

# Generating arbitrary rhythmic patterns with purely inhibitory neural networks \*

Zhijun Yang and Felipe M.G. França

Universidade Federal do Rio de Janeiro  
COPPE - Programa de Engenharia de Sistemas e Computação  
21945-970, Caixa Postal 68511, Rio de Janeiro, RJ, Brazil  
Emails: {felipe, yang}@cos.ufrj.br

**Abstract.** A novel approach for the prediction and generation of coupled neural oscillation among arbitrarily connected inhibitory neurons is proposed. Based on Scheduling by Multiple Edge Reversal (SMER), a very simple distributed algorithm, neural network building blocks can be configured for the generation of complex rhythmic patterns with a very high independence from individual neuronal models. A method for the organization and simulation of the new approach is illustrated by mimicking the main rhythmic gait patterns of an hexapodal animal.

## 1. Introduction

Many research approaches towards modelling mechanisms of coupled neural oscillations are based on dynamical system theory or mathematical analysis such as symmetric and symmetry-breaking Hopf bifurcation [4][5]. However, in those studies a specific mathematical dynamic model of the target system, i.e., the set of individual neuron models, is required for each particular pattern of connectivity among neurons. This paper proposes an alternative, macroscopic, approach to the modelling of the collective behaviour of purely inhibitory neuronal networks. *Scheduling by Edge Reversal* (SER) [3][2] and its generalization, *Scheduling by Multiple Edge Reversal* (SMER) [1][6] distributed algorithms can be applied to predict or reproduce the interesting behaviour of many biological oscillatory neuronal networks, specially *central pattern generators* (CPGs), just assuming some form of *postinhibitory rebound* (PIR) at the neuron's model level.

It is shown how different neuronal network building blocks under SER or SMER can be used to model biological motor systems much more simply and effectively. In order to illustrate the new approach, cockroach's three rhythmic gait patterns are chosen as case study in this paper. Nevertheless, it is suggested how the technique can be extended to reproduce most invertebrate and vertebrate rhythmic movements provided that they can be described topologically.

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## 2. Inhibitory neuronal networks as neighborhood-constrained systems

### 2.1. Scheduling by Edge Reversal (SER)

Consider a neighborhood-constrained system composed of a set of processes and a set of atomic shared resources represented by a connected graph  $G = (N, E)$  where  $N$  is the set of processes, and  $E$ , the set of edges defining the interconnection topology. An edge is present between any two nodes if and only if the two corresponding processes share at least one atomic resource.

SER works in the following way: starting from any acyclic orientation  $\omega$  on  $G$  there is at least one sink node, i.e., a node that has all its edges directed to itself. All sink nodes are allowed to operate while other nodes remain idle. This obviously ensures mutual exclusion at any access made to shared resources by sink nodes. After operation a sink node will reverse the orientation of its edges, becoming a source and thus releasing the access to resources to its neighbors. A new acyclic orientation is defined and the whole process is then repeated for the new set of sinks [3][2]. Let  $\omega' = g(\omega)$  denote this greedy operation, SER can be regarded as the endless repetition of the application of  $g(\omega)$  upon  $G$ . Assuming that  $G$  is finite, it is easy to see that eventually a set of acyclic orientations will be repeated defining a *period* of length  $p$ . This simple dynamics ensures that no deadlock or starvation will ever occur since at every acyclic orientation there is at least one sink, i.e., one node allowed to operate. Also, it is proved that inside any period every node operates exactly  $m$  times [3][2].

SER is a fully distributed graph dynamics algorithm. A very interesting property of this algorithm lies in its generality in the sense that any topology will have its own set of possible SER dynamics [3][2]. Figure 1 illustrates the SER dynamics.

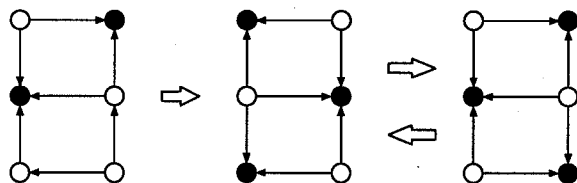


Figure 1: A graph  $G$  under SER, with  $m=1$ , operation cycle  $p=2$ .

### 2.2. Scheduling by Multiple Edge Reversal (SMER)

SMER is a generalization of SER where pre-specified access rates to atomic resources are imposed to processes in a distributed resource-sharing system which is represented by a multigraph  $M(N, E)$ . Differently from SER, with SMER a number of oriented edges can exist between any two nodes. Between any two nodes  $i$  and  $j$ ,  $i, j \in N$ , there can exist  $e_{ij}$  unidirected edges,  $e_{ij} \geq 0$ . The

reversability of node  $i$  is  $r_i$ , i.e., the number of edges that shall be reversed by  $i$  towards each of its neighbouring nodes, indiscriminately, at the end of operation. Node  $i$  is an r-sink if it has at least  $r_i$  edges directed to itself from each of its neighbours. Each r-sink node  $i$  operate and reverse  $r_i$  edges towards its neighbours, the new set of r-sinks will operate and so on. Similarly to sinks under SER, only r-sink nodes are allowed to operate under SMER. It is easy to see that with SMER, nodes are allowed to operate more than once consecutively.

The following lemma states a basic topologic constraint towards the definition of  $M$ , where gcd is the greatest common divisor.

**Lemma 1** [1][6] *Let nodes  $i$  and  $j$  be two neighbors in  $M$ . If no deadlock arises for any initial orientation of the little circles between  $i$  and  $j$ , then  $e_{ij} = r_i + r_j - \text{gcd}(i, j)$ . ■*

Finally, it is important to know that there is always at least one SMER solution for any target system's topology having arbitrary pre-specified reversabilities at any of its nodes [1]. In the next section SMER will be employed on the definition of building blocks used on the construction of artificial CPGs where operating sinks can be seen as firing neurons in purely inhibitory neuronal networks.

### 3. Mimicking the neurolocomotor network of an hexapodal animal

Of long-standing interest are questions about rhythm generation in networks of nonoscillatory neurons, where the driving force is not provided by endogenous pacemaking cells. A simple mechanism for this is based on reciprocal inhibition between neurons, if they exhibit the property of *postinhibitory rebound* (PIR) [9]. Instead of focusing on low-level neuronal features, e.g., membrane potential functions, the next step is to choose and analyse a representative case study and build a corresponding SER- or SMER-driven artificial CPG network; the three common gait patterns of cockroach, i.e., slow walk, medium speed walk and fast walk are investigated. Figure 2 shows the CPG's mutual inhibition structure and corresponding gait phase relationship between six legs.

The understanding of the changes in topology and internal parameters between different gaits is inspired by the contribution of P.A. Getting [7], who argued that modulation of building blocks can greatly alter network operation, even generate a totally new network. This modulation is normally induced by command signals from central nervous system (CNS) or the intrinsic characteristic of building block itself.

In the case of cockroach's fast walk pattern, SER could be directly applied to initiate oscillation and coordinate the movement of the six legs (see Figure 1). For the more complicated rhythmic leg movements of slow and medium speed, in which the fire of neighbor nodes is not exactly out of phase and some phase

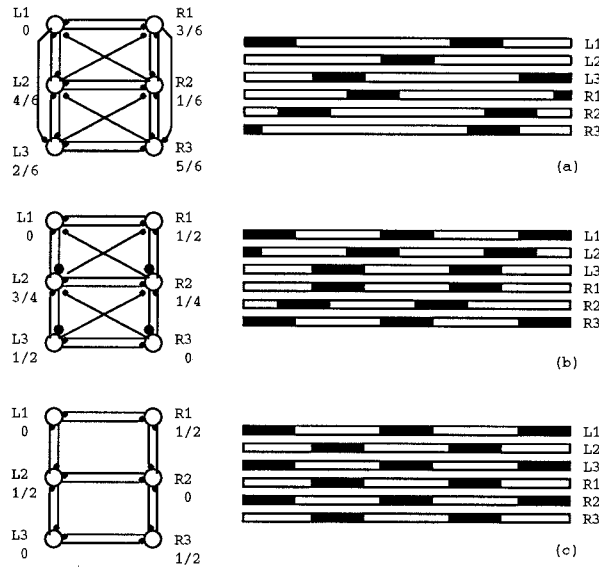


Figure 2: Mutual inhibition structures and phase relations between six legs, each leg represented by one electrically compact nodes, filled circle and its size denote inhibition and its strength (a) Slow walk (b) Medium speed walk (c) Fast walk.

overlapping exist, firstly one has to construct the corresponding building block under SMER, then organize the artificial CPG network with building blocks. A graphic expression of two typical cockroach gait patterns is formulated in Figure 4.

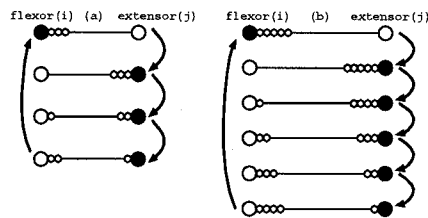


Figure 3: One possible scheme of firing circulation patterns of building blocks (a) Four possible configurations for medium speed gait pattern and; (b) Six possible configurations for slow gait pattern.

From the phase relationship presented in Figure 2 (a) and (b), one can choose a suitable configuration from the corresponding firing circulation patterns introduced in Figure 3, for each of the six nodes in the relative speed model, in order to construct the six-leg rhythmic movement shown in Figure 4 (a) and (b) respectively. Then, self-organized circulation patterns in cockroach's gait on slow and medium speed can be generated by building blocks under SMER.

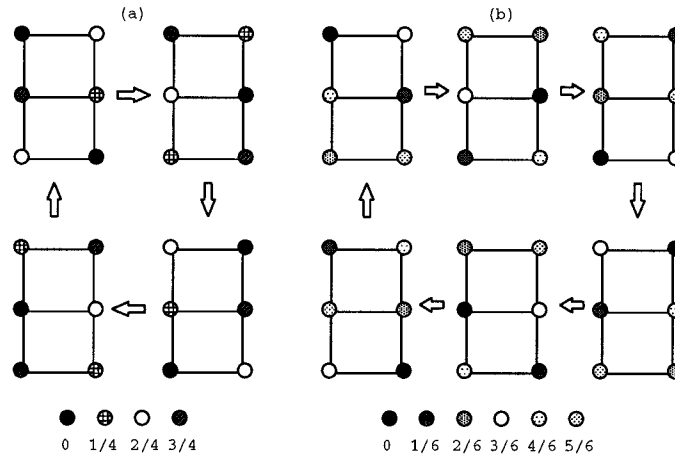


Figure 4: The coordinated rhythmic patterns among six legs (a) medium speed, 0 - exciting, 2/4 - inhibited (b) slow speed, 0 - exciting, 3/6 - inhibited.

It is important to understand the concept of a building block, since it is the building block which should obey SMER, rather than the constructed model of gait patterns. A mutually inhibitory relation between neighboring flexor neurons is assumed so that building blocks are solely responsible for gait pattern generation and transition. By anatomical view, cockroach's leg movement is driven by flexor and extensor motor neurons, the flexor will lift a leg from the ground while extensor does the opposite. This can be mapped into our building block perfectly, taking neuron  $i$  and  $j$  in a building block as flexor and extensor respectively (see Figure 3). Now, a rough insight is apparent, i.e., there is an interesting timing relationship between flexor and extensor during different speed models. As cockroach's walking speed increase, the firing time for extensor (corresponding stance) will decrease dramatically, while firing time for flexor (corresponding swing) keep basically constant, what matches exactly with biological experiments [8]. This insight confirm that cockroach's speed is determined largely by extensor firing, i.e., the time duration of a leg on ground.

Next, an experiment with cockroach's medium speed gait is offered. The rhythmic order exhibited is:  $(L1R3) R2 (L3R1) L2 \dots$

#### 4. Conclusion

Neural network building blocks based on SMER can be configured for the generation of complex rhythmic patterns much independently from individual neuronal models. Gait patterns of hexapodal animals have been generated through a distributed macroscopic approach, which may provide a new and convenient pathway for further VLSI synthesis.

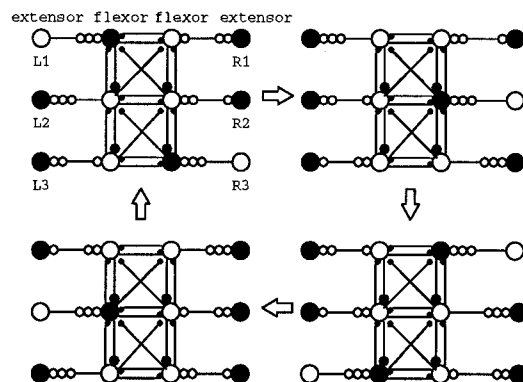


Figure 5: The cockroach's medium speed gait pattern reconstructed with building block (a) from Figure 3; the six flexor neurons' firing threshold is 3, six extensor neurons' is 1.

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