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Evaluation of a Hydrological Drought Index

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Abstract: Indices for characterising hydrological drought are, in general, data demanding and computationally intensive. A very simple and effective index, the Streamflow Drought Index (SDI), has been recently proposed. It is based on cumulative streamflow volumes for overlapping periods of three, six, nine and twelve months within each hydrological year. It allows defining drought states which are modelled as a non-stationary Markov chain. The methodology is validated using data from two river basins in Greece (Evinos and Boeoticos Kephisos). Water from these basins is diverted for water supply of the Athens Metropolitan Area. Thus, the methodology is tested on a real-world system which allows for assessing its applicability within a Drought Watch System in river basins with significant storage works.

Key words: Hydrological drought; drought prediction; SDI; Markov chain; Evinos basin, Boeoticos Kephisos basin.

1. INTRODUCTION

Drought is a naturally occurring phenomenon related to a significant decrease of water availability during a significant period of time and over a large area. It affects man's economic activities, human lives and various elements of the environment such as the Earth's ecosystems. The origin of drought is impossible to define much as the starting point of the global hydrological cycle. Conventionally, decrease of precipitation is considered as the origin of drought. This leads to a reduction of storage volumes and fluxes involved in the hydrological cycle. Depending on the choice of the hydrological variable or variables of interest, drought is characterised as meteorological, hydrological or agricultural (Beran and Rodier, 1985).

Hydrological drought is defined as a significant decrease in the availability of water in all its forms appearing in the land phase of the hydrological cycle. Various hydrological variables are used to describe these forms but streamflow is, by far, the most significant variable from the viewpoint of quantity of water. Hence, a hydrological drought episode is related to streamflow deficit with respect to normal conditions. Each drought event is characterised through four attributes: (a) its severity expressed by a drought index, (b) its time of onset and its duration, (c) its areal extent, and (d) its frequency of occurrence.

According to the methodology proposed by Nalbantis and Tsakiris (2008) the four-dimensional relationship of drought severity-duration-frequency-area is reduced into a much simpler twodimensional relationship of severity versus frequency. First, an index called Streamflow Drought Index (SDI) was proposed which characterises drought severity while fulfilling all requirements of such indices. Second, the time of onset and duration of drought events was eliminated through properly treating time. Third, the frequency of drought occurrence was kept as a significant parameter. Last, the areal extent of a drought event, although very useful for meteorological droughts, is not of interest for hydrological droughts since water managers are interested on streamflow only at a small number of points in space (basin outlets, reservoir inlets and outlets etc.); evidently, streamflow at these points provides an integrated measure of spatially distributed runoff; furthermore, the river basin is proposed by the Water Framework Directive as the unit for applying measures for water resources protection and management. The aim of this paper is to validate the methodology based on SDI through the application on data from two river basins in Greece (Evinos and Boeoticos Kephisos). Water from these basins is diverted for water supply of the Athens Metropolitan Area. So, application on a real-world system serves assessing the applicability of the methodology within a Drought Watch System (Wilhite et al., 2007). To achieve the above goal, a suitable testing framework is set up.

2. METHODOLOGY

2.1 General

Indices for characterising a hydrological drought such as Palmer Hydrological Drought Index (PHDI), Surface Water Supply Index (SWSI) or the index proposed by Palfai (2002) are, in general, data demanding and computationally intensive. On the contrary, the proposed index SDI keeps the advantages of simplicity and effectiveness found in indices of meteorological droughts such as the Standardised Precipitation Index (SPI) (McKee et al., 1993; Tsakiris and Vangelis, 2004; Cancelliere et al., 2007) or the Reconnaissance Drought Index (RDI) (Tsakiris and Vangelis, 2005; Tsakiris et al., 2007). Exclusive use of streamflow is made as the key variable for assessing hydrological droughts.

2.2 The treatment of time

The fact that drought is a natural phenomenon which is slowly developing and is identified only after it has been well established dictates the use of coarse time steps for drought assessments. The typical time step used is monthly which is also employed in this study.

The onset of a drought episode is defined as the time when a drought index falls below a certain truncation level. The truncation level has been defined in various ways. For stationary processes a fixed value has been used while for periodic processes a set of seasonally varying values is more appropriate. The mean over a long period of time was chosen in this work.

In the classical approach in treating time, successive non-overlapping time intervals are used. In the proposed methodology, time is treated as follows: (1) October the first is considered the beginning of the hydrological year which is typical in the Mediterranean region; (2) every three months (31st December, 31st March, 30th June, 30th September) a drought assessment is made regarding the time interval from the start of the hydrological year up to that time; thus time intervals of duration of three, six, nine and twelve months are used; (3) at the above dates, predictions are issued regarding drought conditions for future time intervals.

The overlapping time periods used within each hydrological year are reported as reference periods. These are October-December, October-March, October-June, and October-September (one complete hydrological year).

2.3 The Streamflow Drought Index (SDI)

It is assumed that a time series of monthly streamflow volumes $Q_{i,j}$ is available where i denotes the hydrological year and j the month within that hydrological year (j = 1 for October and j = 12 for September). Based on this series we obtain

$$V_{i,k} = \sum_{j=1}^{3k} Q_{i,j} \quad i = 1, 2, \dots, j = 1, 2, \dots, 12 \quad k = 1, 2, 3, 4$$
(1)

where $V_{i,k}$ is the cumulative streamflow volume for the *i*-th hydrological year and the *k*-th reference period, k = 1 for October-December, k = 2 for October-March, k = 3 for October-June, and k = 4 for October-September.

Based on cumulative streamflow volumes $V_{i,k}$ the Streamflow Drought Index (SDI) is defined for each reference period k of the *i*-th hydrological year as follows

$$\text{SDI}_{i,k} = \frac{V_{i,k} - \overline{V}_k}{s_k}$$
 $i = 1, 2, ..., k = 1, 2, 3, 4$ (2)

where \overline{V}_k and s_k are respectively the mean and the standard deviation of cumulative streamflow volumes of reference period k as these are estimated over a long period of time. In this definition the truncation level is set to \overline{V}_k although other values could be used.

The hydrological drought index of equation 2 is identical to the standardised streamflow volume. This is not entirely new since Ben-Zvi (1987) made use of the standardised annual streamflow volumes.

Generally, for small basins, streamflow may follow a skewed probability distribution which can well be approximated by the family of the Gamma distribution functions. The distribution is then transformed into normal. In this work we use the two-parameter log-normal distribution for which the normalisation is simple: it suffices taking natural logarithms of streamflow. The SDI index is defined as

$$\text{SDI}_{i,k} = \frac{y_{i,k} - y_k}{s_{y,k}}$$
 $i = 1, 2, ..., k = 1, 2, 3, 4$ (3)

where

$$y_{i,k} = \ln(V_{i,k}), i = 1, 2, ..., k = 1, 2, 3, 4$$
 (4)

are the natural logarithms of cumulative streamflow with mean \overline{y}_k and standard deviation $s_{y,k}$ as these statistics are estimated over a long period of time.

Based on SDI, states of hydrological drought are defined which are identical to those used in the meteorological drought indices SPI and RDI. Five states are considered which are denoted by an integer number ranging from 0 (non-drought) to 4 (extreme drought) and are defined through the criteria of Table 1.

State	Description	Criterion	Probability (%)		
0	Non-drought	$SDI \ge 0.0$	50.0		
1	Mild drought	$-1.0 \le \text{SDI} < 0.0$	34.1		
2	Moderate drought	$-1.5 \le \text{SDI} < -1.0$	9.2		
3	Severe drought	$-2.0 \le \text{SDI} < -1.5$	4.4		
4	Extreme drought	SDI < -2.0	2.3		

Table 1: Definition of states of hydrological drought with the aid of SDI

The problem of treating intermittent or ephemeral flows is of importance when dealing with hydrological droughts. Three cases can be distinguished: (1) watercourse with perennial flow, (2) watercourse with ephemeral flow and without complete dryness throughout a whole hydrological year, (3) watercourse with no flow in some hydrological years. According to our definition of SDI, case 2 becomes irrelevant since cumulative streamflow will always possess some positive value. Only the case of completely dry hydrological years (case 3) remains which is arbitrarily classified as extreme drought (state equal to 4).

2.4 The methodological steps

Starting from historical streamflow series, an SDI series is computed which yields a series of drought states. The underlying state process is assumed to possess the structure of a non-stationary Markov chain. Markov chains have been widely applied to predicting droughts (mainly meteorological ones) (Lohani and Loganathan, 1997; Lohani et al., 1998; Ochola and Kerkides, 2003; Paulo and Pereira, 2006).

Let Q_{ij} (i = 1, 2, ..., N; j = 1, 2, ..., 12) be the observed time series of monthly streamflow volumes for the river basin under study, where N is the number of hydrological years.

First, the cumulative streamflow volumes $V_{i,k}$ (i = 1, 2, ..., N; k = 1, 2, 3, 4) are calculated via equation 1. Second, the series SDI_{*i*,*k*} of the SDI index is calculated based on equation 2 or 3. Third, the series of states $x_{i,k}$ (i = 1, 2, ..., N; k = 1, 2, 3, 4) is obtained according to the criteria of Table 1. For each *k*, the related state process $X_{i,k}$ takes discrete values $m \in [0, 1, 2, 3, 4]$. Fourth, the frequency of appearance of each state *m* in each reference period *k*, $F_{m,k}$, is estimated as

$$F_{m,k} = \frac{n_{m,k}}{N} \tag{5}$$

where $n_{m,k}$ is the number of occurrences of state *m* in reference period *k* within the available sample of *N* years. This is an estimate of the marginal probability $p_{m,k}$ of appearance of state *m* in reference period *k*, i.e.

$$p_{m,k} = P(X_{i,k} = m) \ m \in [0, 1, 2, 3, 4] \ \forall i$$
(6)

where *P*(.) denotes probability. For each *k*, probabilities $p_{m,k}$ (m = 0, 1, 2, 3, 4) form a 5×1 column vector p_k .

Fifth, the frequency of state transition $F_{m,m',k}$ from state *m* in reference period *k* to state *m'* in reference period *k*+1 is

$$F_{m,m',k} = \frac{n_{m,m',k}}{\sum_{m'} n_{m,m',k}}$$
(7)

where $n_{m,m',k}$ is the number of occurrences of state *m* in reference period *k* and state *m'* in reference period *k*+1. This is an estimate of the transition probability $p_{m,m',k}$ which is defined as

$$p_{m,m',k} = P(X_{i,k+1} = m' \mid X_{i,k} = m) \quad m \in [0, 1, 2, 3, 4] \quad m' \in [0, 1, 2, 3, 4] \quad \forall i$$
(8)

where P(.|.) denotes conditional probability. For each k, transition probabilities form a 5×5 matrix denoted as P_k .

Assume now that the current time interval is (i,k). Before characterising current drought state, one can predict the marginal probabilities for the next reference period k+1 as

$$\mathbf{p}_{k+1} = \mathbf{P}_k \mathbf{p}_k \tag{9}$$

Within an operational context, at the end of time interval (i,k), all historical data up to that time are assumed to become available. This allows classifying the current interval. Thus, vector p_k of equation 9 has now one element equal to one and all other elements zero. It follows from equation 9 that the only information needed for drought prediction is the matrix P_k as this is approximated by its estimate, the matrix of state transition frequency. This is the main output of the methodology when working off-line on historical series. In real-time situations, the output is (a) a single value of current state and (b) the probabilities of future states as obtained from a stored matrix of state transition probability.

Nalbantis and Tsakiris (2008) have also treated the case with lack of streamflow information. For this they devised a special methodology based on SPI. This was not used in this paper.

A rigorous testing framework was set up to further validate the above methodology.

3. CASE STUDIES

3.1 The study area

The methodology proposed was applied to two river basins from which water is diverted for supplying the Athens Metropolitan Area. The first basin is the Evinos river basin which is located in the West Sterea Hellas Water District in central Greece (Figure 1). At the site of the Agios Demetrios dam it has an upstream basin area of 352 km2, mean elevation of 990 m above the mean sea level, and steep slopes. Its average annual streamflow is 297×106 m3 (Efstratiadis et al. 2000). Water from the Agios Demetrios reservoir is diverted eastwards to the adjacent Mornos reservoir for the water supply of the Greater Athens area. The reservoir possesses a surface area of 3.6 km2 (at the elevation of the spillway crest) and an active storage capacity of 112.1×106 m3. The Agios Demetrios reservoir and the adjacent Mornos reservoir form the main storage facilities of the water supply system of Athens.

The second basin is the Boeoticos Kephisos River Basin which is located in the East Sterea Hellas Water District (Figure 1). It has an upstream area of 1956 km² and an average annual streamflow 285×10^6 m³. Water is collected in the natural Lake Hyliki and thereafter is pumped to Athens. The lake suffers from considerable leakage losses which may reach up to 50% of its total annual inflow due to karstic background. Spills are directed to the adjacent Lake Paralimni. The lake shows a surface area of 27.74 km² (at the elevation of the spillway crest) and an active storage capacity of 584.8×10^6 m³.

Monthly streamflow data at the Agios Demetrios dam have been obtained through processing raw data of concurrent velocity and stage measurements combined with stage recordings at the daily or the hourly time step, mainly at the Poros Reganiou hydrometric station, downstream of the dam site. Data covering the period from 1970-71 to 1999-2000 have been used by Nalbantis and Tsakiris (2008). To include the drought around 2000-01, the record is now extended up to 2001-2002. Regarding the Boeoticos Kephisos River Basin, monthly streamflows from 1907-08 were available. To minimise the effect of any kind of hydrological changes and make a fair comparison of the results from the two basins, the period 1970-71 up to 2002-03 is finally selected for the second basin.



Figure 1: The Evinos river at the Agios Demetrios Reservoir and the Boeoticos Kephisos river at the Karditsa Tunnel (outlet)

3.2 Results

In the earlier work by Nalbantis and Tsakiris (2008) the statistical significance of the skeweness coefficient of the cumulative streamflow volumes was tested. The authors concluded that using natural logarithms of volume was necessary only for the October-December reference period. This led to using the definition of equation 3 for SDI of October-December while keeping equation 2 for the other reference periods. The tests were repeated in this paper also. The skewness coefficients of the initial data, of the natural logarithms of the data and of the final data are shown in Table 2 for the two basins. According to the test of Snedecor and Cochran (1967) the critical upper limits of the absolute value of the skewness coefficient are equal to 0.986 and 0.662 respectively at 0.02 and 0.10 significance level. Hence, taking logarithms was only necessary for the October-December reference period and the Evinos basin. No logarithms were used for the Boeoticos Kephisos streamflows.

Basin	Calculation basis	Oct-Dec	Oct-Mar	Oct-Jun	Year
Evinos	Initial data	0.857	-0.230	-0.207	-0.240
	Logarithms	-0.550	-1.127	-0.952	-0.987
	Final data	-0.550	-0.230	-0.207	-0.240
B. Kephisos	Initial data	0.384	0.143	0.034	0.118
	Logarithms	-0.982	-1.051	-1.145	-1.051
	Final data	0.384	0.143	0.034	0.118

Table 2: Skewness coefficient of cumulative streamflow and of its natural logarithm.

Statistically significant values (at the 0.10 probability) are in italics.

For the Evinos basin the evolution of SDI from one hydrological year to another and each reference period is depicted in Figures 2, 3, and 4 where the SDI series were grouped by two. Similar graphs are given in Figures 5, 6 and 7 for the Boeoticos Kephisos basin. As expected, significant discrepancies are observed only when passing from the first three-month period (October-December) to the first semester (October-March). This is due to the typical Mediterranean hydrological regime which is manifested as a wet period of six months of the hydrological year and a mostly dry period thereafter. As a result, high predictive capacity of drought state is expected when, in the end of March the following question is raised: will the nine-month period of the running year be considered as drought period? The same holds for the assessment in the end of June. The above behaviour was similar in both test basins. The SDI series of the two basins were graphically compared for each reference period separately. Significant differences were shown in the hydrological regime between the western and the eastern part of the Sterea Hellas Region. For the three historical droughts that affected the water supply of Athens one can observe the following:

- a. In the hydrological year 1976-77 the B. Kephisos basin has undergone a moderate drought while in the Evinos basin no-drought conditions prevailed.
- b. During the drought from 1988-89 to 1994-95, we observe that: (1) in 1988-89 the drought was mild in both basins; (2) in 1989-90 the drought became extreme in the Evinos basin and severe in the B. Kephisos basin; (3) in 1990-91 a mild drought in B. Kephisos is combined with no-drought conditions in the west; (4) in 1991-92 the drought was severe in Evinos and moderate in B. Kephisos; (5) in 1992-93 the situation of 1991-92 was reversed; (6) in the next two hydrological years 1993-94 and 1994-95 drought can be characterised as mild.
- c. In 1999-2000 a mild drought was observed in B. Kephisos; in 2000-2001 the drought became severe in both basins while in 2001-2002 a moderate drought in the west is accompanied with a mild drought in the east.

The above drought assessments confirm the results obtained at the time of drought in a more or less qualitative manner (Nalbantis et al, 1994). The conclusions are drawn from comparisons for all

reference periods which show a similar behaviour to that of the whole hydrological year (Figure 9) with the exception of the October-December period (Figure 8) for which the differences between the two basins are clearly more significant.



Figure 2: SDI series for the Evinos basin and the reference periods October-December and October-March.



Figure 3: SDI series for the Evinos basin and the reference periods October-March and October-June.



Figure 4: SDI series for the Evinos basin and the reference periods October-June and the hydrological year (October-September).



Figure 5: SDI series for the Boeoticos Kephisos basin and the reference periods October-December and October-March.



Figure 6: SDI series for the Boeoticos Kephisos basin and the reference periods October-March and October-June.



Figure 7: SDI series for the Boeoticos Kephisos and the reference periods October-June and the hydrological year (October-September).



Figure 8: Comparison of SDI series for the reference period October-December and the two test basins.



Figure 9: Comparison of SDI series for the reference period October-September and the two test basins.

Regarding the frequency of state transition, preliminary tests showed very small observed number of occurrences of states 2 (moderate drought), 3 (severe drought) and 4 (extreme drought). To remedy this problem, the above three states were grouped into one state to which we assigned number 2. In Table 3 we present the matrices of state transition frequency for all pairs of reference periods when passing from one period to the next lengthier period (three pairs in all). These matrices are the main tools for predicting drought state in real-time in the case of availability of streamflow data (see subsection 2.4. In both basins the state transition frequency shows a significant "dispersion" between different states when the starting period is October-December. When passing to other starting periods, the state transition frequency tends to be stabilised to a value close to 1 when staying in the same state and 0 for all state changes. In other words, the respective matrix tends to the identity matrix. Differences were encountered for the Evinos basin between frequencies found by Nalbantis and Tsakiris (2008) and by the present analyses which are due to the extension of the observation period in this work.

4. CONCLUDING REMARKS

This paper aims at validating a methodology for forecasting hydrological droughts within an operational context regarding river basins with works of large total storage capacity.

(a) Evinos Basin					(b) Boeoticos Kephisos basin				
State for	State for Oct-Mar			S	State for State for Oct-Mar			ar	
Oct-Dec	0	1	2	C	Oct-Dec	0	1	2	
0	0.706	0.294	0.000		0	0.579	0.368	0.053	
1	0.455	0.273	0.273		1	0.500	0.400	0.100	
2	0.000	0.500	0.500		2	0.000	0.250	0.750	
State for	State for Oct-Jun			S	tate for	State for Oct-Jun			
Oct-Mar	0	1	2	C	oct-Mar	0	1	2	
0	1.000	0.000	0.000		0	0.875	0.125	0.000	
1	0.100	0.900	0.000		1	0.000	1.000	0.000	
2	0.000	0.000	1.000		2	0.000	0.000	1.000	
State for	State for Oct-Sep			S	tate for	State for Oct-Sep			
Oct-Jun	0	1	2	0	Oct-Jun	0	1	2	
0	0.944	0.056	0.000		0	1.000	0.000	0.000	
1	0.000	1.000	0.000		1	0.000	0.929	0.071	
2	0.000	0.000	1.000		2	0.000	0.000	1.000	

Table 3: Frequency of state transition as estimated from data of the test basins.

The methodology is based on a proposed index called Streamflow Drought Index (SDI) which is calculated for overlapping reference periods within each hydrological year thus allowing for categorising droughts into a small number of categories of drought state. The latter is modelled through a Markov chain which allows for estimating state transition frequency within an off-line context and predicting future drought state within a real-time context.

A rigorous testing framework was set up which allowed for validating the methodology on a real-world water resources system. Two river basins from those contributing to water supply of Athens, Greece were selected as the test basins. These cover both the western part (Evinos basin) and the eastern part (Boeoticos Kephisos basin) of the system and have different hydrological regimes in three respects: precipitation regime, the geological background and the degree of anthropogenic intervention. Analysis has shown that SDI can well discover the main droughts known to have occurred in the region: 1976-77, 1988-95 and 1999-2001. The matrix of frequency of state transition proved a useful tool which allows high predictive capacity of drought when the end of wet season is close (end of March) as well as in later times within the hydrological year. In the middle of the wet season predictions are far less certain.

The methodology used can be easily applied within a Drought Watch System. However, it requires streamflow data of high quality and of sufficient length to accurately estimate the frequency of rare drought phenomena.

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