drawings. Section 3 introduces some definitions and presents preliminary results. Section 4 deals with box-rectangular drawings of G for a special case where some vertices of G are designated as corners of the rectangle corresponding to the contour of the outer face. Section 5 deals with the general case where no vertex is designated as a corner.

2 Applications of Box-Rectangular Drawings

In this section we mention some applications of box-rectangular drawings.

As mentioned in Section 1, rectangular drawings have practical applications in VLSI floorplanning. In a VLSI floorplanning problem, an input is a plane graph F as illustrated in Fig. 2(a); F represents the functional entities of a chip, called *modules*, and interconnections among the modules; each vertex of F represents a module, and an edge between two vertices of F represents the interconnections between the two corresponding modules. An output of the problem for the input graph F is a partition of a rectangular chip area into smaller rectangles as illustrated in Fig. 2(d); each module is assigned to a smaller rectangle, and furthermore, if two modules have interconnections, then their corresponding rectangles must be adjacent, that is, must have a common boundary.

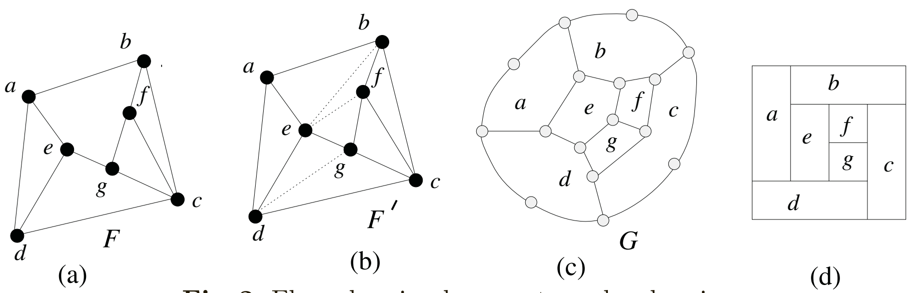

Fig. 2. Floorplanning by a rectangular drawing.

A conventional floorplanning algorithm using rectangular drawings is outlined as follows. First, obtain a graph F' by triangulating all inner faces of F as illustrated in Fig. 2(b), where dotted lines indicate new edges added to F. Then obtain a dual-like graph G of F' as illustrated in Fig. 2(c), where the four vertices of degree 2 drawn by white circles correspond to the four corners of the rectangular chip area. Finally, by finding a rectangular drawing of G, obtain a possible floorplan for F as illustrated in Fig. 2(d).

In the conventional floorplan above, two rectangles are always adjacent if the modules corresponding to them have interconnections. However, two rectangles may be adjacent even if the modules corresponding to them have no interconnections. For example, module e and module f have no interconnection in Fig. 2(a), but their corresponding rectangles are adjacent in the floorplan in Fig. 2(d). Such unwanted adjacencies are not desirable in some other floorplanning problems. In floorplanning of a MultiChip Module (MCM), two chips generating excessive heat should not be adjacent, or two chips operating on high frequency should not be adjacent to avoid malfunctioning due to their interference [S95]. Unwanted adjacencies may cause a dangerous situation in some architectural floorplanning, too [FW74]. For example, in a chemical industry, a processing unit that deals with poisonous chemicals should not be adjacent to a cafeteria.

We can avoid the unwanted adjacencies if we obtain a floorplan for F by using a box-rectangular drawing instead of a rectangular drawing, as follows. First, without triangulating the inner faces of F, find a dual-like graph G of F as illustrated in Fig. 3(b). Then, by finding a box-rectangular drawing of G, obtain a possible floorplan for F as illustrated in Fig. 3(c). In Fig. 3(c) rectangles e and f are not adjacent although there is a dead space corresponding to a vertex of G drawn by a rectangular box. Such a dead space to separate two rectangles in floorplanning is desirable for dissipating excessive heat in an MCM or for ensuring safety in a chemical industry.

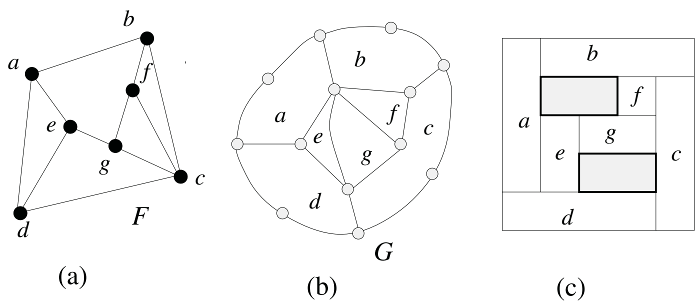

Fig. 3. Floorplanning by a box-rectangular drawing.

3 Preliminaries

In this section we give some definitions and present preliminary results.

Throughout the paper we assume that a *graph* G is a so-called multigraph which has no self loops but may have *multiple edges*, i.e., edges sharing both ends. If G has no multiple edges, then G is called a *simple* graph. We denote the set of vertices of G by $V(G)$, and the set of edges of G by $E(G)$. Let $n = |V(G)|$

and $m = |E(G)|$. The *degree* of a vertex v, denoted by $d(v)$, is the number of edges incident to v in G. We denote the maximum degree of a graph G by $\Delta(G)$ or simply by Δ.

A *cycle* C is an alternating sequence $v_0, e_0, v_1, e_1, \cdots, e_{l-1}, v_l (= v_0)$ of vertices and edges where $e_i = v_i v_{i+1}$ for each i and all edges $e_0, e_1, \cdots, e_{l-1}$ are distinct each other. If no vertex appears twice or more on C, then C is called a *simple cycle*. Thus a pair of multiple edges is a simple cycle. We hereafter call a simple cycle a cycle unless otherwise specified.

A graph is *planar* if it can be embedded in the plane so that no two edges intersect geometrically except at a vertex to which they are both incident. A *plane* graph is a planar graph with a fixed embedding in the plane. A plane graph G divides the plane into connected regions called *faces*. The unbounded region is called the *outer face*. We regard the *contour* of a face as a cycle formed by the edges on the boundary of the face. We denote the contour of the outer face of G by $C_o(G)$.

Let G be a plane connected graph. For a cycle C in G, we denote by $G(C)$ the plane subgraph of G inside C (including C). An edge which is incident to exactly one vertex of a cycle C and located outside of C is called a *leg* of C, and the vertex of C to which the leg is incident is called a *leg-vertex* of C. A cycle C in G is called a *k-legged cycle* if C has exactly k legs. We say that cycles C and C' in a plane graph G are *independent* if $G(C)$ and $G(C')$ have no common vertex. A set $\mathcal{S}$ of cycles is *independent* if any pair of cycles in $\mathcal{S}$ are independent.

We often use the following operation on a plane graph G [O67]. Let v be a vertex of degree d in a plane graph G, let $e_1 = vw_1, e_2 = vw_2, \cdots, e_d = vw_d$ be the edges incident to v, and assume that these edges $e_1, e_2, \cdots, e_d$ appear clockwise around v in this order. Replace v with a cycle $v_1, v_1v_2, v_2, v_2v_3, \cdots, v_dv_1, v_1$, and replace the edges vw_i with v_iw_i for $i = 1, 2, \cdots, d$. We call the operation above the *replacement of a vertex by a cycle*. The cycle $v_1, v_1v_2, v_2, v_2v_3, \cdots, v_dv_1, v_1$ in the resulting graph is called the *replaced cycle* corresponding to vertex v of G.

We often construct a new graph from a graph as follows. Let v be a vertex of degree 2 in a connected graph G. We replace the two edges u_1v and u_2v incident to v with a single edge u_1u_2, and delete v. We call the operation above *the removal of a vertex of degree 2* from G.

The *width* and the *height* of a rectangular drawing D of G is the width and the height of the rectangle corresponding to $C_o(G)$. The following result on rectangular drawings is known.

Lemma 1. *Let G be a connected plane graph such that all vertices have degree 3 except four vertices of degree 2 on $C_o(G)$. Then G has a rectangular drawing if and only if G has none of the following three types of cycles [T84]: (r1) 1-legged cycles, (r2) 2-legged cycles which contain at most one vertex of degree 2, and (r3) 3-legged cycles which contain no vertex of degree 2. Furthermore one can check in linear time whether G satisfies the condition above, and if G does then one can find a rectangular drawing of G in linear time. The sum of the width and the height of the produced rectangular drawing is bounded by $\frac{n}{2}$, where n is the number of vertices in G [RNN98a].* □

We now give some definitions regarding box-rectangular drawings. We say that a vertex of graph G is drawn as a *degenerated box* in a box-rectangular drawing D if the vertex is drawn as a point in D. We often call a degenerated box in D a *point* and call a non-degenerated box a *real box*. We call the rectangle corresponding to $C_o(G)$ the *outer rectangle*, and we call a corner of the outer rectangle simply a *corner*. A box in D containing at least one corner is called a *corner box*. A corner box may be degenerated. The *width* of a box-rectangular drawing D is the width of the outer rectangle. The *height* of D is defined in a similar manner.

If $n = 1$, that is, G has exactly one vertex, then the box-rectangular drawing of G is trivial; the drawing is just a degenerated box corresponding to the vertex.

Thus in the rest of the paper, we may assume that $n \geq 2$. We now have the following four facts and a lemma.

Fact 31 *Any corner box in a box-rectangular drawing contains either one or two corners.* □

Fact 32 *Any box-rectangular drawing has either two, three, or four corner boxes.* □

Fact 33 *In a box-rectangular drawing D of G, any vertex v of degree 2 or 3 satisfies the following (i), (ii) or (iii). (i) Vertex v is drawn as a point containing no corner; (ii) v is drawn as a corner box containing exactly one corner; and (iii) v is drawn as a real (corner) box containing exactly two corners.* □

Fact 34 *In a box-rectangular drawing D of G, every vertex of degree 5 or more is drawn as a real box.* □

Lemma 2. *If G has a box-rectangular drawing, then G has a box-rectangular drawing in which every vertex of degree 4 or more is drawn as a real box.* □

The choice of vertices as corner boxes plays an important role in finding a box-rectangular drawing. For example, the graph in Fig. 4(a) has a box-rectangular drawing if we choose vertices a, b, c and d as corner boxes as illustrated in Fig. 4(g). However, the graph has no box-rectangular drawing if we choose vertices p, q, r and s as corner boxes. If all vertices corresponding to corner boxes are designated for a drawing, then it is rather easy to determine whether G has a box-rectangular drawing with the designated corner boxes. We deal this case in Section 4. In Section 5 we deal with the general case where no vertex of G is designated as corner boxes.

4 Box-Rectangular Drawing with Designated Corner Boxes

In this section we establish a necessary and sufficient condition for the existence of a box-rectangular drawing of a plane graph G when all vertices of G corresponding to corner boxes are designated, and we also give a simple linear-time algorithm to find such a box-rectangular drawing of G if it exists.

By Fact 32, any box-rectangular drawing has either two, three or four corner boxes. In Section 4.1 we consider the case where exactly four vertices are designated as corner boxes, and in Section 4.2 we consider the case where two or three vertices are designated as corner boxes.

4.1 Drawing with Exactly Four Designated Corner Boxes

In this section we assume that exactly four vertices a, b, c and d in a given plane graph G are designated as corner boxes. We construct a new graph G'' from G through an intermediate graph G', and reduce the problem of finding a *box-rectangular drawing* of G with four designated vertices to a problem of finding a *rectangular drawing* of G''.

A plane graph having a vertex of degree 1 has no box-rectangular drawing. Also, a plane graph having a cycle with exactly one leg-vertex has no box-rectangular drawing. Thus we may assume that our input plane graph G has no vertex of degree 1 and has no cycle with exactly one leg-vertex.

We first construct G' from G as follows. If a vertex of degree 2 in G, as vertex d in Fig. 4(a), is a designated vertex, then it is drawn as a corner point in a box-rectangular drawing of G. On the other hand, if a vertex of degree 2, as vertex t in Fig. 4(a), is not a designated vertex, then it is drawn as a point on a vertical or horizontal line segment corresponding to the two edges incident to it, as illustrated in Fig. 4(g). Thus we remove all non-designated vertices of degree 2 one by one from G. The resulting graph is G'. Thus all vertices of degree 2 in G' are designated vertices. Note that some new multiple edges may appear in G'. Clearly G has a box-rectangular drawing with the four designated corner boxes if and only if G' has a box-rectangular drawing with the four designated corner boxes. Fig. 4(a) illustrates a plane graph G with four designated vertices a, b, c and d, and Fig. 4(b) illustrates G'. Fig. 4(f) illustrates a box-rectangular drawing D' of G', and Fig. 4(g) illustrates a box-rectangular drawing D of G.

Since every vertex of degree 2 in G' is a designated vertex, it is drawn as a (corner) point in any box-rectangular drawing of G'. Every designated vertex of degree 3 in G', as vertex a in Fig. 4(b), is drawn as a real box since it is a corner. On the other hand, every non-designated vertex of degree 3 in G' is drawn as a point. These facts together with Lemma 2 imply that if G' has a box-rectangular drawing then G' has a box-rectangular drawing D' in which all designated vertices of degree 3 and all vertices of degree 4 or more in G' are drawn as real boxes.

We now construct G'' from G'. Replace by a cycle each of the designated vertices of degree 3 and the vertices of degree 4 or more, as illustrated in Fig. 4(c). The replaced cycle corresponding to a designated vertex x of degree 3 or more contains exactly one edge, say e_x, on the contour of the outer face, where $x = a, b, c$ or d. Put a dummy vertex x' of degree 2 on e_x. The resulting graph is G''. We let $x' = x$ if a designated vertex x has degree 2. (See Fig. 4(d).) Now G'' has exactly four vertices a', b', c', and d' of degree 2 on $C_o(G'')$, and all other vertices have degree 3.

We now have the following theorem.

Theorem 1. *Let G be a connected plane graph with four designated vertices a, b, c and d on $C_o(G)$, and let G'' be the graph transformed from G as mentioned above. Then G has a box-rectangular drawing with four corner boxes corresponding to a, b, c and d if and only if G'' has a rectangular drawing.*

Proof. Necessity. Assume that G has a box-rectangular drawing with the four designated vertices a, b, c and d as corner boxes. Then by Fact 33 and Lemma 2 G has a box-rectangular drawing D in which all designated vertices of degree 3 and all vertices of degree 4 or more are drawn as real boxes, as illustrated in Fig. 4(g). The drawing D immediately yields a box-rectangular drawing D' of G', and D' immediately gives a rectangular drawing D'' of G''.

Sufficiency. Assume that G'' has a rectangular drawing D'' as illustrated in Fig. 4(e). In D'', each replaced cycle is drawn as a rectangle, since it is a face in G''. Furthermore, the four vertices a', b', c' and d' of degree 2 in G'' are drawn as corners of the rectangle corresponding to $C_o(G'')$. Therefore, D'' immediately gives a box-rectangular drawing D' of G' having the four vertices a, b, c and d as corner boxes, as illustrated in Fig. 4(f). Then, inserting non-designated vertices of degree 2 on horizontal or vertical line segments in D', one can immediately obtain from D' a box-rectangular drawing D of G having the designated vertices a, b, c and d as corner boxes, as illustrated in Fig. 4(g). □

We now have the following theorem.

Theorem 2. *Given a plane graph G with m edges and four designated vertices a, b, c and d on $C_o(G)$, one can determine in $O(m)$ time whether G has a box-rectangular drawing with a, b, c and d as corner boxes, and if G has, then one can find a box-rectangular drawing of G in $O(m)$ time. The sum of the width and the height of a produced box-rectangular drawing of G is bounded by $m + 2$.*

Proof. Time Complexity. Clearly one can construct G'' from G in time $O(m)$. G'' is a connected plane graph such that all vertices have degree 3 except vertices a', b', c' and d' of degree 2. Therefore, by Lemma 1 one can determine in linear time whether G'' has a rectangular drawing or not and find a rectangular drawing D'' of G'' if it exists. One can easily obtain a box-rectangular drawing D of G from D'' in linear time.

Grid size. Let n_2 be the non-designated vertices of degree 2 in G. Let $n' = |V(G')|$ and $m' = |E(G')|$. Then $m' = m - n_2$. We replace some vertices of G' by cycles and add at most 4 dummy vertices to construct G'' from G'. Therefore G'' has at most $2m' + 4$ vertices. From Lemma 1, the sum of the width and the height of the produced rectangular drawing of G'' is bounded by $\frac{2m'+4}{2} = m' + 2$. Now the insertion of a vertex of degree 2 on a horizontal line segment or a vertical line segment increases the width or the height of the box-rectangular drawing by at most one. Thus the sum of the width and the height of the produced box-rectangular drawing of G is bounded by $m' + 2 + n_2 = m + 2$. □

There are infinitely many cycles with four designated vertices for which the sum of the width and the height of any box-rectangular drawing of the cycles is $m - 2$.

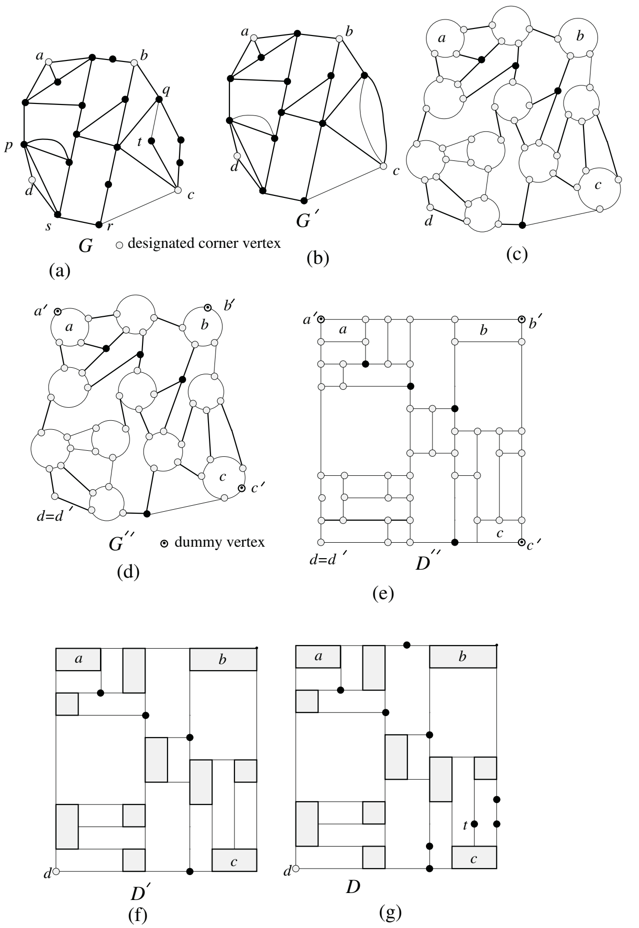

Fig. 4. Illustration of G, G', G'', D'', D' and D.

4.2 Drawing with Two or Three Designated Corner Boxes

If two or three vertices in G are designated as corner boxes and no other vertex can be a corner box, then we can easily reduce this case to the case where exactly four vertices are designated as corner boxes. The details are omitted in this extended abstract.

5 Box-Rectangular Drawing with No Designated Corner Boxes

In this section we consider the general case where no vertex of G is designated as a corner box.

By Fact 32 there are either two, three or four corner boxes in any box-rectangular drawing of G. Therefore, considering all combinations of two, three and four vertices on $C_o(G)$ as corner boxes and applying the algorithm in the previous section for each of the combinations, one can determine whether G has a box rectangular drawing. Such a straightforward method requires time $O(n^5)$ since there are $O(n^4)$ combinations and the algorithm in Section 4 can examine in time $O(n)$ whether G has a box-rectangular drawing for each of them. In this section we first establish a necessary and sufficient condition for the existence of a box-rectangular drawing of G when no vertex is designated as a corner box, and then show that the characterization leads to a linear-time algorithm.

In Section 5.1, we first derive a necessary and sufficient condition for a plane graph G with the maximum degree $\Delta \leq 3$ to have a box-rectangular drawing, and then give a linear-time algorithm to obtain a box-rectangular drawing of such a plane graph G if it exists. In Section 5.2 we give a linear-time algorithm for a plane graph G with the maximum degree $\Delta \geq 4$ by modifying the algorithm in Section 5.1.

5.1 Box-Rectangular Drawing of G with $\Delta \leq 3$.

Let G be a plane graph with the maximum degree at most 3. As in Section 4, we can assume that G is connected and has neither a vertex of degree 1 nor a 1-legged cycle. The following theorem is a main result of Section 5.1.

Theorem 3. *A plane connected graph G with $\Delta \leq 3$ has a box-rectangular drawing if and only if G satisfies the following two conditions: (br1) every 2-legged or 3-legged cycle in G contains an edge on $C_o(G)$; and (br2) any set $\mathcal{S}$ of independent cycles in G satisfies $2 \cdot |\mathcal{S}_2| + |\mathcal{S}_3| \leq 4$, where $\mathcal{S}_2$ is the set of 2-legged cycles in $\mathcal{S}$ and $\mathcal{S}_3$ is the set of 3-legged cycles in $\mathcal{S}$.* □

Before proving the necessity of Theorem 3, we observe the following fact.

Fact 51 *In a box-rectangular drawing D of G, any 2-legged cycle of G contains at least two corners, any 3-legged cycle of G contains at least one corner, and any cycle with four or more legs may contain no corner.* □

We now prove the necessity of Theorem 3.

Necessity of Theorem 3. Assume that G has a box-rectangular drawing D. By Fact 51 any 2-legged or 3-legged cycle in D contains a corner, and hence contain an edge on $C_o(G)$.

Let $\mathcal{S}$ be any set of independent cycles in G. Then by Fact 51 any 2-legged cycle in $\mathcal{S}$ contains at least two corners and any 3-legged cycle in $\mathcal{S}$ contains at least one corner. Since all these cycles in $\mathcal{S}$ are independent, they are vertex-disjoint each other. Therefore there are at least $2 \cdot |\mathcal{S}_2| + |\mathcal{S}_3|$ corners in D. Since there are exactly four corners in D, we have $2 \cdot |\mathcal{S}_2| + |\mathcal{S}_3| \leq 4$. □

We give a constructive proof for the sufficiency of Theorem 3. Our proof, which is omitted in this extended abstract, implies the following corollary.

Corollary 1. *A plane connected graph G with $\Delta \leq 3$ has a box-rectangular drawing if and only if G satisfies the following four conditions: (c1) every 2-legged or 3-legged cycle in G contains an edge on $C_o(G)$; (c2) at most two 2-legged cycles of G are independent each other; (c3) at most four 3-legged cycles of G are independent of each other; and (c4) if G has a pair of independent 2-legged cycles C_1 and C_2, then $\{C_1, C_2, C_3\}$ is not independent for any 3-legged cycle C_3 in G, and neither $G(C_1)$ nor $G(C_2)$ has more than two independent 3-legged cycles of G.* □

Using a method similar to ones in [RNN98a, RNN98b], one can determine whether a given plane graph with $\Delta \leq 3$ satisfies the conditions in Corollary 1 in time $O(m)$. If G satisfies the conditions in Corollary 1, then one can construct a box-rectangular drawing of G in linear time, following the method described in the constructive proof of the sufficiency of Theorem 3 and using the algorithm in [RNN98a]. (We omit the proof in this extended abstract.) We, thus, have the following theorem.

Theorem 4. *Given a plane graph with the maximum degree $\Delta \leq 3$, one can determine in time $O(m)$ whether G has a box-rectangular drawing or not, and if G has, one can find a box-rectangular drawing of G in time $O(m)$, where m is the number of edges in G. The sum of the width and the height of a produced box-rectangular drawing is bounded by $m + 2$.* □

5.2 Box-Rectangular Drawings of Graphs with $\Delta \geq 4$.

In this section we give a necessary and sufficient condition for a plane graph with $\Delta \geq 4$ to have a a box-rectangular drawing when no vertex is designated as corner boxes.

Let G be a plane connected graph with $\Delta \geq 4$. We first transform G into a graph H with $\Delta(H) \leq 3$, and then obtain a box-rectangular drawing of G by applying the algorithm in section 5.1 to H with appropriately choosing designated vertices.

We construct H from G by replacing each vertex v of degree four or more in G by a cycle. Each replaced cycle corresponds to a real box in a box-rectangular drawing. We do not replace a vertex of degree 2 or 3 by a cycle since such a vertex may be drawn as a point by Fact 33. Thus $\Delta(H) \leq 3$. We now have the following theorem.

Theorem 5. *Let G be a connected plane graph with no vertex of degree 1, and let H be the graph transformed from G as above. Then G has a box-rectangular drawing if and only if H has a box-rectangular drawing.* □

It is rather easy to prove the necessity of Theorem 5; one can easily transform any box-rectangular drawing of G to a box-rectangular drawing of H. On the other hand, it is not trivial to prove the sufficiency. However, we give a method to find a box-rectangular drawing of G in linear time if H has a box-rectangular drawing. The detail is omitted in this extended abstract.

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