COMPRESSIVE BLIND SOURCE RECOVERY WITH RANDOM DEMODULATION

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ABSTRACT

Distributed Compressive Sensing (DCS) theory effectively reduces the number of measurements of each signal, by exploiting both intra- and inter-signal correlation structures, which saves on the costs of sampling devices as well as of communication and data processing. In many fields, only the mixtures of source signals are available for compressive sampling, without prior information on both the source signals and the mixing process. However, people are still interested in the source signal rather than the mixing signals. There is a basic solution which reconstructs the mixing signals from the compressive measurements first and then separates the source signals by estimating mixing matrix. However, the reconstruction process takes considerable time and also introduces error into the estimation step. A novel method is proposed in this paper, which directly separates the mixing compressive measurements by estimating the mixing matrix first and then reconstruct the interesting source signals. At the same time, in most situations, the source signals are analog signals. In this paper, Random Demodulation (RD) system is introduced to compressively sample the analog signal. We also verify the independence and non-Gaussian property of the compressive measurement. The experimental results proves that the proposed method is feasible and compared to the basic method, the estimation accuracy is improved.

***Index Terms*—**Distributed compressive sensing (DCS), independent component analysis (ICA), random demodulation (RD), mixing matrix estimating

1. INTRODUCTION

Compressed sensing (CS) is a novel signal acquisition framework which has attracted growing interests in signal processing [1-3]. And Distributed CS (DCS) extends single-signal CS to multiple signals [4,5]. By exploiting both intra- and inter-signal correlation structures, DCS effectively reduces the number of measurements of each signal, which saves on the costs of sampling devices as well as of communication and data processing.

In many fields, such as speech recognition, network anomaly detection, and medical signal processing, only the mixtures of source signals are available for compressive sampling. The recovery of the mixing matrix and source signals from the compressive measurements of mixing signals is the main tasks. Recovering mixing parameters and source signals only from the mixed observations without having prior information on both source signals and the mixing process is often referred to as blind source separation (BSS) [6].

There is a basic algorithm, which can be separated to two procedures. Step 1 is to reconstruct the mixture signals from the observed compressive measurements, while step 2 is estimating the mixing matrix and reconstructing the source signal. However, the reconstruction process takes considerable time and also introduces error into the estimation step [7].

This paper proposes a novel algorithm that estimates the mixing matrix of signals directly in the compressive measurement domain before reconstructing the signals. In the compressive measurement domain, we use independent component analysis (ICA) method to separate the measurement of original signals from measurement of mixing signals, and then we reconstruct part or all of the original signals by classical CS reconstruction algorithm.

In many situations, the input signals are analog signals. In 2005, analog to information converter (AIC) is proposed, and recently several structures for analog signal sub-Nyquist sampling have been developed, in which Random Demodulation (RD) is a typical one. In this paper, the input signals are chosen as analog signals. The RD system is used as the front-end compressive sampling part, then using the proposed algorithm to reconstruct the compressive measurements. The experimental results proves that the proposed method is feasible and compared to the basic method, the estimation accuracy is improved.

The rest of this paper is organized as follows. Section 2 provides the theoretical background of this paper. A detailed description of the proposed algorithm is presented in Section 3. Section 4 provides the RD system properties. The performance of the proposed algorithm is demonstrated in

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Section 5 in comparison to the basic algorithm. Finally, the conclusion is given in Section 6.

2. PROBLEM DESCRIPTION

2.1. Mixing signal model

The source signals are assumed to be independent or uncorrelated with each other. In this paper we just consider the instantaneous linear mixture model, which is shown as follows.

$$\begin{cases} x_1(t) = a_{11}s_1(t) + a_{21}s_2(t) + \cdots + a_{m1}s_m(t) \\ \vdots \\ x_m(t) = a_{1m}s_1(t) + a_{2m}s_2(t) + \cdots + a_{mm}s_m(t) \end{cases} \tag{1}$$

Where t=1,2,...,N denotes the discrete time sequence, and $s_1(t),\ldots,s_m(t)$ denotes the signal set of m sources, x_1(t),...,$x_m(t)$ represents the m linear mixture set of source signals, a_{ij} ($1 \le i \le m, 1 \le j \le m$) denotes the mixing parameters. The equation (1) can be represented in matrix form

$$\mathbf{x} = \mathbf{sA} \tag{2}$$

Where A can be called as the mixing matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mm} \end{bmatrix} \in \mathbb{R}^{m \times m} \tag{3}$$

2.2. Distributed Compressive sensing model

Assume that mixture signals are compressively sensed individually via a same compressed sample framework. A compressed sensing of mixture signals can be given by

$$y_i = \Phi x_i, \quad 1 \le i \le m \tag{4}$$

Where $y_i \in \mathbb{R}^{M \times 1}, M \ll N$ is the compressive measurement of the i-th mixture signals. $\Phi \in \mathbb{R}^{M \times N}$ is the measurement matrix which present the compressed sensing process. For simplicity but without loss of generality we consider the case of m=2 source signals and mixture signals. Then, the compressive observation set of mixture signals can be denoted as follows:

$$\begin{aligned} [y_1 \quad y_2] &= [a_{11}\Phi s_1 + a_{21}\Phi s_2 \quad a_{12}\Phi s_1 + a_{22}\Phi s_2] \\ &= [\Phi s_1 \quad \Phi s_2] \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ &= [y_{s1} \quad y_{s2}]\mathbf{A} \end{aligned} \tag{5}$$

Where y_{s1}, y_{s2} are the compressive measurements of the source signals. In the source mixing situation, y_{s1} and y_{s2} cannot be measured. According to (4) and (5), the equation also can be represented as follows.

$$\mathbf{y} = \Phi \mathbf{sA} \tag{6}$$

2.3. Introduction of RD system

In most compressive sensing situations, the input signals are analog signals, recently several structures are proposed for analog signal in compressive sensing field [8-10]. In this paper, the random demodulation architecture is chosen as the front-end sampling part which is widely used analog signals compressive sampling.

The structure of RD system is shown as follows [8]. As shown in Fig. 1, $x(t)$ is an input analog signal. $p(t)$ is a square pulse with pseudo-random values of {±1} and it alternates between values at or faster than the Nyquist frequency of the input signal $x(t)$. $x(t)$ is multiplied by $p(t)$ in a mixer. An analog low-pass filter follows the mixer and then a traditional ADC samples the output signal at low rate, $y(m)$ represents the compressive sampled data. Finally, the input signal can be reconstructed in computer from the sampled data using some optimization algorithms.

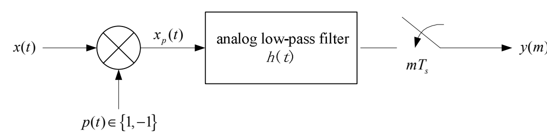

Fig. 1.Structure of the RD system.

Once the parameters of RD system are determined, we can obtain the measurement matrix Φ as follows.

$$\Phi = \mathrm{HP}\Psi \tag{7}$$

Where Ψ is the base of Discrete Fourier Transform, P can be obtained by pseudo-random pulse, and H is determined by the transfer function of low-pass filter.

To summarize, the task of compressive blind source recovery with random demodulation can be described as: given the mixture signals' compressive measurements **y**, as well as the parameters of the adopted RD system, estimate the mixing matrix **A** and recover the source signals **s** without prior information of **A** and **s**.

2.4. Basic algorithm

There is a basic algorithm to solve the above mentioned problem. Fig.2. shows the framework of this method. As can be seen, the basic algorithm reconstructs the mixture signals first and then separates the source signals by estimating mixing matrix **A**. However, as mentioned in paper [7], in some signal processing problems, reconstruction of signals is not a necessary step. Instead, it is possible to deduce attributes of the signal from the compressive measurements. According to this idea, we try to estimate mixing matrix **A** directly from the compressive measurements of mixtures instead of recovering the mixture signals at first. Since it does not need to reconstruct mixture signals, it will greatly reduce the complexity of computation.

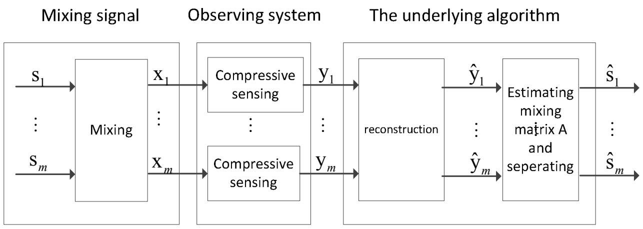

Fig. 2. Framework of the basic algorithm

3. PROPOSED ALGORITHM

3.1. Framework description

In this paper, a novel algorithm is proposed, which is concentrated on mixing matrix estimating and the source signal recovery. In this work, mixture signals are compressively obtained based on compressed sensing. Then, the mixing matrix is recovered directly from the compressive measurements of mixture signals. The framework of the mentioned problem via this proposed algorithm is showed in Fig. 3.

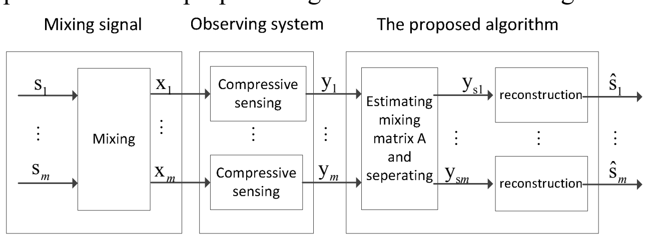

Fig.3. Framework of the proposed algorithm

Here we use independent component analysis (ICA) to estimate the mixing matrix **A**. ICA is one of the most popular approaches to solve the mixing signals problems. It takes advantage of the high-order statistical properties of signals to estimate the mixing matrix and to retrieve the source signals without resorting to any a priori information about the mixing matrix. It exploits only the information carried by the observations of the mixing signals themselves. When using ICA, the signals must satisfy two important properties:

1) all of the signals are independent of each other;

2) at most, one of the signals is Gaussian.

So before using ICA algorithm to estimate the mixing matrix, we must prove that whether the compressive measurements of the mixing signal sampled by RD system satisfy the requirement of the ICA methods. We will verify the feasibility of the proposed method in section 4.

3.2. Algorithm description

In the problem of blind mixing matrix recovery, gradient ascent algorithm can extract source signals from the mixture signals if the source signals are independent or uncorrelated with each other. In the problem of ICA, given a set of mixture signals x, we seek an unmixing matrix W which maximizes the entropy H(Y) of the signals Y=g(**Wx**). We can find an estimate **W** using gradient ascent method to iteratively adjust **W** in order to maximize the entropy of Y=g(**Wx**). H(Y) is described as follows:

$$H(\mathrm{Y})=H(\mathbf{x})+\mathrm{E}\left[\sum_{i=1}^{M}\ln g'(\mathbf{Wx})\right]+\ln\left|\mathbf{W}\right| \quad (8)$$

In the proposed algorithm, we use the gradient ascent method to solve the problem described in (7) and estimate the mixing matrix **A** directly without recovering the mixing signals. Once the mixing matrix is obtained, we can get the separated compressive measurement y_s, and then using orthogonal matching pursuit (OMP) algorithm to recover the source signals [11]. The flowchart of this proposed algorithm is shown in Fig. 4.

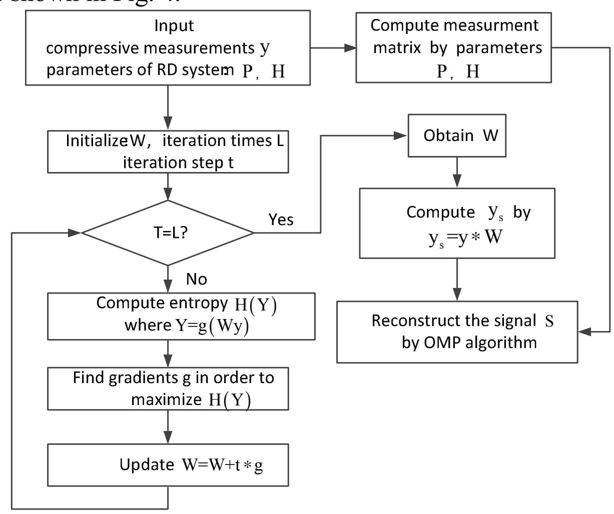

Fig. 4. The flowchart of the proposed algorithm

4. THE RD SYSTEM PROPERTIES

If we want to use the proposed method to separate mixing signal and recovery the original signal, we must verify whether the RD system compressive measurements of the mixing signal satisfy the requirement of the ICA methods, including independence and non-Gaussian properties.

4.1. Independence Properties of RD system compressive measurements

Mutual Information (MI) is a classical measure of the independence property of a signal. The mutual information of signal can be represented as follows.

$$I(\mathbf{x})=\sum_{i=1}^{M}H(\mathbf{x}_i)-H(\mathbf{x}) \quad (9)$$

Where $\mathbf{x}=[\mathbf{x}_1,\mathbf{x}_2,\cdots\mathbf{x}_m]$ and $I(\mathbf{x})$ represents the mutual information, the signal is more independence when the mutual information is smaller.

The chosen signals are the speech signals source1~source6 in [12] and the music signals aguittest, accordiontest, pianotest, saxtest and so on in [13]. In all there are 20 signals, then every two signals form a group. RD system is used to compressively measure the 10 group signals segment by segment. In RD system the low-pass filter is chosen as 2-order low-pass filter which transfer function is as follows.

$$H(s) = A\omega_c^2 \Big/ s^2 + \frac{\omega_c}{Q}s + \omega_c^2 \qquad (10)$$

Where A=2， $f_c = \omega_c / 2\pi \approx 725$ Hz, and the sampling rate f_s=1/3f_o .

The experimental procedure is as follows with MATLAB software.

1) Every group signal s mixed by matrix A to form mixing signal x, where **x=sA** and $\mathbf{A} = \begin{bmatrix} 0.9222 & -0.3867 \\ 0.4509 & -0.8926 \end{bmatrix}$.

2) RD system is used to compressively measure x and s segment by segment. The segment length N=500, and the number of compressive measurements M=300.

3) Calculate the mutual information for each group, the mutual information of original signals, mixing signals, and their compressive measurement can be recorded as MIs, MIx, MIys, MIyx respectively.

Steps 2 and 3 are repeated 100 times and then used to calculate the average values. The results are listed in Table 1.

According to the results as shown in Table1, the mutual information MIys is much smaller than MIyx, which means after RD system, the independence of original signal compressive measurement is greater than the mixing signal compressive measurement. So we can use ICA method to separate original signal compressive measurement ys from the mixing signal compressive measurement yx.

No.	MIs	MIx	MIys	MIyx
1	0	0.0139	5.4026e-7	0.0211
2	0	0.0038	0	0.0277
3	0	0.0030	6.5997e-6	0.0551
4	0	0.0818	4.9140e-5	0.2585
5	7.45e-4	0.6288	0.0115	0.5899
6	0	0.0378	1.9967e-6	0.2061
7	0	0.1205	1.3798e-6	0.2760
8	0	4.01e-4	7.2244e-4	0.1268
9	0	0.0011	3.4611e-4	0.1596
10	0	0.0928	3.1459e-4	0.2721

Table1 The Mutual Information of signals

4.2. Non-Gaussian Properties of RD system compressive measurements

In addition to the independence, the signal must satisfy the non-Gaussian properties before using ICA method. Kurtosis is a classical measure of the non-Gaussian property of a signal. The normalized kurtosis of the signal s is denoted as follows [14].

$$kurt(s) = \frac{E\{s^4\}}{\left(E\{s^2\}\right)^2} - 3 \qquad (11)$$

Where $E\{s^2\}$ is the second-order moment, and $E\{s^4\}$ is the fourth-order moment, When kurt(s) = 0, the signal s is Gaussian. If kurt(s) >0, s follows a super-Gaussian distribution, and if kurt(s) <0, s follows a sub-Gaussian.

The 10 signals are speech signals source1~source6 in [11] and the music signals aguittest, accordiontest, pianotest, and saxtest in [12]. In addition to the above 10 signals, we also use two generated Gaussian signals called s11 and s12. RD system is used to compressively measure the 12 signals segment by segment which is the same as that in section 4.1. The sampling rate f_s=1/3f_o .

The experimental procedure is as follows with MATLAB software.

1) Generate two Gaussian signals s11 and s12 by the “randn ()” function.

2) Using RD system to compressively measure the 12 signals segment by segment. The segment length N=500, and the number of compressive measurements M=300.

3) Use “kurtosis()-3” to calculate the normalized kurtosis for each signal and its compressive measurement, whose values are recorded in the vectors ks and kys , respectively. “Kurtosis()” is a function in MATLAB which can calculate kurtosis of a signal.

Steps 2 and 3 are repeated 100 times and then used to calculate the average of each normalized kurtosis. The results are listed in Table 2.

No.	ks	kys
1	8.9692	14.5005
2	14.6225	23.5536
3	5.9851	11.1058
4	6.3123	10.1212
5	13.6846	24.4518
6	8.3409	14.8368
7	0.4473	3.1643
8	0.4265	3.0116
9	0.4473	3.1643
10	1.9610	5.1906
11	-0.0154	0.0376
12	0.0024	0.0887

Table 2 The kurtosis of source signals and measurements

According to the experiments as shown in Table 2, we can deduce that the compressive measurement of analog super-Gaussian signal through RD system also follows the super-Gaussian distribution. Thus, we can use the proposed method to solve the blind mixing signals problem if the source signals satisfy the properties of ICA method.

5. SIMULATION RESULTS

In the experiment, music signals aguittest, pianotest in [12] are chosen as the input source signal. $\mathbf{s_1}$, $\mathbf{s_2}$,both signal duration time is 10s, and sampling rate f_o=8kHz . The mixing matrix is the same as that in section 4.1, and the mixing signals $\mathbf{x_1}$, $\mathbf{x_2}$ is formed by mixing $\mathbf{s_1}$, $\mathbf{s_2}$ with $\mathbf{A}$. RD system is used to compressively measure the signals segment by segment. In RD system the low-pass filter is the same as that in section 4.1. The segment length N=500. The sampling rate f_s is taken as $\{\frac{1}{10},\frac{1}{9},\frac{1}{8},\frac{1}{7},\frac{1}{6},\frac{1}{5},\frac{1}{4},\frac{1}{3},\frac{1}{2},1\}f_o$.

Here, we use Signal-to-Noise(SNR) to measure the recovery accuracy, which can be calculated as

$$SNR_i = 20\times\log_{10}\left(\frac{\|s_i\|_2}{\|s_i-\hat{s}_i\|_2}\right) \quad (12)$$

Where SNR_i represents the signal to noise of the i-th signal, and s_i represents the i-th signal, $\hat{s}_i$ denotes the recovery signal, here we use SNR donates the average value of all the signal-to-noise.

In the basic method and proposed method, we both use OMP algorithm to reconstruct mixing signals, and separate the signals by FastICA algorithm. Here we call the basic method as OMP-SS method.

For every value of f_s , run the proposed method, OMP-SS method 20 time respectively. Then calculate the average value. The results are shown in Fig. 5.

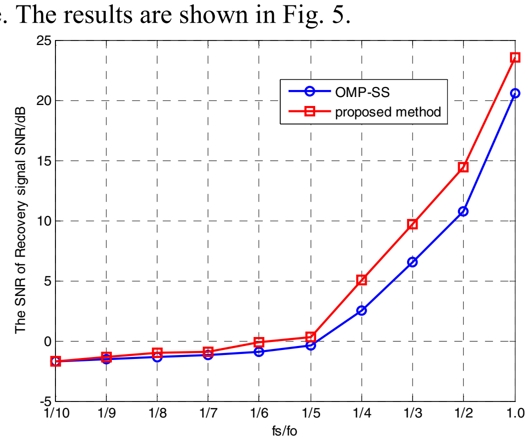

Fig.5 . SNR of two different algorithms

As shown in Fig. 5, with the increase of sampling rate f_s, the SNR of recovery signal becomes better. For all the condition, the SNR of proposed method is better than the OMP-SS method. The experimental results proves that the proposed method is feasible for the analog mixing signal recovery through RD system and compared to the basic method, the estimation accuracy is improved.

6. CONCLUSION

In this paper, a novel blind source recovery method for distributed compressive sensing with random demodulation is proposed, which uses RD system to compressively sample the analog signals and estimates the mixing matrix from the mixing compressive measurement directly by ICA algorithm and then reconstruct the interest signals by OMP algorithm. Numerical simulations show that the compressive measurements of RD system satisfy the independence and non-Gaussian property. The experimental results demonstrated that the proposed method outperforms the basic algorithm with better recovery accuracy.

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