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On a Hopfield net arising in the modelling and control of over-saturated signalized intersections

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Abstract. We present a neural method - based on the Hopfield net - for the modelling and control of over-saturated signalized intersections. The original Hopfield algorithm is modified to take into account proper constraints of the traffic problem. This approach is illustrated by numerical examples of traffic conditions generated by a simulator. We extend the method to urban nets of several interconnected intersections.

1. Introduction

Adaptive traffic control systems[1] by means of traffic lights, are generally based on a mathematical model which propagates the traffic flows through the urban network during the time. Then one realizes a short term forecasting of traffic demand and looks, in real-time for lights setting which minimize a performance criterion, function of delays or waiting time.

In this paper, we investigate the optimization capabilities of the well known Hopfield net [3], to resolve the traffic control problem. Other kinds of Artificial Neural Networks [2] can also be used to give solutions to the traffic adaptive control problem [7], as a particular case of a general optimal control problem [1]. In section 2 of this paper, we formulate the general traffic control problem and the simplifications induced by oversaturating assumption [9]. In section 3, we present the -Hopfield's solution- and finally section 4 is devoted to numerical results.

2. Traffic modelling - The control problem

In this paper, we deal only with elementary intersections with four links and two lights phases. However, complex intersections can be subdivided in two or

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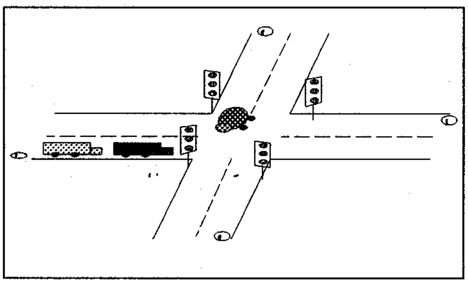

Figure 1: Elementary signallized junction

more elementary intersections[2].

Under oversaturating assumption, the rate of vehicles leaving a light line is constant in time, and so the traffic behavior is described by the following linear dynamic state equation (the time is discrete with a path about 5 seconds in practice):

$$L_i(t+1) = L_i(t) + A_i(t) - S_i \times u_i(t) \tag{1}$$

where: $L_i(t)$ is the queue's length at time t, in link i, $A_i(t)$ are the vehicles arriving on link i during the time interval $[t, t+1[$, $u_i(t)$ is the signal setting for the link i on the same interval and S_i is the saturation flow, i.e. the maximum flow evacuated from the link i in one time path. Since we neglect the yellow's duration in traffic lights, u_i is then a binary variable taking the values 0 for red and 1 for green traffic lights. We can so describe the signal setting for the whole intersection with only one variable denoted u ($u = 1$ if the traffic lights are green for links 1 and 3).

To compute the command on an optimization horizon$[0, T]$ (about a few minutes in practice), on which the arrivals are known, we use a criterion of the form:

$$J(u) = \sum_{t=0}^{T} \sum_{i=1}^{4} \varphi(L_i(t)) \tag{2}$$

J depends on u through the queues L_i, i is an index on the links incoming the controlled junction and generally $\varphi(x) = x$ or x^2: linear or quadratic criterion. The problem has two constraints: a spatial one related to the maximum queue expansion and a security one about the minimum and maximum green time. The command space is of cardinal 2^T but, because of the green time constraints, the admissible solutions are notably less. However, the problem becomes quickly harder for any urban net of several junctions.

[2]See [8] for an introduction to the concepts of traffic flow control and modelling

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3. The neural method

Let consider the quadratic criterion associated to an elementary intersection, as defined in section 2: $J(u) = \sum_{i=0}^{4}\sum_{t=0}^{T} L_i^2(t)) = \sum_{i=1}^{4} J_i$. In the following, we express J as a quadratic function of the $T-$components vector control u and we identify it to the energy function of a Hopfield net [3] of T units: the k^{th} unit encoding $u(k)$, the traffic lights state at the k^{th} time step of the optimization horizon. We then exploit the optimization capabilities of the Hopfield model [4] to compute a (local) minimum control u^*.

Starting from to the linear traffic equation (1), we obtain an analytic expression of the partial criterion J_1 (see [6] for more details):

$$J_1 = \beta_1 + {}^t\Theta_1 u + S_1^2\ {}^t u W^0 u \tag{3}$$

where $\beta_1 = \sum_{t=1}^{T}(C_1(t))^2$ and $C_i(t) = L_i(0) + \sum_{s=0}^{t-1} A_i(s)$, for $i = 1, ..., 4$.

$\Theta_1 = {}^t[\Theta_1(1), \Theta_1(2), ..., \Theta_1(T)]$ is the $T-$components vector given by, for $t = 1, ..., T$:

$$\Theta_1(t) = (T-t+1)(S_1^2 - 2S_1C_1(t)) - 2S_1 \sum_{s=t+1}^{T}(T-s+1)\times A_1(s) \tag{4}$$

W^0 is the null diagonal symmetric matrix given by

$$W^0 = \begin{pmatrix} 0 & T-1 & T-2 & ... & 2 & 1 \\ T-1 & 0 & T-2 & ... & 2 & 1 \\ T-2 & T-2 & 0 & ... & 2 & 1 \\ ... & ... & ... & ... & ... & ... \\ 2 & 2 & ... & 2 & 0 & 1 \\ 1 & 1 & ... & 1 & 1 & 0 \end{pmatrix} \tag{5}$$

For symmetry reasons J_3 is given by an analog equation to (3). From $u_2 = u_4 = 1 - u$, we obtain:

$$J_2 = \beta_2 + {}^t\Theta_2 u + S_2^2\ {}^t u W^0 u \tag{6}$$

where

$$\begin{aligned}\beta_2 &= \sum_{t=1}^{T}(C_2(t))^2 + \tfrac{T(T+1)}{2}S_2^2 - 2S_2\sum_{t=1}^{T}(T-t+1)C_2(t) \\ &+S_2^2(\tfrac{T(T-1)}{2} + \tfrac{T(T-1)(2T-1)}{6} - 2S_2^2\sum_{t=1}^{T}\sum_{s=t+1}^{T}(T-s+1)A_2(s)\end{aligned} \tag{7}$$

Θ_2 is the $T-$components vector:

$$\begin{aligned}\Theta_2(t) = &-(T-t+1)(S_2^2 - 2S_2C_2(t)) \\ &+2S_2\sum_{s=t+1}^{T}(T-s+1)\times A_2(s) - 2S_2\sum_{s=1}^{T} W_{ts}^0\end{aligned} \tag{8}$$

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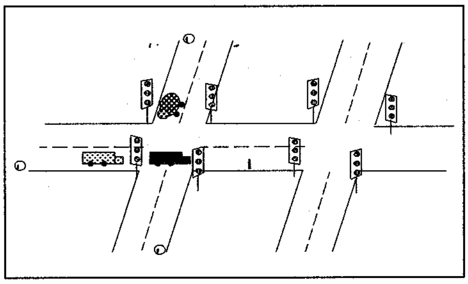

Figure 2: Arrivals to an internal link

We have obviously a similar expression for J_4. The main criterion J is then a quadratic function of the command u:

$$J = \beta + {}^t\Theta u - \frac{1}{2}{}^t uWu \tag{9}$$

with $\beta = \sum_{i=1}^{4} \beta_i$, $\Theta = \sum_{i=1}^{4} \Theta_i$ and $W = -2(\sum_{i=1}^{4} S_i^2) \times W^0$.

The minimum green duration constraint: When controlling an urban net, the arrivals to an internal link of a given intersection depend on the upstream intersection. Let i be an internal link of an urban net, the neighbors of i are numbered 1, 2 and 4. We denote α_{ji} ,j = 1,2 or 4 the directional coefficients, i.e. the rate of vehicles leaving link j for link i.The arrivals to the internal link i are then given by:

$$A_i(t) = \sum_{j \in V(i)} \alpha_{ji} \times S_j u_j(t) \tag{10}$$

where $V(i) = \{\text{neighbors of link } i\}$, $S_j u_j(t)$ are the departures from the upstream link j. Let now suppose that the minimum green duration is of two steps. This assumption is realistic since in practice the time step is about five seconds and the minimum green duration is about ten seconds. We add then to the energy function J defined by (9), the following term which advantages the constraint:

$$E_c = -A \sum_{t=0}^{T-1} (u_t u_{t+1} + \bar{u}_t \bar{u}_{t+1}) \tag{11}$$

where A is a positive constant and $\bar{u}_t = 1 - u_t$.

We prove [6] that $E = J + E_c$ is decreasing on the Hopfield's net trajectories

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which dynamic is given by the equations:

$$\begin{aligned} &u_t(k+1) = H(\sum_{s=1}^{T-1} W_{ts}u_s - \Theta_t + 2A(u_t u_{t+1} - \bar{u}_t\bar{u}_{t+1})) \\ &\forall t = 1,...,T-1 \text{ and for } t = T: \\ &u_T(k+1) = H(\sum_{s=1}^{T-1} W_{Ts}u_s - \Theta_T + A(2u_{T-1} - 1)) \end{aligned} \quad (12)$$

This approach is naturally extensible to urban nets of several intersections. The main difficulty is to obtain an analytic expression of the quadratic criterion associated to an internal link of the net. In fact, we show [6] that this criterion is a quadratic function of the traffic lights states upon the horizon optimization of both the considered link and the upstream intersection. However the interaction matrix is still symmetric and null diagonal which insures the convergence of the optimization process.

4. Numerical results

The simulations were performed on the semi-macroscopic traffic simulator *SSMT* [5]. We consider an urban net of one controlled central junction surrounded by four peripheral intersections, which lights states behave in an a priori fixed way. The main parameters of a simulation are then the rates of arrivals on each incoming link of the urban net (12 in total). The results are presented for three kinds of traffic situations: fluid, saturated and a third one where the arrivals are randomly generated. In the following table, we indicate the relative gain of the constraint Hopfield model relatively to the basic Hopfield model (without minimum green duration constraint), The comparisons are done upon the two following criterions: $\Sigma_1 = \sum_{t=0}^{T}\sum_{i=1}^{4} L_i(t)$ which measures the waiting time for all vehicles crossing the central intersection. The second criterion is the standard quadratic $\Sigma_2 = \sum_{t=0}^{T}\sum_{i=1}^{4} L_i^2(t)$, it has no physical significance, however its optimization tends to equalize queues on antagonist links.

	Fluid	Saturate	Random
% of $V_{\min}$ violation	8.03	6.29	8.40
% of Σ_1	5.13	2.24	4.74
% of Σ_2	10.30	4.07	9.22

Compared results of the constraint and non constraint models

We also compare with the optimal model, which consists in a complete enumeration of the solutions space.

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	Fluid		Saturate		Random	
	$\%\Sigma_1$	$\%\Sigma_2$	$\%\Sigma_1$	$\%\Sigma_2$	$\%\Sigma_1$	$\%\Sigma_2$
Constraint model	5.13	9.30	2.24	4.07	4.59	9.27
Optimal	6.57	9.55	10.84	17.14	12.54	21.64

Compared relative gaps of the two criterions for three traffic situations

In conclusion, we present a neural algorithm for the modelling and control of over-saturated urban intersections. We exploit the optimizations capabilities of the Hopfield net. We modify the genuine algorithm to take into account particular constraints of our own problem and we prove the convergence of the modified algorithm. We illustrate this method by several examples of traffic states generated by a traffic simulator. Finally, we show that this method is naturally extensible to urban nets of several intersections.

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