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NEURAL NETWORK BASED ONE-STEP AHEAD CONTROL AND ITS STABILITY

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Abstract: In this paper, the stability analysis of neural network based one-step ahead control system is presented. First, a brief introduction about the neural control approach is given. Then, the stability condition for the neural control system is obtained from the stability investigation. Finally, an example on simulation is illustrated.

1. Introduction

Nonlinear control systems often are developed with complete knowledge of the controlled process. In the past, it is difficult to find an appropriate approach which can provide us with a general means to describe nonlinear processes. Recently, neural networks have been proved to be able to offer interesting possibilities for modelling an arbitrarily nonlinear process. Thus, the application of neural networks to nonlinear control becomes very attractive.

Suppose a discrete nonlinear dynamic process which is described by

$$y(t)=f[y(t-1),...,y(t-n-1),u(t-1),....,u(t-m-1)] \quad (1)$$

where y is the output of the process, u is the input of the process. The control purpose is to use an one-step ahead control action at time t to drive the output of the process at time t+1 to be equal to the desired output at time t+1, i.e. $y(t+1)=r(t+1)$[3], where r is the desired output of the process. In this paper, we use a feedforward neural network with external feedback inputs for the implementation of this one-step ahead controller.

In control engineering, stability is one of the most important aspects in design of a control system. For neural network based one-step ahead control system, we must also consider the stability design of the controller to ensure the closed-loop stability. Since the neural network based control system is nonlinear, the convenient tool for its stability analysis is the well-known Lyapunov stability theory which has an inherent relevance to nonlinear systems.

In this paper, we will first give a brief introduction of a neural network based one-step

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ahead control approach. Then, the investigation on the stability of the control system is presented. It leads to a sufficient condition for closed-loop stability. A simulation example will finally show an evaluation of the proposed stability design scheme.

2. Neural network based one-step ahead control

The neural network based one-step ahead control system employs a neural network model as an one-step ahead predictor. It is assumed that the neural model is accurate. Based on the neural model, the one-step ahead control is considered as a direct optimization of the following cost function of the control system, i.e.

$$J=[r(t+1)-y_m(t+1)]^2+\alpha[\Delta u(t)]^2 \quad (2)$$

where α is the weighting factor, $\Delta u(t)=u(t)-u(t-1)$, and $y_m(t+1)$ is the one-step ahead predictive output of the neural model. As the controller tries to bring $y_m(t+1)$ to a desired value $r(t+1)$ in one step, this may result in an excessive control effort. Therefore, we consider the minimization of the cost function shown in (2) to achieve a compromise between perfect one-step ahead control and the variation in the amount of control effort.

The neural network model used for description of the controlled process is a multilayer feedforward network with external recurrent inputs, i.e.

$$y_m(t+1)=\sum_{i=1}^{H} w_i s\Big[\sum_{j=1}^{n} w_{ij} y_m(t-j+1)+\sum_{j=1}^{m} w_{i,n+j} u(t-j+1)\Big] \quad (3)$$

where w_i and w_{ij} are the synaptic weights of the network, $s(.)$ is a sigmoid function with the form

$$s(x)=\frac{1-e^{-x}}{1+e^{-x}} \quad (4)$$

Suppose the neural model is well pre-specified by training. Using the neural model described in (3), we can simulate the dynamic behaviour of a nonlinear dynamic process[1]. Thus, we can obtain the one-step ahead control from the optimization of the cost function (2) based upon the neural model (3) by using the gradient descent optimizing technique, i.e.

$$\Delta u(t)=-\frac{\lambda}{1+\alpha\lambda} e(t+1)\frac{\partial e(t+1)}{\partial u(t)} \quad (5)$$

where λ is the optimizing step, $e(t+1)=r(t+1)-y_m(t+1)$, the sensitivity $\partial e(t+1)/\partial u(t)$ can be derived from the neural network model, i.e.

$$\frac{\partial e(t+1)}{\partial u(t)}=-\sum_{i=1}^{H} w_i s'(.) w_{i,n+1} \quad (6)$$

For simplification, we denote

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$$\beta = \frac{\lambda}{1+\alpha\lambda} \tag{7}$$

Here, we have the following theorem:

Theorem 1:
If the parameters satisfy $\alpha\beta \le 1$, the control algorithm given in (5) will be convergent.

Proof:
Consider the derivative of the cost function (2) with respect to t

$$\frac{\partial J}{\partial t} = 2e(t+1)\frac{\partial e(t+1)}{\partial u(t)}\frac{\partial u(t)}{\partial t} + 2\alpha\Delta u(t)\frac{\partial u(t)}{\partial t} \tag{8}$$

Approximately, we have

$$\frac{\Delta J}{\Delta t} = 2e(t+1)\frac{\partial e(t+1)}{\partial u(t)}\frac{\Delta u(t)}{\Delta t} + 2\alpha\Delta u(t)\frac{\Delta u(t)}{\Delta t} \tag{9}$$

From (5) and (7), we have

$$\Delta J = 2(-\frac{1}{\beta}+\alpha)[\Delta u(t)]^2 \tag{10}$$

To ensure the algorithm to be convergent, (2) should be non-increasing, i.e. the right hand side of (10) should be negative. Thus, we obtain the condition for convergence, i.e.

$$\alpha\beta \le 1 \tag{11}$$

When α and λ are chosen as positive values, the convergence condition is always satisfied. If the control algorithm converges, through iteration, we can at least find a local extremum of (2).

The neural network based one-step ahead control is very simple. It only needs one neural network for both prediction and control. The design of the controller is straightforward based on the optimization of the weighted cost function defined in (2).

3. Stability analysis

The stability analysis of the neural network based one-step ahead control system is based upon the well known Lyapunov approach. Firstly, we should define an appropriate Lyapunov function which has relevance to the performance of the control system. It is well known that the purpose of control is to force the output of the controlled process to track the desired trajectory of the system accurately. From this point of view, we can define a Lyapunov function as follows:

$$V = e^2(t+1) + \Delta e^2(t+1) \tag{12}$$

Obviously, the equilibrium point of the control sysem is $(e(t+1), \Delta e(t+1))=(0,0)$. This means that the output of the controlled process will accurately track the desired output

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and remain on the desired trajectory if the energy function is attracted to the equilibrium point. In order to achieve this target, we must apply the corresponding control action to the process to ensure

$$\Delta V \leq 0 \tag{13}$$

Hence, we have theorem 2:

Theorem 2:
Suppose the candidate Lyapunov function is defined as (12), the sufficient condition for the stability of the one-step ahead neural control system is

$$\beta \leq \frac{1}{[\max[s'(.)]\sum_{i=1}^{H} w_i w_{i,n+1}]^2} \tag{14}$$

Proof:
Considering

$$\frac{\partial V}{\partial t} = 2e(t+1)\frac{\partial e(t+1)}{\partial u(t)}\frac{\partial u(t)}{\partial t} + 2\Delta e(t+1)\frac{\partial e(t+1)}{\partial u(t)}\frac{\partial u(t)}{\partial t} \tag{15}$$

approximately, it leads to

$$\Delta V = 2e(t+1)\frac{\partial e(t+1)}{\partial u(t)}\Delta u(t) + 2\Delta e(t+1)\frac{\partial e(t+1)}{\partial u(t)}\Delta u(t) \tag{16}$$

Since (5) and (7) as well as

$$\Delta e(t+1) = \frac{\partial e(t+1)}{\partial u(t)}\Delta u(t) \tag{17}$$

it yields

$$\Delta V = 2e^2(t+1)[\frac{\partial e(t+1)}{\partial u(t)}]^2\beta[-1+\beta[\frac{\partial e(t+1)}{\partial u(t)}]^2] \tag{18}$$

In order to satisfy the condition presented in (13), we should confine β to a certain extent, i.e.

$$\beta \leq \frac{1}{[\frac{\partial e(t+1)}{\partial u(t)}]^2} \tag{19}$$

Referring to (6), we obtain the corresponding sufficient condition for the stability of the neural control system:

$$\beta \leq \frac{1}{[\max[s'(.)]\sum_{i=1}^{H} w_i w_{i,n+1}]^2} \leq \frac{1}{[\frac{\partial e(t+1)}{\partial u(t)}]^2} \tag{20}$$

For the neural network model with the sigmoid function defined in (4), we derive corollary 1.

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Corollary 1:

If the one-step ahead controller is designed based on the neural network with a sigmoid function shown in (4), the sufficient condition for closed-loop stability is that the parameter β should be less than the following bound:

$$\beta \leq \frac{4}{[\sum_{i=1}^{H} w_i w_{i,n+1}]^2} \quad (21)$$

It can be easily satisfied by considering the maximum derivative of the sigmoid function defined in (4) to be 0.5. From theorem 2, we notice that the closed-loop stability of the neural control system is significantly influenced by the parameter β. As the neural network model is pre-determined, in the design of the neural controller, we only select the parameter β to meet the demand of the stability given in (21) to gain a stable control performance.

4. An example

In this section, we will present an example to test the stability result obtained in the previous section. Suppose the controlled nonlinear dynamic process described by[1]:

$$y(t) = \frac{y(t-1)y(t-2)u(t-1)[y(t-3)-1]+u(t-2)}{1+y^2(t-1)+y^2(t-2)}$$

We use a neural network with 5-6-1 structure to model the process. Based on the neural network model, we obtain the corresponding stability upper bound according to (21), i.e.

$$\beta = \frac{\lambda}{1+\alpha\lambda} \leq 5.12$$

In this example, we use different values of β for controller design to test the stability of the neural control system.

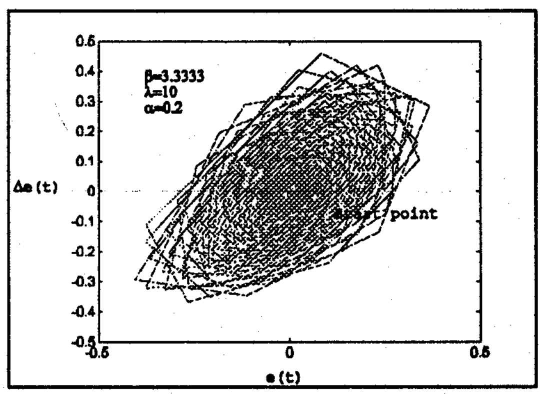

Figure 1 System response in phase-plane (β=3.3333)

Firstly, we fix the value of the control weighting factor, i.e. let α=0.2. Then, we only change the value of λ to see the corresponding system response. Fig. 1 shows the case of λ=10 which leads to β=3.3333. In this test, we see that the violent oscillation around the equilibrium point happens in the system response since β is close to its upper bound. Although the system finally converges to the equilibrium point, the system response is very poor. When we choose λ=4, we have β=2.2222. The corresponding system response is illustrated in Fig. 2. We notice that the control performance is greatly improved though the overshoot exists. In Fig. 3, we demonstrate a satisfactory system response obtained for λ=1 and β=0.8333. In this situation, the system can track the desired trajectory

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without any overshoot or oscillation.

From this example, we see the effect of both λ and α on the closed-loop stability of the one-step ahead neural control system. If we select the parameters based on the stability constraint given in (20) or (21), we can obtain a stable control performance. The given stability condition has been proved by the presented example.

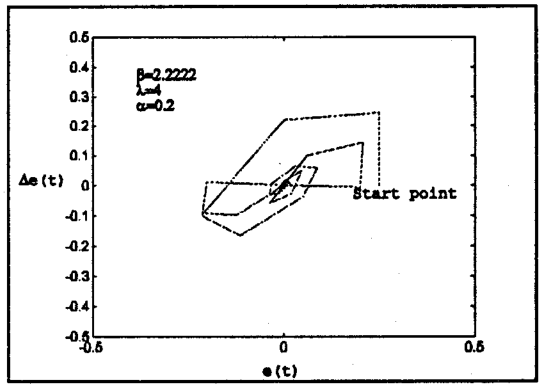

Figure 2 System response in phase-plane (β=2.2222)

5. Conclusions

A neural network based one-step ahead control approach has been presented in this paper. Unlike the other kinds of neural control schemes which usually need more than one neural network for modelling and control, the presented control strategy only uses one for both modelling and control.

The stability analysis based on Lyapunov theory has been given. It results in the corresponding sufficient condition for the closed-loop stability of the neural control system. When the neural model is pre-determined, the closed-loop stability only depends upon the choice of parameter β. The simulation example has proved this stability criterion is available.

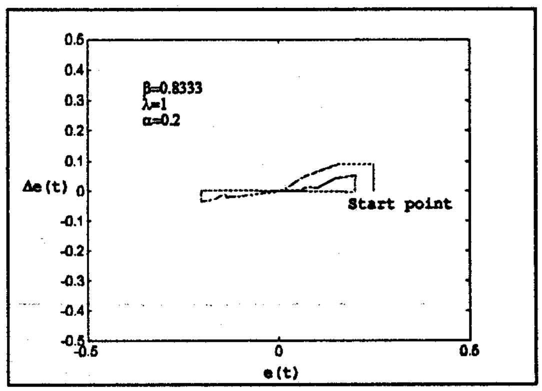

Figure 3 System response in phase-plane (β=0.8333)

In terms of this obtained stability criterion, we can design a stable neural controller to obtain satisfactory control performance.

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